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Approximate ternary quadratic derivations on ternary Banach algebras and C^*-ternary rings

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Abstract

In the current article, we use a fixed point alternative theorem to establish the Hyers-Ulam stability and also the superstability of a ternary quadratic derivation on ternary Banach algebras and C^*-ternary rings which is introduced in Shagholi et al.

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3 Introduction

A basic question in the theory of functional equations is as follows: when is it true that a function, which approximately satisfies a *functional equation*, must be close to an exact solution of the equation? If the problem accepts a unique solution, we say the equation is *stable*. Also, if every approximately solution is an exact solution of it, we say the functional equation is *superstable* (see, [1]). The first stability problem concerning group homomorphisms was raised by Ulam [2] and affirmatively solved by Hyers [3]. In [4], Rassias generalized the Hyers result to approximately linear mappings. Lastly, Gajda [5] answered the question for another case of linear mapping, which was rased by Rassias. This new concept is known as Hyers-Ulam-Rassias stability of functional equations (see, [6]).

The functional equation $f(x + y) + f(x - y) = 2f(x) + 2f(y)$ is called *quadratic* functional equation. In addition, every solution of the above equation is said to be a *quadratic mapping*. A Hyers-Ulam stability problem for the quadratic functional equation was proved by Skof [7] for mappings $f: X \to Y$, where X is a normed space and Y is a Banach space. Later, Czerwik [8] proved the Cauchy-Rassias stability of the quadratic functional equation. Since then, the stability problems of various functional equation have been extensively investigated by a number of authors (for instances, [9,10]).

As it is extensively discussed in [11], the full description of a physical system $\mathbf{S}$ implies the knowledge of three basic ingredients: the set of the observables, the set of the states and the dynamics that describes the time evolution of the system by means of the time dependence of the expectation value of a given observable on a given statue. Originally the set of the observables were considered to be a C^*-algebra [12]. In many applications, however, this was shown not to be the most convenient choice, and so the C^*-algebra was replaced by a Von Neumann algebra. This is because the role of the representation turns out to be crucial, mainly when long range interactions are involved. Here we used a different algebraic structure. A ternary Banach algebra is a complex Banach space $\mathcal{A}$ equipped with a ternary product $(x, y, z) \to [x, y, z]$ of $\mathcal{A}^3$ into $\mathcal{A}$, which is trilinear in the variables, associative in the sense that $[x, y, [z, w, v]] = [x, [w, z, y], v] = [[x, y, z], w, v]$, and satisfies $||[x, y, z]|| \le ||x||\,||y||\,||z||$. A C^*-ternary ring is a complex Banach space $\mathcal{A}$ equipped with a ternary product which is associative and linear in the outer variables, conjugate linear in the middle variable, and $||[x, x, x]|| = ||x||^3$ (see, [13]). If a C^*-ternary algebra $(\mathcal{A}, [\cdot, \cdot, \cdot])$ has an identity, i.e., an element $e \in \mathcal{A}$ such that $x = [x, e, e] = [e, e, x]$ for all $x \in \mathcal{A}$, then it is routine to verify that $\mathcal{A}$, endowed with $x \bullet y := [x, e, y]$ and $x^* := [e, x, e]$, is a unital C^*-algebra. Conversely, if $(\mathcal{A}, \bullet)$ is a unital C^*-algebra, then $[x, y, z] := x \bullet y \bullet z$ makes $\mathcal{A}$ into a C^*-ternary ring.

Recently, Shagholi et al. [14] proved the stability of ternary quadratic derivations on ternary Banach algebras. Also Moslehian had investigated the stability and the superstability of ternary derivations on C^*-ternary rings [15]. Zhou Xu et al. [16] used the fixed point alternative (Theorem 4.2 of current article) to establish Hyers-Ulam-Rassias stability of the general mixed additive-cubic functional equation, where functions map a linear space into a complete quasi fuzzy p-normed space. The generalized Hyers-Ulam stability of an additive-cubic-quartic functional equation in NAN-spaces is also proved by using the mentioned theorem in [17].

In this article, we prove the Hyers-Ulam stability and the superstability of ternary quadratic derivations on ternary Banach algebras and C^*-ternary rings associated with the quadratic functional equation $f(x + y) + f(x - y) = 2f(x) + 2f(y)$ using this fixed point theorem.

4 Stability of ternary quadratic derivations

Throughout this article, for a ternary Banach algebra (or C^*-ternary ring) $\mathcal{A}$, we denote $\overbrace{\mathcal{A} \times \mathcal{A} \times \cdots \times \mathcal{A}}^{n-times}$ by $\mathcal{A}^n$.

Definition 4.1 *Let $\mathcal{A}$ be a ternary Banach algebra or C^*-ternary ring. Then a mapping $D : \mathcal{A} \to \mathcal{A}$ is called a ternary quadratic derivation if it is a quadratic mapping that satisfies*

$$D([x, y, z]) = [D(x), y^2, z^2] + [x^2, D(y), z^2] + [x^2, y^2, D(z)],$$

for all $x, y, z \in \mathcal{A}$.

It is proved in [18] that for the vector spaces X and Y and the fixed positive integer k, the map f: $X \to Y$ is quadratic if and only if the following equality holds:

$$2f\left(\frac{kx + ky}{2}\right) + 2f\left(\frac{kx - ky}{2}\right) = k^2 f(x) + k^2 f(y),$$

for all $x, y \in X$. Also, we can show that f is quadratic if and only if for a fixed positive integer k, we have

$$f(kx + ky) + f(kx - ky) = 2k^2 f(x) + 2k^2 f(y).$$

for all $x, y \in X$. Before proceeding to the main results, to achieve our aim, we need the following known fixed point theorem which has been proven in [19].

Theorem 4.2 *(The fixed point alternative) Suppose that (Ω, d) is a complete generalized metric space and let J: $\Omega \to \Omega$ be a strictly contractive mapping with Lipschitz* *constant $L < 1$. Then, for each element $x \in \Omega$, either $d(J^n x, J^{n+1}x) = \infty$ for all $n \geq 0$, or there exists a natural number n_0 such that:*

(i) $d(J^n x, J^{n+1}x) < \infty$ for all $n \geq n_0$;
(ii) the sequence $\{J^n x\}$ converges to a fixed point y^* of J;
(iii) y^* is the unique fixed point of J in the set $\Lambda = \{y \in \Omega : d(J^{n_0}x, y) < \infty\}$;
(iv) $d(y, y^*) \leq \frac{1}{1-L} d(y, Jy)$ for all $y \in \Lambda$.

In the following theorem, we prove the Hyers-Ulam stability of ternary quadratic derivation on C^*-ternary rings.

Theorem 4.3 *Let $\mathcal{A}$ be a C^*-ternary ring, $f : \mathcal{A} \to \mathcal{A}$ be a mapping with $f(0) = 0$, and also let $\varphi : \mathcal{A}^5 \to [0, \infty)$ be a function such that*

$$\left\| 2f\left(\frac{\mu a + \mu b}{2}\right) + 2f\left(\frac{\mu a - \mu b}{2}\right) - \mu^2(f(a) + f(b)) \right\| \leq \varphi(a, b, 0, 0, 0) \tag{1}$$

$$\| f([x, y, z]) - ([f(x), y^2, z^2] + [x^2, f(y), z^2] + [x^2, y^2, f(z)]) \| \leq \varphi(0, 0, x, y, z) \tag{2}$$

for all $\mu \in \mathbf{T} = \{\mu \in \mathbf{C}: |\mu| = 1\}$ and for all $a, b, x, y, z \in \mathcal{A}$. If there exists a constant $M \in (0, 1)$ such that

$$\varphi(2a, 2b, 2x, 2y, 2z) \leq 4M\varphi(a, b, x, y, z) \tag{3}$$

for all $a, b, x, y, z \in \mathcal{A}$, then there exists a unique ternary quadratic derivation $\| f(a) - D(a) \| \leq \frac{M}{1-M}\psi(a)$ satisfying

$$\| f(a) - D(a) \| \leq \frac{M}{1-M}\psi(a) \tag{4}$$

for all $a \in \mathcal{A}$, where $\psi(a) = \phi(a, 0, 0, 0, 0)$.

Proof. It follows from (3) that

$$\lim_j \frac{\varphi(2^j a, 2^j b, 2^j x, 2^j y, 2^j z)}{4^j} = 0 \tag{5}$$

for all $a, b, x, y, z \in \mathcal{A}$. Putting $\mu = 1$, $b = 0$ and replacing a by $2a$ in (1), we have

$$\| 4f(a) - f(2a) \| \leq \psi(2a) \leq 4M\psi(a) \tag{6}$$

for all $a \in \mathcal{A}$, and so

$$\left\| f(a) - \frac{1}{4}f(2a) \right\| \leq M\psi(a) \tag{7}$$

for all $a \in \mathcal{A}$. We consider the set $\Omega := \{h : \mathcal{A} \to \mathcal{A} | h(0) = 0\}$ and introduce the generalized metric on X as follows:

$$d(h_1, h_2) := \inf\{K \in (0, \infty) : \| h_1(a) - h_2(a) \| \leq K\psi(a), \forall a \in \mathcal{A}\},$$

if there exist such constant K, and $d(h_1,h_2) = \infty$, otherwise. One can show that (Ω,d) is complete. We now define the linear mapping $J: \Omega \to \Omega$ by

$$J(h)(a) = \frac{1}{4}h(2a) \tag{8}$$

for all $a \in \mathcal{A}$. Given $h_1, h_2 \in \Omega$, let $K \in \mathbf{R}^+$ be an arbitrary constant with $d(h_1, h_2) \leq K$, that is

$$\|h_1(a) - h_2(a)\| \leq C\psi(a) \tag{9}$$

for all $a \in \mathcal{A}$. Substituting a by $2a$ in the inequality (9) and using the equalities (3) and (8), we have

$$\|(Jh_1)(a) - (Jh_2)(a)\| = \frac{1}{4}\|h_1(2a) - h_2(2a)\| \leq \frac{1}{4}K\psi(2a) \leq KM\psi(a),$$

for all $a \in \mathcal{A}$, and thus $d(Jh_1,Jh_2) \leq KM$. Therefore, we conclude that $d(Jh_1,Jh_2) \leq Md(h_1, h_2)$ for all $h_1, h_2 \in \Omega$. It follows from (7) that

$$d(Jf,f) \leq M. \tag{10}$$

By the part (*iv*) of Theorem 4.2, the sequence $\{J^nf\}$ converges to a unique fixed point $D : \mathcal{A} \to \mathcal{A}$ in the set $\Omega_1 = \{h \in \Omega, d(f, h) < \infty\}$, i.e.,

$$\lim_{n\to\infty} \frac{f(2^n a)}{4^n} = D(a) \tag{11}$$

for all $a \in \mathcal{A}$. By Theorem 4.2 and (10), we have

$$d(f,D) \leq \frac{d(Tf,f)}{1-M} \leq \frac{M}{1-M}.$$

The last inequality shows that (4) holds for all $a \in \mathcal{A}$. Replace 2^na and 2^nb by a and b, respectively. Now, dividing both sides of the resulting inequality by 2^n, and letting n goes to infinity, we obtain

$$2D\left(\frac{\mu a + \mu b}{2}\right) + D\left(\frac{\mu a - \mu b}{2}\right) = \mu^2 D(a) + \mu^2 D(b) \tag{12}$$

for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbf{T}$. Putting $\mu = 1$ in (12) we have

$$2D\left(\frac{a+b}{2}\right) + 2D\left(\frac{a-b}{2}\right) = D(a) + D(b) \tag{13}$$

for all $a, b \in \mathcal{A}$. Hence D is a quadratic mapping by [18, Proposition 1]. Replacing 2^nx, 2^ny, 2^nz by x, y, z, respectively, in (2), we obtain

$$\begin{aligned}&\left\|f([2^nx,2^ny,2^nz]) - [f(2^nx),(2^ny)^2,(2^nz)^2] - [(2^nx)^2,f(2^ny),(2^nz)^2] - \left[x^2,y^2,\frac{f(2^nz)}{4^n}\right]\right\| \\ &\leq \frac{1}{4}\frac{\phi(0,0,2^nx,2^ny,2^nz)}{4^{3n}}.\end{aligned} \tag{14}$$

Now, the inequality (14) shows that

$$\left\|\frac{f(2^n[x,y,z])}{4^n} - \left[\frac{f(2^nx)}{4^n},y^2,z^2\right] - \left[x^2,\frac{f(2^ny)}{4^n},z^2\right] - \left[x^2,y^2,\frac{f(2^nz)}{4^n}\right]\right\| \leq \frac{1}{4}\frac{\phi(0,0,2^nx,2^ny,2^nz)}{4^{3n}}. \tag{15}$$

By (5), the right hand side of the above inequality tends to zero as $n \to \infty$. Thus

$$D[x,y,z] = [D(x),y^2,z^2] + [x^2,D(y),z^2] + [x^2,y^2,D(z)],$$

for all $x,y,z \in \mathcal{A}$. Therefore D is a ternary quadratic derivation.

Corollary 4.4 *Let p, θ be non negative real numbers such that $p < 2$ and let f be a mapping on a C^*-ternary ring $\mathcal{A}$ with $f(0) = 0$ and*

$$\left\|2f\left(\frac{\mu a+\mu b}{2}\right)+2f\left(\frac{\mu a-\mu b}{2}\right)-\mu^2(f(a)+f(b))\right\| \le \theta(\|a\|^p+\|b\|^p) \tag{16}$$

$$\|f([x,y,z])-([f(x),y^2,z^2]+[x^2,f(y),z^2]+[x^2,y^2,f(z)])\| \le \theta(\|x\|^p+\|y\|^p+\|z\|^p) \tag{17}$$

for all $\mu \in \mathbf{T} = \{\mu \in \mathbf{C}\colon |\mu| = 1\}$ and for all $a,b,x,y,z \in \mathcal{A}$. Then there exists a unique ternary quadratic derivation $D:\mathcal{A}\to\mathcal{A}$ satisfying

$$\|f(a)-D(a)\| \le \frac{2^p\theta}{4-2^p}\|a\|^p \tag{18}$$

for all $a \in \mathcal{A}$.

Proof. The result follows from Theorem 4.3 by putting $\varphi(a, b, x, y, z) = \theta(||a||^p + ||b||^p + ||x||^p + ||y||^p + ||z||^p)$.

Now, we establish the superstability of ternary quadratic derivations on C^*-ternary rings as follows:

Corollary 4.5 *Let p, θ be the nonnegative real numbers with $3p < 2$ and let f be a mapping on a C^*-ternary ring $\mathcal{A}$ and*

$$\left\|2f\left(\frac{\mu a+\mu b}{2}\right)+2f\left(\frac{\mu a-\mu b}{2}\right)-\mu^2(f(a)+f(b))\right\| \le \theta(\|a\|^p\|b\|^p) \tag{19}$$

$$\|f([x,y,z])-([f(x),y^2,z^2]+[x^2,f(y),z^2]+[x^2,y^2,f(z)])\| \le \theta(\|x\|^p\|y\|^p\|z\|^p) \tag{20}$$

for all $\mu \in \mathbf{T} = \{\mu \in \mathbf{C}\colon |\mu| = 1\}$ and for all $a,b,x,y,z \in \mathcal{A}$. Then f is a ternary quadratic derivation on $\mathcal{A}$.

Proof. Putting $a = b = 0$ in (19), we get $f(0) = 0$. Now, if we put $b = 0$, $\mu = 1$ and replace a by $2a$ in (19), then we have $f(2a) = 4f(a)$ for all $a \in \mathcal{A}$. It is easy to see by induction that $f(2^na) = 4^nf(a)$, and so $f(a) = \frac{f(2^na)}{4^n}$ for all $a \in \mathcal{A}$ and $n \in \mathbf{N}$. It follows from Theorem 4.3 that f is a quadratic mapping. Now, by putting $\varphi(a, b, x, y, z) = \theta||a||^p||b||^p(||x||^p + ||y||^p + ||z||^p) + \theta||x||^p||y||^p||z||^p$ in Theorem 4.3, we can obtain the desired result.

Theorem 4.6 *Let $\mathcal{A}$ be a ternary Banach algebra, and let $f:\mathcal{A}\to\mathcal{A}$ be a mapping with $f(0) = 0$, and also let $\varphi:\mathcal{A}^5\to[0,\infty)$ be a function such that*

$$\|f(\mu a+\mu b)+f(\mu a-\mu b)-2\mu^2(f(a)+f(b))\| \le \varphi(a,b,0,0,0) \tag{21}$$

$$\|f([x,y,z])-([f(x),y^2,z^2]+[x^2,f(y),z^2]+[x^2,y^2,f(z)])\| \le \varphi(0,0,x,y,z) \tag{22}$$

for all $\mu \in \mathbf{T} = \{\mu \in \mathbf{C}: |\mu| = 1\}$ *and for all* $a, b, x, y, z \in \mathcal{A}$. *If there exists a constant* $m \in (0, 1)$ *such that*

$$\varphi(2a, 2b, 2x, 2y, 2z) \leq 4m\varphi(a, b, x, y, z) \tag{23}$$

for all $a, b, x, y, z \in \mathcal{A}$, *then there exists a unique ternary quadratic derivation* $\|f(a) - D(a)\| \leq \frac{1}{4(1-m)}\psi(a)$ *satisfying*

$$\|f(a) - D(a)\| \leq \frac{1}{4(1-m)}\psi(a) \tag{24}$$

for all $a \in \mathcal{A}$, *where* $\psi(a) = \varphi(a, a, 0, 0, 0)$.

Proof. Using condition (23), we obtain

$$\lim_n \frac{\varphi(2^n a, 2^n b, 2^n x, 2^n y, 2^n z)}{4^n} = 0 \tag{25}$$

for all $a, b, x, y, z \in \mathcal{A}$. Letting $\mu = 1$, $a = b$, and replacing a by $2a$ in (21), we get

$$\|f(2a) - 4f(a)\| \leq \psi(a)$$

for all $a \in \mathcal{A}$. By the last inequality, we have

$$\left\|\frac{1}{4}f(2a) - f(a)\right\| \leq \frac{1}{4}\psi(a) \tag{26}$$

for all $a \in \mathcal{A}$. Similar to the proof of Theorem 4.3, we consider the set $\Omega := \{h : \mathcal{A} \to \mathcal{A} | h(0) = 0\}$ and introduce a generalized metric on Ω by

$$d(g, h) := \inf\{C \in (0, \infty) : \|g(a) - h(a)\| \leq C\psi(a)\ \forall a \in \mathcal{A}\},$$

if there exist such constant C, and $d(g, h) = \infty$, otherwise. Again, it is easy to check that (Ω, d) is complete. We define the linear mapping $T: \Omega \to \Omega$ by

$$T(h)(a) = \frac{1}{4}h(2a) \tag{27}$$

for all $a \in \mathcal{A}$. For arbitrary elements $g, h \in \Omega$ and $C \in (0, \infty)$ with $d(g, h) \leq C$, we have

$$\|g(a) - h(a)\| \leq C\psi(a) \tag{28}$$

for all $a \in \mathcal{A}$. Replacing a by $2a$ in the inequality (28) and using (23) and (27), we have

$$\|(Tg)(a) - (Th)(a)\| = \frac{1}{4}\|g(2a) - h(2a)\| \leq \frac{1}{4}C\psi(2a) \leq Cm\psi(a),$$

for all $a \in \mathcal{A}$. Thus, $d(Tg, Th) \leq Cm$. Therefore, we conclude that $d(Tg, Th) \leq md(g, h)$ for all $g, h \in X$. It follows from (26) that

$$d(Tf, f) \leq \frac{1}{4}. \tag{29}$$

Hence T is a strictly contractive mapping on Ω. Now, Theorem 4.2 shows that T has a unique fixed point $D : \mathcal{A} \to \mathcal{A}$ in the set $\Omega_1 = \{h \in \Omega, d(f, h) < \infty\}$. On the other hand,

$$\lim_{n\to\infty} \frac{f(2^n a)}{4^n} = D(a) \tag{30}$$

for all $a \in \mathcal{A}$. Again, by using Theorem 4.2 and (29), we obtain

$$d(f, D) \le \frac{d(Tf, f)}{1-m} \le \frac{1}{4(1-m)}$$

i.e., the inequality (24) is true for all $a \in \mathcal{A}$. Let us replace a and b in (21) by $2^n a$ and $2^n b$ respectively, and then divide both sides by 2^n. Passing to the limit as $n \to \infty$, we get

$$D(\mu a + \mu b) + D(\mu a - \mu b) = 2\mu^2 D(a) + 2\mu^2 D(b) \tag{31}$$

for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbf{T}$. Put $\mu = 1$ in (31) to get

$$D(a + b) + D(a - b) = 2D(a) + 2D(b) \tag{32}$$

for all $a, b \in \mathcal{A}$. Hence D is a quadratic mapping. Replace $2^n x$, $2^n y$, $2^n z$ by x, y, z respectively, we obtain

$$\begin{aligned}&\left\| f([2^n x, 2^n y, 2^n z]) - [f(2^n x), (2^n y)^2, (2^n z)^2] - [(2^n x)^2, f(2^n y), (2^n z)^2] - \left[x^2, y^2, \frac{f(2^n z)}{4^n}\right] \right\| \\ &\le \frac{1}{2} \frac{\varphi(0, 0, 2^n x, 2^n y, 2^n z)}{4^{3n}}.\end{aligned} \tag{33}$$

Now, the inequality (33) shows that

$$\begin{aligned}&\left\| \frac{f(2^n [x, y, z])}{4^n} - \left[\frac{f(2^n x)}{4^n}, y^2, z^2\right] - \left[x^2, \frac{f(2^n y)}{4^n}, z^2\right] - \left[x^2, y^2, \frac{f(2^n z)}{4^n}\right] \right\| \\ &\le \frac{1}{2} \frac{\varphi(0, 0, 2^n x, 2^n y, 2^n z)}{4^{3n}}\end{aligned} \tag{34}$$

Taking the limit in the equality (34) and using (25), one obtain that

$$D[x, y, z] = [D(x), y^2, z^2] + [x^2, D(y), z^2] + [x^2, y^2, D(z)]$$

for all $x, y, z \in \mathcal{A}$. Therefore D is a ternary quadratic derivation. This completes the proof of this theorem.

The following corollaries are some applications to show the stability and super stability of ternary quadratic derivations under some conditions.

Corollary 4.7 *Let $\mathcal{A}$ be a ternary Banach algebra. Let p, θ be the non negative real numbers such that $p < 2$ and let $f : \mathcal{A} \to \mathcal{A}$ be a mapping with* $f(0) = 0$ *and*

$$\| f(\mu a + \mu b) + f(\mu a - \mu b) - \mu^2 (f(a) + f(b)) \| \le \theta(\|a\|^p + \|b\|^p) \tag{35}$$

$$\| f([x, y, z]) - ([f(x), y^2, z^2] + [x^2, f(y), z^2] + [x^2, y^2, f(z)]) \| \le \theta(\|x\|^p + \|y\|^p + \|z\|^p) \tag{36}$$

for all $\mu \in \mathbf{T} = \{\mu \in \mathbf{C}: |\mu| = 1\}$ and for all $a, b, x, y, z \in \mathcal{A}$. Then there exists a unique ternary quadratic derivation $D : \mathcal{A} \to \mathcal{A}$ such that

$$\left\|f(a) - D(a)\right\| \leq \frac{2\theta}{4 - 2^p}\|a\|^p \tag{37}$$

for all $a \in \mathcal{A}$.

Proof. The result follows from Theorem 4.6 by putting

$$\varphi(a, b, c, d, u, v) = \theta(\|a\|^p + \|b\|^p + \|c\|^p + \|u\|^p + \|v\|^p).$$

Corollary 4.8 *Let* $\mathcal{A}$ *be a ternary Banach algebra. Let p,* θ *be the nonnegative real numbers with* $3p < 2$ *and let* $f : \mathcal{A} \to \mathcal{A}$ *be a mapping such that*

$$\left\|f(\mu a + \mu b) + 2f(\mu a - \mu b) - \mu^2(f(a) + f(b))\right\| \leq \theta(\|a\|^p\|b\|^p) \tag{38}$$

$$\left\|f([x, y, z]) - ([f(x), y^2, z^2] + [x^2, f(y), z^2] + [x^2, y^2, f(z)])\right\| \leq \theta(\|x\|^p\left\|y\right\|^p\|z\|^p) \tag{39}$$

for all $\mu \in$ **T** $= \{\mu \in$ **C**: $|\mu| = 1\}$ *and for all* $a, b, x, y, z \in \mathcal{A}$ *. Then f is a ternary quadratic derivation on* $\mathcal{A}$ *.*

Proof. If we put $a = b = 0$ in (38), we have $f(0) = 0$. Moreover, letting $a = b = 0$ and $\mu = 1$ in (38), then we have $f(2a) = 4f(a)$ for all $a \in \mathcal{A}$. Similar to the proof of Corollary 4.5, we can show that f is a quadratic mapping. Now, by putting $\varphi(a,b,x,y,z) = \theta||a||^p||b||^p(||x||^p + ||y||^p + ||z||^p) + \theta||x||^p||y||^p||z||^p$ in Theorem 4.6, we will obtain the desired result.

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Authors' contributions
The study presented here was carried out in collaboration between all authors. AB suggested to write the current article. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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