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Coincidence and fixed point results for generalized weak contractions in the sense of Berinde on partial metric spaces

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Abstract

In this paper, we introduce the notion of a generalized (ϕ, L) -weak contraction and we prove some common fixed point results for self-mappings T and S and some fixed point results for a single mapping T by using a (c) -comparison function and a comparison function in the sense of Berinde in a partial metric space. Also, we introduce an example to support the useability of our results.

MSC: 47H10; 54H25

Keywords: partial metric space; weak contraction; fixed point

1 Introduction and preliminaries

The contraction principle of Banach is one of the most important results in nonlinear analysis. After Banach established his existence and uniqueness result, many authors established important fixed point theorems in the literature. For the development of our research, in this article, the article by Matthews [1] is the background.

In 1994, in his elegant article [1], Matthews introduced the notion of a partial metric space and proved the contraction principle of Banach in this new framework. After then, many fixed point theorems in partial metric spaces have been given by several authors (for example, please see [2–24]).

Following Matthews [1], the notion of a partial metric space is given as follows.

Definition 1.1 [1] A *partial metric* on a nonempty set X is a function $p: X \times X \rightarrow \mathbb{R}^+$ such that for all $x, y, z \in X$:

$$(p_1) \quad x = y \iff p(x, x) = p(x, y) = p(y, y),$$

$$(p_2) \quad p(x, x) \leq p(x, y),$$

$$(p_3) \quad p(x, y) = p(y, x),$$

$$(p_4) \quad p(x, y) \leq p(x, z) + p(z, y) - p(z, z).$$

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X .

It is clear that each partial metric p on X generates a T_0 topology τ_p on X . The set $\{B_p(x, \varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$, forms the base of τ_p .

It is remarkable that if p is a partial metric on X , then the functions

$$p^s : X \times X \rightarrow \mathbb{R}^+, \quad p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y) \tag{1.1}$$

and

$$p^w : X \times X \rightarrow \mathbb{R}^+, \quad p^w(x, y) = 2p(x, y) - \min\{p(x, x), p(y, y)\} \tag{1.2}$$

are ordinary equivalent metrics on X .

Definition 1.2 [1] Let (X, p) be a partial metric space. Then:

- (1) A sequence $\{x_n\}$ in a partial metric space (X, p) converges to a point $x \in X$ if and only if $p(x, x) = \lim_{n \rightarrow \infty} p(x, x_n)$.
- (2) A sequence $\{x_n\}$ in a partial metric space (X, p) is called a *Cauchy sequence* if there exists (and is finite) $\lim_{n, m \rightarrow \infty} p(x_n, x_m)$.
- (3) A partial metric space (X, p) is said to be *complete* if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p , to a point $x \in X$ such that $p(x, x) = \lim_{n, m \rightarrow \infty} p(x_n, x_m)$.

The following lemma is crucial in proving our main results.

Lemma 1.1 [1] Let (X, p) be a partial metric space.

- (1) $\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, p^s) .
- (2) A partial metric space (X, p) is complete if and only if the metric space (X, p^s) is complete. Furthermore, $\lim_{n \rightarrow \infty} p^s(x_n, x) = 0$ if and only if

$$p(x, x) = \lim_{n \rightarrow \infty} p(x_n, x) = \lim_{n, m \rightarrow \infty} p(x_n, x_m).$$

The definition of a 0-complete partial metric space is given by Romaguera [19] as follows.

Definition 1.3 [19] A sequence $\{x_n\}$ in a partial metric space (X, p) is called 0-Cauchy if $\lim_{n, m \rightarrow +\infty} p(x_n, x_m) = 0$. We say that (X, p) is 0-complete if every 0-Cauchy sequence in X converges, with respect to τ_p , to a point $x \in X$ such that $p(x, x) = 0$.

We need the following useful lemma in the proof of our main result.

Lemma 1.2 [2] Assume that $x_n \rightarrow z$ as $n \rightarrow +\infty$ in a partial metric space (X, p) such that $p(z, z) = 0$. Then $\lim_{n \rightarrow +\infty} p(x_n, y) = p(z, y)$ for every $y \in X$.

In [25], Berinde introduced the nonlinear type weak contraction using a comparison function. A map $\phi : [0, +\infty) \rightarrow [0, +\infty)$ is called a comparison function if it satisfies:

- (i) ϕ is monotone increasing,
- (ii) $\lim_{n \rightarrow +\infty} \phi^n(t) = 0$ for all $t \geq 0$.

If ϕ satisfies (i) and

- (iii) $\sum_{n=0}^{\infty} \phi^n(t)$ converges for all $t \geq 0$,

then ϕ is said to be a (c)-comparison function.

It is an easy matter to see that if ϕ is a comparison function or a (c) -comparison function, then $\phi(t) < t$ for all $t > 0$ and $\phi(0) = 0$.

Berinde [26, 27] initiated the concept of weak contraction mappings, the concept of almost contraction mappings and the concepts of (ϕ, L) -weak contractions. Berinde [25–32] studied many interesting fixed point theorems for weak contraction mappings, almost contraction mappings and (ϕ, L) -weak contraction mappings in metric spaces. We have to recall the following definition.

Definition 1.4 [25] A single-valued mapping $f : X \rightarrow X$ is called a *Ćirić strong almost contraction* if there exist a constant $\alpha \in [0, 1)$ and some $L \geq 0$ such that

$$d(fx, fy) \leq \alpha \max \left\{ d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2} \right\} + Ld(y, fx)$$

for all $x, y \in X$.

For some theorems of almost contractive mappings in the sense of Berinde on metric spaces, we refer the reader to [33–42].

Very recently, Ishak Altun and Özlem Acar initiated the notions of a (δ, L) -weak contraction and a (ϕ, L) -weak contraction in partial metric spaces as follows.

Definition 1.5 [43] Let (X, p) be a partial metric space. A map T is called a (δ, L) -weak contraction if there exist a $\delta \in [0, 1)$ and some $L \geq 0$ such that

$$p(Tx, Ty) \leq \delta p(x, y) + Lp^w(y, Tx). \tag{1.3}$$

Because of the symmetry of the distance, the (δ, L) -weak contraction condition implicitly includes the following dual one:

$$p(Tx, Ty) \leq \delta p(x, y) + Lp^w(x, Ty). \tag{1.4}$$

Thus by (1.3) and (1.4), the (δ, L) -weak contraction condition can be replaced by the following condition:

$$p(Tx, Ty) \leq \delta p(x, y) + L \min \{ p^w(y, Tx), p^w(x, Ty) \}. \tag{1.5}$$

Definition 1.6 [43] Let (X, p) be a partial metric space. A map T is called (ϕ, L) -weak contraction if there exist a comparison function ϕ and some $L \geq 0$ such that

$$p(Tx, Ty) \leq \phi(p(x, y)) + Lp^w(y, Tx). \tag{1.6}$$

As above, because of the symmetry of the distance, the (ϕ, L) -weak contraction condition implicitly includes the following dual one:

$$p(Tx, Ty) \leq \phi(p(x, y)) + Lp^w(x, Ty). \tag{1.7}$$

Thus, by (1.6) and (1.7), the (ϕ, L) -weak contraction condition can be replaced by the following condition:

$$p(Tx, Ty) \leq \delta\phi(p(x, y)) + L \min\{p^w(y, Tx), p^w(x, Ty)\}. \tag{1.8}$$

Altun and Acar [43] proved the following interesting theorems.

Theorem 1.1 [43] *Let (X, p) be a 0-complete partial metric space and $T: X \rightarrow X$ be a (ϕ, L) -weak contraction mapping with a (c) -comparison function, then T has a fixed point.*

Theorem 1.2 [43] *Let (X, p) be a 0-complete partial metric space and $T: X \rightarrow X$ be a (ϕ, L) -weak contraction mapping. Suppose T also satisfies the following condition: There exist a comparison function ϕ_1 and some $L_1 \geq 0$ such that*

$$p(Tx, Ty) \leq \phi_1(p(x, y)) + L_1 p^w(x, Tx)$$

for all $x, y \in X$. Then T has a unique fixed point.

In this paper, we introduce the notion of a generalized (δ, L) -weak contraction mapping and a generalized (ϕ, L) -weak contraction mapping in partial metric spaces. Then after, we prove some fixed point results for two mappings S and T and some fixed point results for a single mapping T . Our results generalize Theorems 1.1 and 1.2.

2 The main result

We start our work by introducing the following two concepts.

Definition 2.1 Let (X, p) be a partial metric space and $T, S: X \rightarrow X$ be two mappings. The pair (T, S) is called a *generalized (δ, L) -weak contraction* if there exist $\delta \in [0, 1)$ and some $L \geq 0$ such that

$$p(Tx, Sy) \leq \delta \max\left\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}(p(x, Sy) + p(Tx, x))\right\} + L \min\{p^w(x, Sy), p^w(Tx, y)\} \tag{2.1}$$

for all $x, y \in X$.

Definition 2.2 Let (X, p) be a partial metric space and $T, S: X \rightarrow X$ be two mappings. Then the pair (T, S) is called a *generalized (ϕ, L) -weak contraction* if there exist a control function ϕ and some $L \geq 0$ such that

$$p(Tx, Sy) \leq \phi\left(\max\left\{p(x, y), p(x, Tx), p(y, Sy), \frac{1}{2}(p(Tx, y) + p(x, Sy))\right\}\right) + L \min\{p^w(x, Sy), p^w(Tx, y)\} \tag{2.2}$$

for all $x, y \in X$.

Now, we give and prove our first result.

Theorem 2.1 *Let (X, p) be a 0-complete partial metric space and $T, S: X \rightarrow X$ be two mappings such that the pair (T, S) is a generalized (ϕ, L) -weak contraction. If ϕ is a (c)-comparison function, then T and S have a common fixed point.*

Proof Choose $x_0 \in X$. Put $x_1 = Tx_0$. Again, put $x_2 = Sx_1$. Continuing this process, we construct a sequence (x_n) in X such that $x_{2n+1} = Tx_{2n}$ and $x_{2n+2} = Sx_{2n+1}$. Suppose $p(x_n, x_{n+1}) = 0$ for some $n \in \mathbb{N}$. Without loss of generality, we assume $n = 2k$ for some $k \in \mathbb{N}$. Thus $p(x_{2k}, x_{2k+1}) = 0$. Now, by (2.2), we have

$$\begin{aligned} & p(x_{2k+1}, x_{2k+2}) \\ &= p(Tx_{2k}, Sx_{2k+1}) \\ &\leq \phi \left(\max \left\{ p(x_{2k}, x_{2k+1}), p(x_{2k}, Tx_{2k}), p(x_{2k+1}, Sx_{2k+1}), \right. \right. \\ &\quad \left. \left. \frac{1}{2} (p(x_{2k}, Sx_{2k+1}) + p(Tx_{2k}, x_{2k+1})) \right\} \right) \\ &\quad + L \min \{ p^w(Tx_{2k}, x_{2k+1}) + p^w(x_{2k}, Sx_{2k+1}) \} \\ &= \phi \left(\max \left\{ p(x_{2k+1}, x_{2k+2}), \frac{1}{2} (p(x_{2k}, x_{2k+2}) + p(x_{2k+1}, x_{2k+1})) \right\} \right) \\ &\quad + L \min \{ p^w(x_{2k+1}, x_{2k+1}) + p^w(x_{2k}, x_{2k+2}) \} \\ &\leq \phi \left(\max \left\{ p(x_{2k+1}, x_{2k+2}), \frac{1}{2} (p(x_{2k}, x_{2k+1}) + p(x_{2k+1}, x_{2k+2})) \right\} \right) \\ &\leq \phi(p(x_{2k+1}, x_{2k+2})). \end{aligned}$$

Since $\phi(t) < t$ for all $t > 0$, we conclude that $p(x_{2k+1}, x_{2k+2}) = 0$. By (p₁) and (p₂) of the definition of partial metric spaces, we have $x_{2k+1} = x_{2k+2}$. So, $x_{2k} = x_{2k+1} = x_{2k+2}$. Therefore $x_{2k} = Tx_{2k} = Sx_{2k}$ and hence x_k is a fixed point of T and S . Thus, we may assume that $p(x_n, x_{n+1}) \neq 0$ for all $n \in \mathbb{N}$. Given $n \in \mathbb{N}$. If n is even, then $n = 2t$ for some $t \in \mathbb{N}$. By (2.2), we have

$$\begin{aligned} & p(x_{2t}, x_{2t+1}) \\ &= p(x_{2t+1}, x_{2t}) \\ &= p(Tx_{2t}, Sx_{2t-1}) \\ &\leq \phi \left(\max \left\{ p(x_{2t}, x_{2t-1}), p(x_{2t}, Tx_{2t}), p(x_{2t-1}, Sx_{2t-1}), \right. \right. \\ &\quad \left. \left. \frac{1}{2} (p(x_{2t}, Sx_{2t-1}) + p(Tx_{2t}, x_{2t-1})) \right\} \right) \\ &\quad + L \min \{ p^w(x_{2t}, Sx_{2t-1}), p^w(Tx_{2t}, x_{2t-1}) \} \\ &= \phi \left(\max \left\{ p(x_{2t}, x_{2t-1}), p(x_{2t}, x_{2t+1}), \frac{1}{2} (p(x_{2t}, x_{2t}) + p(x_{2t+1}, x_{2t-1})) \right\} \right) \\ &\quad + L \min \{ p^w(x_{2t}, x_{2t}), p^w(x_{2t+1}, x_{2t-1}) \}. \end{aligned}$$

Using (p₄) of the definition of partial metric spaces and the definition of p^w , we arrive at

$$\begin{aligned}
 p(x_{2t}, x_{2t+1}) &\leq \phi \left(\max \left\{ p(x_{2t}, x_{2t-1}), p(x_{2t}, x_{2t+1}), \frac{1}{2} (p(x_{2t-1}, x_{2t}) + p(x_{2t}, x_{2t+1})) \right\} \right) \\
 &\leq \phi (\max \{ p(x_{2t}, x_{2t-1}), p(x_{2t}, x_{2t+1}) \}).
 \end{aligned}
 \tag{2.3}$$

If $\max \{ p(x_{2t}, x_{2t-1}), p(x_{2t}, x_{2t+1}) \} = p(x_{2t}, x_{2t+1})$, then (2.3) yields a contradiction. Thus, $\max \{ p(x_{2t}, x_{2t-1}), p(x_{2t}, x_{2t+1}) \} = p(x_{2t}, x_{2t-1})$ and hence

$$p(x_{2t}, x_{2t+1}) \leq \phi (p(x_{2t}, x_{2t-1})).
 \tag{2.4}$$

If n is odd, then $n = 2t + 1$ for some $t \in \mathbb{N} \cup \{0\}$. By similar arguments as above, we can show that

$$p(x_{2t+1}, x_{2t+2}) \leq \phi (p(x_{2t}, x_{2t+1})).
 \tag{2.5}$$

By (2.4) and (2.5), we have

$$p(x_n, x_{n+1}) \leq \phi (p(x_{n-1}, x_n)).
 \tag{2.6}$$

By repeating (2.6) n -times, we get $p(x_n, x_{n+1}) \leq \phi^n (p(x_0, x_1))$. For $n, m \in \mathbb{N}$ with $m > n$, we have

$$\begin{aligned}
 p(x_n, x_m) &\leq \sum_{i=n}^{m-1} p(x_i, x_{i+1}) - \sum_{i=n}^{m-2} p(x_{i+1}, x_{i+1}) \leq \sum_{i=n}^{m-1} p(x_i, x_{i+1}) \\
 &\leq \sum_{i=n}^{m-1} \phi^i (p(x_0, x_1)) \leq \sum_{i=n}^{\infty} \phi^i (p(x_0, x_1)).
 \end{aligned}$$

Since ϕ is (c)-comparison, we have $\sum_{i=0}^{\infty} \phi^i (p(x_0, x_1))$ converges and hence

$$\lim_{n \rightarrow +\infty} \sum_{i=n}^{\infty} \phi^i (p(x_0, x_1)) = 0.$$

So, $\lim_{n, m \rightarrow +\infty} p(x_n, x_m) = 0$. Thus (x_n) is a 0-Cauchy sequence in X . Since X is 0-complete, there exists $z \in X$ such that $x_n \rightarrow z$ with $p(z, z) = 0$. So,

$$\lim_{n, m \rightarrow +\infty} p(x_n, x_m) = \lim_{n \rightarrow +\infty} p(x_n, z) = p(z, z) = 0.
 \tag{2.7}$$

Now, we prove that $Sz = z$ and $Tz = z$. Since $p(x_{2n+1}, z) \rightarrow p(z, z) = 0$ and $p(x_{2n+2}, z) \rightarrow p(z, z) = 0$, then by Lemma 1.2 we get

$$\lim_{n \rightarrow +\infty} p(x_{2n+1}, Sz) = p(z, Sz)
 \tag{2.8}$$

and

$$\lim_{n \rightarrow +\infty} p(x_{2n+2}, Tz) = p(z, Tz).
 \tag{2.9}$$

By using (2.2), we have

$$\begin{aligned} p(x_{2n+1}, Sz) &= p(Tx_{2n}, Sz) \\ &\leq \phi \left(\max \left\{ p(x_{2n}, z), p(x_{2n}, Tx_{2n}), p(z, Sz), \frac{1}{2}(p(Tx_{2n}, z) + p(x_{2n}, Sz)) \right\} \right) \\ &\quad + L \min \{ p^w(Tx_{2n}, z), p^w(x_{2n}, Sz) \} \\ &= \phi \left(\max \left\{ p(x_{2n}, z), p(x_{2n}, x_{2n+1}), p(z, Sz), \frac{1}{2}(p(x_{2n+1}, z) + p(x_{2n}, Sz)) \right\} \right) \\ &\quad + L \min \{ p^w(x_{2n+1}, z), p^w(x_{2n}, Sz) \}. \end{aligned}$$

On letting $n \rightarrow +\infty$ in the above inequality and using (2.7) and (2.8), we get that $p(z, Sz) \leq \phi(p(z, Sz))$. Since $\phi(t) < t$ for all $t > 0$, we conclude that $p(z, Sz) = 0$. By using (p₁) and (p₂) of the definition of partial metric spaces, we get that $Sz = z$. By similar arguments as above, we may show that $Tz = z$. □

The common fixed point of S and T in Theorem 2.1 is unique if we replaced $\min\{p^w(x, Sy), p^w(Tx, y)\}$ by $\min\{p^w(x, Tx), p^w(x, Sy), p^w(Tx, y)\}$ in (2.2). So, we have the following result.

Theorem 2.2 *Let (X, p) be a 0-complete partial metric space and $T, S: X \rightarrow X$ be two mappings such that*

$$\begin{aligned} p(Tx, Sy) &\leq \phi \left(\max \left\{ p(x, y), p(x, Tx), p(y, Sy), \frac{1}{2}(p(Tx, y) + p(x, Sy)) \right\} \right) \\ &\quad + L \min \{ p^w(x, Tx), p^w(x, Sy), p^w(Tx, y) \} \end{aligned} \tag{2.10}$$

for all $x, y \in X$. If ϕ is a (c)-comparison function, then the common fixed point of T and S is unique.

Proof The existence of the common fixed point of T and S follows from Theorem 2.1. To prove the uniqueness of the common fixed point of T and S , we let u, v be two common fixed points of T and S . Then $Tu = Su = u$ and $Tv = Sv = v$. Now, we will show that $u = v$. By (2.10), we have

$$\begin{aligned} p(u, v) &= p(Tu, Sv) \\ &\leq \phi \left(\max \left\{ p(u, v), p(u, Tu), p(v, Sv), \frac{1}{2}(p(Tu, v) + p(v, Tu)) \right\} \right) \\ &\quad + L \min \{ p^w(u, Tu), p^w(Tu, v), p^w(v, Tu) \} \\ &= \phi \left(\max \left\{ p(u, v), p(u, u), p(v, v), \frac{1}{2}(p(u, v) + p(v, u)) \right\} \right) \\ &\quad + L \min \{ p^w(u, u), p^w(u, v), p^w(v, u) \} \\ &= \phi(p(u, v)). \end{aligned}$$

Since $\phi(t) < t$ for all $t > 0$, we conclude that $p(u, v) = 0$. By (p₁) and (p₂) of the definition of partial metric spaces, we get that $u = v$. □

Taking $T = S$ in Theorems 2.1 and 2.2, we have the following results.

Corollary 2.1 *Let (X, p) be a 0-complete partial metric space and $T: X \rightarrow X$ be a mapping such that*

$$p(Tx, Ty) \leq \phi \left(\max \left\{ p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}(p(Tx, y) + p(x, Ty)) \right\} \right) + L \min \{ p^w(x, Ty), p^w(Tx, y) \}$$

for all $x, y \in X$. If ϕ is a (c)-comparison function, then T has a fixed point.

Corollary 2.2 *Let (X, p) be a 0-complete partial metric space and $T: X \rightarrow X$ be a mapping such that*

$$p(Tx, Ty) \leq \phi \left(\max \left\{ p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}(p(Tx, y) + p(x, Ty)) \right\} \right) + L \min \{ p^w(x, Tx), p^w(x, Ty), p^w(Tx, y) \}$$

for all $x, y \in X$. If ϕ is a (c)-comparison function, then T has a unique fixed point.

By the aid of Lemma 2.1 of Ref. [44], we have the following consequence results of Corollaries 2.1 and 2.2.

Corollary 2.3 *Let (X, p) be a partial metric space and $T, S: X \rightarrow X$ be two mappings such that*

$$p(Tx, Ty) \leq \phi \left(\max \left\{ p(Sx, Sy), p(Sx, Tx), p(Sy, Ty), \frac{1}{2}(p(Tx, Sy) + p(Sx, Ty)) \right\} \right) + L \min \{ p^w(Sx, Ty), p^w(Sy, Tx) \}$$

for all $x, y \in X$. Also, suppose that

- (1) $TX \subseteq SX$.
- (2) SX is a 0-complete subspace of the partial metric space X .

If ϕ is a (c)-comparison function, then T and S have a coincidence point. Moreover, the point of coincidence of T and S is unique.

Corollary 2.4 *Let (X, p) be a partial metric space and $T, S: X \rightarrow X$ be two mappings such that*

$$p(Tx, Ty) \leq \phi \left(\max \left\{ p(Sx, Sy), p(Sx, Tx), p(Sy, Ty), \frac{1}{2}(p(Tx, Sy) + p(Sx, Ty)) \right\} \right) + L \min \{ p^w(Tx, Sx), p^w(Sx, Ty), p^w(Sy, Tx) \}$$

for all $x, y \in X$. Also, suppose that

- (1) $TX \subseteq SX$.
- (2) SX is a 0-complete subspace of the partial metric space X .

If ϕ is a (c)-comparison function, then the point of coincidence of T and S is unique; that is, if $Tu = Su$ and $Tv = Sv$, then $Tu = Tv = Sv = Su$.

By taking $\phi(t) = kt$, $k \in [0, 1)$ in Corollaries 2.1 and 2.2, we have the following results.

Corollary 2.5 Let (X, p) be a 0-complete partial metric space and $T : X \rightarrow X$ be a mapping such that

$$p(Tx, Ty) \leq k \max \left\{ p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}(p(Tx, y) + p(x, Ty)) \right\} + L \min \{ p^w(x, Ty), p^w(Tx, y) \}$$

for all $x, y \in X$. If $0 \leq k < 1$, then T has a fixed point.

Corollary 2.6 Let (X, p) be a 0-complete partial metric space and $T : X \rightarrow X$ be a mapping such that

$$p(Tx, Ty) \leq k \max \left\{ p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}(p(Tx, y) + p(x, Ty)) \right\} + L \min \{ p^w(x, Tx), p^w(x, Ty), p^w(Tx, y) \}$$

for all $x, y \in X$. If $k \in [0, 1)$, then T has a unique fixed point.

By the aid of Lemma 2.1 of Ref. [44], we have the following consequence results of Corollaries 2.5 and 2.6.

Corollary 2.7 Let (X, p) be a partial metric space and $T, S : X \rightarrow X$ be two mappings such that

$$p(Tx, Ty) \leq k \max \left\{ p(Sx, Sy), p(Sx, Tx), p(Sy, Ty), \frac{1}{2}(p(Tx, Sy) + p(Sx, Ty)) \right\} + L \min \{ p^w(Sx, Ty), p^w(Sy, Tx) \}$$

for all $x, y \in X$. Also, suppose that

- (1) $TX \subseteq SX$.
- (2) SX is a 0-complete subspace of the partial metric space X .

If $k \in [0, 1)$, then T and S have a coincidence point.

Corollary 2.8 Let (X, p) be a partial metric space and $T, S : X \rightarrow X$ be two mappings such that

$$p(Tx, Ty) \leq \phi \left(\max \left\{ p(Sx, Sy), p(Sx, Tx), p(Sy, Ty), \frac{1}{2}(p(Tx, Sy) + p(Sx, Ty)) \right\} \right) + L \min \{ p^w(Tx, Sx), p^w(Sx, Ty), p^w(Sy, Tx) \}$$

for all $x, y \in X$. Also, suppose that

- (1) $TX \subseteq SX$.
- (2) SX is a 0-complete subspace of the partial metric space X .

If $k \in [0, 1)$, then the point of coincidence of T and S is unique; that is, if $Tu = Su$ and $Tv = Sv$, then $Tu = Tv = Sv = Su$.

The (c)-comparison function in Theorems 2.1 and 2.2 can be replaced by a comparison function if we formulated the contractive condition to a suitable form. For this instance, we have the following result.

Theorem 2.3 *Let (X, p) be a 0-complete partial metric space and $T: X \rightarrow X$ be a mapping such that*

$$p(Tx, Ty) \leq \phi(\max\{p(x, y), p(x, Tx), p(y, Ty)\}) + L \min\{p^w(x, Tx), p^w(x, Ty), p^w(y, Tx)\} \tag{2.11}$$

for all $x, y \in X$. If ϕ is a comparison function, then T has a unique fixed point.

Proof Choose $x_0 \in X$. Put $x_1 = Tx_0$. Again, put $x_2 \in X$ such that $x_2 = Tx_1$. Continuing the same process, we can construct a sequence (x_n) in X such that $x_{n+1} = Tx_n$. If $p(x_k, x_{k+1}) = 0$ for some $k \in \mathbb{N}$, then by the definition of partial metric spaces, we have $x_k = x_{k+1} = Tx_k$, that is, x_k is a fixed point of T . Thus, we assume that $p(x_n, x_{n+1}) \neq 0$ for all $n \in \mathbb{N}$. By (2.11), we have

$$\begin{aligned} p(x_n, x_{n+1}) &= p(Tx_{n-1}, Tx_n) \\ &\leq \phi(\max\{p(x_{n-1}, x_n), p(x_{n-1}, Tx_{n-1}), p(x_n, Tx_n)\}) \\ &\quad + L \min\{p^w(x_{n-1}, Tx_n), p^w(x_{n-1}, Tx_n), p^w(x_n, Tx_{n-1})\} \\ &= \phi(\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}) + L \min\{p^w(x_{n-1}, x_{n+1}), p^w(x_n, x_n)\} \\ &= \phi(\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}). \end{aligned}$$

If

$$\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_n, x_{n+1}),$$

then

$$p(x_n, x_{n+1}) \leq \phi(p(x_n, x_{n+1})) < p(x_n, x_{n+1}),$$

a contradiction. Thus,

$$\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_{n-1}, x_n)$$

and hence

$$p(x_n, x_{n+1}) \leq \phi(p(x_{n-1}, x_n)) \quad \forall n \in \mathbb{N}. \tag{2.12}$$

Repeating (2.12) n times, we get that

$$p(x_n, x_{n+1}) \leq \phi^n(p(x_0, x_1)).$$

Now, we will prove that (x_n) is a Cauchy sequence in the partial metric space (X, p) . For this, given $\epsilon > 0$, since $\frac{1}{2+L}(\epsilon - \phi(\epsilon)) > 0$ and $\lim_{n \rightarrow +\infty} \phi^n(p(x_0, x_1)) = 0$, there exists $k \in \mathbb{N}$ such that $p(x_n, x_{n+1}) < \frac{1}{2+L}(\epsilon - \phi(\epsilon))$ for all $n \geq k$. Now, given $m, n \in \mathbb{N}$ with $m > n$. Claim: $p(x_n, x_m) < \epsilon$ for all $m > n \geq k$. We prove our claim by induction on m . Since $k + 1 > k$, then

$$p(x_k, x_{k+1}) < \frac{1}{2+L}(\epsilon - \phi(\epsilon)) < \epsilon.$$

The last inequality proves our claim for $m = k + 1$. Assume that our claim holds for $m = k$. To prove our claim for $m = k + 1$, we have

$$\begin{aligned} p(x_n, x_{k+1}) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{k+1}) - p(x_{n+1}, x_{n+1}) \\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{k+1}) \\ &= p(x_n, x_{n+1}) + p(Tx_n, Tx_k). \end{aligned} \tag{2.13}$$

By (2.11), we have

$$\begin{aligned} p(Tx_n, Tx_k) &\leq \phi(\max\{p(x_n, x_k), p(x_n, Tx_n), p(x_k, Tx_k)\}) \\ &\quad + L \min\{p(x_n, Tx_n), p(x_n, Tx_k), p(x_k, Tx_n)\} \\ &= \phi(\max\{p(x_n, x_k), p(x_n, x_{n+1}), p(x_k, x_{k+1})\}) \\ &\quad + L \min\{p(x_n, x_{n+1}), p(x_n, x_{k+1}), p(x_k, x_{n+1})\} \\ &\leq \phi(\max\{p(x_n, x_k), p(x_n, x_{n+1}), p(x_k, x_{k+1})\}) + Lp(x_n, x_{n+1}). \end{aligned}$$

If $\max\{p(x_n, x_k), p(x_n, x_{n+1}), p(x_k, x_{k+1})\} = p(x_n, x_k)$, then by (2.13) we have

$$\begin{aligned} p(x_n, x_{k+1}) &\leq p(x_n, x_{n+1}) + \phi(p(x_n, x_k)) + Lp(x_n, x_{n+1}) \\ &< \frac{1+L}{2+L}(\epsilon - \phi(\epsilon)) + \phi(\epsilon) \\ &< \epsilon. \end{aligned}$$

If $\max\{p(x_n, x_k), p(x_n, x_{n+1}), p(x_k, x_{k+1})\} = p(x_n, x_{n+1})$, then by (2.13) we have

$$\begin{aligned} p(x_n, x_{k+1}) &\leq p(x_n, x_{n+1}) + \phi(p(x_n, x_{n+1})) + Lp(x_n, x_{n+1}) \\ &< (2+L)p(x_n, x_{n+1}) \\ &< \epsilon - \phi(\epsilon) \\ &< \epsilon. \end{aligned}$$

If $\max\{p(x_n, x_k), p(x_n, x_{n+1}), p(x_k, x_{k+1})\} = p(x_k, x_{k+1})$, then by (2.13) we have

$$\begin{aligned} p(x_n, x_{k+1}) &\leq p(x_n, x_{n+1}) + \phi(p(x_k, x_{k+1})) + Lp(x_n, x_{n+1}) \\ &< (1+L)p(x_n, x_{n+1}) + p(x_k, x_{k+1}) \end{aligned}$$

$$\begin{aligned} &< \frac{1+L}{2+L}(\epsilon - \phi(\epsilon)) + \frac{1}{2+L}(\epsilon - \phi(\epsilon)) \\ &< \epsilon. \end{aligned}$$

Thus (x_n) is a 0-Cauchy sequence in X . Since X is 0-complete, then (x_n) converges, with respect to τ_p , to a point z for some $z \in X$ such that

$$\lim_{n,m \rightarrow +\infty} p(x_n, x_m) = \lim_{n \rightarrow +\infty} p(x_n, z) = p(z, z) = 0. \tag{2.14}$$

Now, assume that $p(z, Tz) > 0$. By using (p_4) of the definition of partial metric spaces and (2.11), we have

$$\begin{aligned} p(z, Tz) &\leq p(z, x_{n+1}) + p(x_{n+1}, Tz) \\ &= p(z, x_{n+1}) + p(Tx_n, Tz) \\ &\leq p(z, x_{n+1}) + \phi(\max\{p(x_n, z), p(x_n, Tx_n), p(z, Tz)\}) \\ &\quad + L \min\{p^w(x_n, Tx_n), p^w(x_n, Tz), p^w(Tx_n, z)\} \\ &= p(z, x_{n+1}) + \phi(\max\{p(x_n, z), p(x_n, x_{n+1}), p(z, Tz)\}) \\ &\quad + L \min\{p^w(x_n, x_{n+1}), p^w(x_n, Tz), p^w(x_{n+1}, Sz)\}. \end{aligned} \tag{2.15}$$

Since

$$\lim_{n,m \rightarrow +\infty} p(x_n, x_{n+1}) = \lim_{n \rightarrow +\infty} p(x_n, z) = 0$$

and $p(z, Tz) > 0$, we can choose $n_0 \in \mathbb{N}$ such that

$$\max\{p(x_n, z), p(x_n, x_{n+1}), p(z, Tz)\} = p(z, Tz)$$

for all $n \geq n_0$. Thus (2.15) becomes

$$p(z, Tz) \leq p(z, x_{n+1}) + \phi(p(z, Tz)) + L \min\{p^w(x_n, x_{n+1}), p^w(x_n, Tz), p^w(x_{n+1}, z)\},$$

for all $n \geq n_0$. On letting $n \rightarrow +\infty$ in the above inequality and using 2.14, we get that $p(z, Tz) \leq \phi(p(z, Tz)) < p(z, Tz)$, a contradiction. Thus $p(z, Tz) = 0$. By using (p_1) and (p_2) of the definition of a partial metric space, we get that $z = Tz$; that is, z is a fixed point of T . To prove that the fixed point of T is unique, we assume that u and v are fixed points of T . Thus, we have $Tu = u$ and $Tv = v$. By (2.11), we have

$$\begin{aligned} p(u, v) &= p(Tu, Tv) \\ &\leq \phi(\max\{p(u, v), p(u, Tu), p(v, Tv)\}) \\ &\quad + L \min\{p(u, Tu), p(u, Tv), p(v, Tu)\} \\ &= \phi(p(u, v)). \end{aligned}$$

Since $\phi(t) < t$ for all $t \in \mathbb{N}$, we have $p(u, v) = 0$. By (p_1) and (p_2) , we have $u = v$. □

By the aid of Lemma 2.1 of Ref. [44], we have the following consequence result of Theorem 2.3.

Corollary 2.9 *Let (X, p) be a partial metric space and $T, S: X \rightarrow X$ be two mappings such that for some $L \geq 0$, we have*

$$p(Tx, Ty) \leq \phi(\max\{p(Sx, Sy), p(Sx, Tx), p(Sy, Ty)\}) + L \min\{p^w(Sx, Tx), p^w(Sx, Ty), p^w(Sy, Tx)\}$$

for all $x, y \in X$. Also, suppose that

- (1) $TX \subseteq SX$.
- (2) SX is a 0-complete subspace of the partial metric space X .

If ϕ is a comparison function, then T and S have a coincidence point. Moreover, the point of coincidence of T and S is unique.

The uniqueness of a common fixed point of T and S in Theorem 2.1 can be proved under an additional contractive condition based on a comparison function ϕ_1 .

Corollary 2.10 *Let (X, p) be a 0-complete partial metric space and $T, S: X \rightarrow X$ be two mappings. Assume there exists a (c)-comparison function ϕ such that the pair (T, S) is a generalized (ϕ, L) -weak contraction. Also, suppose that there exist a comparison function ϕ_1 and $L_1 \geq 0$ such that*

$$p(Tx, Sy) \leq \phi_1\left(\max\left\{p(x, y), p(x, Tx), p(y, Sy), \frac{1}{2}(p(Tx, y) + p(x, Sy))\right\}\right) + L_1 p(x, Tx) \tag{2.16}$$

for all $x, y \in X$. Then T and S have a unique common fixed point.

Proof The existence of the common fixed point of T and S follows from Theorem 2.1. To prove the uniqueness of the fixed point, we assume that u and v are two fixed points of T and S . Then by (2.16), we have

$$\begin{aligned} p(u, v) &= p(Tu, Sv) \\ &\leq \phi_1\left(\max\left\{p(u, v), p(u, Tu), p(v, Sv), \frac{1}{2}(p(Tu, v) + p(u, Sv))\right\}\right) + L_1 p(u, Tu) \\ &= \phi_1(p(u, v)). \end{aligned}$$

Since $\phi_1(t) < t$ for all $t > 0$, we get $p(u, v) = 0$ and hence $u = v$. □

Taking $S = T$ in Corollary 2.10, we have the following result.

Corollary 2.11 *Let (X, p) be a 0-complete partial metric space and $T: X \rightarrow X$ be a mapping. Assume there exists a (c)-comparison function ϕ such that*

$$p(Tx, Ty) \leq \phi\left(\max\left\{p(x, y), p(x, Tx), p(y, Ty), \frac{1}{2}(p(Tx, y) + p(x, Ty))\right\}\right) + Lp(x, Tx)$$

for all $x, y \in X$. Also, suppose that there exist a comparison function ϕ_1 and $L_1 \geq 0$ such that

$$p(Tx, Sy) \leq \phi_1 \left(\max \left\{ p(x, y), p(x, Tx), p(y, Sy), \frac{1}{2}(p(Tx, y) + p(x, Sy)) \right\} \right) + Lp(x, Tx)$$

for all $x, y \in X$. Then T has a unique fixed point.

By the aid of Lemma 2.1 of [44], we have the following result.

Corollary 2.12 *Let (X, p) be a partial metric space and $T, S: X \rightarrow X$ be two mappings. Suppose there exist a (c)-comparison function ϕ and $L \geq 0$ such that*

$$p(Tx, Ty) \leq \phi \left(\max \left\{ p(Sx, Sy), p(Sx, Tx), p(Sy, Ty), \frac{1}{2}(p(Tx, Sy) + p(Sx, Ty)) \right\} \right) + L \min \{ p^w(Sx, Ty), p^w(Sy, Tx) \}$$

for all $x, y \in X$. Also, assume that there exist a comparison function ϕ_1 and $L_1 \geq 0$ such that

$$p(Tx, Ty) \leq \phi \left(\max \left\{ p(Sx, Sy), p(Sx, Tx), p(Sy, Ty), \frac{1}{2}(p(Tx, Sy) + p(Sx, Ty)) \right\} \right) + Lp(Sx, Tx)$$

for all $x, y \in X$. Moreover, assume that

- (1) $TX \subseteq SX$.
- (2) SX is a 0-complete subspace of the partial metric space X .

Then the point of coincidence of T and S is unique.

Now, we introduce an example satisfying the hypotheses of Theorem 2.3 to support the useability of our results.

Example 2.1 Let $X = [0, 1]$. Define a partial metric $p: X \times X \rightarrow [0, +\infty)$ by the formula $p(x, y) = \max\{x, y\}$. Also, define $T: X \rightarrow X$ by $Tx = \frac{x}{1+x}$ and the comparison function $\phi: [0, +\infty) \rightarrow [0, +\infty)$ by $\phi(t) = \frac{t}{1+t}$. Then, we have

- (1) (X, p) is a 0-complete partial metric space.
- (2) For any $L \geq 0$, the inequality

$$p(Tx, Ty) \leq \phi(\max\{p(x, y), p(x, Tx), p(y, Ty)\}) + L \min\{p^w(x, Tx), p^w(x, Ty), p^w(y, Tx)\}$$

holds for all $x, y \in X$.

- (3) There are no (c)-comparison function ϕ and $L \geq 0$ such that the inequality

$$p(Tx, Ty) \leq \phi(p(x, y)) + L \min\{p^w(x, Ty), p^w(y, Tx)\}$$

holds for all $x, y \in X$.

Proof To prove (2), given $x, y \in X$. Without loss of generality, we may assume that $y \leq x$. Thus, we have

$$\begin{aligned} p(Tx, Ty) &= p\left(\frac{x}{1+x}, \frac{y}{1+y}\right) \\ &= \frac{x}{1+x} \\ &= \phi(x) \\ &\leq \phi(\max\{p(x, y), p(x, Tx), p(y, Ty)\}) + L \min\{p^w(x, Tx), p^w(x, Ty), p^w(y, Tx)\}. \end{aligned}$$

To prove (3), we assume that there exist a (c)-comparison function ϕ and some $L \geq 0$ such that

$$p(Tx, Ty) \leq \phi(p(x, y)) + L \min\{p^w(x, Ty), p^w(y, Tx)\}$$

holds for all $x, y \in X$.

Thus,

$$p\left(T\frac{1}{n}, T\frac{1}{n+1}\right) \leq \phi\left(p\left(\frac{1}{n}, \frac{1}{1+n}\right)\right) + L \min\left\{p^w\left(\frac{1}{n}, T\frac{1}{n+1}\right), p^w\left(\frac{1}{n+1}, T\frac{1}{n}\right)\right\}$$

holds for all $n \in \mathbb{N}$. Hence

$$p\left(\frac{1}{n+1}, \frac{1}{n+2}\right) \leq \phi\left(p\left(\frac{1}{n}, \frac{1}{1+n}\right)\right) + L \min\left\{p^w\left(\frac{1}{n}, \frac{1}{n+2}\right), p^w\left(\frac{1}{n+1}, \frac{1}{n+1}\right)\right\}$$

holds for all $n \in \mathbb{N}$. Therefore, for $n \in \mathbb{N}$, we have

$$\frac{1}{1+n} \leq \phi\left(\frac{1}{n}\right).$$

By induction on n , we can show that

$$\frac{1}{1+n} \leq \phi^n(1)$$

holds for all $n \in \mathbb{N}$. Since $\sum_{n=0}^{\infty} \frac{1}{1+n}$ diverges, we have $\sum_{n=0}^{\infty} \phi^n(1)$ diverges. So, ϕ is not a (c)-comparison function. \square

Remark Example 2.1 satisfies the hypotheses of Theorem 2.3 and does not satisfy the hypotheses of Theorem 3 and Theorem 4 of [43].

Remarks

- (1) Theorem 1.1 [43, Theorem 3] is a special case of Corollary 2.1.
- (2) [43, Corollary 1] is a special case of Corollary 2.5.
- (3) Theorem 1.2 [43, Theorem 4] is a special case of Corollary 2.10.

3 Conclusion

In this paper, we introduced the notion of a generalized (ϕ, L) -weak contraction. In the first part of this paper, we utilized our definition to derive a common fixed point of two self-mappings T and S under a (c) -comparison function ϕ . Also, we used Lemma 2.1 of Ref. [44] to derive a common fixed point of two self-mappings T and S . In the second part of this paper, we generalized the main result (Theorem 3) of [43] by proving Theorem 2.3 under a comparison function. Also, we utilized Lemma 2.1 of Ref. [44] to derive a common coincidence point of two self-mappings T and S . Finally, we closed our paper by introducing Example 2.1 which satisfies the hypotheses of our result Theorem 2.3 and does not satisfy the hypotheses of Theorems 3, 4 of [43].

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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