Chandok and Postolache *Fixed Point Theory and Applications* 2013, **2013**:28
http://www.fixedpointtheoryandapplications.com/content/2013/1/28

Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH Open Access

Fixed point theorem for weakly Chatterjea-type cyclic contractions

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Abstract

In this article, we introduce the notion of a Chatterjea-type cyclic weakly contraction and derive the existence of a fixed point for such mappings in the setup of complete metric spaces. Our result extends and improves some fixed point theorems in the literature. Example is given to support the usability of the result.

MSC: 41A50; 47H10; 54H25

Keywords: fixed point; cyclic contraction mapping

1 Introduction and preliminaries

It is well known that the fixed point theorem of Banach, for contraction mappings, is one of the pivotal results in analysis. It has been used in many different fields of mathematics but suffers from one major drawback. More accurately, in order to use the contractive condition, a self-mapping T must be Lipschitz continuous, with the Lipschitz constant $L < 1$. In particular, T must be continuous at all points of its domain.

A natural question arises:

Could we find contractive conditions which will imply the existence of a fixed point in a complete metric space but will not imply continuity?

Kannan [1, 2] proved the following result giving an affirmative answer to the above question.

Theorem 1.1 *If (X, d) is a complete metric space and the mapping $T : X \to X$ satisfies*

$$d(Tx, Ty) \le k\big[d(x, Tx) + d(y, Ty)\big], \tag{1.1}$$

where $0 < k < \frac{1}{2}$ and $x, y \in X$, then T has a unique fixed point.

The mappings satisfying (1.1) are called *Kannan-type mappings*.

A similar type of contractive condition has been studied by Chatterjea [3]. He proved the following result.

Theorem 1.2 *If (X, d) is a complete metric space and $T : X \to X$ satisfies*

$$d(Tx, Ty) \le k\big[d(x, Ty) + d(y, Tx)\big], \tag{1.2}$$

where $0 < k < \frac{1}{2}$ and $x, y \in X$, then T has a unique fixed point.

In Theorems 1.1 and 1.2, there is no the requirement for the continuity of T.

Alber and Guerre-Delabriere [4] introduced the concept of weakly contractive mappings and proved the existence of fixed points for single-valued weakly contractive mappings in Hilbert spaces. Thereafter, in 2001, Rhoades [5] proved the fixed point theorem which is one of the generalizations of Banach's contraction mapping principle because the weakly contractions contain contractions as a special case, and he also showed that some results of [4] are true for any Banach space. In fact, weakly contractive mappings are closely related to the mappings of Boyd and Wong [6] and of Reich types [7].

Fixed point problems involving different types of contractive type inequalities have been studied by many authors (see [1–24] and the references cited therein).

In [22], Kirk *et al.* introduced the following notion of a cyclic representation and characterized the Banach contraction principle in the context of a cyclic mapping.

Definition 1.1 [22] Let X be a non-empty set and $T\colon X \to X$ be an operator. By definition, $X = \bigcup_{i=1}^{m} X_i$ is a *cyclic representation* of X with respect to T if

(a) X_i; $i = 1, \ldots, m$ are non-empty sets;

(b) $T(X_1) \subset X_2, \ldots, T(X_{m-1}) \subset X_m$, $T(X_m) \subset X_1$.

It is the aim of this paper to introduce the notion of a *cyclic weakly Chatterjea-type contraction* and then derive a fixed point theorem for such cyclic contractions in the framework of complete metric spaces.

2 Main results

To state and prove our main results, we will introduce our notion of a Chatterjea-type cyclic weakly contraction in a metric space. In this respect, let Φ denote the set of all monotone increasing continuous functions $\mu\colon [0,\infty) \to [0,\infty)$, with $\mu(t) = 0$, if and only if $t = 0$, and let Ψ denote the set of all lower semi-continuous functions $\psi\colon [0,\infty)^2 \to [0,\infty)$, with $\psi(t_1,t_2) > 0$, for $t_1, t_2 \in (0,\infty)$ and $\psi(0,0) = 0$.

Definition 2.1 Let (X,d) be a metric space, m be a natural number, $A_1, A_2, \ldots, A_m$ be non-empty subsets of X and $Y = \bigcup_{i=1}^{m} A_i$. An operator $T\colon Y \to Y$ is called a *Chatterjea-type cyclic weakly contraction* if

(1) $\bigcup_{i=1}^{m} A_i$ is a cyclic representation of Y with respect to T;

(2) $\mu(d(Tx,Ty)) \le \mu(\frac{1}{2}[d(x,Ty) + d(y,Tx)]) - \psi(d(x,Ty), d(y,Tx))$

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1, 2, \ldots, m$, where $A_{m+1} = A_1$, $\mu \in \Phi$ and $\psi \in \Psi$.

Theorem 2.1 *Let (X,d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, \ldots, A_m$ be non-empty closed subsets of X and $Y = \bigcup_{i=1}^{m} A_i$. Suppose that T is a Chatterjea-type cyclic weakly contraction. Then T has a fixed point $z \in \bigcap_{i=1}^{n} A_i$.*

Proof Let $x_0 \in X$. We can construct a sequence $x_{n+1} = Tx_n$, $n = 0, 1, 2, \ldots$.

If there exists $n_0 \in \mathbb{N}$ such that $x_{n_0+1} = x_{n_0}$, hence the result. Indeed, we can see that $Tx_{n_0} = x_{n_0+1} = x_{n_0}$.

Now, we assume that $x_{n+1} \neq x_n$ for any $n = 0, 1, 2, \ldots$. As $X = \bigcup_{i=1}^{m} A_i$, for any $n > 0$, there exists $i_n \in \{1, 2, \ldots, m\}$ such that $x_{n-1} \in A_{i_n}$ and $x_n \in A_{i_{n+1}}$. Since T is a Chatterjea-type cyclic weakly contraction, we have

$$\begin{aligned}
\mu\big(d(x_{n+1},x_n)\big) &= \mu\big(d(Tx_n,Tx_{n-1})\big) \\
&\leq \mu\left(\frac{1}{2}\big[d(x_n,Tx_{n-1})+d(x_{n-1},Tx_n)\big]\right) \\
&\quad - \psi\big(d(x_n,Tx_{n-1}),d(x_{n-1},Tx_n)\big) \\
&= \mu\left(\frac{1}{2}d(x_{n-1},x_{n+1})\right) - \psi\big(0,d(x_{n-1},x_{n+1})\big) \\
&\leq \mu\left(\frac{1}{2}d(x_{n-1},x_{n+1})\right). \qquad (2.1)
\end{aligned}$$

Since μ is a non-decreasing function, for all $n = 1,2,\ldots$, we have

$$d(x_{n+1},x_n) \leq \frac{1}{2}d(x_{n-1},x_{n+1}) \leq \frac{1}{2}\big[d(x_{n-1},x_n)+d(x_n,x_{n+1})\big]. \qquad (2.2)$$

This implies that $d(x_{n+1},x_n) \leq d(x_n,x_{n-1})$. Thus $\{d(x_{n+1},x_n)\}$ is a monotone decreasing sequence of non-negative real numbers and hence is convergent. Therefore, there exists $r \geq 0$ such that $d(x_{n+1},x_n) \to r$. Letting $n \to \infty$ in (2.2), we obtain that $\lim d(x_{n-1},x_{n+1}) = 2r$.

Letting $n \to \infty$ in (2.1) and using the continuity of μ and lower semi-continuity of ψ, we obtain that $\mu(r) \leq \mu(r) - \psi(0,2r)$. This implies that $\psi(2r,0) = 0$, hence $r = 0$. Thus we have proved that

$$d(x_{n+1},x_n) \to 0.$$

Now, we show that $\{x_n\}$ is a Cauchy sequence. For this purpose, we prove the following result first.

Lemma 2.1 *For every positive ϵ, there exists a natural number n such that if $r,q \geq n$ with $r-q \equiv 1 (\operatorname{mod} m)$, then $d(x_r,x_q) < \epsilon$.*

Proof Assume the contrary. Thus there exists $\epsilon > 0$ such that for any $n \in \mathbb{N}$, we can find $r_n > q_n \geq n$ with $r_n - q_n \equiv 1 (\operatorname{mod} m)$ satisfying $d(x_{r_n},x_{q_n}) \geq \epsilon$.

Now, we take $n > 2m$. Then, corresponding to $q_n \geq n$, we can choose r_n such that it is the smallest integer with $r_n > q_n$ satisfying $r_n - q_n \equiv 1 (\operatorname{mod} m)$ and $d(x_{r_n},x_{q_n}) \geq \epsilon$. Therefore, $d(x_{r_{n-m}},x_{q_n}) < \epsilon$. By using the triangular inequality, we have

$$\begin{aligned}
\epsilon &\leq d(x_{q_n},x_{r_n}) \\
&\leq d(x_{q_n},x_{r_{n-m}}) + \sum_{i=1}^{m} d(x_{r_{n-i}},x_{r_{n-i+1}}) \\
&< \epsilon + \sum_{i=1}^{m} d(x_{r_{n-i}},x_{r_{n-i+1}}).
\end{aligned}$$

Letting $n \to \infty$ and using $d(x_{n+1},x_n) \to 0$, we obtain

$$\lim d(x_{q_n},x_{r_n}) = \epsilon. \qquad (2.3)$$

Again, by the triangular inequality,

$$\begin{aligned}\epsilon &\leq d(x_{q_n}, x_{r_n})\\ &\leq d(x_{q_n}, x_{q_{n+1}}) + d(x_{q_{n+1}}, x_{r_{n+1}}) + d(x_{r_{n+1}}, x_{r_n})\\ &\leq d(x_{q_n}, x_{q_{n+1}}) + d(x_{q_{n+1}}, x_{q_n}) + d(x_{q_n}, x_{r_n}) + d(x_{r_n}, x_{r_{n+1}}) + d(x_{r_{n+1}}, x_{r_n}).\end{aligned}$$

Letting $n \to \infty$ and using $d(x_{n+1}, x_n) \to 0$, we get

$$\lim d(x_{q_{n+1}}, x_{r_{n+1}}) = \epsilon. \tag{2.4}$$

Consider

$$\begin{aligned}d(x_{q_n}, Tx_{r_n}) &= d(x_{q_n}, x_{r_{n+1}})\\ &\leq d(x_{q_n}, x_{r_n}) + d(x_{r_n}, x_{r_{n+1}}),\end{aligned} \tag{2.5}$$

and

$$\begin{aligned}d(x_{r_n}, Tx_{q_n}) &= d(x_{r_n}, x_{q_{n+1}})\\ &\leq d(x_{r_n}, x_{q_n}) + d(x_{q_n}, x_{q_{n+1}}).\end{aligned} \tag{2.6}$$

On taking $n \to \infty$ in inequalities (2.5) and (2.6), we have

$$\lim_{n\to\infty} d(x_{q_n}, Tx_{r_n}) = \epsilon, \tag{2.7}$$

and

$$\lim_{n\to\infty} d(x_{r_n}, Tx_{q_n}) = \epsilon. \tag{2.8}$$

As x_{q_n} and x_{r_n} lie in different adjacently labeled sets A_i and A_{i+1} for certain $1 \leq i \leq m$, using the fact that T is a Chatterjea-type cyclic weakly contraction, we obtain

$$\begin{aligned}\mu(\epsilon) &\leq \mu\big(d(x_{q_{n+1}}, x_{r_{n+1}})\big)\\ &= \mu\big(d(Tx_{q_n}, Tx_{r_n})\big)\\ &\leq \mu\left(\frac{1}{2}\big[d(x_{q_n}, Tx_{r_n}) + d(x_{r_n}, Tx_{q_n})\big]\right)\\ &\quad - \psi\big(d(x_{q_n}, Tx_{r_n}), d(x_{r_n}, Tx_{q_n})\big)\\ &= \mu\left(\frac{1}{2}\big[d(x_{q_n}, x_{r_{n+1}}) + d(x_{r_n}, x_{q_{n+1}})\big]\right)\\ &\quad - \psi\big(d(x_{q_n}, x_{r_{n+1}}), d(x_{r_n}, x_{q_{n+1}})\big).\end{aligned} \tag{2.9}$$

On taking $n \to \infty$ in (2.9), using (2.7) and (2.8), the continuity of μ and lower semi-continuity of ψ, we get that

$$\mu(\varepsilon) \leq \mu\left(\frac{1}{2}[\varepsilon + \varepsilon]\right) - \psi(\varepsilon, \varepsilon).$$

Consequently, $\psi(\varepsilon,\varepsilon) \le 0$, which is contradiction with $\varepsilon > 0$. Hence the result is proved. □

Now, using Lemma 2.1, we will show that $\{x_n\}$ is a Cauchy sequence in Y. Fix $\epsilon > 0$. By Lemma 2.1, we can find $n_0 \in \mathbb{N}$ such that $r, q \ge n_0$ with $r - q \equiv 1(\mathrm{mod}\, m)$

$$d(x_r, x_q) \le \frac{\epsilon}{2}. \tag{2.10}$$

Since $\lim d(x_n, x_{n+1}) = 0$, we can also find $n_1 \in \mathbb{N}$ such that

$$d(x_n, x_{n+1}) \le \frac{\epsilon}{2m} \tag{2.11}$$

for any $n \ge n_1$.

Assume that $r, s \ge \max\{n_0, n_1\}$ and $s > r$. Then there exists $k \in \{1, 2, \ldots, m\}$ such that $s - r \equiv k(\mathrm{mod}\, m)$. Hence $s - r + t = 1(\mathrm{mod}\, m)$ for $t = m - k + 1$. So, we have

$$d(x_r, x_s) \le d(x_r, x_{s+j}) + d(x_{s+j}, x_{s+j-1}) + \cdots + d(x_{s+1}, x_s). \tag{2.12}$$

Using (2.10), (2.11) and (2.12), we obtain

$$d(x_r, x_s) \le \frac{\epsilon}{2} + j \times \frac{\epsilon}{2m} \le \frac{\epsilon}{2} + m \times \frac{\epsilon}{2m} = \epsilon. \tag{2.13}$$

Hence $\{x_n\}$ is a Cauchy sequence in Y. Since Y is closed in X, then Y is also complete and there exists $x \in Y$ such that $\lim x_n = x$.

Now, we will prove that x is a fixed point of T.

As $Y = \bigcup_{i=1}^{m} A_i$ is a cyclic representation of Y with respect to T, the sequence $\{x_n\}$ has infinite terms in each A_i for $i = \{1, 2, \ldots, m\}$. Suppose that $x \in A_i$, $Tx \in A_{i+1}$ and we take a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ with $x_{n_k} \in A_i$. By using the contractive condition, we can obtain

$$\begin{aligned}
\mu\bigl(d(x_{n_k+1}, Tx)\bigr) &= \mu\bigl(d(Tx_{n_k}, Tx)\bigr) \\
&\le \mu\left(\frac{1}{2}\bigl[d(x_{n_k}, Tx) + d(x, Tx_{n_k})\bigr]\right) \\
&\quad - \psi\bigl(d(x_{n_k}, Tx), d(x, Tx_{n_k})\bigr) \\
&= \mu\left(\frac{1}{2}\bigl[d(x_{n_k}, Tx) + d(x, x_{n_k+1})\bigr]\right) \\
&\quad - \psi\bigl(d(x_{n_k}, Tx), d(x, x_{n_k+1})\bigr).
\end{aligned}$$

Letting $n \to \infty$ and using the continuity of μ and lower semi-continuity of ψ, we have

$$\mu\bigl(d(x, Tx)\bigr) \le \mu\left(\frac{1}{2}d(x, Tx)\right) - \psi\bigl(d(x, Tx), 0\bigr),$$

which is a contradiction unless $d(x, Tx) = 0$. Hence x is a fixed point of T.

Now, we will prove the uniqueness of the fixed point.

Suppose that x_1 and x_2 ($x_1 \neq x_2$) are two fixed points of T. Using the contractive condition and the continuity of μ and lower semi continuity of ψ, we have

$$\begin{aligned}
\mu\big(d(x_1,x_2)\big) &= \mu\big(d(Tx_1,Tx_2)\big) \\
&\leq \mu\left(\frac{1}{2}\big[d(x_1,Tx_2)+d(x_2,Tx_1)\big]\right) - \psi\big(d(x_1,Tx_2),d(x_2,Tx_1)\big) \\
&= \mu\left(\frac{1}{2}\big[d(x_1,x_2)+d(x_2,x_1)\big]\right) - \psi\big(d(x_1,x_2),d(x_2,x_1)\big) \\
&= \mu\big(d(x_1,x_2)\big) - \psi\big(d(x_1,x_2),d(x_2,x_1)\big) \\
&\leq \mu\big(d(x_1,x_2)\big),
\end{aligned}$$

which is a contradiction unless $x_1 = x_2$. Hence the main result is proved. □

If $\mu(a) = a$, then we have the following result.

Corollary 2.1 *Let (X,d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, \dots, A_m$ be non-empty closed subsets of X and $Y = \bigcup_{i=1}^m A_i$. Suppose that $T\colon Y \to Y$ is an operator such that*

(1) *$\bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T;*
(2) *$d(Tx,Ty) \leq \frac{1}{2}[d(x,Ty)+d(y,Tx)] - \psi(d(x,Ty),d(y,Tx))$*

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1,2,\dots,m$, where $A_{m+1} = A_1$ and $\psi \in \Psi$. Then T has a fixed point $z \in \bigcap_{i=1}^n A_i$.

If $\psi(a,b) = (\frac{1}{2} - k)(a+b)$, where $k \in [0, \frac{1}{2})$, we have the following result.

Corollary 2.2 *Let (X,d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, \dots, A_m$ be non-empty closed subsets of X and $Y = \bigcup_{i=1}^m A_i$. Suppose that $T\colon Y \to Y$ is an operator such that*

(1) *$\bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T;*
(2) *there exists $k \in [0, \frac{1}{2})$ such that $d(Tx,Ty) \leq k[d(x,Ty)+d(y,Tx)]$*

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1,2,\dots,m$, where $A_{m+1} = A_1$. Then T has a fixed point $z \in \bigcap_{i=1}^n A_i$.

3 Applications

Other consequences of our results, for mappings involving contractions of integral type, are given in the following. In this respect, denote by Λ the set of functions $\mu\colon [0,\infty) \to [0,\infty)$ satisfying the following hypotheses:

(h1) μ is a Lebesgue-integrable mapping on each compact of $[0,\infty)$;
(h2) for any $\epsilon > 0$, we have $\int_0^\epsilon \mu(t) > 0$.

Corollary 3.1 *Let (X,d) be a complete metric space, $m \in \mathbb{N}$, $A_1, A_2, \dots, A_m$ be non-empty closed subsets of X and $Y = \bigcup_{i=1}^m A_i$. Suppose that $T\colon Y \to Y$ is an operator such that*

(1) *$\bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T;*
(2) *there exists $k \in [0, \frac{1}{2})$ such that*

$$\int_0^{d(Tx,Ty)} \alpha(s)\,ds \leq k \int_0^{d(x,Ty)+d(y,Tx)} \alpha(s)\,ds$$

for any $x \in A_i$, $y \in A_{i+1}$, $i = 1,2,\dots,m$, where $A_{m+1} = A_1$ and $\alpha \in \Lambda$. Then T has a fixed point $z \in \bigcap_{i=1}^n A_i$.

If we take $A_i = X$, $i = 1, 2, \dots, m$, we obtain the following result.

Corollary 3.2 *Let (X,d) be a complete metric space and $T: X \to X$ be a mapping such that*

$$\int_0^{d(Tx,Ty)} \alpha(s)\,ds \le k \int_0^{d(x,Ty)+d(y,Tx)} \alpha(s)\,ds,$$

for any $xy \in X$, $k \in [0, \frac{1}{2})$ and $\alpha \in \Lambda$. Then T has a fixed point $z \in \bigcap_{i=1}^n A_i$.

Example 3.1 Let X be a subset in $\mathbb{R}$ endowed with the usual metric. Suppose $A_1 = [0,1]$, $A_2 = [0, \frac{1}{2}]$ and $Y = \bigcup_{i=1}^2 A_i$. Define $T: Y \to Y$ such that $Tx = \frac{x}{5}$ for all $x \in Y$. It is clear that $\bigcup_{i=1}^2 A_i$ is a cyclic representation of Y with respect to T. Furthermore, if $\mu: [0,\infty) \to [0,\infty)$ is given as $\mu(t) = t$ and $\psi: [0,\infty)^2 \to [0,\infty)$ is given by $\psi(x,y) = \frac{1}{7}(x+y)$, then $\psi \in \Psi$.

Now, we prove that T satisfies the inequality of Chatterjea-type cyclic weakly contraction, *i.e.*, $\mu(d(Tx,Ty)) \le \mu(\frac{1}{2}[d(x,Ty) + d(y,Tx)]) - \psi(d(x,Ty), d(y,Tx))$. To see this fact, we examine three cases.

Case 1. Suppose that $x \ge y$. Then

$$\mu\bigl(d(Tx,Ty)\bigr) = \mu\left(\left|\frac{x}{5} - \frac{y}{5}\right|\right) = \frac{x-y}{5} \tag{3.1}$$

and

$$\begin{aligned}
&\mu\left(\frac{1}{2}\bigl[d(x,Ty) + d(y,Tx)\bigr]\right) - \psi\bigl(d(x,Ty), d(y,Tx)\bigr)\\
&\quad= \mu\left(\frac{1}{2}\left[\left|x - \frac{y}{5}\right| + \left|y - \frac{x}{5}\right|\right]\right) - \psi\left(\left|x - \frac{y}{5}\right|, \left|y - \frac{x}{5}\right|\right)\\
&\quad= \frac{1}{2}\left[\left|x - \frac{y}{5}\right| + \left|y - \frac{x}{5}\right|\right] - \frac{1}{7}\left[\left|x - \frac{y}{5}\right| + \left|y - \frac{x}{5}\right|\right]\\
&\quad= \frac{5}{14}\left[\left|x - \frac{y}{5}\right| + \left|y - \frac{x}{5}\right|\right].
\end{aligned} \tag{3.2}$$

If $y < \frac{x}{5}$, then

$$\begin{aligned}
\frac{x-y}{5} &\le \frac{5}{14}\left[x - \frac{y}{5} + \frac{x}{5} - y\right]\\
&= \frac{3}{7}(x-y).
\end{aligned}$$

Hence, the given inequality is satisfied.

If $y \ge \frac{x}{5}$, then

$$\begin{aligned}
\frac{x-y}{5} &\le \frac{5}{14}\left[x - \frac{y}{5} + y - \frac{x}{5}\right]\\
&= \frac{2}{7}(x+y).
\end{aligned}$$

Hence the given inequality is satisfied.

Case 2. Suppose that $\frac{y}{5} \leq x \leq y$. Then from (3.1) and (3.2), we have

$$\begin{aligned} \frac{x-y}{5} &\leq \frac{5}{14}\left[x - \frac{y}{5} + y - \frac{x}{5}\right] \\ &= \frac{2}{7}(x+y). \end{aligned}$$

Hence the given inequality is satisfied.

Case 3. Finally, suppose that $\frac{y}{5} \geq x$. Then from (3.1) and (3.2), we have

$$\mu\bigl(d(Tx, Ty)\bigr) = \mu\left(\left|\frac{x}{5} - \frac{y}{5}\right|\right) = \frac{y-x}{5}$$

and

$$\begin{aligned} \frac{x-y}{5} &\leq \frac{5}{14}\left[x - \frac{y}{5} + y - \frac{x}{5}\right] \\ &= \frac{2}{7}(x+y). \end{aligned}$$

Hence the given inequality is satisfied.

Therefore, all the conditions of Theorem 2.1 are satisfied, and so T has a fixed point (which is $z = 0 \in \bigcap_{i=1}^{2} A_i$).

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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Acknowledgements

The authors are thankful to learned referee(s) for suggestions.

Received: 28 November 2012 Accepted: 27 January 2013 Published: 11 February 2013

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doi:10.1186/1687-1812-2013-28
Cite this article as: Chandok and Postolache: Fixed point theorem for weakly Chatterjea-type cyclic contractions. *Fixed Point Theory and Applications* 2013 **2013**:28.