Zhang et al. *Advances in Difference Equations* 2014, 2014:250
http://www.advancesindifferenceequations.com/content/2014/1/250

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

Exponential stability for a stochastic delay neural network with impulses

Suping Zhang[1,2*], Wei Jiang[1] and Zhixin Zhang[1]

*Correspondence: zsp606@163.com
[1]School of Mathematical Science, Anhui University, Hefei, Anhui 230039, China
[2]Department of Mathematics and Physics, Anhui Jianzhu University, Hefei, Anhui 230601, China

Abstract
In this paper, we study the exponential stability for a stochastic neural network with impulses. By employing fixed point theory and some analysis techniques, sufficient conditions are derived for achieving the required result.

MSC: 34K20

Keywords: neural network; exponential stability; impulse; fixed point theory

1 Introduction

Neural networks have found important applications in various areas such as combinatorial optimization, signal processing, pattern recognition, and solving nonlinear algebraic equations. We notice that a lot of practical systems have the phenomenon of time delay, and many scholars have paid much attention to time delay systems [1–8]. As is well known, stochastic functional differential systems which include stochastic delay differential systems have been widely used since stochastic modeling plays an important role in many branches of science and engineering [9]. Consequently, the stability analysis of these systems has received a lot of attention [10–12].

In some applications, besides delay and stochastic effects, impulsive effects are also likely to exist [13, 14]; they could stabilize or destabilize the systems. Therefore, it is of interest to take delay effects, stochastic effects, and impulsive effects into account when studying the dynamical behavior of neural networks.

In [11], Guo *et al.* studied the exponential stability for a stochastic neutral cellular neural network without impulses and obtained new criteria for exponential stability in mean square of the considered neutral cellular neural network by using fixed point theory. To the best of the authors' knowledge there are only a few papers where fixed point theory is used to discuss the stability of stochastic neural networks. In this paper, we will study the exponential stability for a stochastic neural network with impulses by the contraction mapping theorem and Krasnoselskii's fixed point theorem.

2 Some preliminaries

Throughout this paper, unless otherwise specified, we let (Ω, F, P) be a complete probability space with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions, *i.e.* it is right continuous and F_0 contains all P-null sets, $C^b_{F_0}([-\tau, 0]; R^n)$ be the family of all bounded, F_0-measurable functions. Let R^n denote the n-dimensional real space equipped with Euclidean norm $|\cdot|_1$. $B = [b_{ij}(t)]_{n\times n}$ denote a matrix, its norm is denoted by $\|B(t)\|_3 = \sum_{i,j=1}^n |b_{i,j}(t)|$.

In this paper, by using fixed point theory, we discuss the stability of the impulsive stochastic delayed neural networks:

$$\begin{cases} dx(t) = [-(A + \triangle A(t))x(t) + f(t, x(t), x(t-\tau_1))]\,dt \\ \qquad\qquad + \sigma(t, x(t), x(t-\tau_2))\,d\omega(t), \quad t \neq t_k, \\ \triangle x(t_k) = I_k(x(t_k)), \quad t = t_k, k \in Z_+, \\ x(t) = \psi(t), \quad -\tau \le t \le 0, \end{cases} \tag{2.1}$$

where $\tau = \max\{\tau_1, \tau_2\}$, τ_1 and τ_2 are positive constant. $t \to \psi(t) \in C([-\tau, 0], L^p_{F_0}(\Omega, R^n))$ with the norm defined by $\|\psi\|_2 = \sup_{-\tau \le \theta \le 0} |\psi(\theta)|_1$, $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ is the state vector, $A = \operatorname{diag}(a_1, a_2, \dots, a_n) > 0$ (*i.e.* $a_i > 0$, $i = 1, 2, \dots, n$) is the connection weight constant matrix with appropriate dimensions, and $\triangle A(t)$ represents the time-varying parameter which is uncertain with $|\triangle A(t)|_3$ bounded.

Here $f(t, u_1, u_2) \in C(R \times R^n \times R^n)$ is the neuron activation function with $f(t, 0, 0) = 0$ and $\omega(t) = (\omega_1(t), \omega_2(t), \dots, \omega_m(t))^T \in R^m$ is an m-dimensional Brownian motion defined on (Ω, F, P). The stochastic disturbance term, $\sigma(t, u_1, u_2) \in C(R \times R^n \times R^n)$, can be viewed as stochastic perturbations on the neuron states and delayed neuron states with $\sigma(t, 0, 0) = 0$. $\triangle x(t_k) = I_k(x(t_k)) = x(t_k^+) - x(t_k^-)$ is the impulse at moment t_k, and $t_1 < t_2 < \cdots$ is strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$, $x(t_k^+)$ and $x(t_k^-)$ stand for the right-hand and left-hand limit of $x(t)$ at $t = t_k$, respectively. $I_k(x(t_k))$ shows the abrupt change of $x(t)$ at the impulsive moment t_k and $I_k(\cdot) \in C(L^p_{F_t}(\Omega; R^n), L^p_{F_t}(\Omega; R^n))$.

The local Lipschitz condition and the linear growth condition on the function $f(t, \cdot, \cdot)$ and $\sigma(t, \cdot, \cdot)$ guarantee the existence and uniqueness of a global solution for system (2.1); we refer to [9] for detailed information. Clearly, system (2.1) admits a trivial solution $x(t; 0, 0) \equiv 0$.

Definition 2.1 System (2.1) is said to be exponentially stable in mean square for all admissible uncertainties if there exists a solution x of (2.1) and there exist a pair of positive constants β and μ with

$$E\big|x(t)\big|_1^2 \le \mu E\|\psi\|_2^2 e^{-\beta t}, \quad t \ge 0.$$

In order to prove the exponentially stability in mean square of system (2.1), we need the following lemma.

Lemma 2.1 (Krasnoselskii) *Suppose that Ω is Banach space and X is a bounded, convex, and closed subset of Ω. Let $U, S : X \to \Omega$ satisfy the following conditions*:

(1) *$Ux + Sy \in X$ for any $x, y \in X$*;

(2) *U is contraction mapping*;

(3) *S is continuous and compact.*

Then $U + S$ has a fixed point in X.

Lemma 2.2 (C_p inequality) *If $X \in L^p(\Omega, R^n)$, then*

$$E\left|\sum_{i=1}^n X_i\right|^p \le \sum_{i=1}^n E|X_i|^p, \quad \textit{for } 0 < p \le 1$$

and

$$E\left|\sum_{i=1}^{n} X_i\right|^p \le n^{p-1}\sum_{i=1}^{n} E|X_i|^p, \quad \textit{for } p > 1.$$

3 Main results

Let $(ß, \|\cdot\|_ß)$ be the Banach space of all F_0-adapted processes $\varphi(t,\omega) : [-\tau,\infty) \times \Omega \to R^n$ such that $\varphi : [-\tau,\infty) \to L^p_{F_0}(\Omega, R^n)$ is continuous on $t \neq t_k$, $\lim_{t\to t_k^-}\varphi(t,\cdot)$ and $\lim_{t\to t_k^+}\varphi(t,\cdot)$ exist, and $\lim_{t\to t_k^-}\varphi(t,\cdot) = \varphi(t_k,\cdot)$, $k = 1,2,\ldots$; we have

$$\|\varphi\|_ß := \sup_{t\ge 0} E\left|\varphi(t)\right|_1^2, \quad \text{for } \varphi \in ß.$$

Let Λ be the set of functions $\varphi \in ß$ such that $\varphi(s) = \psi(s)$ on $s \in [-\tau,0]$ and $e^{\alpha t}E|\varphi(t,\omega)|_1^2 \to 0$ as $t \to \infty$. It is clear that Λ is a bounded, convex, and closed subset of ß.

To obtain our results, we suppose the following conditions are satisfied:

(H_1) there exist $\mu_1, \mu_2 > 0$ such that $|f(t,u,v) - f(t,\overline{u},\overline{v})|_1 \le \mu_1|u-\overline{u}|_1 + \mu_2|v-\overline{v}|_1$;

(H_2) there exist $\nu_1, \nu_2 > 0$ such that $|\sigma(t,u,v) - \sigma(t,\overline{u},\overline{v})|_1^2 \le \nu_1|u-\overline{u}|_1^2 + \nu_2|v-\overline{v}|_1^2$;

(H_3) there exists an $\alpha > 0$ such that $\alpha < \min\{a_1, a_2, \ldots, a_n\}$;

(H_4) for $k = 1,2,3,\ldots$, the mapping $I_k(\cdot)$ satisfies $I_k(0) \equiv 0$ and is globally Lipschitz function with Lipschitz constants p_k;

(H_5) there exists a constant ρ such that $\inf_{k=1,2,\ldots}\{t_k - t_{k-1}\} \ge \rho$;

(H_6) there exists constant p such that $p_k \le p\rho$, for $i \in N$ and $k = 1,2,\ldots$.

The solution $x(t) := x(t;0,\psi)$ of system (2.1) is, for the time t, a piecewise continuous vector-valued function with the first kind discontinuity at the points t_k ($k = 1,2,\ldots$), where it is left continuous, *i.e.*,

$$x\left(t_k^-\right) = x(t_k), \qquad x\left(t_k^+\right) = x(t_k) + I_k\left(x(t_k)\right), \quad k = 1,2,\ldots.$$

Theorem 3.1 *Assume* (H_1)-(H_6) *hold and the following condition is satisfied*:

$$(\mathrm{P}_1)\quad 3\left\{\frac{n^2}{2\lambda_{\min}(A)}\left[\left(\left\|\triangle A(t)\right\|_3 + \mu_1 + \mu_2\right)^2 + \nu_1 + \nu_2\right] + n^2p^2\left(\frac{1}{\lambda_{\min}(A)} + \rho\right)^2\right\} < 1,$$

then system (2.1) *is exponentially stable in mean square for all admissible uncertainties, that is,* $e^{\alpha t}E|x(t)|_1^2 \to 0$, *as* $t \to \infty$.

Proof System (2.1) is equivalent to

$$\begin{aligned} x(t) = {} & \exp(-At)\psi(0) + \int_0^t \exp A(s-t)\left[-\triangle A(s)x(s) + f\left(s,x(s),x(s-\tau_1)\right)\right]ds \\ & + \int_0^t \exp A(s-t)\sigma\left(s,x(s),x(s-\tau_2)\right)d\omega(s) \\ & + \sum_{0<t_k<t} \exp\left[-A(t-t_k)\right]I_k\left(x(t_k)\right). \end{aligned} \tag{3.1}$$

Let

$$
\begin{aligned}
&J_1(t) := \exp(-At)\psi(0),\\
&J_2(t) := \int_0^t \exp A(s-t)\big[-\triangle A(s)x(s) + f\big(s, x(s), x(s-\tau_1)\big)\big]\,ds,\\
&J_3 := \int_0^t \exp A(s-t)\sigma\big(s, x(s), x(s-\tau_2)\big)\,d\omega(s),\\
&J_4(t) := \sum_{0<t_k<t} \exp\big[-A(t-t_k)\big] I_k\big(x(t_k)\big).
\end{aligned}
$$

Define an operator by $(Qx)(t) = \psi(t)$ for $t \in [-\tau, 0]$, and for $t \geq 0$, we define $(Qx)(t) := J_1(t) + J_2(t) + J_3(t) + J_4(t)$ (*i.e.* the right-hand side of (3.1)). From the definition of Λ, we have $E|\varphi(t)|^2 < \infty$, for all $t \geq 0$, and $\varphi \in \Lambda$.

Next, we prove that $Q\Lambda \subseteq \Lambda$. It is clear that $(Qx)(t)$ is continuous on $[-\tau, 0]$. For a fixed time $t > 0$, it is easy to check that $J_1(t)$, $J_2(t)$, $J_4(t)$ are continuous in mean square on the fixed time $t \neq t_k$, for $k = 1, 2, \ldots$. In the following, we check the mean square continuity of $J_3(t)$ on the fixed time $t \neq t_k$ $(k = 1, 2, \ldots)$.

Let $x \in \Lambda$ and $r \in R$ such that $|r|$ sufficiently small; we obtain

$$
\begin{aligned}
E\big|J_3(t+r) - J_3(t)\big|_1^2 &= E\bigg|\int_0^{t+r} \exp A(s-t-r)\sigma\big(s, x(s), x(s-\tau_2)\big)\,d\omega(s)\\
&\quad - \int_0^t \exp A(s-t)\sigma\big(s, x(s), x(s-\tau_2)\big)\,d\omega(s)\bigg|_1^2\\
&= E\bigg|\int_0^t \big[\exp A(s-t-r) - \exp A(s-t)\big]\sigma\big(s, x(s), x(s-\tau_2)\big)\,d\omega(s)\\
&\quad + \int_t^{t+r} \exp A(s-t-r)\sigma\big(s, x(s), x(s-\tau_2)\big)\,d\omega(s)\bigg|_1^2\\
&\leq 2\bigg(\int_0^t \big\|\big[\exp A(s-t-r) - \exp A(s-t)\big]\big\|_3^2\\
&\quad \times E\big|\sigma\big(s, x(s), x(s-\tau_2)\big)\big|_1^2\,ds\\
&\quad + \int_t^{t+r} \big\|\exp A(s-t-r)\big\|_3^2 E\big|\sigma\big(s, x(s), x(s-\tau_2)\big)\big|_1^2\,ds\bigg)\\
&\to 0 \quad \text{as } r \to 0.
\end{aligned}
$$

Hence, $(Qx)(t)$ is continuous in mean square on the fixed time $t \neq t_k$, for $k = 1, 2, \ldots$. On the other hand, as $t = t_k$, it is easy to check that $J_1(t)$, $J_2(t)$, $J_3(t)$ are continuous in mean square on the fixed time $t = t_k$.

Let $r < 0$ be small enough; we have

$$
\begin{aligned}
E\big|J_4(t_k+r) - J_4(t_k)\big|_1^2 &= E\bigg|\sum_{0<t_m<t_k+r} \exp\big[-A(t_k+r-t_m)\big] I_m\big(x(t_m)\big)\\
&\quad - \sum_{0<t_m<t_k} \exp\big[-A(t_k-t_m)\big] I_m\big(x(t_m)\big)\bigg|_1^2
\end{aligned}
$$

$$\begin{aligned} &\leq E\bigg|\big[\exp\big(-A(t_k+r)\big)-\exp(-At_k)\big] \\ &\quad\times\sum_{0<t_m<t_k}\exp(At_m)I_m\big(x(t_m)\big)\bigg|_1^2, \end{aligned}$$

which implies that $\lim_{r\to 0^-} E|J_4(t_k+r)-J_4(t_k)|_1^2 = 0$.

Let $r>0$ be small enough, we have

$$\begin{aligned} E\big|J_4(t_k+r)-J_4(t_k)\big|_1^2 &= E\bigg|\big[\exp\big(-A(t_k+r)\big)-\exp(-At_k)\big] \\ &\quad\times\sum_{0<t_m<t_k}\exp(At_m)I_m\big(x(t_m)\big)+\exp(-Ar)I_k\big(x(t_k)\big)\bigg|_1^2, \end{aligned}$$

which implies that $\lim_{r\to 0^+} E|J_4(t_k+r)-J_4(t_k)|_1^2 = E|I_k(x(t_k))|_1^2$.

Hence, we see that $(Qx)(t):[-\tau,\infty)\to L^p_{F_0}(\Omega,R^n)$ is continuous in mean square on $t\neq t_k$, and for $t=t_k$, $\lim_{t\to t_k^+}(Qx)(t)$ and $\lim_{t\to t_k^-}(Qx)(t)$ exist. Furthermore, we also obtain $\lim_{t\to t_k^-}(Qx)(t)=(Qx)(t_k)\neq\lim_{t\to t_k^+}(Qx)(t)$.

It follows from (3.1) that

$$e^{\alpha t}E\big|(Qx)(t)\big|_1^2\leq 4e^{\alpha t}\sum_{i=1}^4 E\big|J_i(t)\big|_1^2.$$

By (H_3), it is easy to see $e^{\alpha t}E|J_1(t)|_1^2\to 0$, as $t\to\infty$. Now, we prove $e^{\alpha t}E|J_2(t)|_1^2\to 0$, $e^{\alpha t}E|J_3(t)|_1^2\to 0$, and $e^{\alpha t}E|J_4(t)|_1^2\to 0$, as $t\to\infty$.

Note, for any $\epsilon>0$, there exists $t^*>0$ such that $s\geq t^*-\tau$ implies that $e^{\alpha s}E|x(s)|_1^2<\epsilon$. Hence, we have from ($H_1$), ($H_3$)

$$\begin{aligned} e^{\alpha t}E\big|J_2(t)\big|_1^2 &= e^{\alpha t}E\bigg|\int_0^t \exp A(s-t)\big[-\triangle A(s)x(s)+f\big(s,x(s),x(s-\tau_1)\big)\big]\,ds\bigg|_1^2 \\ &\leq e^{\alpha t}E\int_0^t \big\|\exp A(s-t)\big\|_3^2\big|\big[-\triangle A(s)x(s)+f\big(s,x(s),x(s-\tau_1)\big)\big]\big|_1^2\,ds \\ &= e^{\alpha t}E\int_0^{t^*} \big\|\exp A(s-t)\big\|_3^2\big|\big[-\triangle A(s)x(s)+f\big(s,x(s),x(s-\tau_1)\big)\big]\big|_1^2\,ds \\ &\quad+e^{\alpha t}E\int_{t^*}^t \big\|\exp A(s-t)\big\|_3^2\big|\big[-\triangle A(s)x(s)+f\big(s,x(s),x(s-\tau_1)\big)\big]\big|_1^2\,ds \\ &\leq e^{\alpha t}E\int_0^{t^*} \big\|\exp A(s-t)\big\|_3^2\big[\big\|\triangle A(s)\big\|_3\big|x(s)\big|_1 \\ &\quad+\mu_1\big|x(s)\big|_1+\mu_2\big|x(s-\tau_1)\big|_1\big]^2\,ds \\ &\quad+e^{\alpha t}E\int_{t^*}^t \big\|\exp A(s-t)\big\|_3^2\big[\big\|\triangle A(s)\big\|_3\big|x(s)\big|_1 \\ &\quad+\mu_1\big|x(s)\big|_1+\mu_2\big|x(s-\tau_1)\big|_1\big]^2\,ds \\ &\leq e^{\alpha-2\lambda_{\min}(A)t}n^2\big(\big\|\triangle A(s)\big\|_3+\mu_1+\mu_2\big)^2 \\ &\quad\times E\Big(\sup_{-\tau\leq s\leq t^*}\big|x(s)\big|_1^2\Big)\int_0^{t^*}e^{2\lambda_{\min}(A)s}\,ds \end{aligned}$$

$$
\begin{aligned}
&+ 2e^{\alpha t} n^2 \int_{t^*}^{t} \bigl[\bigl(\| \triangle A(s) \|_3 + \mu_1 \bigr)^2 e^{-\alpha s} e^{\alpha s} E\bigl(|x(s)|_1^2 \bigr) \\
&+ \mu_2^2 e^{-\alpha(s-\tau_1)} e^{\alpha(s-\tau_1)} E \bigl| x(s-\tau_1) \bigr|_1^2 \bigr] e^{2\lambda_{\min}(A)(s-t)}\, ds,
\end{aligned}
$$

where $\| \exp A(s-t) \|_3 = \sum_{i=1}^{n} e^{a_i(s-t)}$, $\lambda_{\min}(A)$ represents the minimal eigenvalue of A. Thus, we have $e^{\alpha t} E |J_2(t)|_1^2 \to 0$ as $t \to \infty$.

From (H_2) and (H_3), we have

$$
\begin{aligned}
e^{\alpha t} E \bigl| J_3(t) \bigr|_1^2 &= e^{\alpha t} E \biggl| \int_0^t \exp A(s-t) \sigma \bigl(s, x(s), x(s-\tau_2) \bigr)\, d\omega(s) \biggr|_1^2 \\
&\le e^{\alpha t} E \int_0^t \bigl\| \exp A(s-t) \bigr\|_3^2 \bigl| \sigma \bigl(s, x(s), x(s-\tau_2) \bigr) \bigr|_1^2\, ds \\
&= e^{\alpha t} E \int_0^{t^*} \bigl\| \exp A(s-t) \bigr\|_3^2 \bigl| \sigma \bigl(s, x(s), x(s-\tau_2) \bigr) \bigr|_1^2\, ds \\
&\quad + e^{\alpha t} E \int_{t^*}^t \bigl\| \exp A(s-t) \bigr\|_3^2 \bigl| \sigma \bigl(s, x(s), x(s-\tau_2) \bigr) \bigr|_1^2\, ds \\
&\le e^{\alpha - 2\lambda_{\min}(A) t} n^2 (\nu_1 + \nu_2) E \Bigl(\sup_{-\tau \le s \le t^*} \bigl| x(s) \bigr|_1^2 \Bigr) \int_0^{t^*} e^{2\lambda_{\min}(A) s}\, ds \\
&\quad + e^{\alpha t} n^2 (\nu_1 + \nu_2) \int_{t^*}^t e^{-\alpha s} e^{\alpha s} E \Bigl(\sup_{-\tau \le s \le t} \bigl| x(s) \bigr|_1^2 \Bigr) e^{2\lambda_{\min}(A)(s-t)}\, ds \\
&\to 0 \quad \text{as } t \to \infty.
\end{aligned}
$$

As $x(t) \in \Lambda$, we have $\lim_{t\to\infty} e^{\alpha t} E |x(t)|_1^2 \to 0$. Then, for any $\epsilon > 0$, there exists a non-impulsive point $T > 0$ such that $s \ge T$ implies $e^{\alpha t} E |x(t)|_1^2 < \epsilon$. It then follows from the conditions (H_4)-(H_6) that

$$
\begin{aligned}
e^{\alpha t} E \bigl| J_4(t) \bigr|_1^2 &= e^{\alpha t} E \biggl| \sum_{0<t_k<t} \exp\bigl[-A(t-t_k) \bigr] I_k \bigl(x(t_k) \bigr) \biggr|_1^2 \\
&\le e^{\alpha t} E \biggl| \sum_{0<t_k<t} \bigl\| \exp A(t_k - t) \bigr\|_3 \bigl| I_k \bigl(x(t_k) \bigr) \bigr|_1 \biggr|^2 \\
&\le e^{\alpha t} E \biggl| \sum_{0<t_k<t} n e^{\lambda_{\min}(A)(t_k - t)} p_k \bigl| x(t_k) \bigr|_1 \biggr|^2 \\
&\le e^{\alpha t} E \biggl| \sum_{0<t_k<T} n e^{\lambda_{\min}(A)(t_k - t)} p_k \bigl| x(t_k) \bigr|_1 \\
&\quad + \sum_{T<t_k<t} n e^{\lambda_{\min}(A)(t_k - t)} p \rho \bigl| x(t_k) \bigr|_1 \biggr|^2 \\
&\le 2e^{\alpha t} \biggl[E \biggl| \sum_{0<t_k<T} n e^{\lambda_{\min}(A)(t_k - t)} p_k \bigl| x(t_k) \bigr|_1 \biggr|^2 \\
&\quad + E \biggl| \sum_{T<t_k<t} n e^{\lambda_{\min}(A)(t_k - t)} p \rho \bigl| x(t_k) \bigr|_1 \biggr|^2 \biggr] \\
&\le 2e^{(\alpha - 2\lambda_{\min}(A)) t} n^2 p_k^2 E \biggl| \sum_{0<t_k<T} e^{\lambda_{\min}(A) t_k} \bigl| x(t_k) \bigr|_1 \biggr|^2
\end{aligned}
$$

$$
\begin{aligned}
&+ 2e^{\alpha t}E\Big|\sum_{T<t_r<t_k} ne^{\lambda_{\min}(A)(t_r-t)}p(t_{r+1}-t_r)\big|x(t_k)\big|_1 \\
&+ ne^{\lambda_{\min}(A)(t_k-t)}p\rho\big|x(t_k)\big|_1\Big|^2 \\
\le{}& 2e^{(\alpha-2\lambda_{\min}(A))t}n^2p_k^2E\Big|\sum_{0<t_k<T} e^{\lambda_{\min}(A)t_k}\big|x(t_k)\big|_1\Big|^2 \\
&+ 2e^{\alpha t}E\Big|\int_T^t ne^{\lambda_{\min}(A)(s-t)}p\big|x(s)\big|_1\,ds + np\rho\big|x(t)\big|_1\Big|^2 \\
\le{}& 2e^{(\alpha-2\lambda_{\min}(A))t}n^2p_k^2E\Big|\sum_{0<t_k<T} e^{\lambda_{\min}(A)t_k}\big|x(t_k)\big|_1\Big|^2 \\
&+ 4e^{\alpha t}E\Big|\int_T^t ne^{\lambda_{\min}(A)(s-t)}p\big|x(s)\big|_1\,ds\Big|^2 + 4e^{\alpha t}E\big|np\rho\big|x(t)\big|_1\big|^2 \\
\le{}& 2e^{(\alpha-2\lambda_{\min}(A))t}n^2p_k^2E\Big|\sum_{0<t_k<T} e^{\lambda_{\min}(A)t_k}\big|x(t_k)\big|_1\Big|^2 \\
&+ 4n^2p^2e^{(\alpha-2\lambda_{\min}(A))t}\int_T^t e^{(2\lambda_{\min}(A)-\alpha)s}e^{\alpha s}E\big|x(s)\big|_1^2\,ds + 4n^2p^2\rho^2\epsilon \\
\le{}& 2e^{(\alpha-2\lambda_{\min}(A))t}n^2p_k^2E\Big|\sum_{0<t_k<T} e^{\lambda_{\min}(A)t_k}\big|x(t_k)\big|_1\Big|^2 \\
&+ 4n^2p^2e^{(\alpha-2\lambda_{\min}(A))t}\epsilon\int_T^t e^{(2\lambda_{\min}(A)-\alpha)s}\,ds + 4n^2p^2\rho^2\epsilon \\
\le{}& 2e^{(\alpha-2\lambda_{\min}(A))t}n^2p_k^2E\Big|\sum_{0<t_k<T} e^{\lambda_{\min}(A)t_k}\big|x(t_k)\big|_1\Big|^2 \\
&+ \frac{4n^2p^2\epsilon}{2\lambda_{\min}(A)-\alpha} + 4n^2p^2\rho^2\epsilon \\
\to{}& 0 \quad \text{as } t\to\infty.
\end{aligned}
$$

Thus we conclude that $Q:\Lambda\to\Lambda$.

Finally, we prove that Q is a contraction mapping. For any $\varphi,\phi\in\Lambda$, we obtain

$$
\begin{aligned}
&\sup_{t\ge-\tau} E\big|(Q\varphi)(t)-(Q\phi)(t)\big|_1^2 \\
&\quad= \sup_{t\ge-\tau}\bigg\{E\bigg|\int_0^t \exp A(s-t)\big[-\triangle A(s)\big(\varphi(s)-\phi(s)\big) \\
&\qquad+ \big(f\big(s,\varphi(s),\varphi(s-\tau_1)\big) - f\big(s,\phi(s),\phi(s-\tau_1)\big)\big)\big]\,ds \\
&\qquad+ \int_0^t \exp A(s-t)\big[\sigma\big(s,\varphi(s),\varphi(s-\tau_2)\big) - \sigma\big(s,\phi(s),\phi(s-\tau_2)\big)\big]\,d\omega(s) \\
&\qquad+ \sum_{0<t_k<t} e^{-A(t-t_k)}\big[I_k\big(\varphi(t_k)\big) - I_k\big(\phi(t_k)\big)\big]\bigg|_1^2\bigg\} \\
&\quad\le 3\sup_{t\ge-\tau}\bigg\{E\int_0^t \big\|\exp A(s-t)\big\|_3^2\big[\big\|\triangle A(s)\big\|_3\big|\varphi(s)-\phi(s)\big|_1 \\
&\qquad+ \mu_1\big|\varphi(s)-\phi(s)\big|_1 + \mu_2\big|\varphi(s-\tau_1)-\phi(s-\tau_1)\big|_1\big]^2\,ds
\end{aligned}
$$

$$
\begin{aligned}
&+ E\int_0^t \|\exp A(s-t)\|_3^2\big[\nu_1|\varphi(s)-\phi(s)|_1^2 + \nu_2|\varphi(s-\tau_2)-\phi(s-\tau_2)|_1^2\big]\,ds \\
&+ E\bigg|\sum_{0<t_k<t}\|\exp A(t_k-t)\|_3 p_k|\varphi(t_k)-\phi(t_k)|_1\bigg|^2\bigg\} \\
\le{}& 3\bigg\{\frac{n^2}{2\lambda_{\min}(A)}\big[\big(\|\triangle A(t)\|_3+\mu_1+\mu_2\big)^2+\nu_1+\nu_2\big] \\
&+ n^2p^2E\bigg|\sum_{0<t_k<t} e^{\lambda_{\min}(A)(t_k-t)}\rho\bigg|_1^2\bigg\}\sup_{t\ge-\tau}E|\varphi(s)-\phi(s)|_1^2 \\
\le{}& 3\bigg\{\frac{n^2}{2\lambda_{\min}(A)}\big[\big(\|\triangle A(t)\|_3+\mu_1+\mu_2\big)^2+\nu_1+\nu_2\big] \\
&+ n^2p^2E\bigg|\sum_{0<t_r<t_k} e^{\lambda_{\min}(A)(t_r-t)}(t_{r+1}-t_r) \\
&+ e^{\lambda_{\min}(A)(t_k-t)}\rho\bigg|_1^2\bigg\}\sup_{t\ge-\tau}E|\varphi(s)-\phi(s)|_1^2 \\
\le{}& 3\bigg\{\frac{n^2}{\lambda_{\min}(A)}\big[\big(\|\triangle A(t)\|_3+\mu_1+\mu_2\big)^2+\nu_1+\nu_2\big] \\
&+ n^2p^2\bigg(\frac{1}{\lambda_{\min}(A)}+\rho\bigg)^2\bigg\}\sup_{t\ge-\tau}E|\varphi(s)-\phi(s)|_1^2.
\end{aligned}
$$

From the condition (P_1), we find that Q is a contraction mapping. Hence, by the contraction mapping principle, we see that Q has a unique fixed point $x(t)$, which is a solution of (2.1) with $x(t)=\psi(t)$ as $t\in[-\tau,0]$ and $e^{\alpha t}E|x(t)|_1^2\to 0$ as $t\to\infty$. □

The second result is established using Krasnoselskii's fixed point theorem.

Theorem 3.2 *Assume* (H_1)-(H_6) *hold and the following condition is satisfied*:

$$(P_2)\quad \frac{n^2}{2\lambda_{\min}(A)}(\nu_1+\nu_2)<1,$$

then system (2.1) *is exponentially stable in mean square for all admissible uncertainties, that is,* $e^{\alpha t}E|x(t)|_1^2\to 0$, *as* $t\to\infty$.

Proof For $\forall x\in X$, define the operators $U: X\to X$ and $S: X\to X$, respectively, by

$$(Ux)(t)=\exp(-At)\psi(0)+\int_0^t \exp A(s-t)\sigma\big(s,x(s),x(s-\tau_2)\big)\,d\omega(s)$$

and

$$
\begin{aligned}
(Sx)(t)={}&\int_0^t \exp A(s-t)\big[-\triangle A(s)x(s)+f\big(s,x(s),x(s-\tau_1)\big)\big]\,ds \\
&+\sum_{0<t_k<t}\exp\big[-A(t-t_k)\big]I_k\big(x(t_k)\big).
\end{aligned}
$$

By the proof of Theorem 3.1, we can verify that $Sx+Uy\in\Lambda$ when $x,y\in\Lambda$ and S is mean square continuous.

Next, we show that U is a contraction mapping. For $x, y \in \Lambda$, we have

$$\begin{aligned}
\sup_{t\geq-\tau}\bigl|U(x)(t)-U(y)(t)\bigr|_1^2 &= \sup_{t\geq-\tau}E\biggl|\int_0^t \exp A(s-t)\bigl[\sigma\bigl(s,x(s),x(s-\tau_2)\bigr)\\
&\quad -\sigma\bigl(s,y(s),y(s-\tau_2)\bigr)\bigr]\,d\omega(s)\biggr|_1^2\\
&\leq \sup_{t\geq-\tau}E\int_0^t \bigl\|\exp A(s-t)\bigr\|_3^2\bigl[\nu_1\bigl|x(s)-y(s)\bigr|_1^2\\
&\quad +\nu_2\bigl|x(s-\tau_2)-y(s-\tau_2)\bigr|_1^2\bigr]\,ds\\
&\leq \frac{n^2}{2\lambda_{\min}(A)}\sup_{t\geq-\tau}E\bigl|x(t)-y(t)\bigr|_1^2.
\end{aligned}$$

From the condition (P_2), we find that U is a contraction mapping.

Finally, we prove that S is compact.

Let $D\subset\Lambda$ be a bounded set:$|x|_1\leq M$, $\forall x\in D$, we have

$$\begin{aligned}
\bigl|(Sx)(t)\bigr|_1 &= \biggl|\int_0^t \exp A(s-t)\bigl[-\triangle A(s)x(s)+f\bigl(s,x(s),x(s-\tau_1)\bigr)\bigr]\,ds\\
&\quad +\sum_{0<t_k<t}\exp\bigl[-A(t-t_k)\bigr]I_k\bigl(x(t_k)\bigr)\biggr|_1\\
&\leq \int_0^t \bigl\|\exp A(s-t)\bigr\|_3\bigl[\bigl\|\triangle A(s)\bigr\|_3\bigl|x(s)\bigr|_1+\mu_1\bigl|x(s)\bigr|_1+\mu_2\bigl|x(s-\tau_1)\bigr|_1\bigr]\,ds\\
&\quad +\sum_{0<t_k<t}\exp A(t_k-t)p\rho\bigl|x(t_k)\bigr|_1\\
&\leq \bigl(\bigl\|\triangle A(s)\bigr\|_3+\mu_1+\mu_2\bigr)M\int_0^t e^{\lambda_{\min}A(s-t)}\,ds+np\rho M\\
&\leq \frac{1}{\lambda_{\min}(A)}\bigl(\bigl\|\triangle A(s)\bigr\|_3+\mu_1+\mu_2+np\rho\bigr)M.
\end{aligned}$$

Therefore, we can conclude that Sx is uniformly bounded.

Further, let $x\in D$ and $\underline{t},\overline{t}\in[t_{k-1},t_k]$, with $\underline{t}<\overline{t}$; we have

$$\begin{aligned}
\bigl|(Sx)(\overline{t})-(Sx)(\underline{t})\bigr|_1 &= \biggl|\int_0^{\overline{t}} \exp A(s-\overline{t})\bigl[-\triangle A(s)x(s)+f\bigl(s,x(s),x(s-\tau_1)\bigr)\bigr]\,ds\\
&\quad +\sum_{0<t_m<\overline{t}}\exp\bigl[-A(\overline{t}-t_m)\bigr]I_m\bigl(x(t_m)\bigr)\\
&\quad -\int_0^{\underline{t}} \exp A(s-\underline{t})\bigl[-\triangle A(s)x(s)+f\bigl(s,x(s),x(s-\tau_1)\bigr)\bigr]\,ds\\
&\quad -\sum_{0<t_m<\underline{t}}\exp\bigl[-A(\underline{t}-t_m)\bigr]I_m\bigl(x(t_m)\bigr)\biggr|_1\\
&= \biggl|\int_0^{\underline{t}}\bigl[\exp A(s-\overline{t})-\exp A(s-\underline{t})\bigr]\\
&\quad \times\bigl[-\triangle A(s)x(s)+f\bigl(s,x(s),x(s-\tau_1)\bigr)\bigr]\,ds
\end{aligned}$$

$$
\begin{aligned}
&\qquad + \int_{\underline{t}}^{\overline{t}} \exp A(s-\overline{t})\bigl[-\triangle A(s)x(s) + f\bigl(s,x(s),x(s-\tau_1)\bigr)\bigr]\,ds \\
&\qquad + \sum_{0<t_m<\overline{t}} \bigl[\exp A(t_m-\overline{t}) - \exp A(t_m-\underline{t})\bigr] I_m\bigl(x(t_m)\bigr)\Big|_1 \\
&\to 0 \quad \text{as } \underline{t} \to \overline{t}.
\end{aligned}
$$

Thus, the equicontinuity of S is obtained. According to the PC-type Ascoli-Arzela lemma [15, Lemma 2.4], $S(D)$ is relatively compact in Λ. Therefore S is compact. By Lemma 2.1, $U+S$ has a fixed point x in Λ and we note $x(s) = (U+S)(s)$ on $[-\tau,0]$ and $e^{\alpha t}E|x(t)|_1^2 \to 0$ as $t \to \infty$. This completes the proof. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Acknowledgements
This research was supported by the National Nature Science Foundation of China (No. 11371027); National Nature Science Foundation of China, Tian Yuan (No. 11326115); Research Project for Academic Innovation (No. yfc100002); Program of Natural Science of Colleges of Anhui Province (No. KJ2013A032); Program of Natural Science Research in Anhui Universities (No. KJ2011A020 and No. KJ2012A019); Special Research Fund for the Doctoral Program of the Ministry of Education of China (No. 20123401120001 and No. 20103401120002).

Received: 11 February 2014 Accepted: 12 September 2014 Published: 24 Sep 2014

References

1. Huang, H, Cao, JD: Exponential stability analysis of uncertain stochastic neural networks with multiple delays. Nonlinear Anal., Real World Appl. **8**, 646-653 (2007)
2. Wang, ZD, Liu, YR, Liu, XH: On global asymptotic stability analysis of neural networks with discrete and distributed delays. Phys. Lett. A **345**(5-6), 299-308 (2005)
3. Wang, ZD, Lauria, S, Fang, JA, Liu, XH: Exponential stability of uncertain stochastic neural networks with mixed time-delays. Chaos Solitons Fractals **32**, 62-72 (2007)
4. Wan, L, Sun, J: Mean square exponential stability of stochastic delayed Hopfield neural networks. Phys. Lett. A **343**, 306-318 (2005)
5. Zheng, Z: Theory of Functional Differential Equations. Anhui Education Press, Hefei (1994)
6. Wei, J: The Degeneration Differential Systems with Delay. Anhui University Press, Hefei (1998)
7. Wei, J: The constant variation formulae for singular fractional differential systems with delay. Comput. Math. Appl. **59**(3), 1184-1190 (2010)
8. Wei, J: Variation formulae for time varying singular fractional delay differential systems. Fract. Differ. Calc. **1**(1), 105-115 (2011)
9. Mao, X: Stochastic Differential Equations and Applications. Horwood, New York (1977)
10. Mao, X: Razumikhin-type theorems on exponential stability of stochastic functional differential equations. Stoch. Process. Appl. **65**, 233-250 (1996)
11. Guo, C, O'Regan, D, Deng, F, Agarwal, RP: Fixed points and exponential stability for a stochastic neutral cellular neural network. Appl. Math. Lett. **26**, 849-853 (2013)
12. Luo, J: Exponential stability for stochastic neutral partial functional differential equations. J. Math. Anal. Appl. **355**, 414-425 (2009)
13. Peng, S, Jia, B: Some criteria on *p*th moment stability of impulsive stochastic functional differential equations. Stat. Probab. Lett. **80**, 1085-1092 (2010)
14. Zhang, YT, Luo, Q: Global exponential stability of impulsive cellular neural networks with time-varying delays via fixed point theory. Adv. Differ. Equ. **2013**, 23 (2013)
15. Bainov, DD, Simeonov, PS: Impulsive Differential Equations: Periodic Solutions and Applications. Longman, New York (1993)

10.1186/1687-1847-2014-250
Cite this article as: Zhang et al.: **Exponential stability for a stochastic delay neural network with impulses.** *Advances in Difference Equations* 2014, 2014:250