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Fault detection and estimation for non-Gaussian stochastic systems with time varying delay

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Abstract
In this paper, fault detection and estimation problem is studied for non-Gaussian stochastic systems with time varying delay. A new approach based on the output probability density function (PDF) and observers technique to detect and estimate time varying faults is presented. Some slack variables and scalars are introduced to design observers' parameters, which can provide more degrees of freedom. A particle distribution example is given to illustrate the design procedures, and the simulation results show the performance of the proposed approaches.

Keywords: fault detection; fault estimation; observer; PDF

1 Introduction

Automatic control systems are widely applied to many industrial processes. However, unexpected faults may destroy the stability of the systems. For such reasons, fault detection and estimation for dynamical systems has received much attention [1–5]. In past two decades, many significant approaches have been presented and applied to practical processes successfully [4]. In general, the fault detection (FD) results can be classified into three types: filter- or observer-based approaches [6–8]; the identification-based FD scheme [9, 10]; and statistic approach [11]. For the dynamic stochastic systems, the filter-based FD approach has been shown as an effective way where generally the variables are supposed to be Gaussian in [12] and [13]. It has been shown that in systems where either the system variables or not, the noise are not Gaussian in [14, 15]. Existing methods may not be sufficient to characterize the closed loop system behavior. As a result, the output PDF rather than the mean variance was proposed [16–18]. Here, we firstly introduce the output PDF definition. For a dynamic stochastic system, suppose that the random process $y \in [a,b]$ is the output of the stochastic system, its output PDFs are defined by $\gamma(z,u(t))$, where $u(t) \in Rm$ is control input. In output PDFs shape control, the B-spline expansion technique has been introduced in the output PDF modeling in [18–20], *i.e.*, the following square root B-spline expansion model has been used to approximate $\gamma(z,u(t))$:

$$\sqrt{\gamma\bigl(z,u(t)\bigr)} = \sum_{i=1}^{n} v_i(u)b_i(z), \tag{1}$$

where $b_i(z)$ $(i = 1, 2, \ldots, n)$ are pre-specified basis functions defined on $[a, b]$, and $v_i(u(t))$ $(i = 1, 2, \ldots, n)$ are the corresponding weights of such an expansion. Denote

$$\begin{aligned} B_0(z) &= \begin{bmatrix} b_1(z) & b_2(z) & \cdots & b_{n-1}(z) \end{bmatrix}^\tau, \\ V(t) := V\bigl(u(t)\bigr) &= \begin{bmatrix} v_1 & v_2 & \cdots & v_{n-1} \end{bmatrix}^\tau. \end{aligned} \tag{2}$$

And let $\Lambda_1 = \int_a^b B_0(z)B_0^\tau(z)\,dz$, $\Lambda_2 = \int_a^b B_0^\tau(z)b_n(z)\,dz$, $\Lambda_3 = \int_a^b b_n^2(z)\,dz \neq 0$, $\Lambda_0 = \Lambda_1\Lambda_3 - \Lambda_2\Lambda_2^\tau$. Furthermore, it can be verified that (1) can be rewritten as:

$$\sqrt{r\bigl(z, u(t)\bigr)} = B^\tau(z)V(t) + h\bigl(V(t)\bigr)b_n(z), \tag{3}$$

where

$$B^\tau(z) = B_0^\tau(z) - \frac{\Lambda_2}{\Lambda_3}b_n(z), \qquad h\bigl(V(t)\bigr) = \frac{\sqrt{\Lambda_3 - V^\tau(t)\Lambda_0 V^\tau(t)}}{\Lambda_3}, \tag{4}$$

where $h(V(t))$ satisfies $\|h(V_1(t)) - h(V_2(t))\|_2 \leq \|U_1(V_1(t) - V_2(t))\|_2$ for any $V_1(t)$ and $V_2(t)$, and U_1 is a known matrix.

The motivation of fault detection and estimation via the output PDFs from the retention system in papermaking was first studied in [20–25], where the weight dynamical system was supposed to be a precise linear model. However, linear mappings cannot change the shape of output PDFs, which implies that the fault cannot be detected through the shape change of the PDFs. To meet the requirement in complex processes, nonlinearity should be considered in the weighting dynamic behavior [18, 26–31]. For example, the following nonlinear dynamic model was considered in [18]:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t - d(t)) + Gg(x(t)) + Hu(t) + Jf(t), \\ V(t) = Ex(t), \end{cases} \tag{5}$$

where $x(t) \in R^m$ is the unmeasured state, $f(t)$ is the fault to be detected and be assumed $\|f(t)\| \leq \alpha$ and $\|\dot{f}(t)\| \leq \beta$. A, A_d, G, H, D and E represent the known parametric matrices of the dynamic part of the weight system. $d(t)$ is time varying delay and satisfies $0 < d(t) \leq h$ and $\dot{d}(t) \leq \mu$. The nonlinear function $g(x(t))$ is assumed to be Lipschitz with respect to the state x, *i.e.*, $\|g(x_1(t)) - g(x_2(t))\|_2 \leq \|U_2(x_1(t) - x_2(t))\|_2$, where U_2 is a known matrix.

Recently, a fault detection algorithm has been established by using the output PDFs in [16, 18, 32–36]. However, the algorithms in [16] did not consider time delay information in the designed fault detection observer and the threshold. The method in [18] provides less conservative fault detection algorithms than [16] by designing delay-dependent observer and minimizing the threshold. To further improve the previous results, in this paper, a new delay-dependent observer design is presented such that the estimation error system is stable, and the fault can be detected and estimated through a threshold by introducing the tuning parameter and slack variable. Finally, particle distribution process example is given to demonstrate the applicability of the proposed approach.

Notation 1 Throughout this paper, for a vector $\omega(t)$, its Euclidean norm is defined by $\|\omega(t)\|_2 = \sqrt{\omega^\tau(t)\omega(t)}$. A real symmetric matrix $P > 0$ (≥ 0) denotes P being a positive

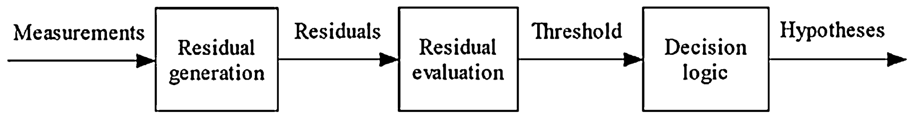

Figure 1 Fault-detection system.

definite (positive semi-definite) matrix, and $A > (\geq)B$ means $A - B > (\geq)0$. I is used to denote an identity matrix with proper dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symmetric terms in a symmetric matrix are denoted by $*$.

2 Fault detection

Generally speaking, a fault-detection system consists of a residual generator and a residual evaluator including an evaluation function and a threshold as in Figure 1 [33–42]. We will consider two parts of fault detection systems by using the information of PDF in the following section.

2.1 Residual generator

For the purpose of residual generation, we construct the following nonlinear observer:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}\big(t - d(t)\big) + Gg\big(\hat{x}(t)\big) + Hu(t) + L\xi(t), \tag{6}$$

where $\hat{x}(t)$ is the estimated state, $L \in R^{m\times p}$ is the gain to be determined, $\xi(t)$ is output PDF's estimation error defined as

$$\xi(t) = \int_a^b \sigma(z)\Big(\sqrt{\gamma\big(z, u(t), f(t)\big)} - \sqrt{\hat{\gamma}\big(z, u(t)\big)}\Big)\,dz$$

and

$$\sqrt{\hat{\gamma}\big(z, u(t)\big)} = B^\tau(z)E\hat{x}(t) + h\big(E\hat{x}(t)\big)b_n(z).$$

Define a state estimation error as $e(t) = x(t) - \hat{x}(t)$ and $\xi(t)$, it can be shown that

$$\begin{aligned} \dot{e}(t) &= (A - L\Gamma_1)e(t) + A_d e\big(t - d(t)\big) + G\big[g\big(x(t)\big) - g\big(\hat{x}(t)\big)\big] \\ &\quad - L\Gamma_2\big[h\big(Ex(t)\big) - h\big(E\hat{x}(t)\big)\big] + Jf(t), \end{aligned} \tag{7}$$

$$\xi(t) = \Gamma_1 e(t) + \Gamma_2\Big(h\big(Ex(t)\big) - h\big(E\hat{x}(t)\big)\Big), \tag{8}$$

where $\Gamma_1 = \int_a^b \sigma(z)B^\tau(z)E\,dz$, $\Gamma_2 = \int_a^b \sigma(z)b_n(z)\,dz$.

Thus, the problem of designing an observer-based fault detection can be described as designing a matrix L such that the error system (7) is asymptotically stable and the fault can be detected.

In order to formulate some practically computable criteria to check the stability of the error system described by (7) and provide a feasible observer design method, the following lemma is needed.

Lemma 1 [1] *For any matrix $M > 0$, scalars $b > a$ and $c < d \leq 0$, if there exists a Lebesgue vector function $\omega(s)$, then the following inequalities hold*:

$$-\int_a^b \omega^{\mathsf{T}}(s)M\omega(s)\,ds \leq -\frac{1}{b-a}\tilde{\omega}^{\mathsf{T}}(s)M\tilde{\omega}(s-), \tag{9}$$

$$-\int_c^d \int_{t+\theta}^t \omega^{\mathsf{T}}(s)M\omega(s)\,ds\,d\theta \leq -\frac{2}{c^2-d^2}\bar{\omega}^{\mathsf{T}}(s)M\bar{\omega}(s), \tag{10}$$

where $\tilde{\omega}(s) = \int_a^b \omega^{\mathsf{T}}(s)\,ds$, $\bar{\omega}(s) = \int_c^d \int_{t+\theta}^t \omega(s)\,ds\,d\theta$.

Based on the above lemma, a new delay-dependent fault detection observer can be designed by using the following result.

Theorem 1 *Given the scalars $\lambda_i > 0$ $(i = h$ and $\mu)$, if there exist matrices $P > 0$, $Q > 0$, $R_1 > 0$, $R_2 > 0$, any matrices Z and N, satisfying*

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \frac{1}{h}R_1 - \varepsilon NA_d & 0 & \frac{2}{h}R_2 & \varepsilon NG & \varepsilon Z\Gamma_2 \\ * & \Xi_{22} & -NA_d & 0 & 0 & -NG & Z\Gamma_2 \\ * & * & \Xi_{33} & \frac{1}{h}R_1 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{h}R_1 & 0 & 0 & 0 \\ * & * & * & * & -\frac{2}{h^2}R_2 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{\lambda_2^2}I & 0 \\ * & * & * & * & * & * & -\frac{1}{\lambda_1^2}I \end{bmatrix} < 0, \tag{11}$$

where

$$\Xi_{11} = -\frac{1}{h}R_1 - 2R_2 + Q + \frac{1}{\lambda_1^2}E^{\tau}U_1^{\tau}U_1E + \frac{1}{\lambda_2^2}U_2^{\tau}U_2 - \varepsilon\bigl(NA - Z\Gamma_1 + A^{\mathsf{T}}N^{\mathsf{T}} - \Gamma_1^{\mathsf{T}}Z^{\mathsf{T}}\bigr),$$

$$\Xi_{12} = P_1 - A^{\mathsf{T}}N^{\mathsf{T}} + \Gamma_1^{\mathsf{T}}Z^{\mathsf{T}} + \varepsilon N,$$

$$\Xi_{22} = N + N^{\mathsf{T}} + hR_1 + \frac{h^2}{2}R_2,$$

$$\Xi_{33} = -\frac{2}{h}R_1 - (1-\mu)Q,$$

then in the absence of the fault $f(t)$, the error system (7) *with gain $L = N^{-1}Z$ is stable.*

Proof Define $\tilde{g} := g(x(s)) - g(\hat{x}(s))$, $\tilde{h} := h(Ex(s) - h(E\hat{x}(s)))$ and denote the Lyapunov function candidate as follows:

$$\begin{aligned} V_1(t) &= e^{\tau}(t)P_1e(t) + \int_{-h}^0 \int_{t+\theta}^t \dot{e}^{\tau}(s)R_1\dot{e}(s)\,ds\,d\theta \\ &\quad + \int_{-h}^0 \int_{\theta}^0 \int_{t+v}^t \dot{e}^{\tau}(s)R_2\dot{e}(s)\,ds\,dv\,d\theta + \int_{t-d(t)}^t e^{\tau}(s)Qe(s)\,ds \\ &\quad + \frac{1}{\lambda_1^2}\int_0^t \bigl[\bigl\|U_1Ee(s)\bigr\|^2 - \|\tilde{h}\|^2\bigr]\,ds + \frac{1}{\lambda_2^2}\int_0^t \bigl[\bigl\|U_2e(s)\bigr\|^2 - \|\tilde{g}\|^2\bigr]\,ds \end{aligned} \tag{12}$$

with $P > 0$, $T > 0$, $Q > 0$. Then following (5) and (6) gives $V(t) \geq 0$. Along the trajectories of (8) in the absence of $f(t)$ and by using the completion-of-square method, it can be shown that

$$\begin{aligned}\dot{V}_1(t) &\le 2e^\tau(t)P\dot{e}(t) + \dot{e}^\tau(t)\left(hR_1 + \frac{h^2}{2}R_2\right)\dot{e}(t) - \int_{t-h}^{t}\dot{e}^\tau(s)R_1\dot{e}(s)\,ds \\ &\quad - \int_{-h}^{0}\int_{t+\theta}^{t}\dot{e}^\tau(s)R_2\dot{e}(s)\,ds\,d\theta + e^\tau(t)Qe(t) - (1-\mu)e^\tau\big(t-d(t)\big)Qe\big(t-d(t)\big) \\ &\quad + \frac{1}{\lambda_1^2}e^\tau(t)\big(E^\tau U_1^\tau U_1E\big)e(t) + \frac{1}{\lambda_2^2}e^\tau(t)\big(U_2^\tau U_2\big)e(t) - \frac{1}{\lambda_1^2}\tilde{h}^\tau\tilde{h} - \frac{1}{\lambda_2^2}\tilde{g}^\tau\tilde{g} \\ &\quad + \varpi_1(t).\end{aligned} \tag{13}$$

It is noted that $\varpi_1(t) = \dot{e}(t) - (A - L\Gamma_1)e(t) - A_d e(t-d(t)) - G\tilde{g} + L\Gamma_2\tilde{h} = 0$ in the absence of $f(t)$. According to the free weighting matrix method in [3], for any matrix N, the following equality holds:

$$2\big(\varepsilon e^\tau(t)N + \dot{e}^\tau(t)N\big)\big(\dot{e}(t) - (A - L\Gamma_1)e(t) - A_d e\big(t-d(t)\big) - G\tilde{g} + L\Gamma_2\tilde{h}\big) = 0. \tag{14}$$

From Lemma 1, it is easily shown that

$$\begin{aligned}&-\int_{t-h}^{t}\dot{e}^\tau(s)R_1\dot{e}(s)\,ds \\ &\quad = -\int_{t-h}^{t-d(t)}\dot{e}^\tau(s)R_1\dot{e}(s)\,ds - \int_{t-d(t)}^{t}\dot{e}^\tau(s)R_1\dot{e}(s)\,ds \\ &\quad \le \begin{bmatrix} e^\mathsf{T}(t) & e^\mathsf{T}(t-d(t)) & e^\mathsf{T}(t-h)\end{bmatrix}\begin{bmatrix} -\frac{R_1}{h} & \frac{R_1}{h} & 0 \\ * & -2\frac{R_1}{h} & \frac{R_1}{h} \\ * & * & -\frac{R_1}{h}\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-d(t)) \\ e(t-h)\end{bmatrix},\end{aligned} \tag{15}$$

$$\begin{aligned}&-\int_{-h}^{0}\int_{t+\theta}^{t}\dot{e}^\tau(s)R_2\dot{e}(s)\,ds\,d\theta \\ &\quad \le -\frac{2}{h^2}\left(\int_{-h}^{0}\int_{t+\theta}^{t}\dot{e}(s)\,ds\,d\theta\right)^\mathsf{T} R_2\left(\int_{-h}^{0}\int_{t+\theta}^{t}\dot{e}(s)\,ds\,d\theta\right) \\ &\quad = \begin{bmatrix} e^\mathsf{T}(t) & \int_{t-h}^{t}e^\mathsf{T}(s)\,ds\end{bmatrix}\begin{bmatrix} -2R_2 & \frac{2}{h}R_2 \\ * & -\frac{2}{h^2}R_2\end{bmatrix}\begin{bmatrix} e(t) \\ \int_{t-h}^{t}e(s)\,ds\end{bmatrix}.\end{aligned} \tag{16}$$

From (13) and (15), we can have $\Xi < 0$, which implies $\dot{V}_1(t) \le \eta_1^\mathsf{T}(t)\Xi\eta_1(t) < 0$, where $\eta_1(t) = [e^\mathsf{T}(t)\ \dot{e}^\mathsf{T}(t)\ e^\mathsf{T}(t-d(t))\ e^\mathsf{T}(t-h)\ \int_{t-h}^{t} e^\mathsf{T}(s)\,ds\ \tilde{g}^\mathsf{T}\ \tilde{h}^\mathsf{T}]^\mathsf{T}$ and the error system (7) is asymptotically stable. This completes the proof. □

Compared with the result in [18], time varying delay is considered and a new method in [1] to deal with time delay is also used in Theorem 1. Meanwhile, to reduce complex computations, some free weighting matrices Y, W in [18] are not introduced in this paper.

2.2 Residual evaluator

After the fault detection observer is designed, the next important task for fault detection is the evaluation of the generated residual, including a threshold and a decision logic unit

[43–46]. In this case, we choose

$$J_r = \sqrt{\int_{t_o}^{t_0+t} \xi^{\mathsf{T}}(s)\xi(s)\,ds} \tag{17}$$

as the residual evaluation function, where t_0 denotes the initial evaluation time instant and t stands for the evaluation time, and $\xi(t)$ is defined in (8). Let

$$J_{th} = \sup_{f(t)=0} J_r \tag{18}$$

be the threshold. Based on this, the following logical relationship is used for fault detection:

$$J_r > J_{th} \Rightarrow alarm \Rightarrow fault,$$
$$J_r < J_{th} \Rightarrow no\ fault.$$

3 Fault estimation

For the purpose of estimation, we construct the following nonlinear observer:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + A_d\hat{x}\big(t-d(t)\big) + Gg\big(\hat{x}(t)\big) + Hu(t) + J\hat{f}(t) + L_2\xi(t), \\ \dot{\hat{f}}(t) &= \Upsilon_1\hat{f}(t) + \Upsilon_2\xi(t), \end{aligned} \tag{19}$$

where $\hat{x}(t)$ and $\hat{f}(t)$ are estimation of $x(t)$ and $f(t)$. L_2, Υ_1 and Υ_2 are the gain parameters to be determined. $\xi(t)$ has been denoted in (8).

By using $e(t) = x(t) - \hat{x}(t)$ and $e_f(t) = f(t) - \hat{f}(t)$, the estimation error system can be formulated to give

$$\begin{aligned} \dot{e}(t) &= (A - L_2\Gamma_1)e(t) + A_d e\big(t-d(t)\big) + G\big[g\big(x(t)\big) - g\big(\hat{x}(t)\big)\big] \\ &\quad - L\Gamma_2\big[h\big(Ex(t)\big) - h\big(E\hat{x}(t)\big)\big] + Je_f(t). \end{aligned} \tag{20}$$

Theorem 2 *Given the scalars $\lambda_i > 0$ $(i = 1,2)$, h, μ and γ, if there exist scalars $u_i > 0$ $(i = 1,2,\ldots,8)$, matrices $P_1 > 0$, $P_2 > 0$, $Q > 0$, $R_1 > 0$, $R_2 > 0$, and any matrices Z, N, W_1 and W_2 satisfying*

$$\begin{bmatrix} \Xi & \bar{\Xi}_0^{\mathsf{T}} \\ \bar{\Xi}_0 & \Upsilon_1^{\mathsf{T}} + \Upsilon_1 \end{bmatrix} + \operatorname{diag}\{u_1I, u_2I, u_3I, u_4I, u_5I, u_6I, u_7I, u_8I\} < 0, \tag{21}$$

where

$$\bar{\Xi}_0 = \begin{bmatrix} -(\gamma\Upsilon_2\Gamma_1 + \varepsilon J^{\mathsf{T}}N^{\mathsf{T}})J^{\mathsf{T}}N^{\mathsf{T}} & 0 & 0 & 0 & 0 & -\gamma\Upsilon_2\Gamma_2 \end{bmatrix} \tag{22}$$

and Ξ is defined in (11). When $\|e_f(t)\| \geq \frac{2\alpha|P_2|+2\beta|W_1|}{u_8}$, the error system (20) with gain $L_2 = N^{-1}Z$, $\Upsilon_1 = P_2^{-1}W_1$ and $\Upsilon_2 = P_2^{-1}W_2$ is asymptotically stable in the presence of $f(t)$.

Proof Denote the Lyapunov function candidate as follows:

$$V_2(t) = V_1(t) + \gamma e_f^{\mathsf{T}}(t)e_f(t) \tag{23}$$

with $\gamma > 0$. It can be shown that

$$\dot{V}_2(t) = \dot{V}_1(t) + 2\gamma e_f^{\mathsf{T}}(t)\dot{e}_f(t) + \varpi_2(t). \tag{24}$$

It is noted that $\varpi_2(t) = \dot{e}(t) - (A - L_2\Gamma_1)e(t) - A_d e(t - d(t)) - G\tilde{g} + L_2\Gamma_2\tilde{h} - Je_f(t) = 0$. According to the free weighting matrix method in [3], for any matrix N, the following equality holds:

$$2\bigl(\varepsilon e^{\mathsf{T}}(t)N + \dot{e}^{\tau}(t)N\bigr)\bigl[\dot{e}(t) - (A - L\Gamma_1)e(t) - A_d e\bigl(t - d(t)\bigr) - G\tilde{g} + L\Gamma_2\tilde{h} - Je_f(t)\bigr] = 0. \tag{25}$$

Then we have

$$\begin{aligned}
\dot{V}_2(t) &\le \eta_1^{\mathsf{T}}(t)\Xi\eta_1(t) + 2\varepsilon e^{\mathsf{T}}(t)NJe_f(t) + 2\dot{e}^{\mathsf{T}}(t)NJe_f(t) + 2\gamma e_f^{\mathsf{T}}(t)\dot{e}_f(t)\\
&= \eta_1^{\mathsf{T}}(t)\Xi\eta_1(t) + 2\varepsilon e^{\mathsf{T}}(t)NJe_f(t) + 2\dot{e}^{\mathsf{T}}(t)NJe_f(t) + 2\gamma e_f^{\mathsf{T}}(t)\dot{f}(t) - 2\gamma e_f^{\mathsf{T}}(t)\Upsilon_1 f(t)\\
&\quad + 2\gamma e_f^{\mathsf{T}}(t)\Upsilon_1 e_f(t) - 2\gamma e_f^{\mathsf{T}}(t)\Upsilon_2\Gamma_1 e(t) - 2\gamma e_f^{\mathsf{T}}(t)\Upsilon_2\Gamma_2\tilde{h}\\
&= \eta_2^{\mathsf{T}}(t)\begin{bmatrix}\Xi & \bar{\Xi}_0^{\mathsf{T}}\\ \bar{\Xi}_0 & W_1^{\mathsf{T}} + W_1\end{bmatrix}\eta_2(t) + 2\gamma e_f^{\mathsf{T}}(t)\dot{f}(t) - 2\gamma e_f^{\mathsf{T}}(t)\Upsilon_1 f(t)\\
&\le -u_1\bigl\|e(t)\bigr\|_2^2 - u_2\bigl\|\dot{e}(t)\bigr\|_2^2 - u_3\bigl\|e\bigl(t - d(t)\bigr)\bigr\|_2^2 - u_4\bigl\|e(t-h)\bigr\|_2^2 - u_5\left\|\int_{t-h}^{t} e(s)\,ds\right\|_2^2\\
&\quad - u_6\|\tilde{g}\|_2^2 - u_7\|\tilde{h}\|_2^2 - u_8\bigl\|e_f(t)\bigr\|_2^2 + 2\gamma\bigl\|e_f(t)\bigr\|\bigl\|\dot{f}(t)\bigr\| + 2\gamma\bigl\|e_f(t)\bigr\|\|\Upsilon_1\|\bigl\|f(t)\bigr\|\\
&\le -u_8\bigl\|e_f(t)\bigr\|_2^2 + 2\gamma\alpha\bigl\|e_f(t)\bigr\| + 2\gamma\beta\bigl\|e_f(t)\bigr\||W_1|\\
&= \bigl(-u_8\bigl\|e_f(t)\bigr\| + 2\gamma\alpha + 2\gamma\beta|W_1|\bigr)\bigl\|e_f(t)\bigr\|,
\end{aligned} \tag{26}$$

where $\eta_2(t) = [\eta_1^{\mathsf{T}}(t)\ e_f^{\mathsf{T}}(t)]^{\mathsf{T}}$, if $\|e_f(t)\| \ge \frac{2\gamma\alpha + 2\gamma\beta|W_1|}{u_8}$, then the above (26) has the form of $\dot{V}_2(t) < 0$. That is to say, the estimation error of the fault is asymptotically stable. □

In Theorem 2, some parameters u_i ($i = 1, 2, \ldots, 8$) and $\gamma$are introduced. These parameters may provide more degrees of freedom in fault estimation observer design and estimation performance.

4 Simulations

In this section, we consider a simple example related to the particle distribution control problems, where the shapes of measured output PDF usually have two or three peaks (see [18–22]). Suppose these output PDFs can be approximated using a square root B-spline model as $\sqrt{\gamma(z, u(t))} = \sum_{i=1}^{3} v_i(u(t), F)b_i(z)$, where z is defined in $[0, 1.5]$ and

$$b_i = \begin{cases} |\sin 2\pi z|, & z \in [0.5(i-1), 0.5i],\\ 0, & \textit{others}.\end{cases}$$

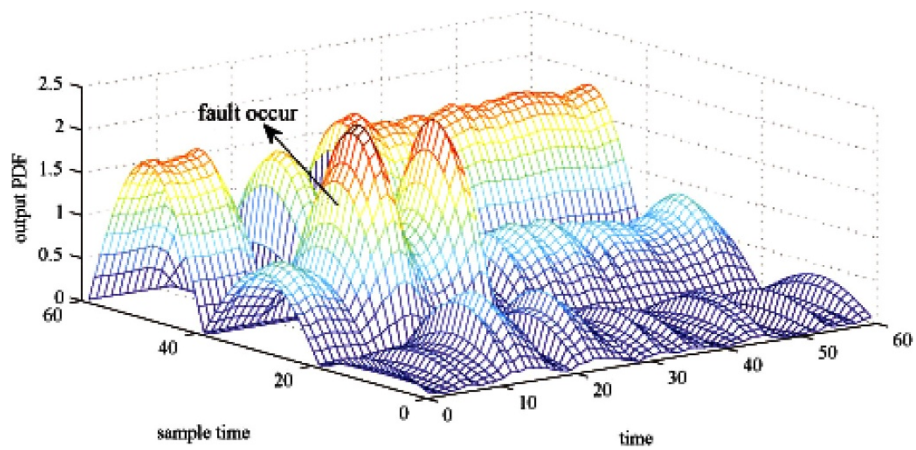

Figure 2 The 3-D mesh plot of the output PDF when the fault occurs.

For $i = 1, 2, 3$, it can be verified that $\Lambda_1 = \mathrm{diag}\{0.25, 0.25\}$, $\Lambda_2 = [0, 0]$, $\Lambda_3 = 0.25$. It is assumed that the identified weighting system is formulated by (5) with the following coefficient matrices:

$$A = \begin{bmatrix} -1.5 & 0 \\ 0 & -2 \end{bmatrix}, \qquad A_d = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.5 \end{bmatrix}, \qquad G = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},$$

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad E = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}, \qquad J = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

The upper bounds of nonlinearity are denoted by $U_1 = \mathrm{diag}\{0.1, 0.1\}$, $U_2 = \mathrm{diag}\{0.9, 0.9\}$. It can be tested that $\Gamma_1 = [0.9549\ 1.2732]$, $\Gamma_2 = 0.3183$ for $\sigma(z) = 1$. In the simulation, the initial condition of the system state and its estimation are selected as

$$x(t) = \begin{bmatrix} 0.1 + \exp(t-5) - 0.1 + \exp(t-5) \end{bmatrix}^{\tau}, \quad t \in [-2.5, 0], \hat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}, t \in [-2.5, 0]$$

with the parameters being given as $\lambda_1 = 1$, $\lambda_2 = 1$, $\mu = 0.5$ The fault is supposed as

$$f(t) = \begin{cases} 0, & t < 5, \\ 0.5 + 0.3 \times \sin(t), & 0 \le t \ge 15, \\ 0, & t > 15. \end{cases}$$

By using Theorem 1 and Theorem 2, we can obtain Figures 2, 3, 4, 5, 6, the three-dimensional (3-D) mesh plot shows the changes of the measured output PDFs and Figure 3 demonstrates the responses of residual signal, Figure 4 shows the threshold and the evaluation function. Figures 5 and 6 demonstrate the response of the error system and fault estimation, when the fault occurs at 5 seconds to 15.

5 Conclusion

In this paper, a new fault detection and estimation scheme has been developed for the stochastic dynamic systems with time varying delay by using stochastic distribution of

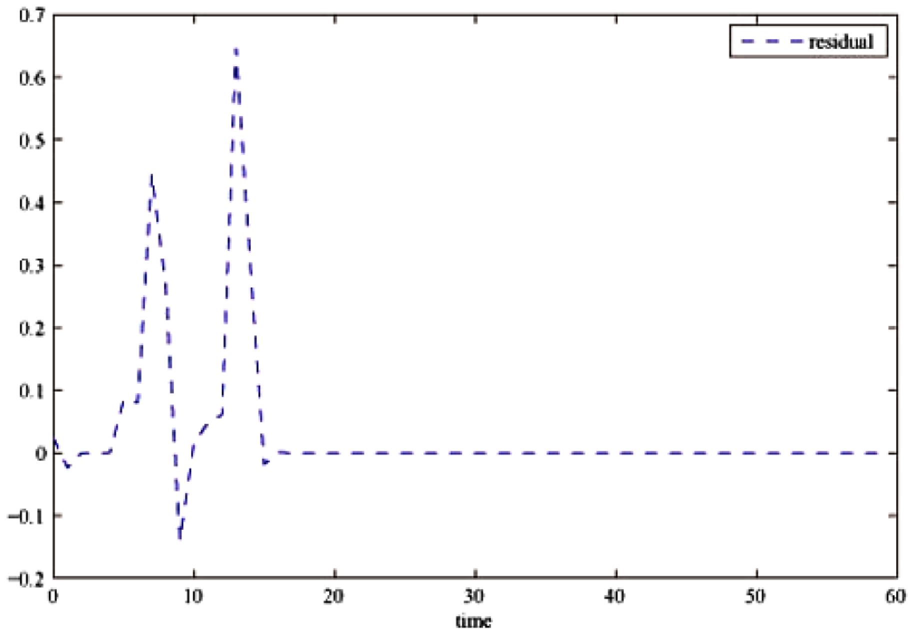

Figure 3 The response of the residual signal.

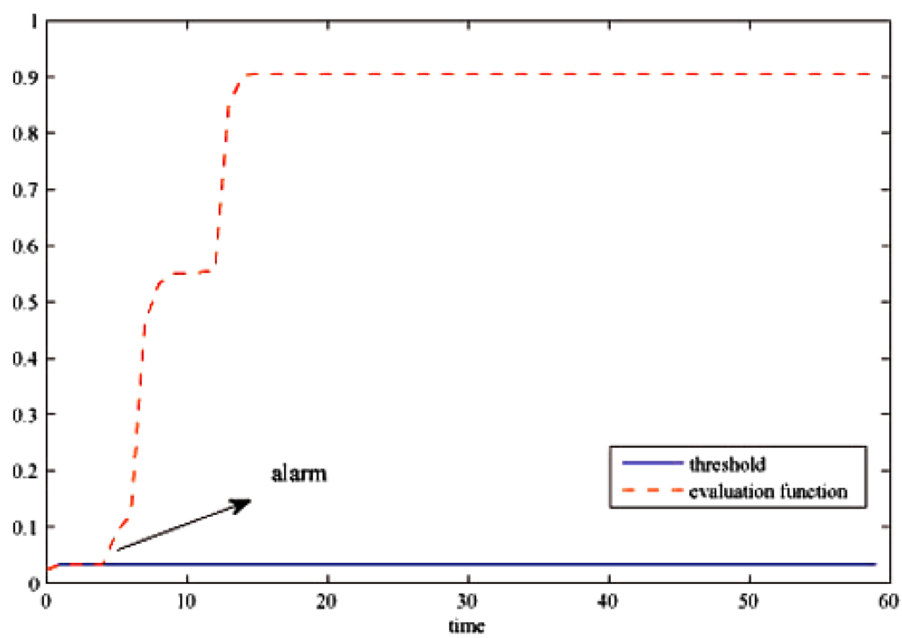

Figure 4 Threshold and the evaluation function.

system output. Based on LMI techniques and by using the slack variables, a new delay-dependent fault detection observer is designed to detect the system fault with a threshold. Furthermore, an observer-based fault estimation method is provided to estimate the size of the fault. Particle distribution example is to show the efficiency of the proposed approach.

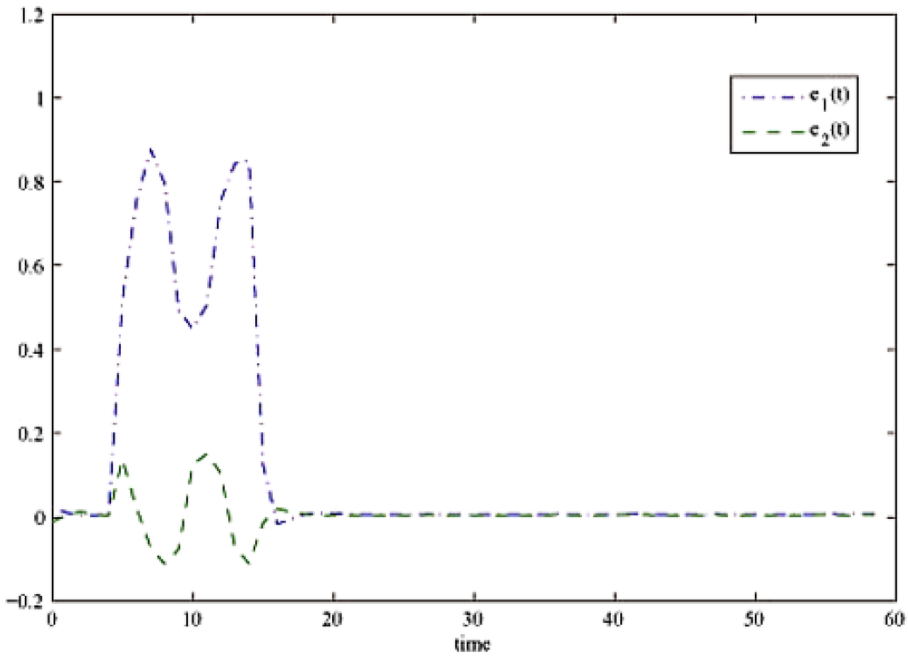

Figure 5 The response of the error system when the fault occurs.

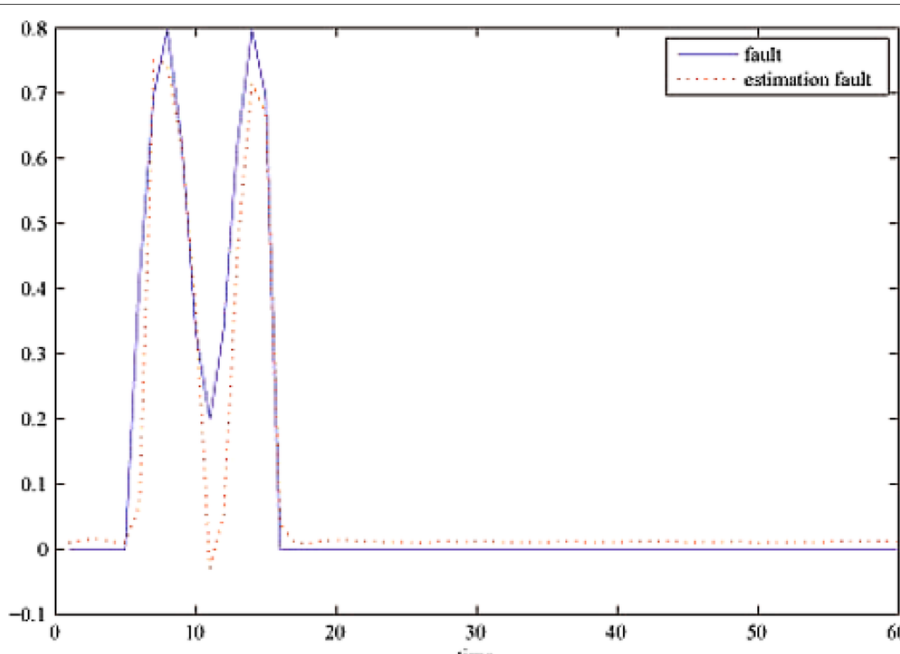

Figure 6 The fault and its estimation.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors drafted the manuscript, read and approved the final manuscript.

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