Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Oscillation criteria for second-order nonlinear neutral dynamic equations with distributed deviating arguments on time scales

Tuncay Candan*

*Correspondence:
tcandan@nigde.edu.tr
Department of Mathematics,
Faculty of Art and Science, Niğde
University, Niğde, 51200, Turkey

Abstract

In this article, we establish some new oscillation criteria and give sufficient conditions to ensure that all solutions of nonlinear neutral dynamic equation of the form

$$(r(t)((y(t)+p(t)y(\tau(t)))^{\Delta})^{\gamma})^{\Delta}+\int_a^b f(t,y(\delta(t,\xi)))\Delta\xi=0$$

are oscillatory on a time scale $\mathbb{T}$, where $\gamma \geq 1$ is a quotient of odd positive integers.

Keywords: oscillation; dynamic equations; time scales; distributed deviating arguments

1 Introduction

The aim of this article is to develop some oscillation theorems for a second-order nonlinear neutral dynamic equation

$$\bigl(r(t)\bigl(\bigl(y(t)+p(t)y\bigl(\tau(t)\bigr)\bigr)^{\Delta}\bigr)^{\gamma}\bigr)^{\Delta}+\int_a^b f\bigl(t,y\bigl(\delta(t,\xi)\bigr)\bigr)\Delta\xi=0 \tag{1}$$

on a time scale $\mathbb{T}$. Throughout this paper, it is assumed that $\gamma \geq 1$ is a quotient of odd positive integers, $0 < a < b$, $\tau(t):\mathbb{T}\to\mathbb{T}$, is rd-continuous function such that $\tau(t) \leq t$ and $\tau(t)\to\infty$ as $t\to\infty$, $\delta(t,\xi):\mathbb{T}\times[a,b]\to\mathbb{T}$ is rd-continuous function such that decreasing with respect to ξ, $\delta(t,\xi)\leq t$ for $\xi\in[a,b]$, $\delta(t,\xi)\to\infty$ as $t\to\infty$, $r(t)>0$ and $0\leq p(t)<1$ are real valued rd-continuous functions defined on $\mathbb{T}$, $p(t)$ is increasing and

(H$_1$) $\int_{t_0}^{\infty}(\frac{1}{r(t)})^{\frac{1}{\gamma}}\Delta t=\infty$,

(H$_2$) $f:\mathbb{T}\times\mathbb{R}\to\mathbb{R}$ is a continuous function such that $uf(t,u)>0$ for all $u\neq 0$ and there exists a positive function $q(t)$ defined on $\mathbb{T}$ such that $|f(t,u)|\geq q(t)|u^{\gamma}|$.

A nontrivial function $y(t)$ is said to be a solution of (1) if $y(t)+p(t)y(\tau(t))\in C^1_{rd}[t_y,\infty]$ and $r(t)((y(t)+p(t)y(\tau(t)))^{\Delta})^{\gamma}\in C^1_{rd}[t_y,\infty]$ for $t_y\geq t_0$ and $y(t)$ satisfies equation (1) for $t_y\geq t_0$. A solution of (1), which is nontrivial for all large t, is called oscillatory if it has no last zero. Otherwise, a solution is called nonoscillatory.

We note that if $\mathbb{T} = \mathbb{R}$, we have $\sigma(t) = t$, $\mu(t) = 0$, $y^{\triangle}(t) = y'(t)$ and, therefore, (1) becomes a second-order neutral differential equation with distributed deviating arguments

$$\left(r(t)\left(\left(y(t) + p(t)y\left(\tau(t)\right)\right)'\right)^{\gamma}\right)' + \int_a^b f\left(t, y\left(\delta(t,\xi)\right)\right) d\xi = 0.$$

If $\mathbb{T} = \mathbb{N}$, we have $\sigma(t) = t + 1$, $\mu(t) = 1$, $y^{\triangle}(t) = \triangle y(t) = y(t+1) - y(t)$ and therefore (1) becomes a second-order neutral difference equation with distributed deviating arguments

$$\triangle\left(r(t)\left(\triangle\left(y(t) + p(t)y\left(\tau(t)\right)\right)\right)^{\gamma}\right) + \sum_{\xi=a}^{b-1} f\left(t, y\left(\delta(t,\xi)\right)\right) = 0$$

and if $\mathbb{T} = h\mathbb{N}$, $h > 0$, we have $\sigma(t) = t + h$, $\mu(t) = h$, $y^{\triangle}(t) = \triangle_h y(t) = \frac{y(t+h)-y(t)}{h}$ and, therefore, (1) becomes a second-order neutral difference equation with distributed deviating arguments

$$\triangle_h\left(r(t)\left(\triangle_h\left(y(t) + p(t)y\left(\tau(t)\right)\right)\right)^{\gamma}\right) + \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} f\left(t, y\left(\delta(t,kh)\right)\right)h = 0.$$

In recent years, there has been important research activity about the oscillatory behavior of second-order neutral differential, difference and dynamic equations. For example, Grace and Lalli [1] considered the following second-order neutral delay equation

$$\left(a(t)\left(x(t) + p(t)x(t-\tau)\right)'\right)' + q(t)f\left(x(t-\tau)\right) = 0, \quad t \geq t_0$$

and Graef *et al.* [2] considered the nonlinear second-order neutral delay equation

$$\left(y(t) + p(t)y\left(\tau(t)\right)\right)'' + q(t)f\left(y(t-\delta)\right) = 0, \quad t \geq t_0.$$

Recently, Agarwal *et al.* [3] considered second-order nonlinear neutral delay dynamic equation

$$\left(r(t)\left(\left(y(t) + p(t)y\left(\tau(t)\right)\right)^{\triangle}\right)^{\gamma}\right)^{\triangle} + f\left(t, y(t-\delta)\right) = 0. \tag{2}$$

Later, Saker [4] considered (2) but he used different technique to prove his results. In [5] and [6], the authors considered the second order neutral functional dynamic equation of the form

$$\left(r(t)\left(\left(y(t) + p(t)y\left(\tau(t)\right)\right)^{\triangle}\right)^{\gamma}\right)^{\triangle} + f\left(t, y\left(\delta(t)\right)\right) = 0,$$

which is more general than (2). For more papers related to oscillation of second-order nonlinear neutral delay dynamic equation on time scales, we refer the reader to [7–10]. For neutral equations with distributed deviating arguments, we refer the reader to the paper by Candan [11]. To the best of our knowledge, [12] is the only paper regarding to the distributed deviating arguments on time scales. The books [13, 14] gives time scale calculus and some applications.

2 Main results

Throughout the paper, we use the following notations for simplicity:

$$x(t) = y(t) + p(t)y\big(\tau(t)\big), \qquad x^{[1]} = r\big(x^{\triangle}\big)^{\gamma}, \qquad x^{[2]} = \big(x^{[1]}\big)^{\triangle} \tag{3}$$

and $\theta_1(t) = \delta(t,a)$ and $\theta_2(t) = \delta(t,b)$.

Theorem 2.1 *Assume that* (H_1) *and* (H_2) *hold. In addition, assume that* $r^{\triangle}(t) \geq 0$. *Then every solution of* (1) *oscillates if the inequality*

$$x^{[2]}(t) + A(t)x^{[1]}\big(\theta_1(t)\big) \leq 0, \tag{4}$$

where

$$A(t) = \frac{(b-a)q(t)(1-p(\theta_1(t)))^{\gamma}}{r(\theta_1(t))}\left(\frac{\theta_2(t)}{2}\right)^{\gamma}$$

has no eventually positive solution.

Proof Let $y(t)$ be a nonoscillatory solution of (1), without loss of generality, we assume that $y(t) > 0$ for $t \geq t_0$, then $y(\tau(t)) > 0$ and $y(\delta(t,\xi)) > 0$ for $t \geq t_1 > t_0$ and $b \geq \xi \geq a$. In the case when $y(t)$ is negative, the proof is similar. In view of (1), (H_2) and (3)

$$x^{[2]}(t) + \int_a^b q(t)y^{\gamma}\big(\delta(t,\xi)\big)\triangle\xi \leq 0 \tag{5}$$

for all $t \geq t_1$, and we see that $x^{[1]}(t)$ is an eventually decreasing function. We claim that $x^{[1]}(t) > 0$ eventually. Assume not then there exists a $t_2 \geq t_1$ such that $x^{[1]}(t_2) = c < 0$, then we have $x^{[1]}(t) \leq c$ for $t \geq t_2$ and it follows that

$$x^{\triangle}(t) \leq \left(\frac{c}{r(t)}\right)^{1/\gamma}. \tag{6}$$

Now integrating (6) from t_2 to t and using (H_1), we obtain

$$x(t) \leq x(t_2) + c^{1/\gamma}\int_{t_2}^{t}\left(\frac{1}{r(s)}\right)^{1/\gamma}\triangle s \to -\infty \quad \text{as } t \to \infty$$

which contradicts the fact that $x(t) > 0$ for all $t \geq t_0$. Hence, $x^{[1]}(t)$ is positive. Therefore, one sees that there is a $t_2 \geq t_1$ such that

$$x(t) > 0, \qquad x^{\triangle}(t) > 0, \qquad x^{[1]}(t) > 0, \qquad x^{[2]}(t) < 0, \quad t \geq t_2. \tag{7}$$

For $t \geq t_3 \geq t_2$, this implies that

$$y(t) \geq x(t) - p(t)x\big(\tau(t)\big) \geq \big(1 - p(t)\big)x(t)$$

then we conclude that

$$y^\gamma\big(\delta(t,\xi)\big) \geq \big(1 - p\big(\delta(t,\xi)\big)\big)^\gamma x^\gamma\big(\delta(t,\xi)\big), \quad t \geq t_4 \geq t_3, \xi \in [a,b]. \tag{8}$$

Multiplying (8) by $q(t)$ and integrating both sides from a to b, we have

$$\int_a^b q(t) y^\gamma\big(\delta(t,\xi)\big)\triangle\xi \geq \int_a^b q(t)\big(1 - p\big(\delta(t,\xi)\big)\big)^\gamma x^\gamma\big(\delta(t,\xi)\big)\triangle\xi. \tag{9}$$

Substituting (9) into (5), we obtain

$$x^{[2]}(t) + \int_a^b q(t)\big(1 - p\big(\delta(t,\xi)\big)\big)^\gamma x^\gamma\big(\delta(t,\xi)\big)\triangle\xi \leq 0. \tag{10}$$

On the other hand, we can verify that $x^{\triangle\triangle}(t) \leq 0$ for $t \geq t_4$ and, therefore, we obtain

$$x(t) = x(t_4) + \int_{t_4}^t x^\triangle(s)\triangle s \geq (t - t_4)x^\triangle(t) \geq \frac{t}{2}x^\triangle(t), \quad t \geq t_5 \geq 2t_4.$$

From the last inequality, it can be easily seen that

$$x\big(\delta(t,\xi)\big) \geq \left(\frac{\delta(t,\xi)}{2}\right)x^\triangle\big(\delta(t,\xi)\big) \geq \left(\frac{\theta_2(t)}{2}\right)x^\triangle\big(\delta(t,\xi)\big), \quad t \geq t_6 \geq t_5, \xi \in [a,b].$$

Substituting the last inequality into (10), we have

$$x^{[2]}(t) + \int_a^b q(t)\big(1 - p\big(\delta(t,\xi)\big)\big)^\gamma \left(\frac{\theta_2(t)}{2}\right)^\gamma \big(x^\triangle\big(\delta(t,\xi)\big)\big)^\gamma \triangle\xi \leq 0$$

and it can be found

$$x^{[2]}(t) + (b-a)q(t)\big(1 - p\big(\theta_1(t)\big)\big)^\gamma \left(\frac{\theta_2(t)}{2}\right)^\gamma \big(x^\triangle\big(\theta_1(t)\big)\big)^\gamma \leq 0,$$

or

$$x^{[2]}(t) + \frac{(b-a)q(t)(1 - p(\theta_1(t)))^\gamma}{r(\theta_1(t))}\left(\frac{\theta_2(t)}{2}\right)^\gamma x^{[1]}\big(\theta_1(t)\big) \leq 0,$$

which is the inequality (4). As a consequence of this, we have a contradiction and therefore every solution of (1) oscillates. □

Theorem 2.2 *Assume that* (H_1) *and* (H_2) *hold. In addition, assume that* $r^\triangle(t) \geq 0$, $\delta(t,\xi)$ *is increasing with respect to* t *and that the inequality*

$$\limsup_{t\to\infty} \int_{\theta_1(t)}^t A(s)\triangle s > 1 \tag{11}$$

holds. Then every solution of (1) *oscillates.*

Proof Let $y(t)$ be a nonoscillatory solution of (1). We can proceed as in the proof of Theorem 2.1 to get (4). Integrating (4) from $\theta_1(t)$ to t for sufficiently large t, we have

$$\begin{aligned}
0 &\geq \int_{\theta_1(t)}^{t} \left(x^{[2]}(s) + A(s)x^{[1]}\left(\theta_1(s)\right)\right)\triangle s \\
&= x^{[1]}(t) - x^{[1]}\left(\theta_1(t)\right) + \int_{\theta_1(t)}^{t} A(s)x^{[1]}\left(\theta_1(s)\right)\triangle s \\
&\geq x^{[1]}(t) - x^{[1]}\left(\theta_1(t)\right) + x^{[1]}\left(\theta_1(t)\right)\int_{\theta_1(t)}^{t} A(s)\triangle s \\
&= x^{[1]}(t) + x^{[1]}\left(\theta_1(t)\right)\left(\int_{\theta_1(t)}^{t} A(s)\triangle s - 1\right) > 0.
\end{aligned}$$

By making use of (11), we reach to a contradiction therefore the proof is complete. □

Theorem 2.3 *Assume that* (H_1) *and* (H_2) *hold. In addition, assume that* $r^{\triangle}(t) \geq 0$, $\delta(t,\xi)$ *is increasing with respect to t and there exists a positive rd-continuous* $\triangle$*-differentiable function* $\alpha(t)$ *such that*

$$\limsup_{t\to\infty} \int_{t_0}^{t} \left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{4\gamma(\frac{\theta_2(s)}{2})^{\gamma-1}\alpha(s)}\right)\triangle s = \infty, \tag{12}$$

where $(\alpha^{\triangle}(s))_+ = \max\{0, \alpha^{\triangle}(s)\}$ *and* $Q(s) = (b-a)q(s)(1-p(\theta_1(s)))^{\gamma}$. *Then every solution of* (1) *is oscillatory on* $[t_0,\infty)$.

Proof Suppose to the contrary that $y(t)$ is nonoscillatory solution of (1). We may assume without loss of generality that $y(t) > 0$ for $t \geq t_0$, then $y(\tau(t)) > 0$ and $y(\delta(t,\xi)) > 0$ for $t \geq t_1 > t_0$ and $b \geq \xi \geq a$. Proceeding as in the proof of Theorem 2.1, we obtain (7) and the inequality (10). Using (7) and Pötzsche's chain rule [15, Theorem 1], we obtain

$$\begin{aligned}
\left(x^{\gamma}(t)\right)^{\triangle} &= \gamma \int_0^1 \left[x(t) + h\mu(t)x^{\triangle}(t)\right]^{\gamma-1}\,dh x^{\triangle}(t) \\
&\geq \gamma \int_0^1 \left(x(t)\right)^{\gamma-1}\,dh x^{\triangle}(t) = \gamma\left(x(t)\right)^{\gamma-1}x^{\triangle}(t) > 0.
\end{aligned} \tag{13}$$

From (10) and (13), we obtain

$$x^{[2]}(t) \leq -(b-a)q(t)\left(1 - p\left(\theta_1(t)\right)\right)^{\gamma}x^{\gamma}\left(\theta_2(t)\right) = -Q(t)x^{\gamma}\left(\theta_2(t)\right), \quad t \geq t_4. \tag{14}$$

Define the function

$$z(t) = \alpha(t)\frac{x^{[1]}(t)}{x^{\gamma}(\theta_2(t))}, \quad t \geq t_4. \tag{15}$$

It is obvious that $z(t) > 0$. Taking the derivative of $z(t)$, we see that

$$\begin{aligned}
z^{\triangle}(t) &= \left(x^{[1]}\right)^{\sigma}(t)\left(\frac{\alpha(t)}{x^{\gamma}(\theta_2(t))}\right)^{\triangle} + \frac{\alpha(t)}{x^{\gamma}(\theta_2(t))}x^{[2]}(t) \\
&= \frac{\alpha(t)x^{[2]}(t)}{x^{\gamma}(\theta_2(t))} + \left(x^{[1]}\right)^{\sigma}(t)\left(\frac{x^{\gamma}(\theta_2(t))\alpha^{\triangle}(t) - \alpha(t)(x^{\gamma}(\theta_2(t)))^{\triangle}}{x^{\gamma}(\theta_2(t))(x^{\sigma}(\theta_2(t)))^{\gamma}}\right).
\end{aligned} \tag{16}$$

Now using (14) in (16), we obtain

$$z^{\triangle}(t) \leq -\alpha(t)Q(t) + \frac{\alpha^{\triangle}(t)z^{\sigma}(t)}{\alpha^{\sigma}(t)} - \frac{\alpha(t)(x^{[1]})^{\sigma}(t)(x^{\gamma}(\theta_2(t)))^{\triangle}}{x^{\gamma}(\theta_2(t))(x^{\sigma}(\theta_2(t)))^{\gamma}}. \tag{17}$$

On the other hand, as in the proof of Theorem 2.1, it can be shown that for sufficiently large $t \geq t_5$

$$x(t) \geq \left(\frac{t}{2}\right)x^{\triangle}(t), \quad t \geq t_5 \geq 2t_4$$

and then

$$\gamma x^{\gamma-1}(t) \geq \gamma \left(\frac{t}{2}\right)^{\gamma-1} \left(x^{\triangle}(t)\right)^{\gamma-1}$$

or

$$\gamma x^{\gamma-1}\bigl(\theta_2(t)\bigr) \geq \gamma \left(\frac{\theta_2(t)}{2}\right)^{\gamma-1} \left(x^{\triangle}\bigl(\theta_2(t)\bigr)\right)^{\gamma-1}, \quad t \geq t_6 \geq t_5. \tag{18}$$

Since $x^{[2]}(t) < 0$, we have

$$x^{[1]}(t) > x^{[1]}\bigl(\sigma(t)\bigr). \tag{19}$$

Multiplying (18) by $x^{\triangle}(\theta_2(t))$ and using (19), it follows that

$$\begin{aligned}
\gamma x^{\gamma-1}\bigl(\theta_2(t)\bigr)x^{\triangle}\bigl(\theta_2(t)\bigr) &\geq \gamma \left(\frac{\theta_2(t)}{2}\right)^{\gamma-1} \left(x^{\triangle}\bigl(\theta_2(t)\bigr)\right)^{\gamma} \\
&\geq \gamma \left(\frac{\theta_2(t)}{2}\right)^{\gamma-1} \frac{r(\theta_2(\sigma(t)))}{r(\theta_2(t))} \left(x^{\triangle}\bigl(\theta_2\bigl(\sigma(t)\bigr)\bigr)\right)^{\gamma} \\
&\geq \gamma \left(\frac{\theta_2(t)}{2}\right)^{\gamma-1} \frac{(x^{[1]})^{\sigma}(\theta_2(t))}{r(\theta_2(t))}.
\end{aligned} \tag{20}$$

From (13), for sufficiently large $t \geq t_7 \geq t_6$, we have

$$\left(x^{\gamma}\bigl(\theta_2(t)\bigr)\right)^{\triangle} \geq \gamma x^{\gamma-1}\bigl(\theta_2(t)\bigr)x^{\triangle}\bigl(\theta_2(t)\bigr). \tag{21}$$

From (20) and (21), it follows that

$$\left(x^{\gamma}\bigl(\theta_2(t)\bigr)\right)^{\triangle} \geq \gamma \left(\frac{\theta_2(t)}{2}\right)^{\gamma-1} \frac{(x^{[1]})^{\sigma}(\theta_2(t))}{r(\theta_2(t))}. \tag{22}$$

Substituting (22) into (17), we obtain

$$z^{\triangle}(t) \leq -\alpha(t)Q(t) + \frac{\alpha^{\triangle}(t)z^{\sigma}(t)}{\alpha^{\sigma}(t)} - \frac{\gamma(\frac{\theta_2(t)}{2})^{\gamma-1}\alpha(t)}{(\alpha^{\sigma}(t))^2 r(\theta_2(t))}\left(z^{\sigma}(t)\right)^2.$$

Using the fact $u - mu^2 \le \frac{1}{4m}$, $m > 0$, we have

$$\begin{aligned} z^{\triangle}(t) &\le -\alpha(t)Q(t) + \frac{(\alpha^{\triangle}(t))_+}{\alpha^{\sigma}(t)}\left(z^{\sigma}(t) - \frac{\gamma(\frac{\theta_2(t)}{2})^{\gamma-1}\alpha(t)}{((\alpha^{\triangle}(t))_+)\alpha^{\sigma}(t)r(\theta_2(t))}\left(z^{\sigma}(t)\right)^2\right) \\ &\le -\left(\alpha(t)Q(t) - \frac{((\alpha^{\triangle}(t))_+)^2 r(\theta_2(t))}{4\gamma(\frac{\theta_2(t)}{2})^{\gamma-1}\alpha(t)}\right). \end{aligned}$$

Integrating the last inequality from t_7 to t, we obtain

$$-z(t_7) < z(t) - z(t_7) \le -\int_{t_7}^{t}\left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{4\gamma(\frac{\theta_2(s)}{2})^{\gamma-1}\alpha(s)}\right)\triangle s$$

or

$$z(t_7) > \int_{t_7}^{t}\left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{4\gamma(\frac{\theta_2(s)}{2})^{\gamma-1}\alpha(s)}\right)\triangle s$$

which contradicts (12). Therefore, the proof is complete. □

Theorem 2.4 *Assume that* (H_1) *and* (H_2) *hold and* $\sigma(t) \neq t$ *for each* $t \in \mathbb{T}$. *Let* $\alpha(t)$, $\delta(t,\xi)$, *and* $Q(s)$ *be as defined in Theorem* 2.3. *If*

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{2^{3-\gamma}(\mu(\theta_2(s)))^{\gamma-1}\alpha(s)}\right)\triangle s = \infty,$$

then every solution of (1) *is oscillatory on* $[t_0,\infty)$.

Proof Following the same lines as in the proof of Theorem 2.1, we get (7) and (10). Using the inequality,

$$x^{\gamma} - y^{\gamma} \ge 2^{1-\gamma}(x-y)^{\gamma}, \quad \gamma \ge 1,$$

we have

$$\begin{aligned} \left(x^{\gamma}(t)\right)^{\triangle} &= \frac{x^{\gamma}(\sigma(t)) - x^{\gamma}(t)}{\mu(t)} \ge 2^{1-\gamma}\frac{(x(\sigma(t)) - x(t))^{\gamma}}{\mu(t)} \\ &= 2^{1-\gamma}\left(\mu(t)\right)^{\gamma-1}\left(\frac{x(\sigma(t)) - x(t)}{\mu(t)}\right)^{\gamma} = 2^{1-\gamma}\left(\mu(t)\right)^{\gamma-1}\left(x^{\triangle}(t)\right)^{\gamma}. \end{aligned} \tag{23}$$

Now setting $z(t)$ by (15), using (17) and (23) we see that

$$z^{\triangle}(t) \le -\alpha(t)Q(t) + \frac{(\alpha^{\triangle}(t))_+ z^{\sigma}(t)}{\alpha^{\sigma}(t)} - \frac{2^{1-\gamma}(\mu(\theta_2(t)))^{\gamma-1}\alpha(t)}{(\alpha^{\sigma}(t))^2 r(\theta_2(t))}\left(z^{\sigma}(t)\right)^2.$$

The remaining part of the proof is similar to that of Theorem 2.3, hence it is omitted. □

Example 2.5 Consider the following second-order neutral nonlinear dynamic equation

$$\left(\left(\left(y(t) + \left(\frac{t+a-1}{t+a}\right)y\left(\tau(t)\right)\right)^{\triangle}\right)^{5/3}\right)^{\triangle} + \int_a^b t^{-1/3}y(t-\xi)\triangle\xi = 0, \quad t \in \mathbb{T}$$

where $\gamma = \frac{5}{3}$, $r(t) = 1$, $p(t) = (\frac{t+a-1}{t+a})$, $q(t) = t^{-1/3}$. One can verify that the conditions of Theorem 2.3 are satisfied. Note that taking $\alpha(s) = s$, we see that

$$\begin{aligned}
&\limsup_{t\to\infty} \int_{t_0}^{t} \left(\alpha(s)Q(s) - \frac{((\alpha^{\triangle}(s))_+)^2 r(\theta_2(s))}{4\gamma(\frac{\theta_2(s)}{2})^{\gamma-1}\alpha(s)} \right) \triangle s \\
&\quad = \limsup_{t\to\infty} \int_{t_0}^{t} \left((b-a)s^{-1} - \frac{1}{\frac{20}{3}(\frac{s-b}{2})^{2/3} s} \right) \triangle s = \infty.
\end{aligned}$$

Therefore, (1) is oscillatory.

Competing interests
The author declares that they have no competing interests.

Received: 26 April 2012 Accepted: 3 April 2013 Published: 18 April 2013

References

1. Grace, SR, Lalli, BS: Oscillations of nonlinear second order neutral delay differential equations. Rad. Mat. **3**, 77-84 (1987)
2. Graef, JR, Grammatikopoulos, MK, Spikes, PW: Asymptotic properties of solutions of nonlinear neutral delay differential equations of the second order. Rad. Mat. **4**(1), 133-149 (1988)
3. Agarwal, RP, O'Regan, D, Saker, SH: Oscillation criteria for second-order nonlinear neutral delay dynamic equations. J. Math. Anal. Appl. **300**, 203-217 (2004)
4. Saker, SH: Oscillation of second-order nonlinear neutral delay dynamic equations on time scales. J. Comput. Appl. Math. **187**, 123-141 (2006)
5. Saker, SH: Oscillation criteria for a second-order quasilinear neutral functional dynamic equation on time scales. Nonlinear Oscil. **13**, 407-428 (2011)
6. Saker, SH, O'Regan, D: New oscillation criteria for second-order neutral functional dynamic equations via the generalized Riccati substitution. Commun. Nonlinear Sci. Numer. Simul. **16**(1), 423-434 (2011)
7. Saker, SH: Oscillation of superlinear and sublinear neutral delay dynamic equations. Commun. Appl. Anal. **12**(2), 173-187 (2008)
8. Saker, SH, Agarwal, RP, O'Regan, D: Oscillation results for second-order nonlinear neutral delay dynamic equations on time scales. Appl. Anal. **86**, 1-17 (2007)
9. Saker, SH, O'Regan, D, Agarwal, RP: Oscillation theorems for second-order nonlinear neutral delay dynamic equations on time scales. Acta Math. Sin. Engl. Ser. **24**, 1409-1432 (2008)
10. Saker, SH: Hille and Nehari types oscillation criteria for second-order neutral delay dynamic equations. Dyn. Contin. Discrete Impuls. Syst., Ser. B, Appl. Algorithms **16**(3), 349-360 (2009)
11. Candan, T, Dahiya, RS: On the oscillation of certain mixed neutral equations. Appl. Math. Lett. **21**(3), 222-226 (2008)
12. Candan, T: Oscillation of second-order nonlinear neutral dynamic equations on time scales with distributed deviating arguments. Comput. Math. Appl. **62**(11), 4118-4125 (2011)
13. Bohner, M, Peterson, A: Dynamic Equations on Time Scales: An Introduction with Applications. Birkhäuser, Boston (2001)
14. Bohner, M, Peterson, A: Advances in Dynamic Equations on Time Scales. Birkhäuser, Boston (2003)
15. Pötzsche, C: Chain rule and invariance principle on measure chains. J. Comput. Appl. Math. **141**, 249-254 (2002)

doi:10.1186/1687-1847-2013-112
Cite this article as: Candan: Oscillation criteria for second-order nonlinear neutral dynamic equations with distributed deviating arguments on time scales. *Advances in Difference Equations* 2013 **2013**:112.