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# Local existence and uniqueness of solutions of a degenerate parabolic system

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**Abstract**

This article deals with a degenerate parabolic system coupled with general nonlinear terms. Using the method of regularization and monotone iteration technique, we obtain the local existence of solutions to the Dirichlet initial boundary value problem. We also establish the uniqueness of the solution if the reaction terms satisfy the Lipschitz condition.

**Keywords:** Existence, Uniqueness, Degenerate, Monotone iteration

## 1 Introduction

In this article, we consider the following degenerate parabolic system

$$\frac{\partial u_i}{\partial t} = \Delta u_i^{m_i} + f_i(x, t, u_1, u_2), \qquad (x, t) \in Q_T, \tag{1.1}$$

$$u_i(x, t) = 0, \qquad (x, t) \in \partial\Omega \times (0, T), \tag{1.2}$$

$$u_i(x, 0) = u_{i0}(x), \qquad x \in \Omega, \tag{1.3}$$

where $m_i > 1$, $i = 1, 2$, $Q_T = \Omega \times (0, T)$, $\Omega$ is a bounded domain in $\mathbb{R}^N$ with smooth boundary, $f_i(x, t, u_1, u_2) \in C(\bar{\Omega} \times [0, T] \times \mathbb{R}^2)$ and $0 \le u_{i0} \in L^\infty(\Omega) \cap H_0^1(\Omega)$.

The coupled equations in (1.1) provide a class of quasilinear degenerate parabolic systems. Problems of this form arise in a number of areas of science. For instance, in models for gas or fluid flow in porous media [1-3] and for the spread of certain biological populations [4-6]. When $m_1 = m_2 = 1$, the system (1.1) models the Newtonian fluids, which is couples with Laplace equations. For various initial boundary problems to this kind system, many articles have been devoted to the existence of the solutions and blowup properties of the solutions [7-9].

In recent years, degenerate parabolic systems are of particular interests since they can take into account nonlinear diffusion occurring in the phenomena appearing in the models, and have been extensively studied by many researchers (see e.g., [3,10-13] and the references therein). The degeneracy and coupled with nonlinear terms of this systems cause great difficulties to study them. In this article, we will establish the local existence and uniqueness results under some special cases for the nonlinear reaction terms. First, by making use the method of regularization and monotone iteration technique, we obtain a sequence of approximation solutions. Then a weak solution is obtained as the limit of the solutions of such problems. Executing this program one encounters two difficulties. The first is proving that the approximating problems which are nondegenerate admits a solution, the second difficulty is to establish uniform estimates for these solutions. At last, we establish the uniqueness results when the reaction terms satisfy the Lipschitz condition.

Since the system (1.1) is degenerate whenever $u_1$, $u_2$ vanish, there is no classical solution in general. So we focus our main efforts on the discussion of weak solutions in the sense of the following.

**Definition 1.1**. A nonnegative vector-valued function $u = (u_1, u_2)$ is called to be a weak solution of the problem (1.1)-(1.3) provided that $u_i^{m_i} \in L^2(0,T;H_0^1(\Omega)) \cap L^\infty(Q_T)$, $\partial u_i^{m_i}/\partial t \in L^2(Q_T)$, and

$$\iint\limits_{Q_T} -u_i\frac{\partial \varphi_i}{\partial t} + \nabla u_i^{m_i}\nabla\varphi_i \mathrm{d}x\mathrm{d}t - \int\limits_{\Omega} u_{i0}(x)\varphi_i(x,0)\,\mathrm{d}x = \iint\limits_{Q_T} f_i(x,t,u_1,u_2)\varphi_i \mathrm{d}x\mathrm{d}t,$$

for any test function $\varphi_i \in C^2(\bar{Q}_T)$ with $\phi_i|_{\partial\Omega\times(0,\ T)} = 0$, $\phi_i(x,\ T) = 0$, $i = 1, 2$. The above equation also implies

$$\int\limits_0^t\int\limits_{\Omega} -u_i\frac{\partial \varphi_i}{\partial t} + \nabla u_i^{m_i}\nabla\varphi_i \mathrm{d}x\mathrm{d}t + \int\limits_{\Omega} u_i(x,t)\varphi_i(x,t)\,\mathrm{d}x - \int\limits_{\Omega} u_{i0}(x)\varphi_i(x,0)\mathrm{d}x$$
$$= \int\limits_0^t\int\limits_{\Omega} f_i(x,t,u_1,u_2)\varphi_i \mathrm{d}x\mathrm{d}t, \quad \text{a.e.} \quad t \in (0,T).$$

**Definition 1.2**. A function $f = f(u_1, u_2)$ is said to be quasimonotone nondecreasing (respectively, nonincreasing) if for fixed $u_1$ (or $u_2$), $f$ is nondecreasing (respectively, nonincreasing) in $u_2$ (or $u_1$).

Throughout this article, we assume $f_i(x, t, u_1, u_2)(i = 1, 2)$ satisfies the following condition:

(A0) $f_i(x, t, u_1, u_2)(i = 1, 2)$ is quasimonotonically nondecreasing for $u_1$, $u_2$.
(A1) There exists a nonnegative function $g(u) \in C^1(\mathbb{R})$ such that

$$\left|f_i(x,t,u_1,u_2)\right| \le \min\left\{g(u_1), g(u_2)\right\} \quad \text{for all } (x,t) \in Q_T,\ u_1,u_2 \in \mathbb{R}.$$

## 2 Existence and uniqueness

In this section, we show the local existence and uniqueness of weak solutions of (1.1)-(1.3). First, we show the local existence results.

**Theorem 2.1**. *Assume (A0), (A1) hold, then there exists a constant* $T_1 \in [0, T]$ *such that (1.1)-(1.3) admits a solution* $(u_1, u_2)$ *in* $Q_{T_1}$.

*Proof.* Due to the degeneracy of the system (1.1), we consider the following regularized problem

$$\frac{\partial u_i}{\partial t} = \mathrm{div}((m_i u_i^{m_1-1} + \varepsilon)\nabla u_i) + f_{i\varepsilon}(x,t,u_1,u_2), \qquad (x,t) \in Q_T, \tag{2.1}$$

$$u_i(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0,T), \tag{2.2}$$

$$u_i(x,0) = u_{i0\varepsilon}(x), \qquad x \in \Omega, \tag{2.3}$$

where $f_{i\varepsilon} \in C^1(\bar{\Omega} \times [0,T] \times \mathbb{R}^2)$; $f_{i\varepsilon} \to f_i$ uniformly on bounded subsets of $\bar{\Omega} \times [0,T] \times \mathbb{R}^2$, and $f_{i\varepsilon}$ satisfies the assumptions (A0), (A1), $u_{i0\varepsilon}(x) \in C_0^\infty(\Omega)$, $u_{i0\varepsilon}^{m_i} \to u_{i0}^{m_i}$, $u_{i0\varepsilon}^{m_i} \to u_{i0}^{m_i}$, strongly in $W_0^{1,2}(\Omega)$ as $\varepsilon \to 0$.

Now we will prove that the regularized problem (2.1)-(2.3) admits a classical solution. Construct a sequence $\{(u_{1\varepsilon}^{(k)}, u_{2\varepsilon}^{(k)})\}_{k=1}^\infty$ from the following iteration process

$$\frac{\partial u_i^{(k)}}{\partial t} - \operatorname{div}((m_i(u_i^{(k)})^{m_i-1} + \varepsilon)\nabla u_i^{(k)}) = f_{i\varepsilon}(x,t,u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}), \qquad (x,t) \in Q_T, \tag{2.4}$$

$$u_{i\varepsilon}^{(k)}(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0,T), \tag{2.5}$$

$$u_{i0\varepsilon}^{(k)}(x,0) = u_{i0\varepsilon}(x), \qquad x \in \Omega, \tag{2.6}$$

with a suitable initial value $(u_{1\varepsilon}^{(0)}, u_{2\varepsilon}^{(0)})$, $i = 1, 2$. By classical results in [14], the problem (2.4)-(2.6) admits a classical solution $(u_{1\varepsilon}^{(k)}, u_{2\varepsilon}^{(k)})$ for fixed $k$ and $\varepsilon$ when $(u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)})$ is smooth. The choice of the initial iteration value which will be obtained by the quasimonotone property of $(f_1, f_2)$ would be crucial to ensure that the above sequence converges to a solution of the generalized problem.

Let $(\underline{u}_{1\varepsilon}^{(0)}(x,t), \underline{u}_{2\varepsilon}^{(0)}(x,t)) = (\inf_{\Omega}\{u_{10\varepsilon}(x)\}, \inf_{\Omega}\{u_{20\varepsilon}(x)\})$, and $(\underline{u}_{1\varepsilon}^{(1)}, \underline{u}_{2\varepsilon}^{(1)})$ be a classical solution of the following problem

$$\begin{aligned}
&\frac{\partial \underline{u}_i^{(1)}}{\partial t} - \operatorname{div}((m_i(\underline{u}_i^{(1)})^{m_i-1} + \varepsilon)\nabla \underline{u}_i^{(1)}) = f_{i\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(0)}, \underline{u}_{2\varepsilon}^{(0)}), && (x,t) \in Q_T,\\
&\underline{u}_{i\varepsilon}^{(1)}(x,t) = 0, && (x,t) \in \partial\Omega \times (0,\ T),\\
&\underline{u}_{i0\varepsilon}^{(1)}(x,0) = u_{i0\varepsilon}(x) \ge \underline{u}_{i\varepsilon}^{(0)}(x), && x \in \Omega.
\end{aligned}$$

By the comparison theorem [15], we have

$$\underline{u}_{1\varepsilon}^{(1)} \ge \underline{u}_{1\varepsilon}^{(0)}, \underline{u}_{2\varepsilon}^{(1)} \ge \underline{u}_{2\varepsilon}^{(0)}.$$

Then the quasimonotone nondecreasing property of $f_{i\varepsilon}$ shows that

$$\begin{aligned}
&f_{1\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(1)}, \underline{u}_{2\varepsilon}^{(1)}) \ge f_{1\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(0)}, \underline{u}_{2\varepsilon}^{(1)}) \ge f_{1\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(0)}, \underline{u}_{2\varepsilon}^{(0)}),\\
&f_{2\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(1)}, \underline{u}_{2\varepsilon}^{(1)}) \ge f_{2\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(1)}, \underline{u}_{2\varepsilon}^{(0)}) \ge f_{2\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(0)}, \underline{u}_{2\varepsilon}^{(0)}).
\end{aligned}$$

Then we can also obtain a classical solution $(\underline{u}_{1\varepsilon}^{(2)}, \underline{u}_{2\varepsilon}^{(2)})$ from (2.4)-(2.6) when $k = 2$, and $\underline{u}_{1\varepsilon}^{(2)} \ge \underline{u}_{1\varepsilon}^{(1)}, \underline{u}_{2\varepsilon}^{(2)} \ge \underline{u}_{2\varepsilon}^{(1)}$. So we can obtain a nondecreasing sequence

$$\underline{u}_{i\varepsilon}^{(0)} \le \underline{u}_{i\varepsilon}^{(1)} \le \underline{u}_{i\varepsilon}^{(2)} \le \cdots \le \underline{u}_{i\varepsilon}^{(k)} \le \cdots.$$

With the similar method, by setting $(\bar{u}_{1\varepsilon}^{(0)}(x,t), \bar{u}_{2\varepsilon}^{(0)}(x,t)) = (\sup_{Q_T}\{u_{10\varepsilon}(x)\}, \sup_{Q_T}\{u_{20\varepsilon}(x)\})$, we obtain a classical solution $(\bar{u}_{1\varepsilon}^{(1)}, \bar{u}_{2\varepsilon}^{(1)})$ of the following problem

$$\begin{aligned}
&\frac{\partial \bar{u}_i^{(1)}}{\partial t} - \operatorname{div}((m_i(\bar{u}_i^{(1)})^{m_i-1} + \varepsilon)\nabla \bar{u}_i^{(1)}) = f_{i\varepsilon}(x,t,\bar{u}_{1\varepsilon}^{(0)},\bar{u}_{2\varepsilon}^{(0)}), && (x,t)\in Q_T,\\
&\bar{u}_{i\varepsilon}^{(1)}(x,t) = 0, && (x,t)\in \partial\Omega\times(0,T),\\
&\bar{u}_{i0\varepsilon}^{(1)}(x,0) = u_{i0\varepsilon}(x) \le \bar{u}_{i\varepsilon}^{(0)}(x), && x\in\Omega,
\end{aligned}$$

and

$$\bar{u}_{1\varepsilon}^{(1)} \le \bar{u}_{1\varepsilon}^{(0)}, \bar{u}_{2\varepsilon}^{(1)} \le \bar{u}_{2\varepsilon}^{(0)}.$$

And the quasimonotone nondecreasing property of $f_{i\varepsilon}$ also shows that

$$\bar{u}_{i\varepsilon}^{(0)} \ge \bar{u}_{i\varepsilon}^{(1)} \ge \bar{u}_{i\varepsilon}^{(2)} \ge \cdots \ge \bar{u}_{i\varepsilon}^{(k)} \ge \cdots.$$

Now we show

$$\underline{u}_{i\varepsilon}^{(0)} \le \underline{u}_{i\varepsilon}^{(1)} \le \underline{u}_{i\varepsilon}^{(2)} \le \cdots \le \underline{u}_{i\varepsilon}^{(k)} \le \underline{u}_{i\varepsilon}^{(k+1)} \le \bar{u}_{i\varepsilon}^{(k+1)} \le \bar{u}_{i\varepsilon}^{(k)} \le \cdots \le \bar{u}_{i\varepsilon}^{(2)} \le \bar{u}_{i\varepsilon}^{(1)} \le \bar{u}_{i\varepsilon}^{(0)}. \tag{2.7}$$

It is obvious that $\underline{u}_{i\varepsilon}^{(0)} \le \bar{u}_{i\varepsilon}^{(0)}$. Assume that $\underline{u}_{i\varepsilon}^{(k)} \le \underline{u}_{i\varepsilon}^{(k)}$, we just need to prove that $\underline{u}_{i\varepsilon}^{(k+1)} \le \underline{u}_{i\varepsilon}^{(k+1)}$. Since $f_{i\varepsilon}$ is quasimonotone nondecreasing, we have

$$\begin{aligned}
&f_{1\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(k)},\underline{u}_{2\varepsilon}^{(k)}) \le f_{1\varepsilon}(x,t,\bar{u}_{1\varepsilon}^{(k)},\underline{u}_{2\varepsilon}^{(k)}) \le f_{1\varepsilon}(x,t,\bar{u}_{1\varepsilon}^{(k)},\bar{u}_{2\varepsilon}^{(k)}),\\
&f_{2\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(k)},\underline{u}_{2\varepsilon}^{(k)}) \le f_{2\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(k)},\bar{u}_{2\varepsilon}^{(k)}) \le f_{2\varepsilon}(x,t,\bar{u}_{1\varepsilon}^{(k)},\bar{u}_{2\varepsilon}^{(k)}).
\end{aligned}$$

From the iteration equations

$$\begin{aligned}
&\frac{\partial \underline{u}_i^{(k+1)}}{\partial t} - \operatorname{div}((m_i(\underline{u}_i^{(k+1)})^{m_1-1} + \varepsilon)\nabla \underline{u}_i^{(k+1)}) = f_{i\varepsilon}(x,t,\underline{u}_{1\varepsilon}^{(k)},\underline{u}_{2\varepsilon}^{(k)}), && (x,t)\in Q_T,\\
&\frac{\partial \bar{u}_i^{(k+1)}}{\partial t} - \operatorname{div}((m_i(\bar{u}_i^{(k+1)})^{m_1-1} + \varepsilon)\nabla \bar{u}_i^{(k+1)}) = f_{i\varepsilon}(x,t,\bar{u}_{1\varepsilon}^{(k)},\bar{u}_{2\varepsilon}^{(k)}), && (x,t)\in Q_T,\\
&\underline{u}_{i\varepsilon}^{(k+1)}(x,t) = 0 = \bar{u}_{i\varepsilon}^{(k+1)}(x,t), && (x,t)\in\partial\Omega\times(0,T),\\
&\underline{u}_{i0\varepsilon}^{(k+1)}(x,0) = u_{i0\varepsilon}(x) = \bar{u}_{i\varepsilon}^{(k+1)}(x,0), && x\in\Omega,
\end{aligned}$$

and the comparison theorem, we have $\underline{u}_{i\varepsilon}^{(k+1)} \le \underline{u}_{i\varepsilon}^{(k+1)}$. Further we can obtain (2.7).

Let $(u_{1\varepsilon}^{(k)}, u_{2\varepsilon}^{(k)}) = (\underline{u}_{1\varepsilon}^{(k)}, \underline{u}_{2\varepsilon}^{(k)})$, then $\{(u_{1\varepsilon}^{(k)}, u_{2\varepsilon}^{(k)})\}_{k=1}^{\infty}$ is a nondecreasing bounded sequence. Then there exist functions $u_{i\varepsilon}$ ($i$ = 1, 2) such that

$$\lim_{k\to\infty} u_{i\varepsilon}^{(k)} = u_{i\varepsilon}, \quad \text{a.e.} \quad \text{in} \quad Q_T. \tag{2.8}$$

The continuity of function $f_{i\varepsilon}$ ($i$ = 1, 2) also shows that

$$\lim_{k\to\infty} f_{i\varepsilon}(x,t,u_{1\varepsilon}^{(k)},u_{2\varepsilon}^{(k)}) = f_{i\varepsilon}(x,t,u_{1\varepsilon},u_{2\varepsilon}), \quad \text{a.e.} \quad \text{in} \quad Q_T. \tag{2.9}$$

Therefore, we claim that there exist $T_1 \in (0, T]$ and a positive constant $M$ (independent of $\varepsilon$ and $k$), such that for all $k$,

$$|u_{i\varepsilon}^{(k)}|_{L^\infty(Q_{T_1})} \le M, \quad i = 1, 2. \tag{2.10}$$

Let $v_i^{\pm}(t)$ be the solutions of the ordinary differential equations

$$\frac{\mathrm{d}v_i^{\pm}(t)}{\mathrm{d}t} = \pm g(v_i), \quad v_i^{\pm}(0) = \pm|u_{i0}|_{L^\infty(\Omega)}, \quad i = 1, 2.$$

The results in [16] show that there exists $T_i^* \in (0, T)$, $i$ = 1, 2, such that $v_i^{\pm}(t)$ exists on $[0, T_i^*]$ with $T_i^*$ depends only on $|u_{i0}|_{L^\infty(\Omega)}$. By the comparison theorem, we have

$$\left|u_{i\varepsilon}^{(k)}(x,t)\right| \le \max\{v_i^+(t), -v_i^-(t)\}, \quad i = 1, 2.$$

Then by setting $T_1 = \frac{1}{2}\min\{T_1^*, T_2^*\}$ and $M = \max\{v_i^+(T_1), -v_i^-(T_1)\}$, we obtain (2.10).

Now we show that $(u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)} \rightharpoonup u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon}$ in $L^2(0, T_1; H_0^1(\Omega))$, $(u_{i\varepsilon}^{(k)})^{m_i}_t \rightharpoonup (u_{i\varepsilon}^{m_i})_t$, $u_{i\varepsilon t}^{(k)} \rightharpoonup u_{i\varepsilon t}$ in $L^2(Q_{T_1})$ as $k \to \infty$, where $\rightharpoonup$ stands for weak convergence.

Multiplying (2.4) by $(u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}$ and integrating over $Q_{T_1} = \Omega \times (0, T_1)$, we have

$$\iint_{Q_{T_1}} \left[\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right]\frac{\partial u_{i\varepsilon}^{(k)}}{\partial t}\mathrm{d}t\,\mathrm{d}x + \iint_{Q_{T_1}} \left|\nabla\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon\nabla u_{i\varepsilon}^{(k)}\right|^2 \mathrm{d}x\mathrm{d}t$$
$$= \iint_{Q_{T_1}} f_{i\varepsilon}(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)})\left[\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right]\mathrm{d}x\mathrm{d}t,$$

that is

$$\iint_{Q_{T_1}} \left|\nabla\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon\nabla u_{i\varepsilon}^{(k)}\right|^2 \mathrm{d}x\mathrm{d}t$$
$$= \iint_{Q_{T_1}} f_{i\varepsilon}(x,\ t,\ u_{1\varepsilon}^{(k-1)},\ u_{2\varepsilon}^{(k-1)})\left[\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right]\ \mathrm{d}x\mathrm{d}t$$
$$-\frac{1}{m_i+1}\int_\Omega \left[\left(u_{i\varepsilon}^{(k)}(x, T_1)\right)^{m_i+1} - \left(u_{i\varepsilon}^{(k)}(x, 0)\right)^{m_i+1}\right]\mathrm{d}x$$
$$-\frac{1}{2}\int_\Omega \left[\left(u_{i\varepsilon}^{(k)}(x, T_1)\right)^2 - \left(u_{i\varepsilon}^{(k)}(x, 0)\right)^2\right]\mathrm{d}x.$$

Then by (2.10) and the property of $f_{i\varepsilon}$ , we have

$$\iint_{Q_{T_1}} \left|\nabla\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon\nabla u_{i\varepsilon}^{(k)}\right|^2 \mathrm{d}x\mathrm{d}t \le C, \tag{2.11}$$

where C is a constant independent of $k$, $\varepsilon$.

Multiplying (2.4) by $\frac{\partial}{\partial t}\left[\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right]$ and integrating over $Q_{T_1}$, by Young's inequality we have

$$\begin{aligned}
&\iint\limits_{Q_{T_1}} \frac{\partial u_{i\varepsilon}^{(k)}}{\partial t} \frac{\partial \left(u_{i\varepsilon}^{(k)}\right)^{m_i}}{\partial t} \mathrm{d}x\mathrm{d}t + \varepsilon \iint\limits_{Q_{T_1}} \left| \frac{\partial u_{i\varepsilon}^{(k)}}{\partial t} \right|^2 \mathrm{d}x\mathrm{d}t \\
&= -\frac{1}{2} \int_0^{T_1} \frac{\partial}{\partial t} \int_\Omega \left| \nabla (u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon \nabla u_{i\varepsilon}^{(k)} \right|^2 \mathrm{d}x\mathrm{d}t + \varepsilon \iint\limits_{Q_{T_1}} f_{i\varepsilon}(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \frac{\partial u_{i\varepsilon}^{(k)}}{\partial t} \mathrm{d}x\mathrm{d}t \\
&\quad + \iint\limits_{Q_{T_1}} f_{i\varepsilon}(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \frac{\partial \left(u_{i\varepsilon}^{(k)}\right)^{m_i}}{\partial t} \mathrm{d}x\mathrm{d}t \\
&= -\frac{1}{2} \int_0^{T_1} \frac{\partial}{\partial t} \int_\Omega \left| \nabla (u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon \nabla u_{i\varepsilon}^{(k)} \right|^2 \mathrm{d}x\mathrm{d}t + \varepsilon \iint\limits_{Q_{T_1}} f_{i\varepsilon}(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \frac{\partial u_{i\varepsilon}^{(k)}}{\partial t} \mathrm{d}x\mathrm{d}t \\
&\quad + \frac{2m_i}{m_i + 1} \iint\limits_{Q_{T_1}} f_{i\varepsilon}(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \left(u_{i\varepsilon}^{(k)}\right)^{(m_i-1)/2} \frac{\partial \left(u_{i\varepsilon}^{(k)}\right)^{(m_i+1)/2}}{\partial t} \mathrm{d}x\mathrm{d}t \\
&\le \frac{1}{2} \int_\Omega \left| \nabla \left(u_{i0\varepsilon}^{(k)}\right)^{m_i} + \varepsilon \nabla u_{i0\varepsilon}^{(k)} \right|^2 \mathrm{d}x - \frac{1}{2} \int_\Omega \left| \nabla \left(u_{i\varepsilon}^{(k)}(x, T_1)\right)^{m_i} + \varepsilon \nabla u_{i\varepsilon}^{(k)}(x, T_1) \right|^2 \mathrm{d}x \\
&\quad + \frac{1}{4} \iint\limits_{Q_{T_1}} f_{i\varepsilon}^2(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \,\mathrm{d}x\mathrm{d}t + \frac{m_i}{2} \iint\limits_{Q_{T_1}} f_{i\varepsilon}^2(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \left| u_{i\varepsilon}^{(k)} \right|^{m_i-1} \mathrm{d}x\mathrm{d}t \\
&\quad + \frac{2m_i}{(m_i + 1)^2} \iint\limits_{Q_{T_1}} \left| \frac{\partial}{\partial t} \left(u_{i\varepsilon}^{(k)}\right)^{(m_i+1)/2} \right|^2 \mathrm{d}x\mathrm{d}t + \varepsilon^2 \iint\limits_{Q_{T_1}} \left| \frac{\partial u_{i\varepsilon}^{(k)}}{\partial t} \right|^2 \mathrm{d}x\mathrm{d}t.
\end{aligned}$$

Noticing that the first term of the left side of the above inequality can be rewritten as

$$\iint\limits_{Q_{T_1}} \frac{\partial u_{i\varepsilon}^{(k)}}{\partial t} \frac{\partial (u_{i\varepsilon}^{(k)})^{m_i}}{\partial t} \mathrm{d}x\mathrm{d}t = \frac{4m_i}{(m_i + 1)^2} \iint\limits_{Q_{T_1}} \left| \frac{\partial}{\partial t} \left(u_{i\varepsilon}^{(k)}\right)^{(m_i+1)/2} \right|^2 \mathrm{d}x\mathrm{d}t.$$

Then we have

$$\begin{aligned}
&\frac{2m_i}{(m_i + 1)^2} \iint\limits_{Q_{T_1}} \left| \frac{\partial}{\partial t} \left(u_{i\varepsilon}^{(k)}\right)^{(m_i+1)/2} \right|^2 \mathrm{d}x\mathrm{d}t + (\varepsilon - \varepsilon^2) \iint\limits_{Q_{T_1}} \left| \frac{\partial u_{i\varepsilon}^{(k)}}{\partial t} \right|^2 \mathrm{d}x\mathrm{d}t \\
&\le \frac{1}{2} \int_\Omega \left| \nabla \left(u_{i0\varepsilon}^{(k)}\right)^{m_i} + \varepsilon \nabla u_{i0\varepsilon}^{(k)} \right|^2 \mathrm{d}x + \frac{1}{4} \iint\limits_{Q_{T_1}} f_{i\varepsilon}^2(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \,\mathrm{d}x\mathrm{d}t \\
&\quad + \frac{m_i}{2} \iint\limits_{Q_{T_1}} f_{i\varepsilon}^2(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \left| u_{i\varepsilon}^{(k)} \right|^{m_i-1} \mathrm{d}x\mathrm{d}t.
\end{aligned}$$

Therefore

$$\iint\limits_{Q_{T_1}} \left| \frac{\partial}{\partial t} \left(u_{i\varepsilon}^{(k)}\right)^{(m_i+1)/2} \right|^2 \mathrm{d}x\mathrm{d}t \le C.$$

Furthermore, we can obtain

$$\iint\limits_{Q_{T_1}} \left| \frac{\partial}{\partial t}\left(u_{i\varepsilon}^{(k)}\right)^{m_i} \right|^2 \mathrm{d}x\mathrm{d}t = \frac{4m_i}{(m_i+1)^2} \iint\limits_{Q_{T_1}} (u_{i\varepsilon}^{(k)})^{m_i-1} \left| \frac{\partial}{\partial t}\left(u_{i\varepsilon}^{(k)}\right)^{(m_i+1)/2} \right|^2 \le C,$$
$$\iint\limits_{Q_{T_1}} \left| \frac{\partial}{\partial t} u_{i\varepsilon}^{(k)} \right|^2 \mathrm{d}x\mathrm{d}t \le C. \tag{2.12}$$

Following (2.8), (2.9), (2.12) and the uniqueness of the weak limits, it is easy to know that, as $k \to \infty$,

$$u_{i\varepsilon}^{(k)} \to u_{i\varepsilon}, f_{i\varepsilon}(x,t,u_{1\varepsilon}^{(k)},u_{2\varepsilon}^{(k)}) \to f_{i\varepsilon}(x,t,u_{1\varepsilon},u_{2\varepsilon}), \quad \text{a.e.} \quad \text{in} \quad Q_{T_1}, \tag{2.13}$$

$$\frac{\partial u_{i\varepsilon}^{(k)}}{\partial t} \rightharpoonup \frac{\partial u_{i\varepsilon}}{\partial t}, \frac{\partial (u_{i\varepsilon}^{(k)})^{m_i}}{\partial t} \rightharpoonup \frac{\partial u_{i\varepsilon}^{m_i}}{\partial t}, \quad \text{in} \quad L^2(Q_{T_1}), \tag{2.14}$$

where $\rightharpoonup$ stands for weak convergence, $i = 1, 2$. Furthermore (2.11) implies that there exists $\nu_s \in L^2(Q_{T_1})$, $s = 1, ..., n$, such that

$$\frac{\partial \left( (u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)} \right)}{\partial x_s} \rightharpoonup \nu_s \quad \text{a.e.} \quad \text{in} \quad L^2(Q_{T_1}).$$

Hence,

$$\iint\limits_{Q_{T_1}} -u_{i\varepsilon}\frac{\partial \varphi_i}{\partial t} + \nu \nabla \varphi_i \mathrm{d}x\mathrm{d}t - \int\limits_{\Omega} u_{i0\varepsilon}(x)\varphi_i(x,0)\,\mathrm{d}x = \iint\limits_{Q_{T_1}} f_i(x,t,u_{1\varepsilon},u_{2\varepsilon})\varphi_i\,\mathrm{d}x\mathrm{d}t, \tag{2.15}$$

where $\nu = (\nu_1, ..., \nu_n)$, $\varphi_i \in C^2(\bar{Q}_{T_1})$ with $\varphi_i|_{\partial\Omega\times(0,T_1)} = 0$, $\phi_i(x, T_1) = 0$, $i = 1, 2$.

Now for any $\phi_i$ given as before, we show

$$\iint\limits_{Q_{T_1}} \nabla\left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right)\nabla\varphi_i\,\mathrm{d}x\mathrm{d}t = \iint\limits_{Q_{T_1}} \nu\nabla\varphi_i \mathrm{d}x\mathrm{d}t, \quad \text{as } k \to \infty. \tag{2.16}$$

For any $w \in L^2(0, T_1; H_0^1(\Omega))$, $\zeta \in C^1(\bar{Q}_{T_1})$, $0 \le \zeta \le 1$, $\zeta|_{\partial\Omega\times(0,T_1)} = 0$ with $\zeta(x, T_1) = 0$, multiplying (2.4) by $\zeta\left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right)$ and integrating over $Q_{T_1}$, we have

$$\begin{aligned}
&\iint\limits_{Q_{T_1}} \zeta \left|\nabla\left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right)\right|^2 \mathrm{d}x\mathrm{d}t \\
&\quad= \iint\limits_{Q_{T_1}} \zeta\left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right) f_i(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)})\,\mathrm{d}x\mathrm{d}t \\
&\qquad+ \int\limits_{\Omega} \zeta(x,0)\left(\frac{1}{m_i+1}\left(u_{i0\varepsilon}^{(k)}\right)^{m_i+1} + \frac{\varepsilon}{2}(u_{i0\varepsilon}^{(k)})^2\right)\mathrm{d}x \\
&\qquad+ \iint\limits_{Q_{T_1}} \left(\frac{1}{m_i+1}(u_{i\varepsilon}^{(k)})^{m_i+1} + \frac{\varepsilon}{2}(u_{i\varepsilon}^{(k)})^2\right)\zeta_t\,\mathrm{d}x\mathrm{d}t \\
&\qquad- \iint\limits_{Q_{T_1}} \left((u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right)\nabla\left((u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right)\nabla\zeta\,\mathrm{d}x\mathrm{d}t.
\end{aligned} \tag{2.17}$$

Notice that

$$\iint_{Q_{T_1}} \zeta \left|\nabla \left((u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right)\right|^2 \mathrm{d}x\mathrm{d}t$$
$$- \iint_{Q_{T_1}} \zeta \nabla \left((u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right) \nabla w \,\mathrm{d}x\mathrm{d}t - \iint_{Q_{T_1}} \zeta \nabla w \nabla \left((u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)} - w\right) \mathrm{d}x\mathrm{d}t$$
$$= \iint_{Q_{T_1}} \zeta \nabla \left((u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)} - w\right) \nabla \left((u_{i\varepsilon}^{(k)})^{m_i} + \varepsilon u_{i\varepsilon}^{(k)} - w\right) \mathrm{d}x\mathrm{d}t \geq 0,$$

from (2.17), we get

$$\iint_{Q_{T_1}} \zeta \left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right) f_i(x, t, u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}) \,\mathrm{d}x\mathrm{d}t$$
$$+ \int_\Omega \zeta(x, 0) \left(\frac{1}{m_i + 1} (u_{i0\varepsilon}^{(k)})^{m_i+1} + \frac{\varepsilon}{2} (u_{i0\varepsilon}^{(k)})^2\right) \mathrm{d}x$$
$$+ \iint_{Q_{T_1}} \left(\frac{1}{m_i + 1} (u_{i\varepsilon}^{(k)})^{m_i+1} + \frac{1}{2} (u_{i\varepsilon}^{(k)})^2\right) \zeta_t \,\mathrm{d}x\mathrm{d}t$$
$$- \iint_{Q_{T_1}} \left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right) \nabla \left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right) \nabla \zeta \,\mathrm{d}x\mathrm{d}t$$
$$- \iint_{Q_{T_1}} \zeta \nabla \left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)}\right)\big) \nabla w \,\mathrm{d}x\mathrm{d}t - \iint_{Q_{T_1}} \zeta \nabla w \nabla \left(\left(u_{i\varepsilon}^{(k)}\right)^{m_i} + \varepsilon u_{i\varepsilon}^{(k)} - w\right) \mathrm{d}x\mathrm{d}t \geq 0.$$

Letting $k \to \infty$, then

$$\begin{aligned} &\iint_{Q_{T_1}} \zeta \left((u_{i\varepsilon})^{m_i} + \varepsilon u_{i\varepsilon}\right) f_i(x, t, u_{1\varepsilon}, u_{2\varepsilon}) \varphi_i \,\mathrm{d}x\mathrm{d}t \\ &\quad + \int_\Omega \zeta(x, 0) \left(\frac{u_{i0\varepsilon}^{m_i+1}}{m_i + 1} + \frac{\varepsilon}{2} u_{i0\varepsilon}^2\right) \mathrm{d}x \\ &\quad + \iint_{Q_{T_1}} \left(\frac{u_{i\varepsilon}^{m_i+1}}{m_i + 1} + \frac{1}{2} u_{i\varepsilon}^2\right) \zeta_t \,\mathrm{d}x\mathrm{d}t - \iint_{Q_{T_1}} (u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon}) \nu \nabla \zeta \,\mathrm{d}x\mathrm{d}t \\ &\quad - \iint_{Q_{T_1}} \zeta \nu \nabla w \,\mathrm{d}x\mathrm{d}t - \iint_{Q_{T_1}} \zeta \nabla w \nabla (u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon} - w) \,\mathrm{d}x\mathrm{d}t \geq 0. \end{aligned} \tag{2.18}$$

Set $\varphi_i = \zeta (u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon})$ in (2.15), we obtain

$$\iint_{Q_{T_1}} \zeta (u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon}) f_i(x, t, u_{1\varepsilon}, u_{2\varepsilon}) \,\mathrm{d}x\mathrm{d}t$$
$$+ \int_\Omega \zeta(x, 0) \left(\frac{u_{i0\varepsilon}^{m_i+1}}{m_i + 1} + \frac{\varepsilon}{2} u_{i0\varepsilon}^2\right) \mathrm{d}x + \iint_{Q_{T_1}} \left(\frac{u_{i\varepsilon}^{m_i+1}}{m_i + 1} + \frac{u_{i\varepsilon}^2}{2}\right) \zeta_t \,\mathrm{d}x\mathrm{d}t$$
$$= \iint_{Q_{T_1}} (u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon}) \nu \nabla \zeta \,\mathrm{d}x\mathrm{d}t + \iint_{Q_{T_1}} \zeta \nu \nabla (u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon}) \,\mathrm{d}x\mathrm{d}t.$$

Substituting the above equation into (2.18), we get

$$\iint\limits_{Q_{T_1}} \zeta(\nu - \nabla w)\nabla(u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon} - w)\,\mathrm{d}x\mathrm{d}t \geq 0. \tag{2.19}$$

Taking $w = u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon} - \delta\varphi_i$, $\delta \geq 0$ in (2.19) and then let $\delta \to 0$, we obtain

$$\iint\limits_{Q_{T_1}} \zeta(\nu - \nabla(u_{i\varepsilon}^{m_i} + \varepsilon u_{i\varepsilon}))\nabla\varphi_i\,\mathrm{d}x\mathrm{d}t \geq 0,$$

where $\varphi_i \in C^1(\bar{Q}_{T_1})$ with $\varphi_i|_{\partial\Omega\times(0,T_1)} = 0$. Obviously, if we let $\delta \leq 0$, we can get the inverted inequality. So we can obtain (2.16) by choosing suitable $\zeta$, s.t. supp$\phi_i \subset$ supp$\zeta$ and $\zeta = 1$ on supp$\phi_i$.

In summary, we have proved that $u_\varepsilon = (u_{1\varepsilon}, u_{2\varepsilon})$ is a weak solution of (2.1)-(2.3).

Now, we will prove that the limit of $u_\varepsilon = (u_{1\varepsilon}, u_{2\varepsilon})$ is a weak solution of (1.1)-(1.3). Since $u_\varepsilon = (u_{1\varepsilon}, u_{2\varepsilon})$ satisfies similar estimates as (2.10)-(2.12), combining the property of $f_{i\varepsilon}$, we know that there are functions $u_i^{m_i} \in L^2(0, T_1; H_0^1(\Omega))$, $u_{it}, u_{it}^{m_i} \in L^2(Q_{T_1})$, $i = 1$, 2, such that for some subsequence of $(u_{1\varepsilon}, u_{2\varepsilon})$, denoted by itself for simplicity, when $\varepsilon \to 0$

$$u_{i\varepsilon} \to u_i,\ f_{i\varepsilon}(x,t,u_{1\varepsilon},u_{2\varepsilon}) \to f_i(x,t,u_1,u_2), \quad \text{a.e.} \quad \text{in} \quad Q_{T_1},$$
$$\frac{\partial u_{i\varepsilon}}{\partial t} \rightharpoonup \frac{\partial u_i}{\partial t},\ \frac{\partial u_{i\varepsilon}^{m_i}}{\partial t} \rightharpoonup \frac{\partial u_i^{m_i}}{\partial t}, \quad \text{in} \quad L^2(Q_{T_1}).$$

Then a similar argument as above shows that $u = (u_1, u_2)$ is a weak solution of (1.1)-(1.3). □

The following is the uniqueness result to the solution of the system.

**Theorem 2.2**. *Assume that $f = (f_1, f_2)$ is Lipschitz continuous in $(u_1, u_2)$, then* (1.1)-(1.3) *has a unique solution.*

*Proof*. Assume that $u = (u_1, u_2)$, $v = (v_1, v_2)$ are two solutions of (1.1)-(1.3). Form Definition 1, we see that

$$\begin{aligned}&\int_0^t\int_\Omega -u_i\frac{\partial\varphi_i}{\partial t} + \nabla u_i^{m_i}\nabla\varphi_i\,\mathrm{d}x\mathrm{d}t + \int_\Omega u_i(x,t)\varphi_i(x,t)\mathrm{d}x - \int_\Omega u_{i0}(x)\varphi_i(x,0)\mathrm{d}x\\&\quad = \int_0^t\int_\Omega f_i(x,t,u_1,u_2)\varphi_i\,\mathrm{d}x\mathrm{d}t, \quad \text{a.e.} \quad t \in (0,T).\end{aligned} \tag{2.20}$$

$$\begin{aligned}&\int_0^t\int_\Omega -v_i\frac{\partial\varphi_i}{\partial t} + \nabla v_i^{m_i}\nabla\varphi_i\,\mathrm{d}x\mathrm{d}t + \int_\Omega v_i(x,t)\varphi_i(x,t)\,\mathrm{d}x - \int_\Omega v_{i0}(x)\varphi_i(x,0)\,\mathrm{d}x\\&\quad = \int_0^t\int_\Omega f_i(x,t,v_1,v_2)\varphi_i\,\mathrm{d}x\mathrm{d}t, \quad \text{a.e.} \quad t \in (0,T).\end{aligned} \tag{2.21}$$

Subtracting the two equations, we get

$$\int_\Omega (u_i(x,t) - v_i(x,t))\varphi_i(x,t)\,\mathrm{d}x = \int_0^t\int_\Omega (u_i - v_i)(\varphi_{it} + \Phi(x,s)\Delta\varphi_i)\,\mathrm{d}x\mathrm{d}s + \int_0^t\int_\Omega (f_i(x,t,u_1,u_2) - f_i(x,t,v_1,v_2))\varphi_i\,\mathrm{d}x\mathrm{d}s, \tag{2.22}$$

where

$$\Phi(x,s) \equiv \int_0^1 m_i(\theta u_i + (1-\theta)v_i)^{m_1-1}\mathrm{d}\theta.$$

Since $(u_1, u_2)$ and $(v_1, v_2)$ are bounded on $Q_t$, it follows from $m > 1$, $\Phi(x, s)$ is a bounded nonnegative function. Thus, appropriate test function $\phi_i$ may be chosen exactly as in [[17], pp. 118-123] and combined with the Lipschitz continuity of $f_i$ to obtain

$$\int_\Omega |u_i(x,t) - v_i(x,t)|\mathrm{d}x \le C\int_0^t\int_\Omega |u_1 - v_1| + |u_2 - v_2|\mathrm{d}x\mathrm{d}s, \quad i = 1, 2.$$

where $C > 0$ is a bounded constant. Further, we have

$$\int_\Omega |u_1(x,t) - v_1(x,t)| + |u_2(x,t) - v_2(x,t)|\mathrm{d}x \le C\int_0^t\int_\Omega |u_1 - v_1| + |u_2 - v_2|\mathrm{d}x\mathrm{d}s.$$

Combined with the Gronwall's lemma, we see that $u_i \equiv v_i$, $i = 1, 2$. The proof is completed. □

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**Authors' contributions**
DZ and JS carried out the proof of existence, BW conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

**Competing interests**
The authors declare that they have no competing interests.

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