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Coupled coincidence point theorems for contractions without commutative condition in intuitionistic fuzzy normed spaces

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Abstract

Recently, Gordji et al. [Math. Comput. Model. 54, 1897-1906 (2011)] prove the coupled coincidence point theorems for nonlinear contraction mappings satisfying commutative condition in intuitionistic fuzzy normed spaces. The aim of this article is to extend and improve some coupled coincidence point theorems of Gordji et al. Also, we give an example of a nonlinear contraction mapping which is not applied by the results of Gordji et al., but can be applied to our results.

2000 MSC: primary 47H10; secondary 54H25; 34B15.

Keywords: intuitionistic fuzzy normed space, coupled fixed point, coupled coincidence point, partially ordered set, commutative condition

1. Introduction

The classical Banach's contraction mapping principle first appear in [1]. Since this principle is a powerful tool in nonlinear analysis, many mathematicians have much contributed to the improvement and generalization of this principle in many ways (see [2-10] and others).

One of the most interesting is study to other spaces such as probabilistic metric spaces (see [11-15]). The fuzzy theory was introduced simultaneously by Zadeh [16]. The idea of intuitionistic fuzzy set was first published by Atanassov [17]. Since then, Saadati and Park [18] introduced the concept of intuitionistic fuzzy normed spaces (IFNSs). In [19], Saadati et al. have modified the notion of IFNSs of Saadati and Park [18].

Several researchers have applied fuzzy theory to the well-known results in many fields, for example, quantum physics [20], nonlinear dynamical systems [21], population dynamics [22], computer programming [23], fixed point theorem [24-27], fuzzy stability problems [28-30], statistical convergence [31-34], functional equation [35], approximation theory [36], nonlinear equation [37,38] and many others.

In the other hand, coupled fixed points and their applications for binary mappings in partially ordered metric spaces were introduced by Bhaskar and Lakshmikantham [39]. They applied coupled fixed point theorems to show the existence and uniqueness of a solution for a periodic boundary value problem. After that, Lakshmikantham and Ćirić

[40] proved some more generalizations of coupled fixed point theorems in partially ordered sets.

Recently, Gordji et al. [41] proved some coupled coincidence point theorems for contractive mappings satisfying commutative condition in partially complete IFNSs as follows:

Theorem 1.1 (Gordji et al. [41]). *Let (X, \preceq) be a partially ordered set and $(X, \mu, \nu, *, \diamond)$ a complete IFNS such that (μ, ν) has n -property and*

$$a \diamond b \leq ab \leq a * b, \quad \forall a, b \in [0, 1]. \tag{1.1}$$

Let $F: X \times X \rightarrow X$ and $g: X \rightarrow X$ be two mappings such that F has the mixed g -monotone property and

$$\begin{aligned} \mu(F(x, y) - F(u, v), kt) &\geq \mu(gx - gu, t) * \mu(gy - gv, t), \quad \forall x, y, u, v \in X, \\ \nu(F(x, y) - F(u, v), kt) &\leq \nu(gx - gu, t) \diamond \nu(gy - gv, t), \quad \forall x, y, u, v \in X, \end{aligned} \tag{1.2}$$

for which $g(x) \preceq g(u)$ and $g(y) \succeq g(v)$, where $0 < k < 1$, $F(X \times X) \subseteq g(X)$, g is continuous and g commuting with F . Suppose that either

- (1) F is continuous or
- (2) X has the following properties:
 - (a) if $\{x_n\}$ is a non-decreasing sequence with $\{x_n\} \rightarrow x$, then $gx_n \preceq gx$ for all $n \in \mathbb{N}$,
 - (b) if $\{y_n\}$ is a non-increasing sequence with $\{y_n\} \rightarrow y$, then $gy \preceq gy_n$ for all $n \in \mathbb{N}$.

If there exist $x_0, y_0 \in X$ such that

$$g(x_0) \preceq F(x_0, y_0), \quad g(y_0) \succeq F(y_0, x_0),$$

then F and g have a coupled coincidence point in $X \times X$.

In this article, we improve the result given by Gordji et al. [41] without using the commutative condition and also give an example to validate the main results in this article. Our results improve and extend some couple fixed point theorems due to Gordji et al. [41] and other couple fixed point theorems.

2. Preliminaries

Now, we give some definitions, examples and lemmas for our main results in this article.

Definition 2.1 ([42]). A binary operation $*$: $[0,1]^2 \rightarrow [0,1]$ is called a *continuous t -norm* if $([0,1], *)$ is an abelian topological monoid, i.e.,

- (1) $*$ is associative and commutative;
- (2) $*$ is continuous;
- (3) $a * 1 = a$ for all $a \in [0,1]$;
- (4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2.2 ([42]). A binary operation \diamond : $[0,1]^2 \rightarrow [0,1]$ is called a *continuous t -conorm* if $([0,1], \diamond)$ is an abelian topological monoid, i.e.,

- (1) \diamond is associative and commutative;
- (2) \diamond is continuous;
- (3) $a \diamond 0 = a$ for all $a \in [0,1]$;
- (4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Using the continuous t -norm and t -conorm, Saadati and Park [18] introduced the concept of IFNSs.

Definition 2.3 ([18]). The 5-tuple $(X, \mu, \nu, *, \diamond)$ is called an IFNS if X is a vector space, $*$ is a continuous t -norm, \diamond is a continuous t -conorm and μ, ν are fuzzy sets on $X \times (0, \infty)$ satisfying the following conditions: for all $x, y \in X$ and $s, t > 0$,

- (IF₁) $\mu(x, t) + \nu(x, t) \leq 1$;
 - (IF₂) $\mu(x, t) > 0$;
 - (IF₃) $\mu(x, t) = 1$ if and only if $x = 0$;
 - (IF₄) $\mu(\alpha x, t) = \mu\left(x, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;
 - (IF₅) $\mu(x, t) * \mu(y, s) \leq \mu(x + y, t + s)$;
 - (IF₆) $\mu(x, \cdot): (0, \infty) \rightarrow [0,1]$ is continuous;
 - (IF₇) μ is a non-decreasing function on \mathbb{R}^+ ,
- $$\lim_{t \rightarrow \infty} \mu(x, t) = 1, \quad \lim_{t \rightarrow 0} \mu(x, t) = 0;$$

- (IF₈) $\nu(x, t) < 1$;
 - (IF₉) $\nu(x, t) = 0$ if and only if $x = 0$;
 - (IF₁₀) $\nu(\alpha x, t) = \nu\left(x, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;
 - (IF₁₁) $\nu(x, t) \diamond \nu(y, s) \geq \nu(x + y, t + s)$;
 - (IF₁₂) $\nu(x, \cdot): (0, \infty) \rightarrow [0,1]$ is continuous;
 - (IF₁₃) ν is a non-increasing function on \mathbb{R}^+ ,
- $$\lim_{t \rightarrow \infty} \nu(x, t) = 0, \quad \lim_{t \rightarrow 0} \nu(x, t) = 1.$$

In this case, (μ, ν) is called an *intuitionistic fuzzy norm*.

Definition 2.4 ([18]). Let $(X, \mu, \nu, *, \diamond)$ be an IFNS.

(1) A sequence $\{x_n\}$ in X is said to be *convergent* to a point $x \in X$ with respect to the intuitionistic fuzzy norm (μ, ν) if, for any $\varepsilon > 0$ and $t > 0$, there exists $k \in \mathbb{N}$ such that

$$\mu(x_n - x, t) > 1 - \varepsilon, \quad \nu(x_n - x, t) < \varepsilon, \quad \forall n \geq k.$$

In this case, we write $\lim_{n \rightarrow \infty} x_n = x$. In fact that $\lim_{n \rightarrow \infty} x_n = x$ if $\mu(x_n - x, t) \rightarrow 1$ and $\nu(x_n - x, t) \rightarrow 0$ as $n \rightarrow \infty$ for every $t > 0$.

(2) A sequence $\{x_n\}$ in X is called a *Cauchy sequence* with respect to the intuitionistic fuzzy norm (μ, ν) if, for any $\varepsilon > 0$ and $t > 0$, there exists $k \in \mathbb{N}$ such that

$$\mu(x_n - x_m, t) > 1 - \varepsilon, \quad \nu(x_n - x_m, t) < \varepsilon, \quad \forall n, m \geq k.$$

This implies $\{x_n\}$ is Cauchy if $\mu(x_n - x_m, t) \rightarrow 1$ and $\nu(x_n - x_m, t) \rightarrow 0$ as $n, m \rightarrow \infty$ for every $t > 0$.

(3) An IFNS $(X, \mu, \nu, *, \diamond)$ is said to be *complete* if every Cauchy sequence in $(X, \mu, \nu, *, \diamond)$ is convergent.

Definition 2.5 ([43,44]). Let X and Y be two IFNS. A function $g : X \rightarrow Y$ is said to be *continuous* at a point $x_0 \in X$ if, for any sequence $\{x_n\}$ in X converging to a point $x_0 \in X$, the sequence $\{g(x_n)\}$ in Y converges to a point $g(x_0) \in Y$. If $g : X \rightarrow Y$ is continuous at each $x \in X$, then $g : X \rightarrow Y$ is said to be *continuous* on X .

Example 2.6 ([41]). Let $(X, \|\cdot\|)$ be an ordinary normed space and θ an increasing and continuous function from \mathbb{R}^+ into $(0,1)$ such that $\lim_{t \rightarrow \infty} \theta(t) = 1$. Four typical examples of these functions are as follows:

$$\theta(t) = \frac{t}{t+1}, \quad \theta(t) = \sin\left(\frac{\pi t}{2t+1}\right), \quad \theta(t) = 1 - e^{-t}, \quad \theta(t) = e^{-\frac{1}{t}}.$$

Let $a * b = ab$ and $a \diamond b \geq ab$ for all $a, b \in [0,1]$. If, for any $t \in (0, \infty)$, we define

$$\mu(x, t) = [\theta(t)]^{\|x\|}, \quad \nu(x, t) = 1 - [\theta(t)]^{\|x\|}, \quad \forall x \in X,$$

then $(X, \mu, \nu, *, \diamond)$ is an IFNS.

The other basic properties and examples of IFNSs are given in [18].

Definition 2.7 ([41]). Let $(X, \mu, \nu, *, \diamond)$ be an IFNS. (μ, ν) is said to satisfy the *n-property* on $X \times (0, \infty)$ if

$$\lim_{n \rightarrow \infty} [\mu(x, k^n t)]^{n^p} = 1, \quad \lim_{n \rightarrow \infty} [\nu(x, k^n t)]^{n^p} = 0$$

whenever $x \in X, k > 1$ and $p > 0$.

For examples for *n-property* see in [41]. Next, we give some notion in coupled fixed point theory.

Definition 2.8 ([39]). Let X be a non-empty set. An element $(x, y) \in X \times X$ is call a *coupled fixed point* of the mapping $F : X \times X \rightarrow X$ if

$$x = F(x, y), \quad y = F(y, x).$$

Definition 2.9 ([40]). Let X be a non-empty set. An element $(x, y) \in X \times X$ is call a *coupled coincidence point* of the mappings $F : X \times X \rightarrow X$ and $g : X \rightarrow X$ if

$$g(x) = F(x, y), \quad g(y) = F(y, x).$$

Definition 2.10 ([39]). Let (X, \preceq) be a partially ordered set and $F : X \times X \rightarrow X$ be a mapping. The mapping F is said to has the *mixed monotone property* if F is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument, that is, for any $x, y \in X$

$$x_1, x_2 \in X, x_1 \preceq x_2 \quad \Rightarrow \quad F(x_1, y) \preceq F(x_2, y) \tag{2.1}$$

and

$$y_1, y_2 \in X, y_1 \preceq y_2 \quad \Rightarrow \quad F(x, y_1) \succeq F(x, y_2). \tag{2.2}$$

Definition 2.11 ([40]). Let (X, \preceq) be a partially ordered set and $F : X \times X \rightarrow X, g : X \rightarrow X$ be mappings. The mapping F is said to has the *mixed g-monotone property* if F is monotone *g*-non-decreasing in its first argument and is monotone *g*-non-increasing in its second argument, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, g(x_1) \preceq g(x_2) \Rightarrow F(x_1, \gamma) \preceq F(x_2, \gamma) \tag{2.3}$$

and

$$\gamma_1, \gamma_2 \in X, g(\gamma_1) \preceq g(\gamma_2) \Rightarrow F(x, \gamma_1) \succeq F(x, \gamma_2). \tag{2.4}$$

Definition 2.12 ([40]). Let X be a non-empty set and $F : X \times X \rightarrow X, g : X \rightarrow X$ be mappings. The mappings F and g are said to be *commutative* if

$$g(F(x, \gamma)) = F(g(x), g(\gamma)), \quad \forall x, \gamma \in X.$$

The following lemma proved by Haghi et al. [45] is useful for our main results:

Lemma 2.13 ([45]). *Let X be a nonempty set and $g : X \rightarrow X$ be a mapping. Then, there exists a subset $E \subseteq X$ such that $g(E) = g(X)$ and $g : E \rightarrow X$ is one-to-one.*

3. Main Results

First, we prove a coupled fixed point theorem for a mapping $F : X \times X \rightarrow X$ which is an essential tool in the partial order IFNSs to show the existence of coupled fixed point. Although the proof in Theorem 3.1 is not difficult to modify, it is an important theorem which is helpful in proving some coupled coincidence point theorems without commutative condition.

Theorem 3.1. *Let (X, \preceq) be a partially ordered set and $(X, \mu, \nu, *, \diamond)$ a complete IFNS such that (μ, ν) has n -property and*

$$a \diamond b \leq ab \leq a * b, \quad \forall a, b \in [0, 1]. \tag{3.1}$$

Let $F : X \times X \rightarrow X$ be mapping such that F has the mixed monotone property and

$$\begin{aligned} \mu(F(x, \gamma) - F(u, \nu), kt) &\geq \mu(x - u, t) * \mu(\gamma - \nu, t), \quad \forall x, \gamma, u, \nu \in X, \\ \nu(F(x, \gamma) - F(u, \nu), kt) &\leq \nu(x - u, t) \diamond \nu(\gamma - \nu, t), \quad \forall x, \gamma, u, \nu \in X, \end{aligned} \tag{3.2}$$

for which $x \preceq u$ and $\gamma \succeq \nu$, where $0 < k < 1$. Suppose that either

- (1) F is continuous or
- (2) X has the following properties:
 - (a) if $\{x_n\}$ is a non-decreasing sequence with $\{x_n\} \rightarrow x$, then $x_n \preceq x$ for all $n \in \mathbb{N}$,
 - (b) if $\{\gamma_n\}$ is a non-increasing sequence with $\{\gamma_n\} \rightarrow \gamma$, then $\gamma \preceq \gamma_n$ for all $n \in \mathbb{N}$.

If there exist $x_0, \gamma_0 \in X$ such that

$$x_0 \preceq F(x_0, \gamma_0), \quad \gamma_0 \succeq F(\gamma_0, x_0),$$

then F has a coupled fixed point in $X \times X$.

Proof. Let $x_0, \gamma_0 \in X$ be such that

$$x_0 \preceq F(x_0, \gamma_0), \quad \gamma_0 \succeq F(\gamma_0, x_0).$$

Since $F(X \times X) \subseteq X$, we can construct the sequences $\{x_n\}$ and $\{\gamma_n\}$ in X such that

$$x_{n+1} = F(x_n, \gamma_n), \quad \gamma_{n+1} = F(\gamma_n, x_n), \quad \forall n \geq 0. \tag{3.3}$$

Now, we show that

$$x_n \preceq x_{n+1}, \quad \gamma_n \succeq \gamma_{n+1}, \quad \forall n \geq 0. \tag{3.4}$$

In fact, by induction, we prove this. For $n = 0$, since $x_0 \preceq F(x_0, y_0) = x_1$ and $y_0 = F(y_0, x_0) \succcurlyeq y_1$, we show that (3.4) holds for $n = 0$. Suppose that (3.4) holds for any $n \geq 0$. Then, we have

$$x_n \preceq x_{n+1}, \quad y_n \succcurlyeq y_{n+1}. \tag{3.5}$$

Since F has the mixed monotone property, it follows from (3.5) and (2.1) that

$$F(x_n, y) \preceq F(x_{n+1}, y), \quad F(y_{n+1}, x) \preceq F(y_n, x), \quad \forall x, y \in X, \tag{3.6}$$

and also it follows from (3.5) and (2.2) that

$$F(y, x_n) \succcurlyeq F(y, x_{n+1}), \quad F(x, y_{n+1}) \succcurlyeq F(x, y_n), \quad \forall x, y \in X. \tag{3.7}$$

If we take $y = y_n$ and $x = x_n$ in (3.6), then we get

$$x_{n+1} = F(x_n, y_n) \preceq F(x_{n+1}, y_n), \quad F(y_{n+1}, x_n) \preceq F(y_n, x_n) = y_{n+1}. \tag{3.8}$$

If we take $y = y_{n+1}$ and $x = x_{n+1}$ in (3.7), then we get

$$F(y_{n+1}, x_n) \succcurlyeq F(y_{n+1}, x_{n+1}) = y_{n+2}, \quad x_{n+2} = F(x_{n+1}, y_{n+1}) \succcurlyeq F(x_{n+1}, y_n). \tag{3.9}$$

Hence, it follows from (3.8) and (3.9) that

$$x_{n+1} \preceq x_{n+2}, \quad y_{n+1} \succcurlyeq y_{n+2}. \tag{3.10}$$

Therefore, by induction, we conclude that (3.4) holds for all $n \geq 0$, that is,

$$x_0 \preceq x_1 \preceq x_2 \preceq \dots \preceq x_n \preceq x_{n+1} \preceq \dots \tag{3.11}$$

and

$$y_0 \succcurlyeq y_1 \succcurlyeq y_2 \succcurlyeq \dots \succcurlyeq y_n \succcurlyeq y_{n+1} \succcurlyeq \dots \tag{3.12}$$

Define $\alpha_n(t) := \mu(x_n - x_{n+1}, t) * \mu(y_n - y_{n+1}, t)$. Then, using (3.2) and (3.3), we have

$$\begin{aligned} \mu(x_n - x_{n+1}, kt) &= \mu(F(x_{n-1}, y_{n-1}) - F(x_n, y_n), kt) \\ &\geq \mu(x_{n-1} - x_n, t) * \mu(y_{n-1} - y_n, t) \\ &= \alpha_{n-1}(t) \end{aligned} \tag{3.13}$$

and

$$\begin{aligned} \mu(y_n - y_{n+1}, kt) &= \mu(y_{n+1} - y_n, kt) \\ &= \mu(F(y_n, x_n) - F(y_{n-1}, x_{n-1}), kt) \\ &\geq \mu(y_n - y_{n-1}, t) * \mu(x_n - x_{n-1}, t) \\ &= \mu(y_{n-1} - y_n, t) * \mu(x_{n-1} - x_n, t) \\ &= \alpha_{n-1}(t). \end{aligned} \tag{3.14}$$

From the t -norm property, (3.13) and (3.14), it follows that

$$\alpha_n(kt) \geq \alpha_{n-1}(t) * \alpha_{n-1}(t). \tag{3.15}$$

From (3.1), we have

$$\alpha_{n-1}(t) * \alpha_{n-1}(t) \geq [\alpha_{n-1}(t)]^2. \tag{3.16}$$

By (3.15) and (3.16), we get $\alpha_n(kt) \geq [\alpha_{n-1}(t)]^2$ for all $n \geq 1$. Repeating this process, we have

$$\alpha_n(t) \geq \left[\alpha_{n-1} \left(\frac{t}{k} \right) \right]^2 \geq \dots \geq \left[\alpha_0 \left(\frac{t}{k^n} \right) \right]^{2^n}, \tag{3.17}$$

which implies that

$$\mu(x_n - x_{n+1}, kt) * \mu(\gamma_n - \gamma_{n+1}, kt) \geq \left[\mu \left(x_0 - x_1, \frac{t}{k^n} \right) \right]^{2^n} * \left[\mu \left(\gamma_0 - \gamma_1, \frac{t}{k^n} \right) \right]^{2^n}. \tag{3.18}$$

On the other hand, we have

$$t(1 - k)(1 + k + \dots + k^{m-n-1}) < t, \quad \forall m > n, \quad 0 < k < t.$$

By property of t-norm, we get

$$\begin{aligned} & \mu(x_n - x_m, t) * \mu(\gamma_n - \gamma_m, t) \\ & \geq \mu(x_n - x_m, t(1 - k)(1 + k + \dots + k^{m-n-1})) \\ & \quad * \mu(\gamma_n - \gamma_m, t(1 - k)(1 + k + \dots + k^{m-n-1})) \\ & \geq \mu(x_n - x_{n+1}, t(1 - k)) * \mu(\gamma_n - \gamma_{n+1}, t(1 - k)) \\ & \quad * \mu(x_{n+1} - x_{n+2}, t(t - k)k) * \mu(\gamma_{n+1} - \gamma_{n+2}, t(1 - k)k) \\ & \quad * \dots \\ & \quad * \mu(x_{m-1} - x_m, t(1 - k)k^{m-n-1}) * \mu(\gamma_{m-1} - \gamma_m, t(t - k)k^{m-n-1}) \\ & \geq \mu \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) * \mu \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \\ & \quad * \dots \\ & \quad * \mu \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) * \mu \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \\ & \geq \left[\mu \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \right]^{m-n} * \left[\mu \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \right]^{m-n} \\ & \geq \left[\mu \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \right]^m * \left[\mu \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \right]^m \\ & \geq \left[\mu \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \right]^{np} * \left[\mu \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \right]^{np}, \end{aligned} \tag{3.19}$$

where $p > 0$ such that $m < n^p$. Since (μ, ν) has the n -property, we have

$$\lim_{n \rightarrow \infty} \left[\mu \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \right]^{np} = 1$$

and so

$$\lim_{n \rightarrow \infty} \mu(x_n - x_m) * \mu(\gamma_n - \gamma_m) = 1. \tag{3.20}$$

Next, we claim that

$$\lim_{n \rightarrow \infty} \nu(x_n - x_m) \diamond \nu(\gamma_n - \gamma_m) = 0.$$

Define $\beta_n(t) := \nu(x_n - x_{n+1}, t) \diamond \nu(\gamma_n - \gamma_{n+1}, t)$. It follows from (3.2) and (3.3) that

$$\begin{aligned} \nu(x_n - x_{n+1}, kt) &= \nu(F(x_{n-1}, \gamma_{n-1}) - F(x_n, \gamma_n), kt) \\ &\leq \nu(x_{n-1} - x_n, t) \diamond \nu(\gamma_{n-1} - \gamma_n, t) \\ &= \beta_{n-1}(t) \end{aligned} \tag{3.21}$$

and

$$\begin{aligned}
 v(\gamma_n - \gamma_{n+1}, kt) &= v(\gamma_{n+1} - \gamma_n, kt) \\
 &= v(F(\gamma_n, x_n) - F(\gamma_{n-1}, x_{n-1}), kt) \\
 &\leq v(\gamma_n - \gamma_{n-1}, t) \diamond v(x_n - x_{n-1}, t) \\
 &= v(\gamma_{n-1} - \gamma_n, t) \diamond v(x_{n-1} - x_n, t) \\
 &= \beta_{n-1}(t).
 \end{aligned}
 \tag{3.22}$$

Thus, it follows from the notion of t -conorm, (3.21) and (3.22) that

$$\beta_n(kt) \leq \beta_{n-1}(t) \diamond \beta_{n-1}(t).
 \tag{3.23}$$

From (3.1), we have

$$\beta_{n-1}(t) \diamond \beta_{n-1}(t) \leq [\beta_{n-1}(t)]^2.
 \tag{3.24}$$

Thus, by (3.23) and (3.24), we get $\beta_n(kt) \leq [\beta_{n-1}(t)]^2$ for all $n \geq 1$. Repeating this process again, we have

$$\beta_n(t) \leq \left[\beta_{n-1} \left(\frac{t}{k} \right) \right]^2 \leq \dots \leq \left[\beta_0 \left(\frac{t}{k^n} \right) \right]^{2^n},
 \tag{3.25}$$

that is,

$$v(x_n - x_{n+1}, kt) \diamond v(\gamma_n - \gamma_{n+1}, kt) \leq \left[v \left(x_0 - x_1, \frac{t}{k^n} \right) \right] \diamond \left[v \left(\gamma_0 - \gamma_1, \frac{t}{k^n} \right) \right]^{2^n}.
 \tag{3.26}$$

Since we have

$$t(1 - k)(1 + k + \dots + k^{m-n-1}) < t, \quad \forall m > n, \quad 0 < k < 1,$$

by the t -conorm property, we get

$$\begin{aligned}
 &v(x_n - x_m, t) \diamond v(\gamma_n - \gamma_m, t) \\
 &\leq v(x_n - x_m, t(1 - k)(1 + k + \dots + k^{m-n-1})) \\
 &\quad \diamond v(\gamma_n - \gamma_m, t(1 - k)(1 + k + \dots + k^{m-n-1})) \\
 &\leq v(x_n - x_{n+1}, t(1 - k)) \diamond v(\gamma_n - \gamma_{n+1}, t(1 - k)) \\
 &\quad \diamond v(x_{n+1} - x_{n+2}, t(1 - k)k) \diamond v(\gamma_{n+1} - \gamma_{n+2}, t(1 - k)k) \\
 &\quad \diamond \dots \\
 &\quad \diamond v(x_{m-1} - x_m, t(1 - k)k^{m-n-1}) \diamond v(\gamma_{m-1} - \gamma_m, t(1 - k)k^{m-n-1}) \\
 &\leq v \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \diamond \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \\
 &\quad \diamond \dots \\
 &\quad \diamond v \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \diamond v \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \\
 &\leq \left[v \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \right]^{m-n} \diamond \left[v \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \right]^{m-n} \\
 &\leq \left[v \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \right]^m \diamond \left[v \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \right]^m \\
 &\leq \left[v \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \right]^{np} \diamond \left[v \left(\gamma_0 - \gamma_1, (1 - k) \frac{t}{k^n} \right) \right]^{np},
 \end{aligned}
 \tag{3.27}$$

where $p > 0$ such that $m < n^p$. Since (μ, ν) has the n -property, we have

$$\lim_{n \rightarrow \infty} \left[\nu \left(x_0 - x_1, (1 - k) \frac{t}{k^n} \right) \right]^{n^p} = 0$$

and so

$$\lim_{n \rightarrow \infty} \nu(x_n - x_m) \diamond \nu(y_n - y_m) = 0. \tag{3.28}$$

From (3.20) and (3.28), we know that the sequences $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X . Since X complete, there exist $x, y \in X$ such that

$$\lim_{n \rightarrow \infty} x_n = x, \quad \lim_{n \rightarrow \infty} y_n = y. \tag{3.29}$$

Next, we show that $x = F(x, y)$ and $y = F(y, x)$. If the assumption (1) holds, then we have

$$x = \lim_{n \rightarrow \infty} x_{n+1} = \lim_{n \rightarrow \infty} F(x_n, y_n) = F(\lim_{n \rightarrow \infty} x_n, \lim_{n \rightarrow \infty} y_n) = F(x, y) \tag{3.30}$$

and

$$y = \lim_{n \rightarrow \infty} y_{n+1} = \lim_{n \rightarrow \infty} F(y_n, x_n) = F(\lim_{n \rightarrow \infty} y_n, \lim_{n \rightarrow \infty} x_n) = F(y, x). \tag{3.31}$$

Therefore, $x = F(x, y)$ and $y = F(y, x)$, that is, F has a coupled fixed point.

Suppose that the assumption (2) holds. Since $\{x_n\}$ is non-decreasing and $x_n \rightarrow x$, it follows from (a) that $x_n \preceq x$ for all $n \in \mathbb{N}$. Similarly, we can conclude that $y_n \succeq y$ for all $n \in \mathbb{N}$. Then, by (3.2), we get

$$\begin{aligned} \mu(x_{n+1} - F(x, y), kt) &= \mu(F(x_n, y_n) - F(x, y), kt) \\ &\geq \mu(x_n - x, t) * \mu(y_n - y, t). \end{aligned} \tag{3.32}$$

Taking the limit as $n \rightarrow \infty$, we have $\mu(x - F(x, y), kt) = 1$ and so $x = F(x, y)$. Using (3.2) again, we have

$$\begin{aligned} \nu(y_{n+1} - F(y, x), kt) &= \nu(F(y_n, x_n) - F(y, x), kt) \\ &= \nu(F(y, x) - F(y_n, x_n), kt) \\ &\leq \nu(y - y_n, t) \diamond \nu(x - x_n, t) \\ &= \nu(y_n - y, t) \diamond \nu(x_n - x, t). \end{aligned} \tag{3.33}$$

Taking the limit as $n \rightarrow \infty$ in both sides of (3.33), we have $\nu(y - F(y, x), kt) = 0$ and then $y = F(y, x)$. Therefore, F has a coupled fixed point at (x, y) . This completes the proof. \square

Next, we prove the existence of coupled coincidence point theorem, where we do not require that F and g are commuting.

Theorem 3.2. *Let (X, \preceq) be a partially ordered set and $(X, \mu, \nu, *, \diamond)$ a IFNS such that (μ, ν) has n -property and*

$$a \diamond b \leq ab \leq a * b, \quad \forall a, b \in [0, 1]. \tag{3.34}$$

Let $F : X \times X \rightarrow X$ and $g : X \rightarrow X$ be two mappings such that F has the mixed g -monotone property and

$$\begin{aligned} \mu(F(x, y) - F(u, v), kt) &\geq \mu(gx - gu, t) * \mu(gy - gv, t), \quad \forall x, y, u, v \in X, \\ \nu(F(x, y) - F(u, v), kt) &\leq \nu(gx - gu, t) \diamond \nu(gy - gv, t), \quad \forall x, y, u, v \in X, \end{aligned} \tag{3.35}$$

for which $gx \preceq gu$ and $gy \succeq gv$, where $0 < k < 1$, $F(X \times X) \subseteq g(X)$, $g(X)$ is complete and g is continuous. Suppose that either

- (1) F is continuous or
- (2) X has the following property:
 - (a) if $\{x_n\}$ is a non-decreasing sequence with $\{x_n\} \rightarrow x$, then $x_n \preceq x$ for all $n \in \mathbb{N}$,
 - (b) if $\{y_n\}$ is a non-increasing sequence with $\{y_n\} \rightarrow y$, then $y \preceq y_n$ for all $n \in \mathbb{N}$.

If there exist $x_0, y_0 \in X$ such that

$$g(x_0) \preceq F(x_0, y_0), \quad g(y_0) \succeq F(y_0, x_0),$$

then F and g have a coupled coincidence point in $X \times X$.

Proof. Using Lemma 2.13, there exists $E \subseteq X$ such that $g(E) = g(X)$ and $g : E \rightarrow X$ is one-to-one. We define a mapping $\mathcal{A} : g(E) \times g(E) \rightarrow X$ by

$$\mathcal{A}(gx, gy) = F(x, y), \quad \forall gx, gy \in g(E). \tag{3.36}$$

As g is one to one on $g(E)$, so \mathcal{A} is well-defined. Thus, it follows from (3.35) and (3.36) that

$$\mu(\mathcal{A}(gx, gy) - \mathcal{A}(gu, gv), kt) \geq \mu(gx - gu, t) * (gy - gv, t) \tag{3.37}$$

and

$$\nu(\mathcal{A}(gx, gy) - \mathcal{A}(gx, gy), kt) \leq \nu(gx - gu, t) \diamond \nu(gy - gv, t) \tag{3.38}$$

for all $gx, gy, gu, gv \in g(E)$ with $gx \preceq gy$ and $gy \succeq gv$. Since F has the mixed g -monotone property, for all $x, y \in X$, we have

$$x_1, x_2 \in X, gx_1 \preceq gx_2 \Rightarrow F(x_1, y) \preceq F(x_2, y) \tag{3.39}$$

and

$$y_1, y_2 \in X, gy_1 \succeq gy_2 \Rightarrow F(x, y_1) \preceq F(x, y_2). \tag{3.40}$$

Thus, it follows from (3.36), (3.39) and (3.40) that, for all $gx, gy \in g(E)$,

$$gx_1, gx_2 \in g(E), gx_1 \preceq gx_2 \Rightarrow \mathcal{A}(gx_1, gy) \preceq \mathcal{A}(gx_2, gy) \tag{3.41}$$

and

$$gy_1, gy_2 \in g(E), gy_1 \succeq gy_2 \Rightarrow \mathcal{A}(gx, gy_1) \preceq \mathcal{A}(gx, gy_2), \tag{3.42}$$

which implies that \mathcal{A} has the mixed monotone property.

Suppose that the assumption (1) holds. Since F is continuous, \mathcal{A} is also continuous. Using Theorem 3.1 with the mapping \mathcal{A} , it follows that \mathcal{A} has a coupled fixed point $(u, v) \in g(X) \times g(X)$.

Suppose that the assumption (2) holds. We can conclude similarly in the proof of Theorem 3.1 that the mapping \mathcal{A} has a coupled fixed point $(u, v) \in g(X) \times g(X)$.

Finally, we prove that F and g have a coupled coincidence point in X . Since (u, v) is a coupled fixed point of \mathcal{A} , we get

$$u = \mathcal{A}(u, v), \quad v = \mathcal{A}(v, u). \tag{3.43}$$

Since $(u, v) \in g(X) \times g(X)$, there exists a point $(\widehat{u}, \widehat{v}) \in X \times X$ such that

$$u = g\widehat{u}, \quad v = g\widehat{v}. \tag{3.44}$$

Thus, it follows from (3.43) and (3.44) that

$$g\widehat{u} = \mathcal{A}(g\widehat{u}, g\widehat{v}), \quad g\widehat{v} = \mathcal{A}(g\widehat{v}, g\widehat{u}). \tag{3.45}$$

Also, from (3.36) and (3.45), we get

$$g\widehat{u} = F(\widehat{u}, \widehat{v}), \quad g\widehat{v} = F(\widehat{v}, \widehat{u}). \tag{3.46}$$

Therefore, $(\widehat{u}, \widehat{v})$ is a coupled coincidence point of F and g . This completes the proof.

□

Next, we give example to validate Theorem 3.2.

Example 3.3. Let $X = \mathbb{R}$, $a * b = ab \geq a \diamond b$ for all $a, b \in [0,1]$ and $\theta(t) = e^{-\frac{1}{t}}$. Then, $(X, \mu, \nu, *, \diamond)$ is a complete fuzzy normed space, where

$$\mu(x, t) = [\theta(t)]^{|x|}, \quad \nu(x, t) = 1 - [\theta(t)]^{|x|}, \quad \forall x \in X,$$

that (μ, ν) satisfies the n -property on $X \times (0, \infty)$. If X is endowed with the usual order as $x \preceq y \Leftrightarrow y - x \in [0, \infty)$, then (X, \preceq) is a partially ordered set. Define mappings $F : X \times X \rightarrow X$ and $g : X \rightarrow X$ by

$$F(x, y) = 1, \quad \forall (x, y) \in X \times X$$

and

$$g(x) = x - 1, \quad \forall x \in X.$$

Since

$$g(F(x, y)) = g(1) = 0 \neq 1 = F(gx, gy)$$

for all $x, y \in X$, the mappings F and g do not satisfy the commutative condition. Hence, Theorem 2.5 of Gordji et al. [41] cannot be applied to this example. But, by simple calculation, we see that $F(X \times X) \subseteq g(X)$, g and F are continuous and F has the mixed g -monotone property. Moreover, there exist $x_0 = 1$ and $y_0 = 3$ with

$$g(1) = 0 \preceq 1 = F(1, 3)$$

and

$$g(3) = 2 \succeq 1 = F(3, 1).$$

Now, for any $x, y, u, v \in X$ with $gx \preceq gu$ and $gy \succeq gv$, we get

$$\begin{aligned} \mu(F(x, y) - F(u, v), kt) &= \mu(0, kt) \\ &= 1 \\ &\geq \mu(gx - gu, t) * \mu(gy - gv, t) \end{aligned} \tag{3.47}$$

and

$$\begin{aligned} \nu(F(x, y) - F(u, v), kt) &= \nu(0, kt) \\ &= 0 \\ &\leq \nu(gx - gu, t) \diamond \nu(gy - gv, t), \end{aligned} \tag{3.48}$$

where $0 < k < 1$. Therefore, all the conditions of Theorem 3.2 hold and so F and g have a coupled coincidence point in $X \times X$. In fact, a point $(2,2)$ is a coupled coincidence point of F and g .

Remark 3.4. Although Theorem 2.5 of Gordji et al. [41] is essential tool in the partially ordered fuzzy normed spaces to claim the existence of coupled coincidence points of two mappings. However, some mappings do not have the commutative property as in the above example. Therefore, it is very interesting to use Theorem 3.2 as another auxiliary tool to claim the existence of a coupled coincidence point.

Acknowledgements

The first author would like to thank the Research Professional Development Project under the Science Achievement Scholarship of Thailand (SAST) and the third author would like to thank the Higher Education Research Promotion and National Research University Project of Thailand, Office of the Higher Education Commission (under the Project NRU-CSEC No. 54000267) for financial support during the preparation of this manuscript. This project was partially completed while the first and third authors visit Department of Mathematics Education, Gyeongsang National University, Korea. Also, the second author was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant Number: 2011-0021821).

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All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

Received: 4 August 2011 Accepted: 18 November 2011 Published: 18 November 2011

References

1. Banach, S: Sur les opérations dans les ensembles abstraits et leurs applications aux équations intégrales. *Fund Math.* **3**, 133–181 (1922)
2. Agarwal, RP, Meehan, M, O'Regan, D: *Fixed Point Theory and Applications*. Cambridge University Press, Cambridge (2001)
3. Agarwal, RP, El-Gebeily, MA, O'Regan, D: Generalized contractions in partially ordered metric spaces. *Appl Anal.* **87**, 1–8 (2008). doi:10.1080/00036810701714164
4. Aleomraninejad, SMA, Rezapour, Sh, Shahzad, N: On fixed point generalizations of Suzuki's method. *Appl Math Lett.* **24**, 1037–1040 (2011). doi:10.1016/j.aml.2010.12.025
5. Chang, SS, Cho, YJ, Kang, SM, Fan, JX: Common fixed point theorems for multi-valued mappings in Menger *PM-spaces*. *Math Japon.* **40**, 289–293 (1994)
6. Chang, SS, Lee, BS, Cho, YJ, Chen, YQ, Kang, SM, Jung, JS: Generalized contraction mapping principle and differential equations in probabilistic metric spaces. *Proc Amer Math Soc.* **124**, 2367–2376 (1996). doi:10.1090/S0002-9939-96-03289-3
7. Chang, SS, Cho, YJ, Lee, BS, Lee, GM: Fixed degree and fixed point theorems for fuzzy mappings in probabilistic metric spaces. *Fuzzy Sets Syst.* **87**, 325–334 (1997). doi:10.1016/0165-0114(95)00373-8
8. Cho, YJ, Ha, KS, Chang, SS: Common fixed point theorems for compatible mappings of type (A) in non-Archimedean Menger *PM-spaces*. *Math Japon.* **46**, 169–179 (1997)
9. Rezaian, R, Cho, YJ, Saadati, R: A common fixed point theorem in Menger probabilistic quasi-metric spaces. *Chaos Solitons Fractals.* **37**, 1153–1157 (2008). doi:10.1016/j.chaos.2006.10.007
10. Sintunavarat, W, Cho, YJ, Kumam, P: Common fixed point theorems for *c*-distance in ordered cone metric spaces. *Comput Math Appl.* **62**, 1969–1978 (2011). doi:10.1016/j.camwa.2011.06.040
11. Chang, SS, Cho, YJ, Kang, SM: *Nonlinear Operator Theory in Probabilistic Metric Spaces*. Nova Science Publishers, Inc., New York (2001)
12. Chang, SS, Cho, YJ, Wu, X: New version of KKM theorem in probabilistic metric spaces with applications. *Appl Math Mech.* **17**, 1009–1019 (1996). doi:10.1007/BF00119948
13. Cho, YJ, Chang, SS, Jung, JS, Kang, SM, Wu, X: Minimax theorems in probabilistic metric spaces. *Bull Austral Math Soc.* **51**, 103–119 (1995). doi:10.1017/S0004972700013939
14. Grabiec, M, Cho, YJ, Radu, V: *On Nonsymmetric Topological and Probabilistic Structures*. Nova Science Publishers, Inc., New York (2006)
15. Ha, KS, Cho, YJ, Shin, KY: Accretive operators in probabilistic normed spaces. *Bull Korean Math Soc.* **31**, 45–54 (1994)
16. Zadeh, LA: Fuzzy sets. *Inf Control.* **8**, 338–353 (1965). doi:10.1016/S0019-9958(65)90241-X
17. Atanassov, K: Intuitionistic fuzzy sets. In: Sgurev V (ed.) VII ITRK's Session, Sofia (June 1983 Central Science and Technical Library, Bulgarian Academy of Sciences, 1984).

18. Saadati, R, Park, JH: On the intuitionistic fuzzy topological spaces. *Chaos Solitons Fractals*. **27**, 331–344 (2006). doi:10.1016/j.chaos.2005.03.019
19. Saadati, R, Sedghi, S, Shobe, N: Modified intuitionistic fuzzy metric spaces and some fixed point theorems. *Chaos Solitons Fractals*. **38**, 36–47 (2008). doi:10.1016/j.chaos.2006.11.008
20. Elnaschie, MS: On the verifications of heterotic strings theory and $\epsilon^{(m)}$ theory. *Chaos Solitons Fractals*. **11**, 397–407 (2000)
21. Hong, L, Sun, JQ: Bifurcations of fuzzy nonlinear dynamical systems. *Commun Nonlinear Sci Numer Simul*. **1**, 1–12 (2006)
22. Barros, LC, Bassanezi, RC, Tonelli, PA: Fuzzy modelling in population dynamics. *Ecol Model*. **128**, 27–33 (2000). doi:10.1016/S0304-3800(99)00223-9
23. Giles, R: A computer program for fuzzy reasoning. *Fuzzy Sets Syst*. **4**, 221–234 (1980). doi:10.1016/0165-0114(80)90012-3
24. Chang, SS, Cho, YJ, Kim, JK: Ekeland's variational principle and Caristi's coincidence theorem for set-valued mappings in probabilistic metric spaces. *Periodica Math Hungarica*. **33**, 83–92 (1996). doi:10.1007/BF02093505
25. Rezapour, Sh: Common fixed point of self-maps in intuitionistic fuzzy metric spaces. *Math Vesniki*. **60**, 261–268 (2008)
26. Saadati, R, Vaezpour, SM, Cho, YJ: Quicksort algorithm: application of a fixed point theorem in intuitionistic fuzzy quasi-metric spaces at a domain of words. *J Comput Appl Math*. **228**, 219–225 (2009). doi:10.1016/j.cam.2008.09.013
27. Sintunavarat, W, Kumam, P: Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces. *J Appl Math* **2011**, 14 (2011). Article ID 637958
28. Mihet, D: The fixed point method for fuzzy stability of the Jensen functional equation. *Fuzzy Sets Syst*. **160**, 1663–1667 (2009). doi:10.1016/j.fss.2008.06.014
29. Mohiuddine, SA: Stability of Jensen functional equation in intuitionistic fuzzy normed space. *Chaos Solitons Fractals*. **42**, 2989–2996 (2009). doi:10.1016/j.chaos.2009.04.040
30. Mohiuddine, SA, Cananb, M, Şevli, H: Intuitionistic fuzzy stability of a Jensen functional equation via fixed point technique. *Math Comput Model*. **54**, 2403–2409 (2011). doi:10.1016/j.mcm.2011.05.049
31. Mohiuddine, SA, Lohani, QMD: On generalized statistical convergence in intuitionistic fuzzy normed space. *Chaos Solitons and Fractals*. **42**, 1731–1737 (2009). doi:10.1016/j.chaos.2009.03.086
32. Mursaleen, M, Mohiuddine, SA: Statistical convergence of double sequences in intuitionistic fuzzy normed spaces. *Chaos Solitons Fractals*. **41**, 2414–2421 (2009). doi:10.1016/j.chaos.2008.09.018
33. Mursaleen, M, Mohiuddine, SA, Edely, OHH: On the ideal convergence of double sequences in intuitionistic fuzzy normed spaces. *Comput Math Appl*. **59**, 603–611 (2010). doi:10.1016/j.camwa.2009.11.002
34. Mursaleen, M, Mohiuddine, SA: On lacunary statistical convergence with respect to the intuitionistic fuzzy normed space. *J Comput Appl Math*. **233**(2), 142–149 (2009). doi:10.1016/j.cam.2009.07.005
35. Mohiuddine, SA, Şevli, H: Stability of Pexiderized quadratic functional equation in intuitionistic fuzzy normed space. *J Comput Appl Math*. **235**, 2137–2146 (2011). doi:10.1016/j.cam.2010.10.010
36. Mursaleen, M, Karakaya, V, Mohiuddine, SA: Schauder basis, separability and approximation property in intuitionistic fuzzy normed space. *Abstr Appl Anal* **2010**, 14 (2010). Article ID 131868
37. Cho, YJ, Huang, NJ, Kang, SM: Nonlinear equations for fuzzy mappings in probabilistic metric spaces. *Fuzzy Sets Syst*. **110**, 115–122 (2000). doi:10.1016/S0165-0114(98)00009-8
38. Cho, YJ, Lan, HY, Huang, NJ: A system of nonlinear operator equations for a mixed family of fuzzy and crisp operators in probabilistic normed spaces. *J Inequal Appl* **16** (2010). 2010, Article ID 152978
39. Bhaskar, TG, Lakshmikantham, V: Fixed point theorems in partially ordered metric spaces and applications. *Nonlinear Anal*. **65**, 1379–1393 (2006). doi:10.1016/j.na.2005.10.017
40. Lakshmikantham, V, Ćirić, L: Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces. *Nonlinear Anal*. **70**, 4341–4349 (2009). doi:10.1016/j.na.2008.09.020
41. Gordji, ME, Baghani, H, Cho, YJ: Coupled fixed point theorems for contractions in intuitionistic fuzzy normed spaces. *Math Comput Model*. **54**, 1897–1906 (2011). doi:10.1016/j.mcm.2011.04.014
42. Schweizer, B, Sklar, A: Statistical metric spaces. *Pacific J Math*. **10**, 314–334 (1960)
43. Mursaleen, M, Mohiuddine, SA: Nonlinear operators between intuitionistic fuzzy normed spaces and Fréchet differentiation. *Chaos Solitons Fractals*. **42**, 1010–1015 (2009). doi:10.1016/j.chaos.2009.02.041
44. Mursaleen, M, Mohiuddine, SA: On stability of a cubic functional equation in intuitionistic fuzzy normed spaces. *Chaos Solitons Fractals*. **42**, 2997–3005 (2009). doi:10.1016/j.chaos.2009.04.041
45. Haghi, RH, Rezapour, Sh, Shahzad, N: Some fixed point generalizations are not real generalizations. *Nonlinear Anal*. **74**, 1799–1803 (2011). doi:10.1016/j.na.2010.10.052

doi:10.1186/1687-1812-2011-81

Cite this article as: Sintunavarat et al.: Coupled coincidence point theorems for contractions without commutative condition in intuitionistic fuzzy normed spaces. *Fixed Point Theory and Applications* 2011 **2011**:81.