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Some results on difference polynomials sharing values

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Abstract

This article is devoted to studying uniqueness of difference polynomials sharing values. The results improve those given by Liu and Yang and Heittokangas et al.

1 Introduction and main results

In this article, we shall assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna theory (e.g., see [1-3]). In addition, we will use the notations $\lambda(f)$ to denote the exponent of convergence of zero sequences of meromorphic function $f(z)$; $\sigma(f)$ to denote the order of $f(z)$. We say that meromorphic functions f and g share a finite value a *CM* when $f - a$ and $g - a$ have the same zeros with the same multiplicities. For a non-zero constant c, the forward difference $\Delta_c^{n+1}f(z) = \Delta_c^n f(z+c) - \Delta_c^n f(z)$, $\Delta_c^{n+1}f(z) = \Delta_c^n f(z+c) - \Delta_c^n f(z)$, $n = 1, 2, \ldots$. In general, we use the notation C to denote the field of complex numbers.

Currently, there has been an increasing interest in studying difference equations in the complex plane. Halburd and Korhonen [4,5] established a version of Nevanlinna theory based on difference operators. Ishizaki and Yanagihara [6] developed a version of Wiman-Valiron theory for difference equations of entire functions of small growth.

Recently, Liu and Yang [7] establish a counterpart result to the *Brück* conjecture [8] valid for transcendental entire function for which $\sigma(f) < 1$. The result is stated as follows.

Theorem A. *Let f be a transcendental entire function such that $\sigma(f) < 1$. If f and $\Delta_c^n f$ share a finite value a CM, n is a positive integer, and c is a fixed constant, then*

$$\frac{\Delta_c^n f - a}{f - a} = \tau$$

for some non-zero constant τ.

Heittokangas et al. [9], prove the following result which is a shifted analogue of *Brück* conjecture valid for meromorphic functions.

Theorem B. *Let f be a meromorphic function of order of growth $\sigma(f) < 2$, and let $c \in C$. If $f(z)$ and $f(z + c)$ share the values $a \in C$ and ∞ CM, then*

$$\frac{f(z+c) - a}{f(z) - a} = \tau$$

for some constant τ.

Here, we also study the shift analogue of *Brück* conjecture, and obtain the results as follows.

Theorem 1.1. *Let* $f(z)$ *be a non-constant entire function,* $\sigma(f) <1$ *or* $1 < \sigma(f) <2$ *and* $\lambda(f) < \sigma(f) = \sigma$. *Set* $L_1(f) = a_n(z) f(z + n) + a_{n-1}(z) f(z + n - 1) +\ldots + a_1(z) f(z + 1) + a_0(z) f(z)$, *where* $a_j(z)(0 \le j \le n)$ *are entire functions with* $a_n(z)a_0(z) \not\equiv 0$. *Suppose that if* $\sigma(f) <1$, *then* $\max\{\sigma(a_j)\} = \alpha <1$, *and if* $1 < \sigma(f) <2$, *then* $\max\{\sigma(a_j)\} = \alpha < \sigma - 1$. *If f and* $L_1(f)$ *share* 0 *CM, then*

$$L_1(f) = cf,$$

where c is a non-zero constant.

Theorem 1.2. *Let* $f(z)$ *be a non-constant entire function,* $2 < \sigma(f) <\infty$ *and* $\lambda(f) < \sigma(f)$. *Set* $L_2(f) = a_n(z) f(z + n) + a_{n-1}(z) f(z + n - 1) +\ldots + a_1(z) f(z + 1) + e^z f(z)$, $a_j(z)(1 \le j \le n)$ *are entire functions with* $\sigma(a_j) <1$ *and* $a_n(z) \not\equiv 0$. *If f and* $L_2(f)$ *share* 0 *CM, then*

$$L_2(f) = h(z)f,$$

where h(z) is an entire function of order no less than 1.

Theorem 1.3. *Let* $f(z)$ *be a non-constant entire function,* $\sigma(f) <1$ *or* $1 < \sigma(f) <2$, $\lambda(f) < \sigma(f)$. *Set* $L_3(f) = a_n(z) f(z + n) + a_{n-1}(z) f(z + n - 1) + \ldots + a_1(z) f(z + 1) + a_0(z) f(z)$, $a_j(z)$ $(0 \le j \le n)$ *are polynomials and* $a_n(z) \not\equiv 0$. *If f and* $L_3(f)$ *share a polynomial* $P(z)$ *CM, then*

$$L_3(f) - p(z) = c(f(z) - p(z)),$$

where c is a non-zero constant.

Theorem 1.4. *Let* $f(z)$ *be a non-constant entire function,* $\sigma(f) <1$ *or* $1 < \sigma(f) <2$, $\lambda(f) < \sigma(f)$. *Set* $a(z)$ *is an entire function with* $\sigma(a) <1$. *If f and* $a(z)f(z + n)$ *share a polynomial* $P(z)$ *CM, then*

$$a(z)f(z+n) - p(z) = c(f(z) - p(z)),$$

where c is a non-zero constant.

The method of the article is partly from [10].

2 Preliminary lemmas

Lemma 2.1. [11]*Let* $f(z)$ *be a meromorphic function with* $\sigma(f) = \eta <\infty$. *Then for any given* $\varepsilon >0$, *there is a set* $E_1 \subset (1, +\infty)$ *that has finite logarithmic measure, such that*

$$|f(z)| \le \exp\{r^{\eta+\varepsilon}\},$$

holds for $|z| = r \notin [0, 1] \cup E_1$, $r \to \infty$.

Applying Lemma 2.1 *to* $\frac{1}{f}$, *it is easy to see that for any given* $\varepsilon >0$, *there is a set* $E_2 \subset (1, \infty)$ *of finite logarithmic measure, such that*

$$\exp\{-r^{\eta+\epsilon}\} \le |f(z)| \le \exp\{r^{\eta+\varepsilon}\},$$

holds for $|z| = r \notin [0, 1] \cup E_2$, $r \to \infty$.

Lemma 2.2. [11]*Let*

$$Q(z) = b_n z^n + b_{n-1}z^{n-1} + \cdots + b_0,$$

where n is a positive integer and $b_n = \alpha_n e^{i\theta_n}, \alpha_n > 0, \theta_n \in [0, 2\pi)$. *For any given* $\varepsilon(0 < \varepsilon < \frac{\pi}{4n})$, *we introduce 2n open sectors*

$$S_j : -\theta_n + (2j-1)\frac{\pi}{2n} + \varepsilon < \theta < -\theta_n + (2j+1)\frac{\pi}{2n} - \varepsilon (j = 0, 1, \ldots, 2n-1).$$

Then there exists a positive number $R = R(\varepsilon)$ *such that for* $|z| = r > R$,

$$Re\{Q(z)\} > \alpha_n(1-\varepsilon)\sin(n\varepsilon)r^n$$

if $z \in S_j$ *where j is even; while*

$$Re\{Q(z)\} < -\alpha_n(1-\varepsilon)\sin(n\varepsilon)r^n$$

if $z \in S_j$ *where j is odd.*

Now for any given $\theta \in [0, 2\pi)$, *if* $\theta \neq -\frac{\theta_n}{n} + (2j-1)\frac{\pi}{2n}$, $(j = 0, 1, \ldots, 2n - 1)$, *then we take* ε *sufficiently small, there is some* S_j, $j \in \{0, 1, \ldots, 2n - 1\}$ *such that* $\theta \in S_j$.

Lemma 2.3. [12]*Let f(z) be a meromorphic function of order* $\sigma = \sigma(f) < \infty$, *and let* λ' *and* λ'' *be, respectively, the exponent of convergence of the zeros and poles of f. Then for any given* $\varepsilon > 0$, *there exists a set* $E \subset (1, \infty)$ *of* $|z| = r$ *of finite logarithmic measure, so that*

$$2\pi i n_{z,\eta} + \log\frac{f(z+\eta)}{f(z)} = \eta\frac{f'(z)}{f(z)} + O(r^{\beta+\varepsilon}),$$

or equivalently,

$$\frac{f(z+\eta)}{f(z)} = e^{\eta\frac{f'(z)}{f(z)} + O(r^{\beta+\varepsilon})},$$

holds for $r \notin E \cup [0, 1]$, *where* $n_{z,\eta}$ *is an integer depending on both z and* η, $\beta = \max\{\sigma - 2, 2\lambda - 2\}$ *if* $\lambda < 1$ *and* $\beta = \max\{\sigma - 2, \lambda - 1\}$ *if* $\lambda \geq 1$ *and* $\lambda = \max\{\lambda', \lambda''\}$.

Lemma 2.4. [2]*Let f(z) be an entire function of order* σ, *then*

$$\sigma = \limsup_{r\to\infty}\frac{\log\nu(r)}{\log r}$$

where $\nu(r)$ *be the central index of f.*

Lemma 2.5. [2,13,14]*Let f be a transcendental entire function, let* $0 < \delta < \frac{1}{4}$ *and z be such that* $|z| = r$ *and that*

$$|f(z)| > M(r, g)\nu(r, g)^{-\frac{1}{4}+\delta}$$

holds. Then there exists a set $F \subset R_+$ *of finite logarithmic measure, i.e.,* $\int_F \frac{dt}{t} < \infty$, *such that*

$$\frac{f^{(m)}(z)}{f(z)} = \left(\frac{\nu(r,f)}{z}\right)^m (1 + o(1))$$

holds for all $m \geq 0$ *and all* $r \notin F$.

Lemma 2.6. [10]*Let f(z) be a transcendental entire function,* $\sigma(f) = \sigma < \infty$, *and* $G = \{\omega_1, \omega_2, \ldots, \omega_n\}$, *and a set* $E \subset (1, \infty)$ *having logarithmic measure* $lmE < \infty$. *Then there is* *a positive number* $B(\frac{3}{4} \le B \le 1)$, *a point range* $\{z_k = r_k e^{i\omega_k}\}$*such that* $|f(z_k)| \geq BM(r_k, f)$, $\omega_k \in [0, 2\pi)$, $\lim_{k\to\infty} \omega_k = \omega_0 \in [0, 2\pi)$, $r_k \notin E \cup [0, 1]$, $r_k \to \infty$, *for any given* $\varepsilon > 0$, *we have*

$$r_k^{\sigma-\varepsilon} < \nu(r_k, f) < r_k^{\sigma+\varepsilon}.$$

3 Proof of Theorem 1.1

Under the hypothesis of Theorem 1.1, see [3], it is easy to get that

$$\frac{L_1(f)}{f} = e^{Q(z)}, \tag{3.1}$$

where $Q(z)$ is an entire function. If $\sigma(f) < 1$, we get $Q(z)$ is a constant. Then Theorem 1.1 holds. Next, we suppose that $1 < \sigma(f) < 2$ and $\lambda(f) < \sigma(f) = \sigma$. We divide this into two cases ($Q(z)$ is a constant or a polynomial with deg $Q = 1$) to prove.

Case (1): $Q(z)$ is a constant. Then Theorem 1.1 holds.

Case (2): deg $Q = 1$. By Lemma 2.3 and $\lambda(f) < \sigma(f) = \sigma$, for any given $0 < \varepsilon < \min\{\frac{\sigma-1}{2}, \frac{1-\alpha}{2}, \frac{\sigma-\lambda(f)}{2}, \frac{\sigma-1-\alpha}{2}\}$, there exists a set $E_1 \subset (1, \infty)$ of $|z| = r$ of finite logarithmic measure, so that

$$\frac{f(z+j)}{f(z)} = \exp\left\{j\frac{f'(z)}{f(z)} + o\left(r^{\sigma(f)-1-\varepsilon}\right)\right\}, j = 1, 2, \ldots, n \tag{3.2}$$

holds for $r \notin E_1 \cup 0[1]$.

By Lemma 2.5, there exists a set $E_2 \subset (0, \infty)$ of finite logarithmic measure, such that

$$\frac{f'(z)}{f(z)} = (1 + o(1))\frac{\nu(r, f)}{z}, \tag{3.3}$$

holds for $|z| = r \notin E_2 \cup [0, 1]$, where z is chosen as in Lemma 2.5.

By Lemma 2.1, for any given $\varepsilon > 0$, there exists a set $E_3 \subset (1, \infty)$ that has finite logarithmic measure such that

$$\exp\{-r^{\alpha+\varepsilon}\} \le |a_j(z)| \le \exp\{r^{\alpha+\varepsilon}\}(j = 0, 1, \ldots, n) \tag{3.4}$$

holds for $|z| = r \notin [0, 1] \cup E_3$, $r \to \infty$.

Set $E = E_1 \cup E_2 \cup E_3$ and $G = \{-\frac{\varphi_n}{n} + (2j-1)\frac{\pi}{2n} | j = 0, 1\} \cup \{\frac{\pi}{2}, \frac{3\pi}{2}\}$. By Lemma 2.6, there exist a positive number $B \in [\frac{3}{4}, 1]$, a point range $\{z_k = r_k e^{i\theta_k}\}$ such that $|f(z_k)| \geq BM(r_k, f]$, $\theta_k \in [0, 2\pi)$, $\lim_{k\to\infty} \theta_k = \theta_0 \in [0, 2\pi) \setminus G$, $r_k \notin E \cup [0, 1]$, $r_k \to \infty$, for any given $\varepsilon > 0$, as $r_k \to \infty$, we have

$$r_k^{\sigma(f)-\varepsilon} < \nu(r_k, f) < r_k^{\sigma(f)+\varepsilon} \tag{3.5}$$

By (3.1)-(3.3), we have that

$$a_n \exp\left\{n(1 + o(1))\frac{\nu(r_k, f)}{z_k}\right\} + \cdots + a_1 \exp\left\{(1 + o(1))\frac{\nu(r_k, f)}{z_k}\right\} + a_0 = e^{Q(z)}\Bigg\} \tag{3.6}$$

Let $Q(z) = \tau e^{i\theta_1} z + b_0$, $\tau > 0$, $\theta_1 \in [0, 2\pi)$. By Lemma 2.4, there are two opened angles for above ε,

$$S_j : -\theta_1 + (2j-1)\frac{\pi}{2} + \varepsilon < \theta < -\theta_1 + (2j+1)\frac{\pi}{2} + \varepsilon (j = 0, 1)$$

For the above θ_0, there are two cases: (i) $\theta_0 \in S_0$; (ii) $\theta_0 \in S_1$.

Case (i). $\theta_0 \in S_1$. Since S_j is an opened set and $\lim_{k\to\infty} \theta_k = \theta_0$, there is a $K > 0$ such that $\theta_k \in S_j$ when $k > K$. By Lemma 2.2, we have

$$Re\{Q(r_k e^{i\theta_k})\} < -\eta r_k, \tag{3.7}$$

where $\eta = \eta(1 - \varepsilon)\sin(\varepsilon) > 0$. By Lemma 2.2, if $Re z_k > \zeta r_k$ $(0 < \zeta \le 1)$. By (3.4)-(3.7), we have

$$\begin{aligned} &\exp\{r_k^{\sigma(f)-1-\varepsilon} - r_k^{\alpha+\varepsilon}\} \\ &\le \left| a_n \exp\left\{ n(1+o(1))\frac{\nu(r_k,f)}{z_k} \right\} \right| \\ &\le 3\left| a_n \exp\left\{ n(1+o(1))\frac{\nu(r_k,f)}{z_k} \right\} + \cdots + a_1 \exp\{(1+o(1))\frac{\nu(r_k,f)}{z_k}\} + a_0 \right| \\ &= 3\left| e^{Q(z)} \right| \le 3e^{-\eta r_k}, \end{aligned} \tag{3.8}$$

which contradicts that $0 < \sigma(f) - 1 - \alpha - \varepsilon$.

If $Re z_k < -\zeta r_k$ $(0 < \zeta \le 1)$, By (3.4)-(3.7), we have

$$\begin{aligned} 1 &\le \left| \frac{a_n}{a_0} \exp\left\{ n(1+o(1))\frac{\nu(r_k,f)}{z_k} \right\} + \cdots + \frac{a_1}{a_0} \exp\left\{ (1+o(1))\frac{\nu(r_k,f)}{z_k} \right\} \right| + \left| \frac{e^{Q(z)}}{a_0} \right| \\ &\le 2n \exp\left\{ -\eta r_k^{\sigma(f)-1+\varepsilon} + 2r_k^{\alpha+\varepsilon} \right\} + e^{-\eta r_k} \exp\{r_k^{\alpha+\varepsilon}\}, \end{aligned} \tag{3.9}$$

which implies that $1 < 0$, $r \to \infty$, a contradiction.

Case (ii). $\theta_0 \in S_0$. Since S_0 is an opened set and $\lim_{k\to\infty} \theta_k = \theta_0$, there is $K > 0$ such that $\theta_k \in S_j$ when $k > K$. By Lemma 2.2, we have

$$Re\{Q(r_k e^{i\theta_k})\} > \eta r_k, \tag{3.10}$$

where $\eta = \tau(1 - \varepsilon)\sin(\varepsilon) > 0$. By (3.4)-(3.6), (3.9), we obtain

$$\begin{aligned} &(n+1)\exp\{n r_k^{\sigma(f)-1+\varepsilon} + r_k^{\alpha+\varepsilon}\} \\ &\ge |a_n \exp\{n(1+o(1))\frac{\nu(r_k,f)}{z_k}\} + \cdots + a_1 \exp\{(1+o(1))\frac{\nu(r_k,f)}{z_k}\} + a_0| \\ &= |e^{Q(z)}| \ge e^{\eta r_k}. \end{aligned} \tag{3.11}$$

From (3.11), we get that $\sigma(f) \ge 2$, a contradiction. Theorem 1.1 is thus proved.

4 Proof of Theorem 1.2

Under the hypothesis of Theorem 1.2, see [3], it is easy to get that

$$\frac{L_2(f)}{f} = e^{Q(z)}, \tag{4.1}$$

where $Q(z)$ is an entire function. For $Q(z)$, we discuss the following two cases.

Case (1): $Q(z)$ is a polynomial with deg $Q = n \ge 1$. Then Theorem 1.2 is proved.

Case (2): $Q(z)$ is a constant. Using the similar reasoning as in the proof of Theorem 1.1, we get that

$$a_n \exp\left\{n(1+o(1))\frac{\nu(r_k,f)}{z_k}\right\} + \cdots + a_1 \exp\left\{(1+o(1))\frac{\nu(r_k,f)}{z_k}\right\} + a = -e^{z_k}, \quad (4.2)$$

where a is some non-zero constant.

If $Rez_k < -\eta r_k$ ($\eta \in (0, 1]$), By (3.4), (3.5), (4.2), we have

$$\begin{aligned} |a| &\le \left|a_n \exp\left\{n(1+o(1))\frac{\nu(r_k,f)}{z_k}\right\} + \cdots + a_1 \exp\left\{(1+o(1))\frac{\nu(r_k,f)}{z_k}\right\}\right| + |\exp\{z_k\}| \\ &\le \exp\{-\eta r_k\} + n\exp\{-\eta r_k^{\sigma(f)-1+\varepsilon} + 2r_k^{\alpha+\varepsilon}\}, \end{aligned} \quad (4.3)$$

which is impossible.

If $Rez_k > \eta r_k$ ($\eta \in (0, 1]$), By (3.4), (3.5) and (4.2), we get

$$\begin{aligned} &\exp\left\{\eta r_k^{\sigma(f)-1-\varepsilon}\right\} < \exp\left\{n\frac{\nu(r_k,f)}{z_k} - r_k^{\alpha+\varepsilon}\right\} \\ &\le 2\left|a_n \exp\left\{n(1+o(1))\frac{\nu(r_k,f)}{z_k}\right\} + \cdots + a_1 \exp\left\{(1+o(1))\frac{\nu(r_k,f)}{z_k}\right\} + a\right| \\ &= 2|-\exp\{z_k\}| \le 2\exp\{r_k\}, \end{aligned} \quad (4.4)$$

which contradicts that $\sigma(f) > 2$. This completes the proof of Theorem 1.2.

5 Proof of Theorem 1.3

Since f and $L_3(f)$ share P CM, we get

$$\frac{L_3(f)}{f} = e^{Q(z)}, \quad (5.1)$$

where $Q(z)$ is an entire function. If $\sigma(f) < 1$, we get $Q(z)$ is a constant. Then Theorem 1.3 holds. Next, we suppose that $1 < \sigma(f) < 2$ and $\lambda(f) < \sigma(f) = \sigma$. Set $F(z) = f(z) - P(z)$, then $\sigma(F) = \sigma(f)$. Substituting $F(z) = f(z) - p(z)$ into (5.1), we obtain

$$\frac{a_n(z)F(z+n) + a_{n-1}(z)F(z+n-1) + \cdots + a_1(z)F(z+1)}{F(z)} + a_0(z) + \frac{b(z)}{F(z)} = e^{Q(z)}, \quad (5.2)$$

where $b(z) = a_n(z)P(z+n) + \ldots + a_1(z)P(z+1) + a_0(z)p(z)$ is a polynomial. We discuss the following two cases.

Case 1. $Q(z)$ is a complex constant. Then Theorem 1.3 holds.

Case 2. $Q(z)$ is a polynomial with deg $Q = 1$. By Lemma 2.3 and $\lambda(f) < \sigma(f) = \sigma$, for any given $0 < \varepsilon < \min\{\frac{\sigma-1}{2}, \frac{1-\alpha}{2}, \frac{\sigma-\lambda(f)}{2}, \frac{\sigma-1-\alpha}{2}\}$, there exists a set $E_1 \subset (1, \infty)$ of $|z| = r$ of finite logarithmic measure, so that

$$\frac{f(z+j)}{f(z)} = \exp\{j\frac{f'(z)}{f(z)} + o(r^{\sigma(f)-1-\varepsilon})\}, j = 1, 2, \ldots, n \quad (5.3)$$

holds for $r \notin E_1 \cup [0, 1]$.

By Lemma 2.5, there exists a set $E_2 \subset (0, \infty)$ of finite logarithmic measure, such that

$$\frac{f'(z)}{f(z)} = (1+o(1))\frac{\nu(r,f)}{z}, \quad (5.4)$$

holds for $|z| = r \notin E_2 \cup [0, 1]$, where z is chosen as in Lemma 2.5.

Set $E = E_1 \cup E_2$ and $G = \{-\frac{\varphi_n}{n} + (2j-1)\frac{\pi}{2n} | j = 0, 1\} \cup \{\frac{\pi}{2}, \frac{3\pi}{2}\}$. By Lemma 2.6, there exist a positive number $B \in [\frac{3}{4}, 1]$, a point range $\{z_k = r_k e^{i\theta_k}\}$ such that $|f(z_k)| \geq BM(r_k, f)$, $\theta_k \in [0, 2\pi)$, $\lim_{k\to\infty}\theta_k = \theta_0 \in [0, 2\pi) \setminus G$, $r_k \notin E \cup 0[1]$, $r_k \to \infty$, for any given $\varepsilon > 0$, as $r_k \to \infty$, we have

$$r_k^{\sigma(f)-\varepsilon} < \nu(r_k, f) < r_k^{\sigma(f)+\varepsilon}. \tag{5.5}$$

Since F is a transcendental entire function and $|f(z_k)| \geq BM(r_k, f)$, we obtain

$$\frac{b(z_k)}{F(z_k)} \to 0, (r_k \to \infty). \tag{5.6}$$

By (5.2)-(5.6), we have that

$$a_n \exp\left\{n(1+o(1))\frac{\nu(r_k, f)}{z_k}\right\} + \cdots + a_1 \exp\left\{(1+o(1))\frac{\nu(r_k, f)}{z_k}\right\} + a_0 + o(1) = e^{Q(z)}. \tag{5.7}$$

Using similar proof as in proof of Theorem 1.1, we can get a contradiction. Hence, Theorem 1.3 holds.

6 Proof of Theorem 1.4

Using similar proof as in proof of Theorem 1.1, we can get Theorem 1.4 holds.

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Author's contributions

YL completed the main part of this article, YL, XQ and HX corrected the main theorems. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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