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Performance evaluation of max-d_{min} precoding in impulsive noise for train-to-wayside communications in subway tunnels

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Abstract

This paper addresses the performance evaluation of the multiple input multiple output (MIMO) precoding technique, referred to as max-d_{min} precoding, over fading channel with impulsive noise in a railway tunnel. Measurements showed that the received signal at the antenna on the moving train roof near the catenary suffers from electromagnetic noise interference (EMI). This implies that the traditional Gaussian noise model is no longer valid and an impulsive noise model has to be considered. Based on this observation, we investigate the performance of the max-d_{min} MIMO precoding technique, based on the minimum distance criterion, in an impulsive noise modeled as an α-stable distribution. The main contributions are (i) a general approximation of the error probability of the max-d_{min} precoder, in the presence of Cauchy noise for an $n_r \times n_t$ MIMO system, and (ii) the performance evaluation, in terms of bit error rate, of a complete communication system, considering a MIMO channel model in tunnel and impulsive noise, both obtained by measurements. Two soft detection techniques, providing the soft decisions to the channel decoder, are proposed based on the approximation of the probability density function of the impulsive noise by either a Gaussian or a Cauchy law.

Keywords: MIMO systems; Closed loop; Precoding techniques; Impulsive noise; α-stable distribution; Channel correlation; Tunnel environments; Minimal distance density function

1 Introduction

Spatial diversity offered by multiple input multiple output (MIMO) techniques can help to provide efficient transmissions in underground tunnels [1]. In the field of public transport, particularly in subway tunnels, MIMO techniques can improve the system performance in terms of data rate, robustness, and availability [1]. Moreover, when the channel state information at the transmitter (CSI-T) is known, precoding techniques permit to compensate the channel impairments such as spatial correlation between antennas and can improve the overall performance drastically, particularly in confined environments [1]. Indeed, precoding is a processing technique that exploits CSI-T by operating on the signal before transmission. MIMO precoding typically makes use of singular value decomposition (SVD) to convert the MIMO channel, represented by a full matrix, into parallel subchannels, without any interference. MIMO precoding is of great practical interest in wireless communication and remains an active research area, fueled by applications in commercial wireless technology.

In this paper, we consider a closed-loop MIMO precoder based on the maximization of the minimal distance, referred to as max-d_{min} precoder. Indeed, the max-d_{min} precoder outperforms other kinds of MIMO precoders in terms of bit error rate (BER) performance, particularly in correlated propagation scenarios [2-4]. Moreover, it provides a high spectral efficiency compared to the single antenna scheme. In general, the analyses of MIMO system performance so far have been performed assuming an ideal Gaussian noise model. However, this model is not representative of real environments in railway systems, where impulsive noise can be identified. Several models of impulsive noise have been proposed in the literature: mixtures of Gaussian [5,6], the generalized Gaussian [7],

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the generalized t distribution [8], the distributions of Middleton [9,10], and the α-stable laws [11,12]. The effects of these impulsive noise distributions on OFDM systems and power line communications have been largely investigated [13,14]. In [15], the authors considered the effect of a mixture of Gaussian noise and impulsive noise (α-stable distribution) on typical single input single output (SISO) techniques. Some more recent works on MIMO systems also considered impulsive noise distributions. In [16], the performance analysis of three typical MIMO systems - zero forcing (ZF) system, maximum likelihood (ML) system, and space-time block coding (STBC) system - was performed in a mixture of Gaussian noise and impulsive noise. The upper bound of symbol error rate (SER) in this mixed noise was derived for each system. In [17], authors analyzed the symmetric α-stable ($S\alpha S$) noise component after performing ZF filtering in the receiver and deduced a probability density function (pdf) approximation of the $S\alpha S$ noise component by using Cauchy-Gaussian mixture with bi-parameter model. Based on this approximated pdf, they provided a closed-form expression of the BER performance in MIMO systems. Nevertheless, none of those works has considered MIMO precoding.

In this paper, we focus on the max-$d_{\min}$ MIMO precoder in the presence of a specific impulsive noise, whose model was obtained thanks to measurements on the antenna dedicated to Global System for Mobile Communications-Railways (GSM-R) situated on the roof of a running train. The measurement campaigns were carried out on trains running at a speed between 160 and 200 km/h. They showed that an intermittent source of electromagnetic interference (EMI) is received by the GSM-R antennas [18]. The EMI is generated by the electric arc emissions due to the sliding contact between the catenary and the pantograph. The variables that influence the electric arc emissions are the contact wire surface conditions, the pantograph sliding contact conditions, the temperature, the train speed, the amplitude of the collected current, the mechanical suspensions reaction, and in general the mechanical characteristics of the catenary system [19]. The noise distribution measured in [18] is well approximated by a $S\alpha S$ distribution [20].

The contributions of the paper are as follows:

(i) We present a new approximation of the error probability of the max-$d_{\min}$ precoder, in the presence of Cauchy noise. This approximation is available for any number of antennas and any rectangular quadrature amplitude modulation (QAM) modulation order. This is an extension of our previous work only valid for 2 × 2 MIMO system and 4-QAM modulation [21], using the pdf statistic of minimum Euclidean distance of the received constellation of a precoded MIMO system with rectangular QAM modulations.

(ii) We apply our precoded MIMO solution on a practical transmission in a tunnel. The transmission involves a realistic channel model in tunnel, including impulsive noise and MIMO channel, both based on measurements and a communication system close to Wi-Fi technology, including a channel code. Since the channel decoder has to be fed with soft decisions by the MIMO detector, two soft detection techniques are proposed, based on the approximation of the probability density function of the impulsive noise by either a Gaussian or a Cauchy law.

The rest of the paper is organized as follows. After describing the system model with $S\alpha S$ noise in Section 2, Section 3 details the error probability analysis of the max-$d_{\min}$ precoder assuming full channel state information (CSI) at the receiver side and perfect channel estimation in a theoretical Rayleigh channel. Section 4 highlights practical performance results considering a MIMO channel measured in a real tunnel environment. Conclusions and perspectives are presented in Section 5.

Throughout the paper, boldface characters are used for matrices (upper case) and vectors (lower case). Superscript $(\cdot)^*$ denotes conjugate transposition. $\mathbf{I}_M$ stands for the $M \times M$ identity matrix. $\|\cdot\|_2$ and $\|\cdot\|_F$ indicate the two-norm and the Frobenius norm of a matrix, respectively. $\mathbb{C}^m$ is the m-dimensional complex vector space, $\mathcal{N}_c\left(0, \sigma^2\right)$ is the complex normal distribution. λ is the wavelength.

2 System model

We consider the case of a single-user transmission from n_t transmit antennas to n_r receive antennas over a fading channel. We assume that the channel coefficients are known at the receiver and at the transmitter. The amount of information sent from the receiver to the transmitter can be reduced by using partial or quantized CSI [4,22]. In this case, the receiver chooses the precoding matrix from a finite cardinality codebook, designed off-line and known at both sides of the communication link. Since the quantization is a stand-alone problem, in this paper, we focus on full CSI-T. Thus, the general input-output relation of the precoded MIMO scheme is given by

$$\mathbf{y} = \mathbf{HFx} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ is the complex received symbol vector, $\mathbf{x}$ is the complex transmitted symbol vector of b streams such that $b \leq \min(n_t, n_r)$ and $E = [\mathbf{xx}^*]$, $\mathbf{F}$ is the linear precoder respecting $\|\mathbf{F}\|_F^2 = E_t$ where E_t is the total transmit energy, $\mathbf{n} \in \mathbb{C}^{n_r \times 1}$ is a complex $S\alpha S$ noise vector, and $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ is the Rayleigh decorrelated channel matrix.

In Section 4, **H** will be described by a Kronecker model to account for the spatial correlation measured in the tunnel.

2.1 Impulsive noise: α-stable distribution

α-stable random processes provide a suitable model for a wide range of non-Gaussian heavy-tailed impulsive noise encountered in wireless communication channels [12]. This can be justified by the generalized central limit theorem considering that the noise results from a large number of possible impulsive effects [11]. However, there is no closed-form expression of their pdf and cumulative distribution except for the Gaussian ($\alpha = 2$), Cauchy ($\alpha = 1$), and Levy distributions ($\alpha = 1/2$ and $\beta = 1$). They are generally described by their characteristic function

$$\phi(t) = \begin{cases} \exp\left(i\mu t - |\gamma t|^{\alpha}\left[1 - i\beta sgn(t)\tan\frac{\pi\alpha}{2}\right]\right) & \text{for } \alpha \neq 1 \\ \exp\left(i\mu t - |\gamma t|\left[1 + i\beta\frac{2}{\pi}sgn(t)\log|t|\right]\right) & \text{for } \alpha = 1 \end{cases} \tag{2}$$

where $\alpha \in]0,2]$ is the characteristic exponent. It measures the tail heaviness of the distribution. The more α tends to 0, the slower the tail decreases and vice versa. $\gamma > 0$ is the dispersion or scale parameter. γ is similar to the variance of the Gaussian distribution. $\mu \in \mathbb{R}$ is the localization parameter. $\beta \in [-1,1]$ is the symmetry parameter. When $\beta = 0$, the distribution is symmetric about μ, denoted $S\alpha S$, and Equation 2 is reduced to

$$\phi(t) = e^{i\mu t - |\gamma t|^{\alpha}}. \tag{3}$$

The probability density function may be numerically calculated using the inverse Fourier transform of Equation 2 or 3. In [20], a distribution fitting of the measured transient EMI acting on GSM-R antenna on the train roof revealed that the measured data in [18] is well modeled by the $S\alpha S$ distribution. Table 1 gives the parameter values of the distribution, estimated by [20].

2.2 Closed-loop MIMO precoding technique

We consider the max-$d_{\min}$ precoder, based on the maximization of the minimum Euclidean distance of the received constellation [20]. By performing the singular value decomposition (SVD) of the channel matrix **H**, $\mathbf{F}_v$ and $\mathbf{G}_v$ can be obtained, where $\mathbf{H}_v = \mathbf{G}_v\mathbf{H}\mathbf{F}_v = \text{diag}(\sigma_1, \cdots, \sigma_b)$. $\mathbf{H}_v$ is the virtual channel matrix, whose elements represent subchannel gains arranged in a descending order. The precoding and decoding matrix can respectively be written as $\mathbf{F} = \mathbf{F}_v\mathbf{F}_d$ and $\mathbf{G} = \mathbf{G}_d\mathbf{G}_v$. By applying the precoding and the decoding matrix at the transmitter and at the receiver, respectively, the received vector of the precoding-based MIMO scheme is given by

$$\mathbf{y} = \mathbf{G}_d\mathbf{H}_v\mathbf{F}_d\mathbf{x} + \mathbf{G}_d\mathbf{n}_v. \tag{4}$$

The max-$d_{\min}$ precoder solution is given by

$$\begin{aligned} \mathbf{F}_d &= \arg\max_{\mathbf{F}_i} d_{\min}(\mathbf{F}_i), \text{ where } d_{\min}(\mathbf{F}_i) \\ &= \min_{(\mathbf{x}_k,\mathbf{x}_l)\in\mathcal{C}, \mathbf{x}_k\neq\mathbf{x}_l} \|\mathbf{H}_v\mathbf{F}_i(\mathbf{x}_k - \mathbf{x}_l)\|. \end{aligned} \tag{5}$$

In [2], an analytic solution of Equation 5 is given for two independent data streams, $b = 2$ and a 4-QAM. Thanks to a judicious change of variables, the virtual channel matrix can be parameterized as

$$\mathbf{H}_v = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} = \rho\begin{pmatrix} \cos\theta & 0 \\ 0 & \sin\theta \end{pmatrix} \tag{6}$$

where $\theta = \arctan\left(\frac{\sigma_2}{\sigma_1}\right)$ is the channel angle and $\rho = \sqrt{\sigma_1^2 + \sigma_2^2}$ is the channel gain. The solution does not depend on the signal-to-noise ratio (SNR) but rather on the value of the channel angle θ and can take two different forms. The optimized solution for MIMO systems using high-order QAM modulation is hard to find. However, a general form of minimum Euclidean distance based precoders for all rectangular QAM modulations was proposed in [23]. This form is based on two linear precoders, which maximize the Euclidean distance $d_{\min}$ for the two independent data streams according to the following rule: pouring power only on the strongest subchannel or on both subchannels according to the value of the channel angle θ in comparison to θ_0. The threshold $\theta_0(M)$ depends on the modulation level and is defined by

$$\theta_0(M) = \arctan\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}N^2 + \sqrt{6}N\sqrt{2}-1}}, \text{ with } N = \sqrt{M} - 1 \tag{7}$$

The corresponding minimal Euclidean distance for these two precoders, referred to as $\mathbf{F}_{r1}$ and $\mathbf{F}_{\text{octa}}$, is given in Table 2 [23]. Suboptimal extensions for substream number greater than 2 can be found in [3].

Table 1 Estimated parameters values of the $S\alpha S$ distribution

Model	Parameter	Estimated value
	Exponent characteristic α	1.253
α-stable	Dispersion γ	0.419
	Location μ	$3.95 \times 10^{-4} \approx 0$
	Symmetry parameter β	$2.64 \times 10^{-4} \approx 0$

Table 2 Minimum distance

Precoders	$d^2_{\min}$
$\mathbf{F}_{r1}$ $0 \leq \theta \leq \theta_0$	$E_s\rho^2\frac{6}{M-1}\frac{\cos^2\gamma}{N^2+\sqrt{3}N+2}$ with $N = \sqrt{M}-1$
$\mathbf{F}_{\text{octa}}$ $\theta_0 \leq \theta \leq \pi$	$E_s\rho^2\frac{6(2-\sqrt{2})}{M-1}\frac{\cos^2\gamma\sin^2\gamma}{1+(2-2\sqrt{2})\cos^2\gamma}$

2.3 Detection techniques

The lack of analytical form of the probability density makes it difficult to study detection techniques in the presence of impulsive noise. For SISO systems, some good approximations of probability density of the $S\alpha S$ noise as a mixture of noise have been proposed in [15]. However, the maximum likelihood (ML) detection in the general case of $S\alpha S$ noise is difficult to solve for the precoded MIMO scheme. In this study, we will consider a suboptimal detection technique, based on the assumption of a Cauchy detector [24]. This assumption is justified since the estimated error exponent of the considered $S\alpha S$ distribution (Table 1) is close to that of the Cauchy distribution.

The optimal decoding rule is to find $\widehat{\mathbf{x}}$ that maximizes the likelihood

$$\widehat{\mathbf{x}} = \text{argmax}\, p\left(\mathbf{y}|\mathbf{x}\right). \tag{8}$$

If $\mathbf{n}$ follows a Cauchy distribution, the rule of maximum likelihood is then given by

$$\widehat{\mathbf{x}} = \text{argmin} \prod_{k=1}^{n_r} \left(y_k - \sum_i h_{e_{ki}} x_i\right)^2, \text{with } \mathbf{H}_\text{e} = \mathbf{HF}. \tag{9}$$

In the following section we analyze the error probability of the precoded MIMO scheme in the presence of Cauchy noise ($\alpha = 1$) [24].

3 Error probability analysis

Equation 6 shows that the virtual channel is fully characterized by two variables ρ and θ which are the channel gain and the channel angle, respectively. The behavior and performance of the max-$d_{\min}$ precoder depend on these two parameters. Furthermore, ρ and θ are random variables (RV) whose laws depend on the channel. Thus, in this section, we focus on the theoretical laws of these two RV, and especially θ, whose pdf depends directly on the distribution of the two largest eigenvalues of the channel. The main idea is to derive the expression of pdf of the minimum Euclidean distance and then the error probability of the max-$d_{\min}$ precoder. As the channel matrix $\mathbf{H}$ is an uncorrelated Rayleigh matrix, $\mathbf{W} = \mathbf{HH}^*$ is a Wishart matrix. Using the random matrix theory, we can define the joint distribution of nonzero eigenvalues of a Wishart matrix for $\min(n_t, n_r) = m$ [25].

$$f^{(m)}_{\lambda_1,\cdots,\lambda_m}\left(\lambda_1,\cdots,\lambda_m\right) = k_m \prod_{i=1}^{m} \lambda_i^{n_s} e^{-\lambda_i} \times \prod_{i=1,\, i<j}^{m} \left(\lambda_i - \lambda_j\right)^2 \tag{10}$$

where

$$\lambda_i = \sigma_i^2,\ \ n_s = |n_t - n_r| \ \text{ and } \ k_m = \frac{1}{\prod_{i=1}^{m} (n_t - i)!\,(n_r - i)!}$$

In the max-$d_{\min}$ precoder case, we are interested only in the two largest eigenvalues. Thus, the conjoint law of these two largest eigenvalues is expressed by

$$f_{\lambda_1,\lambda_2}\left(\lambda_1,\lambda_2\right) = \int_0^{\lambda_2} \cdots \int_0^{\lambda_{m-1}} f_{\lambda_1,\lambda_2,\cdots,\lambda_m}\left(\lambda_1,\lambda_2,\cdots,\lambda_m\right) \times d\lambda_3 \ldots d\lambda_m. \tag{11}$$

A general form is difficult to obtain, but it can be demonstrated that the conjoint law has the form [26]

$$f_{\lambda_1,\lambda_2}\left(\lambda_1,\lambda_2\right) = k_2\left(\lambda_1,\lambda_2\right)^{n_s} e^{-(\lambda_1+\lambda_2)}\left(\lambda_1-\lambda_2\right)^2 \times \sum_{n=0}^{m-2} e^{-n\lambda_2} \sum_{i,j} p_{n,i,j} \lambda_1^i \lambda_2^j e^{-n\lambda_2} \tag{12}$$

where coefficients $p_{n,i,j}$ are computed by any mathematical computational software program. In order to find the pdf of the square minimum Euclidian distance for the max-$d_{\min}$ precoder, $\text{pdf}_{\max-d_{\min}}$, we consider a second change of variables based on a more tractable couple of RV:

$$\begin{cases} \Gamma = \lambda_1 + \lambda_2 = \rho^2 \\ \beta = \dfrac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \cos 2\theta \end{cases}. \tag{13}$$

Thus, the square minimum Euclidian distance can be written as [26] in a simplest form:

$$d^2_{\min} = \alpha_M \Gamma \delta\left(\beta\right) \tag{14}$$

with

$$\delta\left(\beta\right) = \frac{1+\beta}{2},\ \ \alpha_M = \frac{6}{(M-1)\left(N^2 + \sqrt{3}N + 2\right)} \quad \text{for } \mathbf{F}_{r1} \tag{15}$$

$$\delta\left(\beta\right) = \frac{1-\beta^2}{\left(2-\sqrt{2}\right)\beta},\ \ \alpha_M = \frac{6(2-\sqrt{2})}{(M-1)} \quad \text{for } \mathbf{F}_\text{octa} \tag{16}$$

where δ is a function of β and α_M is a constant depending on the modulation size. The expression of the minimum distance takes into account the two possibilities $\mathbf{F}_{r1}$ and $\mathbf{F}_\text{octa}$. In (14), the determination of the pdf of the square minimum distance requires the computation of the marginal law of $\Gamma\delta$ from the joint law of Γ and δ. Finally, the pdf of the square minimum distance in the case of the max-$d_{\min}$ precoder is

$$\text{pdf}_{\max-d_{\min}}\left(d^2\right) = \frac{1}{\alpha_M}\left(g_{\text{F}_{r1}}\left(\frac{d^2}{\alpha_M}\right) + g_{\text{F}_\text{octa}}\left(\frac{d^2}{\alpha_M}\right)\right) \tag{17}$$

where

$$\begin{cases} g_{\mathbf{F}_{r1}}(z) = ke^{-z}z^{n_s}\sum_{n,i,j} p_{n,i,j} z^i \psi_{n,j}\left(z, \frac{\beta_0\left(M\right)}{1+\beta_0\left(M\right)}\right) \\ g_{\mathbf{F}_{\text{octa}}}(z) = \frac{k}{2\sqrt{2}}\sum_{n,i,j} p_{n,i,j}\left(\frac{z}{2\sqrt{2}}\right)^{2n_s+3+i+j} e^{-z\left(1+\frac{n}{2}\right)} \times \left(e^{-z\frac{2}{\sqrt{2}}n}\phi^{i,j}_{1,n+1}\left(z,\sqrt{2}+1\right)\right) \\ \psi_{n,j}(z,a) = \frac{\gamma_{inc}(n_s+j+3,\xi)}{(n+1)^{n_s+j+3}} - 2z\frac{\gamma_{inc}(n_s+j+2,\xi)}{(n+1)^{n_s+j+2}} + z^2\frac{\gamma_{inc}(n_s+j+1,\xi)}{(n+1)^{n_s+j+1}} \\ \beta_0(M) = \cos(2\theta_0(M)) \\ \phi^{i,j}_{a,b}(x,t_{\text{sup}}) = \left[e^{-\frac{x}{2\sqrt{2}}\left((a+1)t+\frac{b+1}{t}\right)}\sum_l w_l t^l\right]_1^{t_{\text{sup}}} + \beta_{K_0}\int_1^{t_{\text{sup}}} e^{-\frac{x}{2\sqrt{2}}\left(at+\frac{b}{t}\right)}\frac{1}{t}dt \\ \qquad + \beta_{K_1}\frac{x}{\sqrt{2}}\int_1^{t_{\text{sup}}} e^{-\frac{x}{2\sqrt{2}}\left(at+\frac{b}{t}\right)}\frac{\left(t^2-1\right)^2}{4t^3}dt \\ \xi = a(n+1)z \\ \gamma_{inc}(a,x) = \int_0^x t^{\alpha-1}e^{-t}dt. \end{cases} \tag{18}$$

Only the main results are reminded here, and the reader may find more detailed calculus in [26]. The minimum Euclidean distance directly affects the error probability at the output of the ML detector. The closer the two impacts of the received constellation are, the higher the error probability is. Therefore, we use the theoretical error probability approximation, limited to the nearest neighbors as in [25]: considering an impact of the received constellation, the error only comes from the choice of one neighbor at the distance $d_{\min}$. In order to take into account the channel statistics, the expression is averaged by using the integration weighted by the pdf of $d_{\min}$. Figure 1 shows different examples of results for the Gaussian noise. The proposed formula is a good lower bound and a close approximation at high SNR for the Gaussian noise. The Cauchy receiver, which is the optimum in the case of $\alpha = 1$, performs quite closely to the optimum receiver for a wide range of α and γ [27]. Thus, we suggest using the same method as in [25] applied to the Cauchy law. First, we approximate the pdf of the measured $S\alpha S$ noise by a Cauchy law given by

$$f_{\text{Cauchy}}\left(x\right) = \frac{\gamma}{\pi\left(\gamma^2+x^2\right)}. \tag{19}$$

Then, we deduce the complementary cumulative distribution function (ccdf) of the Cauchy noise by

$$\begin{aligned} \overline{F}(x) &= P\left(X > x\right) = 1 - F(x) \\ &= \frac{1}{2} - \frac{1}{\pi}\arctan\left(\frac{x}{\gamma}\right) \end{aligned} \tag{20}$$

where $F(x)$ is the cumulative distribution function (cdf) of the Cauchy distribution. Finally, the ccdf of the Cauchy noise, which is analog to erfc function for the Gaussian case, is averaged over the statistics of the minimum distance considering a decorrelated Rayleigh channel, and the BER expression of the max-$d_{\min}$ precoder is provided:

$$P_e \simeq \int_0^\infty \frac{N(\theta)}{2}\left(\frac{1}{2} - \frac{1}{\pi}\arctan\left(\sqrt{\frac{\text{SNR}\cdot u}{8}}\right)\right)\text{pdf}_{\max -d_{\min}}(u)du \tag{21}$$

where

$$\begin{cases} N(\theta) = \dfrac{N_e(\theta)N_b(\theta)}{b\log_2(M)}, \text{ is the average number of neighbors and different bits for a fixed modulation} \\ \text{SNR} = E_t/\gamma, \text{the signal to noise (SNR) ratio} \end{cases} \tag{22}$$

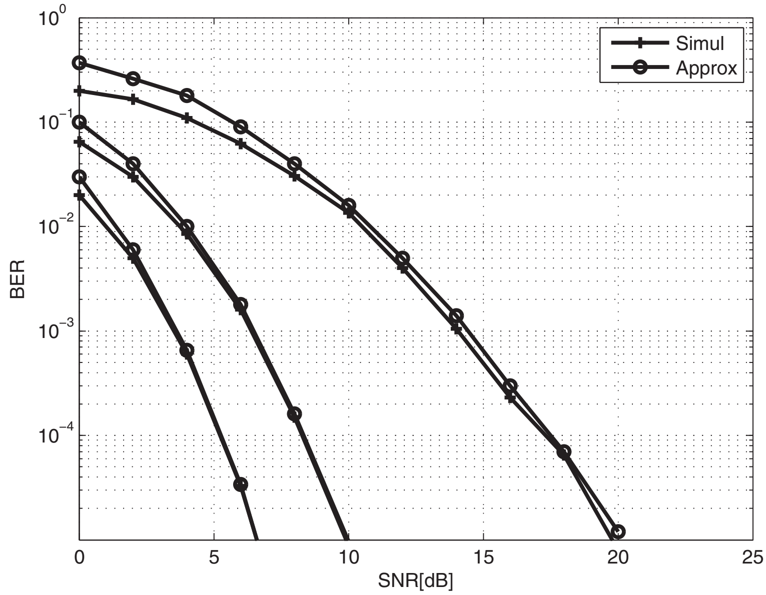

Figure 1 Numerical approximation and simulation - max-d_{min}. Error probability of the max-d_{min} precoder with a Gaussian noise for different MIMO systems: 2 × 2, 6 × 2, and 10 × 2, from right to left, respectively.

$N_e(\theta)$ represents the average number of nearest neighbors at the distance $d_{\min}$, and $N_b(\theta)$ is the average number of errors among the b transmitted bits per symbol. These two parameters can be obtained using the received constellations and depend on the modulation and the max-$d_{\min}$ precoder ($\mathbf{F}_{r1}$ or $\mathbf{F}_{\text{octa}}$). Table 3 presents the results in the special case where $b = 2$, and 4 bits is transmitted using M-QAM modulations. The exact analytical solution of equality 21 is hard to find due to the difficulty to deal with the integration. So, we manage it by numerical integration. We compare these results with the simulation of the whole communication chain in a Rayleigh channel. Simulation parameters are presented in Table 4. $\mathbf{G}_d$ is chosen as an identity matrix, and $\mathbf{n}_\nu$ is a Cauchy noise in this stage. There are several algorithms to simulate stable α-random variables [28]. In particular, for X a Cauchy random variable, we generate a uniform random variable $U \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$, and then we apply the relationship [28]:

$$\mathrm{X} = \gamma \tan U + \mu. \tag{23}$$

Figure 2 provides the theoretical and the simulated BER versus the SNR ratio for a 2 × 2 MIMO system with different modulation order in the presence of an impulsive noise modeled by the Cauchy law. By comparing the theoretical results (line with plus sign) and the simulation results (line with circle), we can readily find that the theoretical results match the simulated results of the communication chain very well at high SNR values (>20 dB or BER $< 10^{-2}$) for 4-QAM and 16-QAM modulations. In these cases, the agreement is not so good for the low SNR values. For the 64-QAM modulation, the gap between theoretical and the simulated results is about 3 dB. This demonstrates the interest of the proposed closed-form BER performance only at high SNR. It may be considered as a good lower bound for 2 × 2 MIMO systems in the presence of impulsive noise. In Figures 3, 4, and 5, we show

Table 3 N_e and N_b values

		$\mathbf{F}_{r1}$	$\mathbf{F}_{\text{octa}}$
4-QAM	N_e	3.5	7
	N_b	1.471	1.488
16-QAM	N_e	4.312	13.875
	N_b	2.032	2.29
64-QAM	N_e	4.32	18.484
	N_b	2.196	2.913

Table 4 Simulation parameters

MIMO systems	**2 × 2, 4 × 4, 8 × 8**
Number of streams	2
Number of frames	10,000
Frame length	800 bits
Modulation	4-QAM, 16-QAM, 64-QAM

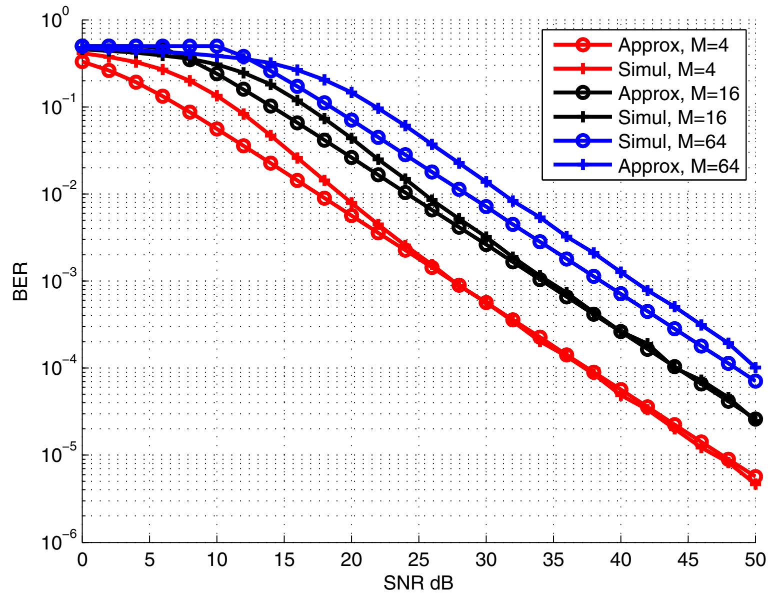

Figure 2 Numerical approximation and simulation - max-$d_{\min}$ $n_t = 2$ $n_r = 2$ $M = (4, 16, 64)$. Error probability of the max-$d_{\min}$ precoder in 2×2 MIMO system with a Cauchy noise ($\alpha = 1$) and 4, 16, 64-QAM.

the performance analysis of 4×4 and 8×8 MIMO systems for 4-QAM, 16-QAM, and 64-QAM, respectively. By comparing the theoretical results (line with plus sign) and the simulation results (line with circle), we can readily find that the theoretical results match the simulated results of the communication chain very well at high SNR values (>30 dB or BER $< 10^{-4}$) for 4-QAM and 16-QAM modulations. For higher modulation, we have to improve our BER approximation by considering more neighbors. With 4-QAM modulation, the 4×4 system provides a high gain

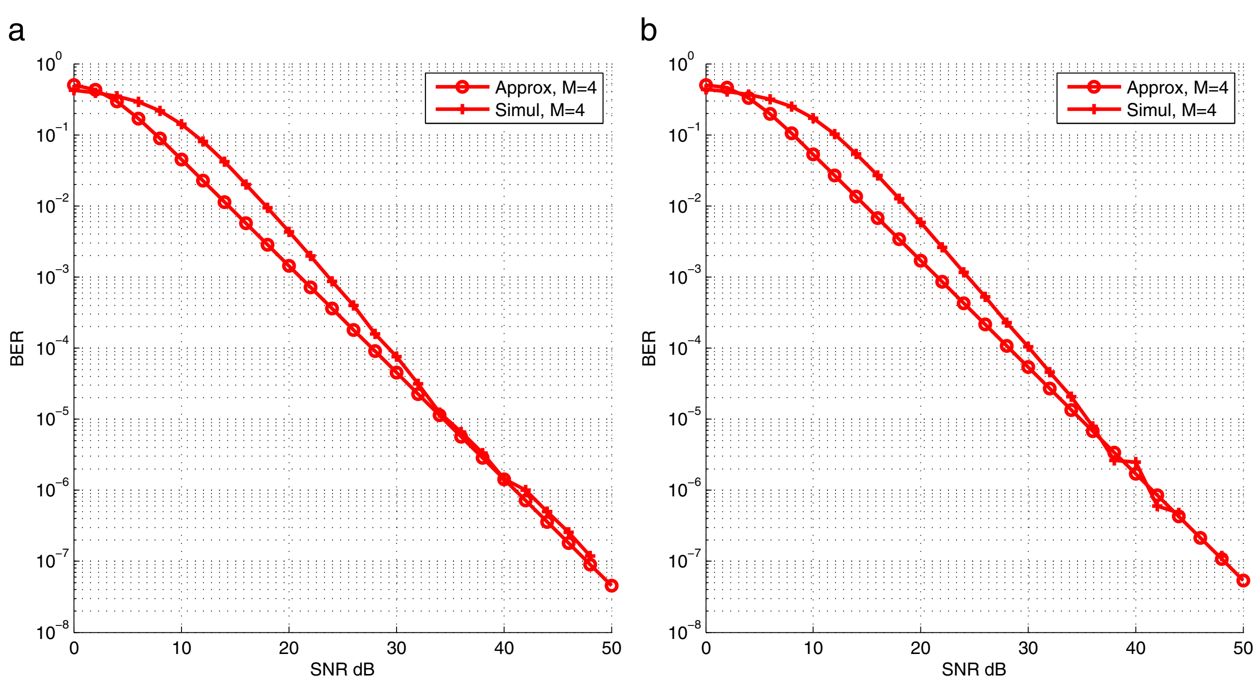

Figure 3 Numerical approximation and simulation - max-$d_{\min}$ $M = 4$. Error probability of the max-$d_{\min}$ precoder in 4×4 and 8×8 MIMO systems with a Cauchy noise ($\alpha = 1$) and 4-QAM. **(a)** 4×4 MIMO system. **(b)** 8×8 MIMO system.

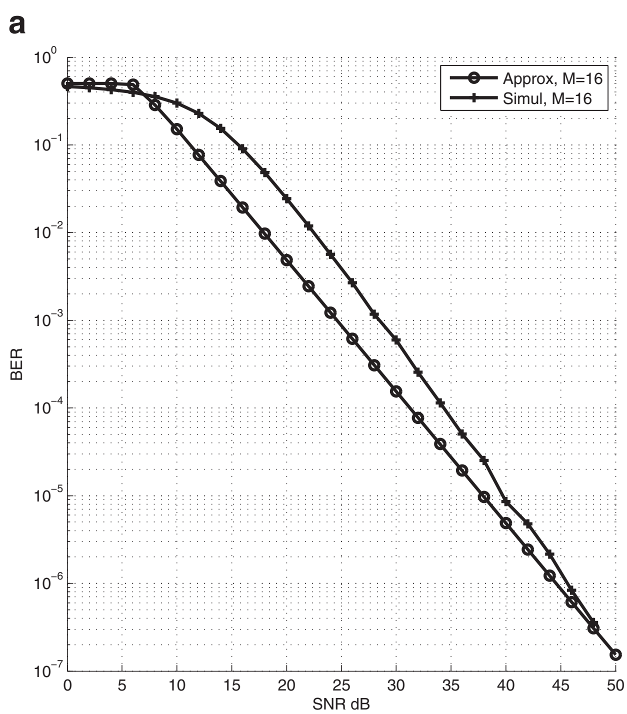

Figure 4 Numerical approximation and simulation - max-$d_{\min}$ $M = 16$. Error probability of the max-$d_{\min}$ precoder in 4 × 4 and 8 × 8 MIMO systems with a Cauchy noise ($\alpha = 1$) and 16-QAM. **(a)** 4 × 4 MIMO system. **(b)** 8 × 8 MIMO system.

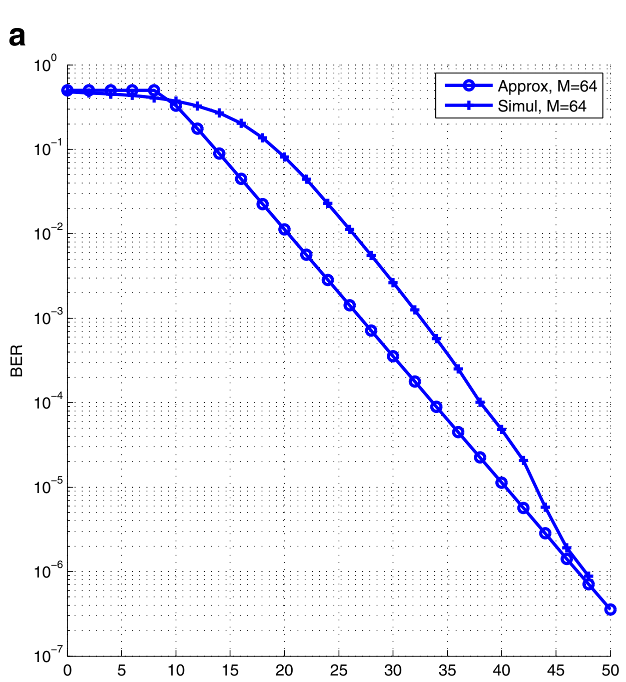

Figure 5 Numerical approximation and simulation - max-$d_{\min}$ $M = 64$. Error probability of the max-$d_{\min}$ precoder in 4 × 4 and 8 × 8 MIMO systems with a Cauchy noise ($\alpha = 1$) and 64-QAM. **(a)** 4 × 4 MIMO system. **(b)** 8 × 8 MIMO system.

at high SNR value compared to the 2×2 system, more than 20 dB at BER = 10^{-5} . However, there is no significant gain when we consider a 8×8 system compared to the 4×4 system with the same modulation order. This approximation may be considered as a good lower bound for any MIMO system in the presence of impulsive noise.

4 Performance in a real tunnel environment

4.1 Realistic model from measured MIMO channels

The considered transmission chain mimics an IEEE 802.11× PHY modem that involves a bit-interleaved coded modulation (BICM) resulting in the concatenation of a channel encoder, a bit interleaver, and a bit-to-symbol mapper. The channel code is a $\frac{1}{2}$ rate convolutional code with constraint length $K = 7$ and defined by the generator polynomials $g_0 = 0133$ and $g_1 = 0171$. The frame of encoded data is then randomly interleaved and converted to complex symbols belonging to the constellation alphabet of 4-QAM modulation. This BICM scheme is followed by the max-$d_{\min}$ precoder, which adapts $b = 2$ streams to the 4×4 MIMO channel. Decoding is performed using a soft detection technique considering two different laws: Gaussian and Cauchy (see Section 4.2). For this simulation, 10,000 frames of 800 bits each were transmitted. The channel is quasi-static, so $\mathbf{H}$ is assumed constant over the transmission of several consecutive vector symbols. The considered noise is $S\alpha S$, and the generation of $S\alpha S$ random variables follows the formula proposed by [25].

A MIMO channel sounding campaign provided the channel matrices $\mathbf{H}$, thanks to the measured complex impulse responses between each couple of transmitting and receiving antennas [29]. Measurements were conducted in the Tunnel of Roux located in the Ardèche region in the south of France. We used the Propsound$^{\text{TM}}$ channel sounder from the Elektrobit company (Oulu, Finland) [30] which is based on the spread-spectrum sounding method for the delay domain. It is a multidimensional channel sounder which measures radio channel in time and spatial domains, and provides complex impulse responses (CIR). The rapid switching of the antennas makes Propsound$^{\text{TM}}$ suitable for MIMO measurements. Measurement configurations and first analyses are presented in [29]. Due to the small RMS delay spread in this environment (few ns) regarding the data frame size, the channel matrices $\mathbf{H}$ are modeled in narrow band using the Kronecker model [31] as indicated in (24):

$$\mathbf{H} = \Sigma_r^{1/2}\mathbf{H}_w\Sigma_t^{1/2} \tag{24}$$

where $\mathbf{H}_w \in \mathbb{C}^{n_r\times n_t}$ is the i.i.d. Rayleigh matrix, and $\Sigma_t \in \mathbb{C}^{n_t\times n_t}$ and $\Sigma_r \in \mathbb{C}^{n_r\times n_r}$ represent the correlation matrices averaged along the tunnel axis at the transmitter and receiver sides, respectively, computed from $\mathbf{H}$. The measurements of the MIMO channel were conducted in the Tunnel of Roux (Ardèche region in the south of France, Figure 6). Two MIMO configurations related to the spacing between the receiving and transmitting antennas, 2λ (average value of correlation along the tunnel $\rho = 0.96$) and 10λ (average value of correlation $\rho = 0.57$), are considered. They correspond to a high and a low correlation scenario, respectively [29]. In such an environment (empty tunnel, no cross-sectional change), there is a geometrical similarity between the environment near the transmitting antennas and that near the receiving antennas. So, the correlation values are equivalent at both sides. All antennas at transmission and reception sides are vertically polar-

Figure 6 Tunnel of Roux environment.

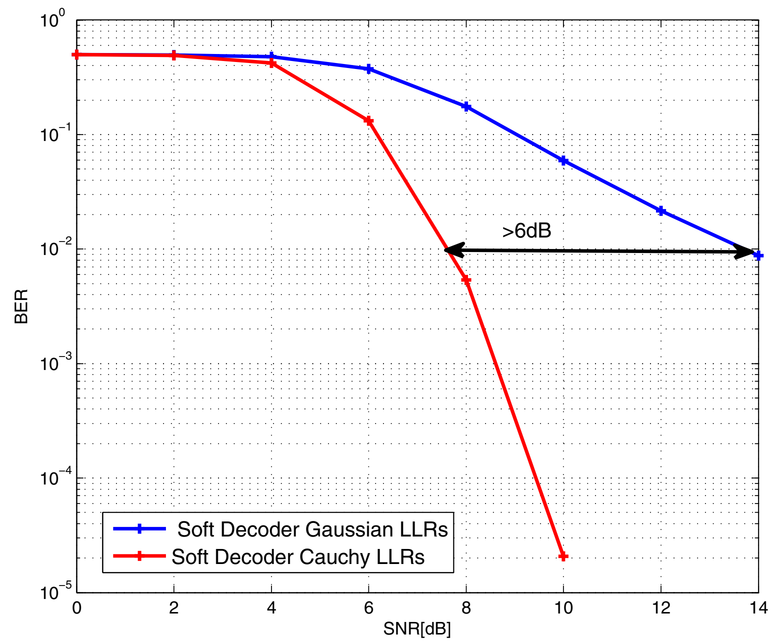

Figure 7 Max-$d_{\min}$ BER performance in the measured 4 × 4 MIMO channel. Perfect estimation of **H**, 4-QAM in coded system with soft decoding in noise environment ($\alpha = 1.253$, $\gamma = 0.419$, $\beta = \mu = 0$).

ized. In the rest of the paper, we will consider only the low correlation configuration

4.2 Soft detection technique

Since channel coding is considered in the communication chain, log-likelihood ratios (LLRs) have to be provided to the channel decoder. For each MIMO received vector $\mathbf{y}$ and assuming a Gaussian noise, LLRs are given by

$$\mathrm{LLR}_{\mathrm{Gaussian}}\left(b|\mathbf{Y}\right) = \ln \frac{\sum\limits_{S \in \mathcal{X}_{b=0}} \exp\left(-\frac{1}{2\sigma^2}\left\|\mathbf{y} - \mathbf{H}_e\mathbf{s}\right\|^2\right)}{\sum\limits_{S \in \mathcal{X}_{b=0}} \exp\left(-\frac{1}{2\sigma^2}\left\|\mathbf{y} - \mathbf{H}_e\mathbf{s}\right\|^2\right)}. \tag{25}$$

Assuming a Cauchy noise, LLRs are given by

$$\mathrm{LLR}_{\mathrm{Cauchy}}\left(b|\mathbf{Y}\right) = \ln \frac{\sum\limits_{S \in \mathcal{X}_{b=0}} 1/\left(\gamma^2 + \left(\left\|\mathbf{y} - \mathbf{H}_e\mathbf{s}\right\|\right)^2\right)}{\sum\limits_{S \in \mathcal{X}_{b=1}} 1/\left(\gamma^2 + \left(\left\|\mathbf{y} - \mathbf{H}_e\mathbf{s}\right\|\right)^2\right)}. \tag{26}$$

4.3 Simulation results and discussion

Figure 7 corresponds to the max-$d_{\min}$ BER performance in the measured 4×4 MIMO channel obtained in the low correlated scenario (antenna spacing equal to 10λ and $\rho = 0.57$). In this figure, we present the comparison between the two different soft detection techniques for the max-$d_{\min}$ precoder. It can be seen that the detector with the Gaussian assumption gives poor performance, compared to the detector with the Cauchy assumption. For a BER of 10^{-2}, the gap between these two detectors is greater than 6 dB. Therefore, the channel decoder based on LLRs obtained, thanks to the Cauchy assumption, can increase the performance potential of the communication system in this type of railway environment.

5 Conclusion

In this paper, we have investigated the performance, in terms of BER, of a precoded MIMO system, based on the max-$d_{\min}$ precoder, in the presence of impulsive noise in the railway environment. First, we have proposed a lower bound on the error probability of the max-$d_{\min}$ precoder in a Cauchy noise environment for any MIMO system dimensions and M-QAM rectangular modulation, which is tight at high SNR relative to the simulation results of the communication chain. The expression is validated by the Monte Carlo evaluation of the minimum distance and the full simulation of the communication chain. The chosen approximation is simple and more accurate, but a more complex form may be found by taking into account all neighbors. Second, we have evaluated the performance of a realistic communication system close to the Wi-Fi PHY layer, including a realistic tunnel channel, based on previous MIMO channel sounding measurements and $S\alpha S$ parameter values derived from distribution fitting of measured transient EMI received at the GSM-R antenna on the roof of the train. These transient EMI are due to bad sliding contacts between the catenary and the pantograph. The log-likelihood ratios, needed at the input of the channel decoder, have been expressed in a Cauchy noise for the MIMO system. We have shown that the soft channel decoder based on those log-likelihood ratios behaves remarkably well in the studied tunnel context, even if $S\alpha S$ impulsive noise is considered.

Research is also underway to provide an approximation in correlated channels and improve the detection technique in other MIMO configurations.

Competing interests
The authors declare that they have no competing interests.

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