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Existence and exponential stability of anti-periodic solutions for HCNNs with time-varying leakage delays

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Abstract
This paper is concerned with a class of high-order cellular neural networks (HCNNs) model with time-varying delays in the leakage terms. By using the Lyapunov functional method and differential inequality techniques, we establish sufficient conditions on the existence and exponential stability of anti-periodic solutions for the model. Our results complement some recent ones.
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1 Introduction

In the past decade, high-order cellular neural networks (HCNNs) have attracted much attention due to their wide range of applications in many fields such as signal and image processing, pattern recognition, optimization, and many other subjects. There have been extensive results on the problem of global stability of periodic solutions and anti-periodic solutions of HCNNs in the literature (see [1–5]). Recently, some attention has been paid to neural networks with time delay in the leakage (or 'forgetting') term (see [6–15]). In particular, Xu [16] considered the existence and exponential stability of the anti-periodic solutions for the following HCNNs with time-varying delays in the leakage terms:

$$\begin{aligned} x_i'(t) &= -c_i(t)x_i\bigl(t-\delta_i(t)\bigr) + \sum_{j=1}^{n} a_{ij}(t)f_j\bigl(x_j\bigl(t-\tau_{ij}(t)\bigr)\bigr) \\ &\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)g_j\bigl(x_j\bigl(t-\alpha_{ijl}(t)\bigr)\bigr)g_l\bigl(x_l\bigl(t-\beta_{ijl}(t)\bigr)\bigr) \\ &\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} d_{ijl}(t)\int_0^{\infty}\sigma_{ijl}(u)h_j\bigl(x_j(t-u)\bigr)\,du\int_0^{\infty}\nu_{ijl}(u)h_l\bigl(x_l(t-u)\bigr)\,du \\ &\quad + I_i(t), \quad i=1,2,\ldots,n, \end{aligned} \tag{1.1}$$

in which n corresponds to the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the ith unit at the time t, $c_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs, $a_{ij}(t)$, $b_{ijl}(t)$ and $d_{ijl}(t)$ are the first- and second-order connection weights of the neural network, $\delta_i(t) \geq 0$ corresponds to the time-varying leakage delays, $\alpha_{ijl}(t) \geq 0$, $\beta_{ijl}(t) \geq 0$ and $\tau_{ij}(t) \geq 0$ correspond to the transmission delays, $\sigma_{ijl}(u)$ and $\nu_{ijl}(u)$ correspond to the transmission delay kernels, $I_i(t)$ denotes the external inputs at time t, f_j, g_j and h_j are the activation functions of signal transmission.

The initial conditions associated with system (1.1) are of the form

$$x_i(s) = \varphi_i(s), \quad s \in (-\infty, 0], i = 1, 2, \ldots, n, \tag{1.2}$$

where $\varphi_i(\cdot)$ denotes a real-valued bounded continuous function defined on $(-\infty, 0]$.

Under some suitable conditions on coefficients of (1.1), the author in [16] derived some new sufficient conditions ensuring that all solutions of system (1.1) converge exponentially to the anti-periodic solution, but the result leaves room for improvement. In fact, in the proof of Lemma 2.1, the expression $x_i(t) - x_i(t - \delta_i(t)) = \int_{t-\delta_i(t)}^t x_i'(u)\,du$ was used, and the author replaced $x_i'(u)$ by the right-hand side of equation (1.1). The case that $t - \delta_i(t) < 0$ is possible, so the integration $\int_{t-\delta_i(t)}^t x_i'(u)\,du$ should be handled as $\int_{t-\delta_i(t)}^t x_i'(u)\,du = \int_{t-\delta_i(t)}^0 x_i'(u)\,du + \int_0^t x_i'(u)\,du$, for $\int_0^t x_i'(u)\,du$ can be replaced by the right-hand side of equation (1.1), but for $\int_{t-\delta_i(t)}^0 x_i'(u)\,du$ cannot be replaced by the right-hand side of equation (1.1). A similar error also occurs in Lemma 2.2 of [16]. For this reason, the course of proof in Lemmas 2.1 and 2.2 is not true. Motivated by this, we shall give a new proof to ensure the existence and exponential stability of the anti-periodic solutions for system (1.1). Moreover, an example is also provided to illustrate the effectiveness of our results.

Let $u(t) : R \to R$ be continuous in t. $u(t)$ is said to be T-anti-periodic on R if

$$u(t + T) = -u(t) \quad \text{for all } t \in R.$$

Throughout this paper, for $i, j, l = 1, 2, \ldots, n$, it will be assumed that $c_i, I_i, a_{ij}, b_{ijl}, d_{ijl} : R \to R$ and $\delta_i, \tau_{ij}, \alpha_{ijl}, \beta_{ijl} : R \to [0, +\infty)$ are bounded continuous functions, $\sigma_{ijl}, \nu_{ijl} : [0, +\infty) \to R$ are continuous functions, c_i is bounded above and below by positive constants, δ_i' is a bounded continuous function, $|\sigma_{ijl}(t)|e^{\kappa t}$ and $|\nu_{ijl}(t)|e^{\kappa t}$ are integrable on $[0, +\infty)$ for a certain positive constant κ, and

$$c_i(t + T) = c_i(t), \qquad a_{ij}(t + T)f_j(\nu) = -a_{ij}(t)f_j(-\nu), \tag{1.3}$$

$$b_{ijl}(t + T)g_j(\nu_j)g_l(\nu_l) = -b_{ijl}(t)g_j(-\nu_j)g_l(-\nu_l), \tag{1.4}$$

$$\begin{aligned} &d_{ijl}(t + T)\int_0^\infty \sigma_{ijl}(u)h_j\big(\nu_j(t-u)\big)\,du \int_0^\infty \nu_{ijl}(u)h_l\big(\nu_l(t-u)\big)\,du \\ &\quad = -d_{ijl}(t)\int_0^\infty \sigma_{ijl}(u)h_j\big(-\nu_j(t-u)\big)\,du \int_0^\infty \nu_{ijl}(u)h_l\big(-\nu_l(t-u)\big)\,du, \end{aligned} \tag{1.5}$$

$$\delta_i(t + T) = \delta_i(t), \qquad \tau_{ij}(t + T) = \tau_{ij}(t), \qquad I_i(t + T) = -I_i(t), \tag{1.6}$$

$$\alpha_{ijl}(t + T) = \alpha_{ijl}(t), \qquad \beta_{ijl}(t + T) = \beta_{ijl}(t), \tag{1.7}$$

where $t, \nu \in R$, ν_j and ν_l are real-valued bounded continuous functions defined on R.

For bounded continuous functions f, we set

$$f^- = \inf_{t\in R}\left|f(t)\right|, \qquad f^+ = \sup_{t\in R}\left|f(t)\right|.$$

In order to investigate the anti-periodic solution of HCNNs (1.1), we also give some usual assumptions.

(H_1) There exist nonnegative constants L_j^f, L_j^g, L_j^h, M_j^g and M_j^h such that

$$\left|f_j(u)-f_j(v)\right| \le L_j^f|u-v|, \qquad \left|g_j(u)-g_j(v)\right| \le L_j^g|u-v|, \qquad \left|h_j(u)-h_j(v)\right| \le L_j^h|u-v|$$

and

$$\left|g_j(u)\right| \le M_j^g, \qquad \left|h_j(u)\right| \le M_j^h,$$

where $u, v \in R$, $j = 1, 2, \ldots, n$.

(H_2) For all $t > 0$ and $i \in \{1, 2, \ldots, n\}$, there exist positive constants $\xi_1, \xi_2, \ldots, \xi_n$ and η^* such that $c_i^+\delta_i^+ < 1$, and

$$\begin{aligned}
&-\left[c_i(t)\left(1-2c_i^+\delta_i^+\right) - \left|c_i(t) - \left(1-\delta_i'(t)\right)c_i\left(t-\delta_i(t)\right)\right|\right]\frac{1}{1-c_i^+\delta_i^+}\xi_i \\
&\quad + \sum_{j=1}^n \left|a_{ij}(t)\right| L_j^f \frac{1}{1-c_j^+\delta_j^+}\xi_j \\
&\quad + \sum_{j=1}^n\sum_{l=1}^n \left|b_{ijl}(t)\right|\left(L_j^g\frac{1}{1-c_j^+\delta_j^+}\xi_j M_l^g + M_j^g\frac{1}{1-c_l^+\delta_l^+}\xi_l L_l^g\right) \\
&\quad + \sum_{j=1}^n\sum_{l=1}^n \left|d_{ijl}(t)\right|\left(\int_0^\infty \left|\sigma_{ijl}(u)\right| L_j^h\,du\frac{1}{1-c_j^+\delta_j^+}\xi_j\int_0^\infty\left|\nu_{ijl}(u)\right|\,du M_l^h \right.\\
&\quad \left. + M_j^h\int_0^\infty\left|\sigma_{ijl}(u)\right|\,du\int_0^\infty\left|\nu_{ijl}(u)\right| L_l^h\,du\frac{1}{1-c_j^+\delta_j^+}\xi_j\right) \\
&< -\eta^*.
\end{aligned}$$

2 Preliminary lemmas and main results

Lemma 2.1 *Let* (H_1) *and* (H_2) *hold. Suppose that* $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ *is a solution of system* (1.1) *with the initial conditions*

$$\left|\varphi_i(t) - \int_{t-\delta_i(t)}^t c_i(s)\varphi_i(s)\,ds\right| < \xi_i\frac{\gamma}{\eta^*}, \quad t\in(-\infty,0], i = 1, 2, \ldots, n, \tag{2.1}$$

where

$$\begin{aligned}
\gamma = \max_{i\in I_n}&\left\{\sum_{j=1}^n a_{ij}^+\left|f_j(0)\right| + \sum_{j=1}^n\sum_{l=1}^n b_{ijl}^+\left|g_j(0)\right|M_l^g\right. \\
&\left. + \sum_{j=1}^n\sum_{l=1}^n d_{ijl}^+\int_0^\infty\left|\sigma_{ijl}(u)\right|\,du h_j(0)\int_0^\infty\left|\nu_{ijl}(u)\right|\,du M_l^h + I_i^+\right\}.
\end{aligned} \tag{2.2}$$

Then

$$\left|x_i(t) - \int_{t-\delta_i(t)}^{t} c_i(s)x_i(s)\,ds\right| < \xi_i \frac{\gamma}{\eta^*} \tag{2.3}$$

and

$$\left|x_i(t)\right| \le \frac{\xi_i \frac{\gamma}{\eta^*}}{1 - c_i^+ \delta_i^+} \tag{2.4}$$

for all $t \ge 0$, $i = 1, 2, \ldots, n$.

Proof Suppose (2.3) holds. Then, for a given $\hat{t} \ge 0$ and $i \in J_n = \{1, 2, \ldots, n\}$, we have

$$\begin{aligned}
\left|x_i(t)\right| &\le \left|x_i(t) - \int_{t-\delta_i(t)}^{t} c_i(s)x_i(s)\,ds\right| + \left|\int_{t-\delta_i(t)}^{t} c_i(s)x_i(s)\,ds\right| \\
&< \xi_i \frac{\gamma}{\eta^*} + c_i^+ \delta_i^+ \sup_{s \in (-\infty, \hat{t}]} \left|x_i(s)\right| \quad \text{for all } t \in (-\infty, \hat{t}]
\end{aligned}$$

and

$$\begin{aligned}
\left|x_i(t)\right| &\le \sup_{s \in (-\infty, \hat{t}]} \left|x_i(s)\right| \\
&< \xi_i \frac{\gamma}{\eta^*} + c_i^+ \delta_i^+ \sup_{s \in (-\infty, \hat{t}]} \left|x_i(s)\right| \quad \text{for all } t \in (-\infty, \hat{t}],
\end{aligned}$$

which combined with (H_2) implies that (2.4) holds. Therefore, it suffices to prove (2.3). We achieve this by way of contradiction. Let

$$X_i(t) = x_i(t) - \int_{t-\delta_i(t)}^{t} c_i(s)x_i(s)\,ds.$$

Suppose that (2.3) does not hold. Then there exist $i_0 \in J_n$ and $t_* > 0$ such that

$$\left|X_{i_0}(t_*)\right| = \xi_{i_0} \frac{\gamma}{\eta^*} \text{ and (2.3) holds for all } t \in (-\infty, t_*) \text{ and } i \in J_n. \tag{2.5}$$

It follows that (2.4) holds for all $t \in (-\infty, t_*)$ and $i \in J_n$. From (1.1), we have

$$\begin{aligned}
&\frac{d}{dt} X_{i_0}(t) \\
&\quad = x_{i_0}'(t) - \bigl[c_{i_0}(t)x_{i_0}(t) - \bigl(1 - \delta_{i_0}'(t)\bigr)c_{i_0}\bigl(t - \delta_{i_0}(t)\bigr)x_{i_0}\bigl(t - \delta_{i_0}(t)\bigr)\bigr] \\
&\quad = -\bigl[c_{i_0}(t)x_{i_0}(t) - \bigl(1 - \delta_{i_0}'(t)\bigr)c_{i_0}\bigl(t - \delta_{i_0}(t)\bigr)x_{i_0}\bigl(t - \delta_{i_0}(t)\bigr)\bigr] \\
&\qquad + \Biggl[-c_{i_0}(t)x_{i_0}\bigl(t - \delta_{i_0}(t)\bigr) + \sum_{j=1}^{n} a_{i_0 j}(t)f_j\bigl(x_j\bigl(t - \tau_{i_0 j}(t)\bigr)\bigr) \\
&\qquad + \sum_{j=1}^{n} \sum_{l=1}^{n} b_{i_0 jl}(t)g_j\bigl(x_j\bigl(t - \alpha_{i_0 jl}(t)\bigr)\bigr)g_l\bigl(x_l\bigl(t - \beta_{i_0 jl}(t)\bigr)\bigr)
\end{aligned}$$

$$\begin{aligned}
&+\sum_{j=1}^{n}\sum_{l=1}^{n} d_{i_0jl}(t)\int_0^\infty \sigma_{i_0jl}(u)h_j\big(x_j(t-u)\big)\,du\int_0^\infty \nu_{i_0jl}(u)h_l\big(x_l(t-u)\big)\,du \\
&+ I_{i_0}(t)\Bigg] \\
={}& -c_{i_0}(t)x_{i_0}(t) - \big[c_{i_0}(t) - \big(1-\delta'_{i_0}(t)\big)c_{i_0}\big(t-\delta_{i_0}(t)\big)\big]x_{i_0}\big(t-\delta_{i_0}(t)\big) \\
&+\sum_{j=1}^{n} a_{i_0j}(t)f_j\big(x_j\big(t-\tau_{i_0j}(t)\big)\big) \\
&+\sum_{j=1}^{n}\sum_{l=1}^{n} b_{i_0jl}(t)g_j\big(x_j\big(t-\alpha_{i_0jl}(t)\big)\big)g_l\big(x_l\big(t-\beta_{i_0jl}(t)\big)\big) \\
&+\sum_{j=1}^{n}\sum_{l=1}^{n} d_{i_0jl}(t)\int_0^\infty \sigma_{i_0jl}(u)h_j\big(x_j(t-u)\big)\,du\int_0^\infty \nu_{i_0jl}(u)h_l\big(x_l(t-u)\big)\,du \\
&+ I_{i_0}(t) \\
={}& -c_{i_0}(t)X_{i_0}(t) - c_{i_0}(t)\int_{t-\delta_{i_0}(t)}^{t} c_{i_0}(s)x_{i_0}(s)\,ds \\
&- \big[c_{i_0}(t) - \big(1-\delta'_{i_0}(t)\big)c_{i_0}\big(t-\delta_{i_0}(t)\big)\big]x_{i_0}\big(t-\delta_{i_0}(t)\big) \\
&+\sum_{j=1}^{n} a_{i_0j}(t)f_j\big(x_j\big(t-\tau_{i_0j}(t)\big)\big) \\
&+\sum_{j=1}^{n}\sum_{l=1}^{n} b_{i_0jl}(t)g_j\big(x_j\big(t-\alpha_{i_0jl}(t)\big)\big)g_l\big(x_l\big(t-\beta_{i_0jl}(t)\big)\big) \\
&+\sum_{j=1}^{n}\sum_{l=1}^{n} d_{i_0jl}(t)\int_0^\infty \sigma_{i_0jl}(u)h_j\big(x_j(t-u)\big)\,du\int_0^\infty \nu_{i_0jl}(u)h_l\big(x_l(t-u)\big)\,du \\
&+ I_{i_0}(t).
\end{aligned}$$

This, together with (2.5), the fact that (2.4) holds for $t \in (-\infty, t_*)$ and $i \in J_n$, (H_1) and (H_2), yields

$$\begin{aligned}
&D^-\big|X_{i_0}(t_*)\big| \\
&\quad\le -c_{i_0}(t_*)\big|X_{i_0}(t_*)\big| + c_{i_0}(t_*)\int_{t_*-\delta_{i_0}(t_*)}^{t_*} c_{i_0}(s)\big|x_{i_0}(s)\big|\,ds \\
&\qquad+\big|c_{i_0}(t_*) - \big(1-\delta'_{i_0}(t_*)\big)c_{i_0}\big(t_*-\delta_{i_0}(t_*)\big)\big|\big|x_{i_0}\big(t_*-\delta_{i_0}(t_*)\big)\big| \\
&\qquad+\left|\sum_{j=1}^{n} a_{i_0j}(t_*)f_j\big(x_j\big(t_*-\tau_{i_0j}(t_*)\big)\big)\right| \\
&\qquad+\left|\sum_{j=1}^{n}\sum_{l=1}^{n} b_{i_0jl}(t_*)g_j\big(x_j\big(t_*-\alpha_{i_0jl}(t_*)\big)\big)g_l\big(x_l\big(t_*-\beta_{i_0jl}(t_*)\big)\big)\right| \\
&\qquad+\sum_{j=1}^{n}\sum_{l=1}^{n} d_{i_0jl}(t_*)\int_0^\infty \sigma_{i_0jl}(u)h_j\big(x_j(t_*-u)\big)\,du\int_0^\infty \nu_{i_0jl}(u)h_l\big(x_l(t_*-u)\big)\,du \\
&\qquad+\big|I_{i_0}(t_*)\big|
\end{aligned}$$

$$\begin{aligned}
&\le -c_{i_0}(t_*)\left|X_{i_0}(t_*)\right| + c_{i_0}(t_*)\int_{t_*-\delta_{i_0}(t_*)}^{t_*} c_{i_0}^+\left|x_{i_0}(s)\right|ds \\
&\quad + \left|c_{i_0}(t_*) - \left(1-\delta_{i_0}'(t_*)\right)c_{i_0}\left(t_*-\delta_{i_0}(t_*)\right)\right|\left|x_{i_0}\left(t_*-\delta_{i_0}(t_*)\right)\right| \\
&\quad + \sum_{j=1}^n \left|a_{i_0j}(t_*)\right|\left(\left|f_j\left(x_j\left(t_*-\tau_{i_0j}(t_*)\right)\right)-f_j(0)\right| + \left|f_j(0)\right|\right) \\
&\quad + \sum_{j=1}^n\sum_{l=1}^n \left|b_{i_0jl}(t_*)\right|\left(\left|g_j\left(x_j\left(t_*-\alpha_{i_0jl}(t_*)\right)\right)-g_j(0)\right| + \left|g_j(0)\right|\right)M_l^g \\
&\quad + \sum_{j=1}^n\sum_{l=1}^n \left|d_{i_0jl}(t_*)\right|\int_0^\infty\left|\sigma_{i_0jl}(u)\right|\left(\left|h_j\left(x_j(t_*-u)\right)-h_j(0)\right| + \left|h_j(0)\right|\right)du \\
&\quad \times \int_0^\infty \left|\nu_{i_0jl}(u)\right|M_l^h\,du + \left|I_{i_0}(t_*)\right| \\
&\le -c_{i_0}(t_*)\xi_{i_0}\frac{\gamma}{\eta^*} + c_{i_0}(t_*)c_{i_0}^+\delta_{i_0}^+\frac{\xi_{i_0}\frac{\gamma}{\eta^*}}{1-c_{i_0}^+\delta_{i_0}^+} \\
&\quad + \left|c_{i_0}(t_*) - \left(1-\delta_{i_0}'(t_*)\right)c_{i_0}\left(t_*-\delta_{i_0}(t_*)\right)\right|\frac{\xi_{i_0}\frac{\gamma}{\eta^*}}{1-c_{i_0}^+\delta_{i_0}^+} \\
&\quad + \sum_{j=1}^n \left|a_{i_0j}(t_*)\right|\left(L_j^f\left|x_j\left(t_*-\tau_{i_0j}(t_*)\right)\right| + \left|f_j(0)\right|\right) \\
&\quad + \sum_{j=1}^n\sum_{l=1}^n \left|b_{i_0jl}(t_*)\right|\left(L_j^g\left|x_j\left(t_*-\alpha_{i_0jl}(t_*)\right)\right| + \left|g_j(0)\right|\right)M_l^g \\
&\quad + \sum_{j=1}^n\sum_{l=1}^n \left|d_{i_0jl}(t_*)\right|\int_0^\infty\left|\sigma_{i_0jl}(u)\right|\left(L_j^h\left|x_j(t_*-u)\right| + \left|h_j(0)\right|\right)du\int_0^\infty\left|\nu_{i_0jl}(u)\right|M_l^h\,du \\
&\quad + \left|I_{i_0}(t_*)\right| \\
&\le -c_{i_0}(t_*)\xi_{i_0}\frac{\gamma}{\eta^*} + c_{i_0}(t_*)c_{i_0}^+\delta_{i_0}^+\frac{\xi_{i_0}\frac{\gamma}{\eta^*}}{1-c_{i_0}^+\delta_{i_0}^+} \\
&\quad + \left|c_{i_0}(t_*) - \left(1-\delta_{i_0}'(t_*)\right)c_{i_0}\left(t_*-\delta_{i_0}(t_*)\right)\right|\frac{\xi_{i_0}\frac{\gamma}{\eta^*}}{1-c_{i_0}^+\delta_{i_0}^+} \\
&\quad + \sum_{j=1}^n \left|a_{i_0j}(t_*)\right|L_j^f\frac{\xi_j\frac{\gamma}{\eta^*}}{1-c_j^+\delta_j^+} + \sum_{j=1}^n\sum_{l=1}^n \left|b_{i_0jl}(t_*)\right|L_j^g\frac{\xi_j\frac{\gamma}{\eta^*}}{1-c_j^+\delta_j^+}M_l^g \\
&\quad + \sum_{j=1}^n\sum_{l=1}^n \left|d_{i_0jl}(t_*)\right|\int_0^\infty\left|\sigma_{i_0jl}(u)\right|du L_j^h\frac{\xi_j\frac{\gamma}{\eta^*}}{1-c_j^+\delta_j^+}\int_0^\infty\left|\nu_{i_0jl}(u)\right|M_l^h\,du \\
&\quad + \left[\sum_{j=1}^n a_{i_0j}^+\left|f_j(0)\right| + \sum_{j=1}^n\sum_{l=1}^n b_{i_0jl}^+\left|g_j(0)\right|M_l^g\right. \\
&\quad \left. + \sum_{j=1}^n\sum_{l=1}^n d_{i_0jl}^+\int_0^\infty\left|\sigma_{i_0jl}(u)\right|du h_j(0)\int_0^\infty\left|\nu_{i_0jl}(u)\right|du M_l^h + I_{i_0}^+\right] \\
&\le \left\{-\left[c_{i_0}(t_*)\left(1-2c_{i_0}^+\delta_{i_0}^+\right) - \left|c_{i_0}(t_*) - \left(1-\delta_{i_0}'(t_*)\right)c_{i_0}\left(t_*-\delta_{i_0}(t_*)\right)\right|\right]\frac{1}{1-c_{i_0}^+\delta_{i_0}^+}\xi_{i_0}\right.
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\left|a_{i_0j}(t_*)\right|L_j^f\frac{1}{1-c_j^+\delta_j^+}\xi_j+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|b_{i_0jl}(t_*)\right|L_j^g\frac{\xi_j}{1-c_j^+\delta_j^+}M_l^g\\
&+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|d_{i_0jl}(t_*)\right|\int_0^\infty\left|\sigma_{i_0jl}(u)\right|du L_j^h\frac{\xi_j}{1-c_j^+\delta_j^+}\int_0^\infty\left|\nu_{i_0jl}(u)\right|M_l^h\,du\Bigg\}\frac{\gamma}{\eta^*}+\gamma\\
&<-\eta^*\frac{\gamma}{\eta^*}+\gamma\\
&=0.
\end{aligned}
$$

This contradicts with $D^-|X_{i_0}(t^*)|\geq 0$ and hence (2.3) is proved. This completes the proof. □

Remark 2.1 In view of the boundedness of this solution in Lemma 2.1, from the theory of functional differential equations with infinite delay in [17], it follows that the solution of system (1.1) with initial conditions satisfying (2.1) can be defined on $[0,+\infty)$.

Lemma 2.2 *Suppose that* (H_1)-(H_2) *are true. Let* $x^*(t)=(x_1^*(t),x_2^*(t),\dots,x_n^*(t))^T$ *be the solution of system* (1.1) *with initial value* $\varphi^*=(\varphi_1^*(t),\varphi_2^*(t),\dots,\varphi_n^*(t))^T$, *and let* $x(t)=(x_1(t),x_2(t),\dots,x_n(t))^T$ *be the solution of system* (1.1) *with initial value* $\varphi=(\varphi_1(t),\varphi_2(t),\dots,\varphi_n(t))^T$. *Then there exists a positive constant r such that*

$$
x_i(t)-x_i^*(t)=O\left(e^{-rt}\right),\quad i\in J_n.
$$

Proof In view of (H_2), using a similar argument as that in the proof of (2.7) in [16], we can choose $\kappa>r>0$ and $\eta>0$ such that $c_i(t)>r$, and

$$
\begin{aligned}
&-\left[\left(c_i(t)-r\right)\left(1-2c_i^+\delta_i^+\right)-\left|c_i(t)e^{r\delta_i(t)}-\left(1-\delta_i'(t)\right)c_i\left(t-\delta_i(t)\right)\right|\right]\frac{1}{1-c_i^+\delta_i^+}\xi_i\\
&\quad+\sum_{j=1}^{n}\left|a_{ij}(t)\right|L_j^f e^{r\tau_{ij}(t)}\frac{1}{1-c_j^+\delta_j^+}\xi_j\\
&\quad+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|b_{ijl}(t)\right|\left(L_j^g e^{r\alpha_{ijl}(t)}\frac{1}{1-c_j^+\delta_j^+}\xi_jM_l^g+M_j^g e^{r\beta_{ijl}(t)}\frac{1}{1-c_l^+\delta_l^+}\xi_lL_l^g\right)\\
&\quad+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|d_{ijl}(t)\right|\left(\int_0^\infty\left|\sigma_{ijl}(u)\right|e^{ru}L_j^h\,du\frac{1}{1-c_j^+\delta_j^+}\xi_j\int_0^\infty\left|\nu_{ijl}(u)\right|du M_l^h\right.\\
&\quad\left.+M_j^h\int_0^\infty\left|\sigma_{ijl}(u)\right|du\int_0^\infty\left|\nu_{ijl}(u)\right|e^{ru}L_l^h\,du\frac{1}{1-c_j^+\delta_j^+}\xi_j\right)\\
&<-\eta,\quad t\geq 0, i\in J_n.
\end{aligned}
\tag{2.6}
$$

Let $y(t)=x(t)-x^*(t)$. Then

$$
\begin{aligned}
&y_i'(t)\\
&\quad=-c_i(t)y_i\left(t-\delta_i(t)\right)+\sum_{j=1}^{n}a_{ij}(t)\left(f_j\left(y_j\left(t-\tau_{ij}(t)\right)+x_j^*\left(t-\tau_{ij}(t)\right)\right)-f_j\left(x_j^*\left(t-\tau_{ij}(t)\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)\bigl(g_j\bigl(y_j\bigl(t-\alpha_{ijl}(t)\bigr)\\
&+x_j^*\bigl(t-\alpha_{ijl}(t)\bigr)\bigr)g_l\bigl(y_l\bigl(t-\beta_{ijl}(t)\bigr)+x_l^*\bigl(t-\beta_{ijl}(t)\bigr)\bigr)\\
&-g_j\bigl(x_j^*\bigl(t-\alpha_{ijl}(t)\bigr)\bigr)g_l\bigl(x_l^*\bigl(t-\beta_{ijl}(t)\bigr)\bigr)\bigr)+\sum_{j=1}^{n}\sum_{l=1}^{n} d_{ijl}(t)\\
&\times\biggl(\int_0^\infty \sigma_{ijl}(u)h_j\bigl(y_j(t-u)+x_j^*(t-u)\bigr)\,du\int_0^\infty \nu_{ijl}(u)h_l\bigl(y_l(t-u)+x_l^*(t-u)\bigr)\,du\\
&-\int_0^\infty \sigma_{ijl}(u)h_j\bigl(x_j^*(t-u)\bigr)\,du\int_0^\infty \nu_{ijl}(u)h_l\bigl(x_l^*(t-u)\bigr)\,du\biggr),\quad i\in J_n,
\end{aligned}
\tag{2.7}
$$

which yields

$$
\begin{aligned}
&\frac{d}{dt}Y_i(t)\\
&\quad=re^{rt}y_i(t)+e^{rt}y_i'(t)\\
&\qquad-\bigl[c_i(t)e^{rt}y_i(t)-\bigl(1-\delta_i'(t)\bigr)c_i\bigl(t-\delta_i(t)\bigr)e^{r(t-\delta_i(t))}y_i\bigl(t-\delta_i(t)\bigr)\bigr]\\
&\quad=re^{rt}y_i(t)-\bigl[c_i(t)e^{rt}y_i(t)-\bigl(1-\delta_i'(t)\bigr)c_i\bigl(t-\delta_i(t)\bigr)e^{r(t-\delta_i(t))}y_i\bigl(t-\delta_i(t)\bigr)\bigr]\\
&\qquad+e^{rt}\Biggl\{-c_i(t)y_i\bigl(t-\delta_i(t)\bigr)\\
&\qquad+\sum_{j=1}^{n} a_{ij}(t)\bigl(f_j\bigl(y_j\bigl(t-\tau_{ij}(t)\bigr)+x_j^*\bigl(t-\tau_{ij}(t)\bigr)\bigr)-f_j\bigl(x_j^*\bigl(t-\tau_{ij}(t)\bigr)\bigr)\bigr)\\
&\qquad+\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)\bigl(g_j\bigl(y_j\bigl(t-\alpha_{ijl}(t)\bigr)+x_j^*\bigl(t-\alpha_{ijl}(t)\bigr)\bigr)\\
&\qquad\times g_l\bigl(y_l\bigl(t-\beta_{ijl}(t)\bigr)+x_l^*\bigl(t-\beta_{ijl}(t)\bigr)\bigr)-g_j\bigl(x_j^*\bigl(t-\alpha_{ijl}(t)\bigr)\bigr)g_l\bigl(x_l^*\bigl(t-\beta_{ijl}(t)\bigr)\bigr)\bigr)\\
&\qquad+\sum_{j=1}^{n}\sum_{l=1}^{n} d_{ijl}(t)\biggl(\int_0^\infty \sigma_{ijl}(u)h_j\bigl(y_j(t-u)+x_j^*(t-u)\bigr)\,du\\
&\qquad\times\int_0^\infty \nu_{ijl}(u)h_l\bigl(y_l(t-u)+x_l^*(t-u)\bigr)\,du\\
&\qquad-\int_0^\infty \sigma_{ijl}(u)h_j\bigl(x_j^*(t-u)\bigr)\,du\int_0^\infty \nu_{ijl}(u)h_l\bigl(x_l^*(t-u)\bigr)\,du\biggr)\Biggr\}\\
&\quad=-\bigl(c_i(t)-r\bigr)Y_i(t)-\bigl(c_i(t)-r\bigr)\int_{t-\delta_i(t)}^{t} c_i(s)e^{rs}y_i(s)\,ds\\
&\qquad-\bigl[c_i(t)-\bigl(1-\delta_i'(t)\bigr)c_i\bigl(t-\delta_i(t)\bigr)e^{-r\delta_i(t)}\bigr]e^{rt}y_i\bigl(t-\delta_i(t)\bigr)\\
&\qquad+e^{rt}\Biggl[\sum_{j=1}^{n} a_{ij}(t)\bigl(f_j\bigl(y_j\bigl(t-\tau_{ij}(t)\bigr)+x_j^*\bigl(t-\tau_{ij}(t)\bigr)\bigr)-f_j\bigl(x_j^*\bigl(t-\tau_{ij}(t)\bigr)\bigr)\bigr)\\
&\qquad+\sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)\bigl(g_j\bigl(y_j\bigl(t-\alpha_{ijl}(t)\bigr)+x_j^*\bigl(t-\alpha_{ijl}(t)\bigr)\bigr)
\end{aligned}
$$

$$\begin{aligned}
&\times g_l\big(y_l\big(t-\beta_{ijl}(t)\big)+x_l^*\big(t-\beta_{ijl}(t)\big)\big)-g_j\big(x_j^*\big(t-\alpha_{ijl}(t)\big)\big)g_l\big(x_l^*\big(t-\beta_{ijl}(t)\big)\big)\big)\\
&+\sum_{j=1}^{n}\sum_{l=1}^{n}d_{ijl}(t)\bigg(\int_0^\infty \sigma_{ijl}(u)h_j\big(y_j(t-u)+x_j^*(t-u)\big)\,du\\
&\times\int_0^\infty \nu_{ijl}(u)h_l\big(y_l(t-u)+x_l^*(t-u)\big)\,du\\
&-\int_0^\infty \sigma_{ijl}(u)h_j\big(x_j^*(t-u)\big)\,du\int_0^\infty \nu_{ijl}(u)h_l\big(x_l^*(t-u)\big)\,du\bigg)\bigg],
\end{aligned}\tag{2.8}$$

where

$$Y_i(t)=e^{rt}y_i(t)-\int_{t-\delta_i(t)}^{t}c_i(s)e^{rs}y_i(s)\,ds,\quad i\in J_n.$$

Denote

$$M=\max_{1\le i\le n}\Big\{\sup_{s\in(-\infty,0]}\big|Y_i(s)\big|\Big\}.$$

There exists $K>0$ such that

$$\big|Y_i(t)\big|\le M<K\xi_i\quad\text{for all } t\in(-\infty,0] \text{ and } i\in I_n.$$

We claim that

$$\big|Y_i(t)\big|<K\xi_i\quad\text{for all } t>0 \text{ and } i\in J_n.\tag{2.9}$$

Otherwise, there exist $i\in J_n$ and $\theta>0$ such that

$$\big|Y_i(\theta)\big|=K\xi_i\quad\text{and}\quad\big|Y_j(t)\big|<K\xi_j\quad\text{for all } t\in(-\infty,\theta) \text{ and } j\in J_n.$$

It follows that for $t\in(-\infty,\theta]$ and $j\in J_n$,

$$\begin{aligned}
e^{rt}\big|y_j(t)\big|&\le\bigg|e^{rt}y_j(t)-\int_{t-\delta_j(t)}^{t}c_j(s)e^{rs}y_j(s)\,ds\bigg|+\bigg|\int_{t-\delta_j(t)}^{t}c_j(s)e^{rs}y_j(s)\,ds\bigg|\\
&\le K\xi_j+c_j^+\delta_j^+\sup_{s\in(-\infty,\theta]}e^{rs}\big|y_j(s)\big|
\end{aligned}\tag{2.10}$$

and hence

$$e^{rt}\big|y_j(t)\big|\le\sup_{s\in(-\infty,\theta]}e^{rs}\big|y_j(s)\big|\le\frac{K\xi_j}{1-c_j^+\delta_j^+}.\tag{2.11}$$

Then, for the upper left derivative of $|Y_i(t)|$, from (2.6), (2.8), (2.11) and (H_1), we have

$$\begin{aligned}
0&\le D^-\big|Y_i(\theta)\big|\\
&\le-\big(c_i(\theta)-r\big)Y_i(\theta)+\bigg|-\big(c_i(\theta)-r\big)\int_{\theta-\delta_i(\theta)}^{\theta}c_i(s)e^{rs}y_i(s)\,ds\bigg|
\end{aligned}$$

$$
\begin{aligned}
&+ \Bigg| -\big[c_i(\theta) - \big(1-\delta_i'(\theta)\big)c_i\big(\theta-\delta_i(\theta)\big)e^{-r\delta_i(\theta)}\big]e^{r\theta}y_i\big(\theta-\delta_i(\theta)\big) \\
&+ e^{r\theta}\Bigg[\sum_{j=1}^{n} a_{ij}(\theta)\big(f_j\big(y_j\big(\theta-\tau_{ij}(\theta)\big)+x_j^*\big(\theta-\tau_{ij}(\theta)\big)\big)-f_j\big(x_j^*\big(\theta-\tau_{ij}(\theta)\big)\big)\big) \\
&+ \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(\theta)\big(g_j\big(y_j\big(\theta-\alpha_{ijl}(\theta)\big)+x_j^*\big(\theta-\alpha_{ijl}(\theta)\big)\big)g_l\big(y_l\big(\theta-\beta_{ijl}(\theta)\big) \\
&+ x_l^*\big(\theta-\beta_{ijl}(\theta)\big)\big)-g_j\big(x_j^*\big(\theta-\alpha_{ijl}(\theta)\big)\big)g_l\big(x_l^*\big(\theta-\beta_{ijl}(\theta)\big)\big)\big) \\
&+ \sum_{j=1}^{n}\sum_{l=1}^{n} d_{ijl}(\theta)\bigg(\int_0^\infty \sigma_{ijl}(u)h_j\big(y_j(\theta-u)+x_j^*(\theta-u)\big)\,du \\
&\times \int_0^\infty \nu_{ijl}(u)h_l\big(y_l(\theta-u)+x_l^*(\theta-u)\big)\,du \\
&- \int_0^\infty \sigma_{ijl}(u)h_j\big(x_j^*(\theta-u)\big)\,du\int_0^\infty \nu_{ijl}(u)h_l\big(x_l^*(\theta-u)\big)\,du\bigg)\Bigg]\Bigg| \\
\le{}& -\big(c_i(\theta)-r\big)\big|Y_i(\theta)\big| + \big(c_i(\theta)-r\big)\frac{K\xi_i}{1-c_i^+\delta_i^+}c_i^+\delta_i^+ \\
&+ \big|c_i(\theta)-\big(1-\delta_i'(\theta)\big)c_i\big(\theta-\delta_i(\theta)\big)e^{-r\delta_i(\theta)}\big|e^{r\delta_i(\theta)}e^{r(\theta-\delta_i(\theta))}\big|y_i\big(\theta-\delta_i(\theta)\big)\big| \\
&+ \sum_{j=1}^{n}\big|a_{ij}(\theta)\big|L_j^f e^{r\tau_{ij}(\theta)}e^{r(\theta-\tau_{ij}(\theta))}\big|y_j\big(\theta-\tau_{ij}(\theta)\big)\big| \\
&+ \sum_{j=1}^{n}\sum_{l=1}^{n}\big|b_{ijl}(\theta)\big|e^{r\theta}\big(\big|g_j\big(y_j\big(\theta-\alpha_{ijl}(\theta)\big)+x_j^*\big(\theta-\alpha_{ijl}(\theta)\big)\big)g_l\big(y_l\big(\theta-\beta_{ijl}(\theta)\big) \\
&+ x_l^*\big(\theta-\beta_{ijl}(\theta)\big)\big)-g_j\big(x_j^*\big(\theta-\alpha_{ijl}(\theta)\big)\big)g_l\big(y_l\big(\theta-\beta_{ijl}(\theta)\big)+x_l^*\big(\theta-\beta_{ijl}(\theta)\big)\big)\big| \\
&+ \big|g_j\big(x_j^*\big(\theta-\alpha_{ijl}(\theta)\big)\big)g_l\big(y_l\big(\theta-\beta_{ijl}(\theta)\big)+x_l^*\big(\theta-\beta_{ijl}(\theta)\big)\big) \\
&- g_j\big(x_j^*\big(\theta-\alpha_{ijl}(\theta)\big)\big)g_l\big(x_l^*\big(\theta-\beta_{ijl}(\theta)\big)\big)\big|\big) \\
&+ \sum_{j=1}^{n}\sum_{l=1}^{n}\big|d_{ijl}(\theta)\big|e^{r\theta}\bigg(\bigg|\int_0^\infty \sigma_{ijl}(u)h_j\big(y_j(\theta-u)+x_j^*(\theta-u)\big)\,du \\
&\times \int_0^\infty \nu_{ijl}(u)h_l\big(y_l(\theta-u)+x_l^*(\theta-u)\big)\,du \\
&- \int_0^\infty \sigma_{ijl}(u)h_j\big(x_j^*(\theta-u)\big)\,du\int_0^\infty \nu_{ijl}(u)h_l\big(y_l(\theta-u)+x_l^*(\theta-u)\big)\,du\bigg| \\
&+ \bigg|\int_0^\infty \sigma_{ijl}(u)h_j\big(x_j^*(\theta-u)\big)\,du\int_0^\infty \nu_{ijl}(u)h_l\big(y_l(\theta-u)+x_l^*(\theta-u)\big)\,du \\
&- \int_0^\infty \sigma_{ijl}(u)h_j\big(x_j^*(\theta-u)\big)\,du\int_0^\infty \nu_{ijl}(u)h_l\big(x_l^*(\theta-u)\big)\,du\bigg|\bigg) \\
\le{}& -\big(c_i(\theta)-r\big)\big|Y_i(\theta)\big| + \big(c_i(\theta)-r\big)\frac{K\xi_i}{1-c_i^+\delta_i^+}c_i^+\delta_i^+ \\
&+ \big|c_i(\theta)-\big(1-\delta_i'(\theta)\big)c_i\big(\theta-\delta_i(\theta)\big)e^{-r\delta_i(\theta)}\big|e^{r\delta_i(\theta)}e^{r(\theta-\delta_i(\theta))}\big|y_i\big(\theta-\delta_i(\theta)\big)\big| \\
&+ \sum_{j=1}^{n}\big|a_{ij}(\theta)\big|L_j^f e^{r\tau_{ij}(\theta)}e^{r(\theta-\tau_{ij}(\theta))}\big|y_j\big(\theta-\tau_{ij}(\theta)\big)\big|
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|b_{ijl}(\theta)\right|\left(L_j^g e^{r\alpha_{ijl}(\theta)} e^{r(\theta-\alpha_{ijl}(\theta))}\left|y_j\left(\theta-\alpha_{ijl}(\theta)\right)\right|M_l^g\right.\\
&\left.+M_j^g e^{r\beta_{ijl}(\theta)} e^{r(\theta-\beta_{ijl}(\theta))}\left|y_l\left(\theta-\beta_{ijl}(\theta)\right)\right|L_l^g\right)\\
&+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|d_{ijl}(\theta)\right|\left(\int_0^\infty\left|\sigma_{ijl}(u)\right|e^{ru}L_j^h e^{r(\theta-u)}\left|y_j(\theta-u)\right|du\int_0^\infty\left|\nu_{ijl}(u)\right|du M_l^h\right.\\
&\left.+M_j^h\int_0^\infty\left|\sigma_{ijl}(u)\right|du\int_0^\infty\left|\nu_{ijl}(u)\right|e^{ru}L_l^h e^{r(\theta-u)}\left|y_l(\theta-u)\right|du\right)\\
\le{}&-\left[\left(c_i(\theta)-r\right)\left(1-2c_i^+\delta_i^+\right)-\left|c_i(\theta)e^{r\delta_i(\theta)}-\left(1-\delta_i'(\theta)\right)c_i\left(\theta-\eta_i(\theta)\right)\right|\right]\frac{1}{1-c_i^+\delta_i^+}K\xi_i\\
&+\sum_{j=1}^{n}\left|a_{ij}(\theta)\right|L_j^f e^{r\tau_{ij}(\theta)}\frac{1}{1-c_j^+\delta_j^+}K\xi_j\\
&+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|b_{ijl}(\theta)\right|\left(L_j^g e^{r\alpha_{ijl}(\theta)}\frac{1}{1-c_j^+\delta_j^+}K\xi_j M_l^g+M_j^g e^{r\beta_{ijl}(\theta)}\frac{1}{1-c_l^+\delta_l^+}K\xi_l L_l^g\right)\\
&+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|d_{ijl}(\theta)\right|\left(\int_0^\infty\left|\sigma_{ijl}(u)\right|e^{ru}L_j^h\,du\frac{1}{1-c_j^+\delta_j^+}K\xi_j\int_0^\infty\left|\nu_{ijl}(u)\right|du M_l^h\right.\\
&\left.+M_j^h\int_0^\infty\left|\sigma_{ijl}(u)\right|du\int_0^\infty\left|\nu_{ijl}(u)\right|e^{ru}L_l^h\,du\frac{1}{1-c_j^+\delta_j^+}K\xi_j\right)\\
={}&\left\{-\left[\left(c_i(\theta)-r\right)\left(1-2c_i^+\delta_i^+\right)-\left|c_i(\theta)e^{r\delta_i(\theta)}-\left(1-\delta_i'(\theta)\right)c_i\left(\theta-\delta_i(\theta)\right)\right|\right]\frac{1}{1-c_i^+\delta_i^+}\xi_i\right.\\
&+\sum_{j=1}^{n}\left|a_{ij}(\theta)\right|L_j^f e^{r\tau_{ij}(\theta)}\frac{1}{1-c_j^+\delta_j^+}\xi_j\\
&+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|b_{ijl}(\theta)\right|\left(L_j^g e^{r\alpha_{ijl}(\theta)}\frac{1}{1-c_j^+\delta_j^+}\xi_j M_l^g+M_j^g e^{\lambda\beta_{ijl}(\theta)}\frac{1}{1-c_l^+\delta_l^+}\xi_l L_l^g\right)\\
&+\sum_{j=1}^{n}\sum_{l=1}^{n}\left|d_{ijl}(\theta)\right|\left(\int_0^\infty\left|\sigma_{ijl}(u)\right|e^{ru}L_j^h\,du\frac{1}{1-c_j^+\delta_j^+}\xi_j\int_0^\infty\left|\nu_{ijl}(u)\right|du M_l^h\right.\\
&\left.\left.+M_j^h\int_0^\infty\left|\sigma_{ijl}(u)\right|du\int_0^\infty\left|\nu_{ijl}(u)\right|e^{ru}L_l^h\,du\frac{1}{1-c_j^+\delta_j^+}\xi_j\right)\right\}K\\
<{}&-\eta K\\
<{}&0,
\end{aligned}
$$

which is a contradiction. This proves (2.9), which produces

$$\left|y_i(t)\right|e^{rt}<\frac{K\xi_i}{1-c_i^+\eta_i^+}$$

or

$$\left|x_i(t)-x_i^*(t)\right|\le\frac{K\xi_i}{1-c_i^+\eta_i^+}e^{-rt}$$

for all $t>0$ and $i\in J_n$. The proof of Lemma 2.2 is completed. □

Remark 2.2 If $x^*(t) = (x_1^*(t), x_2^*(t), \dots, x_n^*(t))^T$ is the T-anti-periodic solution of system (1.1), it follows from Lemma 2.2 that $x^*(t)$ is globally exponentially stable.

Theorem 2.1 *Suppose that* (H_1) *and* (H_2) *are satisfied. Then system* (1.1) *has exactly one* T*-anti-periodic solution* $x^*(t)$. *Moreover,* $x^*(t)$ *is globally exponentially stable.*

Proof The proof proceeds in the same way as in Theorem 3.1 in [16]. □

3 Example and remark

In this section, some examples and remarks are provided to demonstrate the effectiveness of our results.

Example 3.1 Consider the following HCNNs with time-varying delays in the leakage terms:

$$\begin{cases} x_1'(t) = -1.5x_1(t - \frac{1}{1{,}000}\cos^2 t) + \frac{1}{4}\sin t f_1(x_1(t-1)) + \frac{1}{36}\sin^3 t f_2(x_2(t-1)) \\ \qquad + \frac{1}{72}\sin t g_1^2(x_1(t-1)) + \frac{1}{72}\sin t g_1(x_1(t-1))g_2(x_2(t-1)) \\ \qquad + \frac{1}{72}\sin t g_2^2(x_2(t-1)) \\ \qquad + \frac{1}{80}\sin t \int_0^\infty e^{-u}h_1(x_1(t-u))\,du \int_0^\infty e^{-u}h_2(x_2(t-u))\,du + \sin t, \\ x_2'(t) = -1.5x_2(t - \frac{1}{1{,}000}\cos^2 t) + \frac{1}{36}\sin^5 t f_1(x_1(t-1)) + \frac{1}{4}\sin^3 t f_2(x_2(t-1)) \\ \qquad + \frac{1}{72}\sin^3 t g_1^2(x_1(t-1)) + \frac{1}{72}\sin^3 t g_1(x_1(t-1))g_2(x_2(t-1)) \\ \qquad + \frac{1}{72}\sin^3 t g_2^2(x_2(t-1)) \\ \qquad + \frac{1}{80}\sin^7 t \int_0^\infty e^{-u}h_1(x_1(t-u))\,du \int_0^\infty e^{-u}h_2(x_2(t-u))\,du + 2\sin t, \end{cases} \tag{3.1}$$

where

$$f_i(x) = \frac{1}{2}\bigl(|x| + \cos x\bigr), \qquad g_i(x) = h_i(x) = |\arctan x| + \cos x,$$

$$c_i(t) = 1.5, \qquad I_i(t) = i\sin t, \quad i = 1,2,$$

$$\delta_1(t) = \delta_2(t) = \frac{1}{1{,}000}\cos^2 t, \qquad a_{11}(t) = \frac{1}{4}\sin t, \qquad a_{12}(t) = \frac{1}{36}\sin^3 t,$$

$$a_{21}(t) = \frac{1}{36}\sin^5 t, \qquad a_{22}(t) = \frac{1}{4}\sin^3 t, \qquad b_{111}(t) = b_{112}(t) = b_{122}(t) = \frac{1}{72}\sin t,$$

$$b_{211}(t) = b_{212}(t) = b_{222}(t) = \frac{1}{72}\sin^3 t, \qquad d_{112}(t) = \frac{1}{80}\sin t, \qquad d_{212}(t) = \frac{1}{80}\sin^7 t.$$

Note that

$$L_i^f = 1, \qquad L_i^g = L_i^h = 2, \qquad M_i^g = M_i^h = \frac{\pi}{2} + 1, \quad i = 1,2.$$

Therefore,

$$\begin{aligned} &-\bigl[c_i(t)\bigl(1 - 2c_i^+\delta_i^+\bigr) - \bigl|c_i(t) - \bigl(1 - \delta_i'(t)\bigr)c_i\bigl(t - \delta_i(t)\bigr)\bigr|\bigr]\frac{1}{1 - c_i^+\delta_i^+}\xi_i \\ &\quad + \sum_{j=1}^n \bigl|a_{ij}(t)\bigr| L_j^f \frac{1}{1 - c_j^+\delta_j^+}\xi_j \\ &\quad + \sum_{j=1}^n \sum_{l=1}^n \bigl|b_{ijl}(t)\bigr| \biggl(L_j^g \frac{1}{1 - c_j^+\delta_j^+}\xi_j M_l^g + M_j^g \frac{1}{1 - c_l^+\delta_l^+}\xi_l L_l^g\biggr) \end{aligned}$$

$$\begin{aligned}
&+ \sum_{j=1}^{n}\sum_{l=1}^{n}\bigl|d_{ijl}(t)\bigr|\biggl(\int_0^\infty \bigl|\sigma_{ijl}(u)\bigr|L_j^h\,du\frac{1}{1-c_j^+\delta_j^+}\xi_j\int_0^\infty \bigl|\nu_{ijl}(u)\bigr|\,du M_l^h \\
&+ M_j^h\int_0^\infty \bigl|\sigma_{ijl}(u)\bigr|\,du\int_0^\infty \bigl|\nu_{ijl}(u)\bigr|L_l^h\,du\frac{1}{1-c_j^+\delta_j^+}\xi_j\biggr) \\
&< -\biggl[1.5\times\biggl(1-2\times 1.5\times\frac{1}{1{,}000}\biggr)-1.5\times\frac{2}{1{,}000}\biggr]\times\frac{1}{1-1.5\times\frac{1}{1{,}000}} \\
&+ \biggl(\frac{1}{4}+\frac{1}{4}\biggr)\times\frac{1}{1-1.5\times\frac{1}{1{,}000}} \\
&+ 3\times\frac{1}{72}\times 2\times 2\times\biggl(\frac{\pi}{2}+1\biggr)\times\frac{1}{1-1.5\times\frac{1}{1{,}000}} \\
&+ \frac{1}{80}\times 2\times 2\times\biggl(\frac{\pi}{2}+1\biggr)\times\frac{1}{1-1.5\times\frac{1}{1{,}000}} \\
&< -0.2,\quad t\geq 0, \xi_i=1, i=1,2,
\end{aligned}$$

which implies that system (3.1) satisfies all the conditions in Theorem 2.1. Hence, system (3.1) has exactly one π-anti-periodic solution. Moreover, the π-anti-periodic solution is globally exponentially stable.

Remark 3.1 Since

$$t-\delta_i(t) = t-\frac{1}{1{,}000}\cos^2 t < 0$$

is possible for some $t > 0$, $i = 1, 2$, one can find that the results in [16] and the references therein cannot be applicable to prove that all solutions of HRNNs (3.1) converge exponentially to the anti-periodic solution. In this present paper, the expression

$$x_i(t) - x_i\bigl(t-\delta_i(t)\bigr) = \int_{t-\delta_i(t)}^{t} x_i'(u)\,du$$

has not been used in the proof of Theorem 2.1. In particular, by introducing two new transformations

$$X_i(t) = x_i(t) - \int_{t-\delta_i(t)}^{t} c_i(s)x_i(s)\,ds$$

and

$$Y_i(t) = e^{rt}y_i(t) - \int_{t-\delta_i(t)}^{t} c_i(s)e^{rs}y_i(s)\,ds,\quad i\in J_n,$$

we employ a novel proof to establish some criteria to guarantee the global exponential stability of the anti-periodic solution for HRNNs with leakage delays. Moreover, we also find that Theorem 3.1 of [16] holds under the following additional conditions:

$$t-\delta_i(t)\geq 0\quad \text{for all } t\geq 0, i\in J_n.$$

This implies that the results of this paper are new and complement the corresponding ones in [16].

Competing interests
The author declares that they have no competing interests.

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