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Energy-efficient estimation of a MIMO channel

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Abstract

Exploiting the benefits of multiple antenna technologies is strongly conditioned on knowledge of the wireless channel that affects the transmissions. To this end, various channel estimation algorithms have been proposed in the literature for multiple-input multiple-output (MIMO) channels. These algorithms are typically studied from a perspective that does not consider constraints on the energy consumption of their implementation. This article proposes a methodology for evaluating the total energy consumption required for transmitting, receiving, and processing a preamble signal in order to produce a channel estimate in multiple antenna systems. The methodology is used for finding the training signals that minimize the energy consumption for attaining given mean square estimation error. We show that the energy required for processing the preamble signal by executing the estimation algorithms dominates the total energy consumed by the channel estimation process. Therefore, algorithm simplicity is a key factor for achieving energy-efficient channel acquisition. We use our method for analyzing the widely used least squares and minimum mean square error (MSE) estimation algorithms and find that both have a similar energy consumption when the same MSE estimation is targeted.

Keywords: MIMO channel estimation, Energy efficiency

1 Introduction

Multiple-input multiple-output (MIMO) communication techniques have been incorporated into different wireless systems due to their capability for allowing higher data rates (multiplexing gain) or for increasing link reliability (diversity gain). However, recent studies have shown that MIMO techniques can be used alternatively for reducing energy consumption in comparison to a single-input single-output (SISO) link that attains the same data rate and link reliability. In [1], when the link distance is larger than a given threshold, data transmission using a 2×2 MIMO system with Alamouti space–time coding was shown to be more energy-efficient than an equivalent SISO system. A detailed energy consumption model for an $N \times N$ singular value decomposition-based MIMO system is proposed in [2]. The model includes retransmission statistics and shows that for a given link distance and number of channels used exist a single optimal radiation power level at which the mean energy consumption required to transmit a bit correctly is minimized.

But the use of MIMO is strongly conditioned on knowing the wireless channel, which the above contributions

assume perfectly known. This knowledge is typically obtained by transmitting a known training preamble that allows the receiver to estimate the channel by executing an estimation algorithm.

The design of training preambles for channel estimation has not yet been studied well in terms of energy efficiency. Typically, the design of the preamble signals focuses on minimizing the channel estimation error [3] or on maximizing the channel capacity under imperfect channel knowledge [4]. Furthermore, existing models of MIMO energy consumption as the ones in [1,2] ignore the energy required for transmitting, receiving, and processing a preamble signal. In fact, MIMO channel estimation can be a significant part of the baseband processing energy consumption because the algorithms usually perform complex algebraic operations.

In this article, we present a method for comparing the energy efficiency of different channel estimation algorithms. We formulate an energy consumption model that allows to find the training signals that minimize the energy consumption of the algorithms given a mean square error (MSE) of estimation. Particularly, we study the minimum MSE (MMSE) and least squares (LS) channel estimation algorithms and optimize their respective preambles for minimum energy consumption at a given target MSE. We

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show that their optimal energy consumption difference is negligible.

The rest of the article is organized as follows: Section 2 describes the energy consumption model for channel estimation algorithms. Section 3 examines the LS and MMSE algorithms and details their energy consumption and MSE. Section 4 formulates and solves the optimization problem that allows to find the optimal training signals. Section 5 provides numerical results and Section 6 summarizes our conclusions.

Notation: $\mathbf{x}^H$ denotes the conjugate transpose operation over $\mathbf{x}$, $\|\mathbf{x}\|$ is the norm of vector $\mathbf{x}$, $\mathbb{E}\{\cdot\}$ indicates expected value and $\mathbf{I}_M$ is the $M \times M$ identity matrix. The superscript check ˇ denotes that the variable corresponds to a single branch of either the transmitter or receiver.

2 Energy consumption model

Our goal is to minimize the energy consumption required for producing a channel estimate with a given estimation quality. For this purpose, we formulate a model that includes the energy consumption of all the components involved in the channel estimation process.

We consider a generic low-power MIMO transceiver architecture for the preamble communication as shown in Figure 1, with N_t transmit and N_r receive antennas. It is to be noted that data processing blocks such as channel encoder, modulator, channel decoder, and demodulator are not included because they do not participate in the channel estimation process.

In the sequel, we detail each source of the energy consumption.

1. Start-up energy consumption, $\mathcal{E}_{ST}$
   We assume that the transmitter and receiver are by default in a low-power consumption (sleep) mode. Hence, they must be brought online before they can communicate the preamble. We denote as $\mathcal{E}_{ST}$ the energy required to start-up the transceivers, which is dominated by the stabilization of the frequency synthesizer [5]. If this component consumes a power P_{syn} has a settling time T_{tr} and is shared among all branches (either transmitting or receiving), then the start-up energy of two frequency synthesizers can be expressed as

$$\mathcal{E}_{ST} = 2P_{syn}T_{tr}. \qquad (1)$$

2. Energy consumption of the transmitter electronics, $\mathcal{E}_{el,tx}$
   It represents the energy consumption of the digital-to-analog converters (DAC), filters, and mixers of the transmitter. These components consume energy for each transmitted preamble
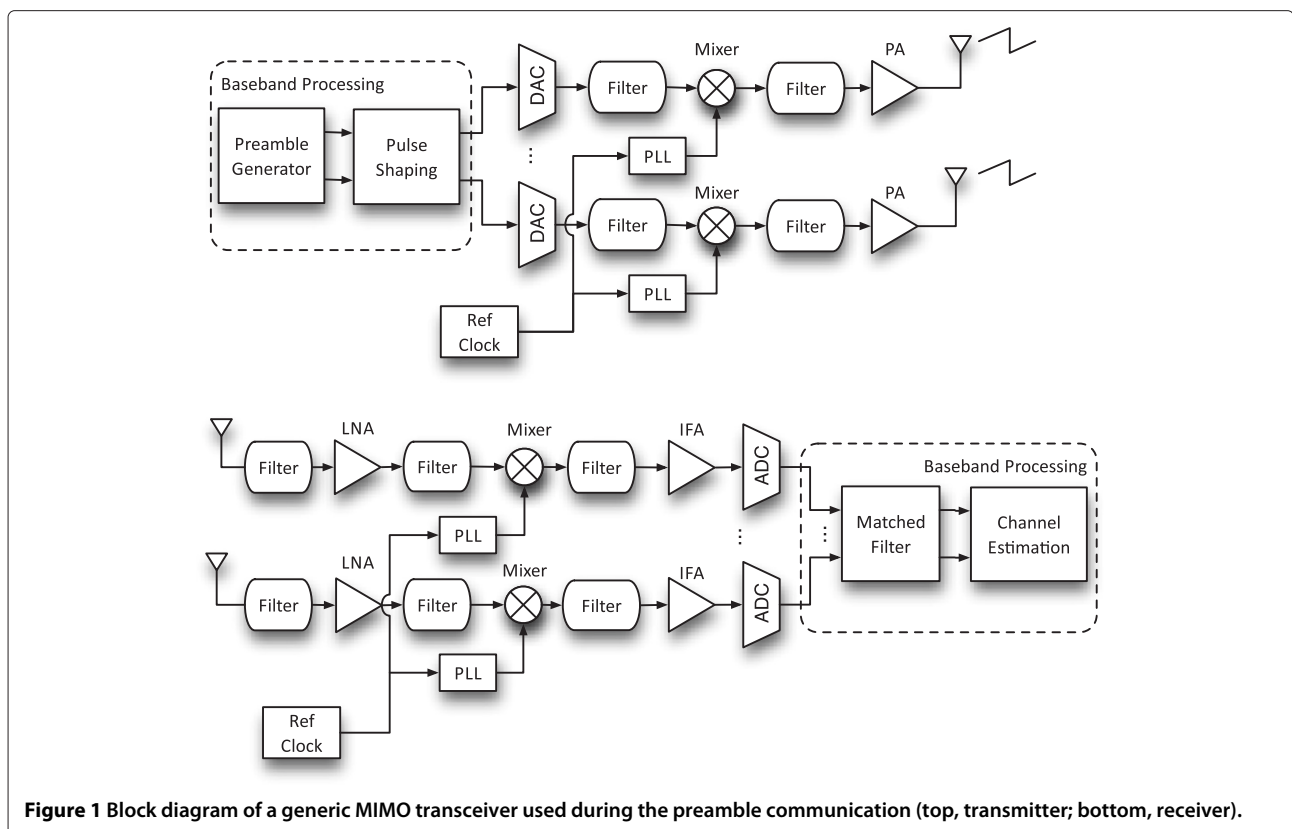

Figure 1 Block diagram of a generic MIMO transceiver used during the preamble communication (top, transmitter; bottom, receiver).

symbol. We define a binary variable $s_{(n,t)}$ that indicates if transmitter branch n transmits a preamble symbol during symbol time t, with $n = 1, \ldots, N_t$ and $t = 1, \ldots, N_p$. Thus, the total energy consumed by these components is given by

$$\mathcal{E}_{\text{el,tx}} = T_s \left(\check{P}_{\text{el,tx}} \sum_{n=1}^{N_t} \sum_{t=1}^{N_p} s_{(n,t)} + P_{\text{syn}} N_p \right), \quad (2)$$

where T_s is the symbol period and $\check{P}_{\text{el,tx}} = \check{P}_{\text{DAC}} + \check{P}_{\text{filter}} + \check{P}_{\text{mixer}}$ represents the power consumption of DAC, filters and mixer of each transmitter branch. This model allows for each antenna to transmit a different and arbitrary sequences of training symbols, which can include silences.

3. Energy consumption due to electromagnetic radiation, $\mathcal{E}_{\text{PA}}$

Each preamble symbol is broadcast from a transmitting antenna with a transmission power $\check{P}_{\text{tx}}$ provided by the respective power amplifier (PA). The PA's power consumption is modeled by

$$\check{P}_{\text{PA}} = \frac{\xi}{\eta} \check{P}_{\text{tx}}, \quad (3)$$

where ξ is the peak-to-average ratio of the transmitted signal and η is the drain efficiency of the PA [5]. The energy consumed jointly by all PA's is

$$\mathcal{E}_{\text{PA}} = \frac{\xi}{\eta} T_s \sum_{n=1}^{N_t} \sum_{t=1}^{N_p} \check{P}_{\text{tx}(n,t)} s_{(n,t)}, \quad (4)$$

where $\check{P}_{\text{tx}(n,t)}$ is the irradiated power of the training symbol transmitted by the antenna n during the symbol time t, with $n = 1, \ldots, N_t$ and $t = 1, \ldots, N_p$. This model allows for arbitrary transmission power per preamble symbol.

4. Energy consumption of the receiver electronics, $\mathcal{E}_{\text{el,rx}}$

It represents the energy consumption of the components that remain energized during the reception time of the preamble, which is equal to $N_p T_s$. Thus,

$$\mathcal{E}_{\text{el,rx}} = N_p T_s \left(\check{P}_{\text{el,rx}} N_r + P_{\text{syn}} \right), \quad (5)$$

where $\check{P}_{\text{el,rx}} = \check{P}_{\text{ADC}} + \check{P}_{\text{filter}} + \check{P}_{\text{mixer}} + \check{P}_{\text{LNA}} + \check{P}_{\text{IFA}}$ represents the power consumption of analog-to-digital converter (ADC), filters, mixer, low-noise amplifier (LNA) and intermediate frequency amplifier (IFA) of each receiving branch. We will assume that filters and mixers at the receiver consume the same power as these components at the transmitter.

5. Energy consumption due to channel estimation, $\mathcal{E}_{\text{estim}}$

Every time a packet is received, the channel estimation engine performs K different kinds of arithmetic operations, each of which has an energy consumption $\mathcal{E}_k$, with $k = 1, \ldots, K$ and is performed n_k times during the execution of the entire channel estimation algorithm. Thus,

$$\mathcal{E}_{\text{estim}} = \sum_{k=1}^{K} n_k \mathcal{E}_k. \quad (6)$$

If the operations are performed by an arithmetic processing unit (APU), the energy consumption of the kth operation can be modeled as [6]

$$\mathcal{E}_k = V_{\text{dd}} I_o \Delta t_k, \quad (7)$$

where V_{dd} is the APU operating voltage, I_o is the average current during the execution time of the arithmetic operations. It is to be noted that I_o depends on V_{dd} and on the clock frequency, f_{APU}. Δt_k is the time required for executing the kth operation. It is related to the number of clock cycles, c_k, required by the operation and to f_{APU} as follows:

$$\Delta t_k = \frac{c_k}{f_{\text{APU}}}. \quad (8)$$

Replacing these terms in (6), the energy required for estimating the channel is given by

$$\mathcal{E}_{\text{estim}} = \frac{V_{\text{dd}} I_o}{f_{\text{APU}}} \sum_{k=1}^{K} n_k c_k. \quad (9)$$

The sum of the energies (1), (2), (4), (5), and (9) gives the total energy consumed by the communication process in order to produce a channel estimate. It is given by

$$\begin{aligned}
\mathcal{E}_{\text{total}} = {} & 2 P_{\text{syn}} T_{\text{tr}} + T_s \left(\check{P}_{\text{el,tx}} \sum_{t=1}^{N_t} \sum_{p=1}^{N_p} s_{(t,p)} + P_{\text{syn}} N_p \right) \\
& + T_s N_p (\check{P}_{\text{el,rx}} N_r + P_{\text{syn}}) + T_s \frac{\xi}{\eta} \sum_{t=1}^{N_t} \sum_{p=1}^{N_p} \check{P}_{\text{tx}(t,p)} s_{(t,p)} \\
& + \frac{V_{\text{dd}} I_o}{f_{\text{APU}}} \sum_{k=1}^{K} n_k c_k.
\end{aligned}$$

$$(10)$$

We now turn our attention to the estimation problem, focusing on the number of arithmetic operations required by various common channel estimation algorithms.

3 Channel estimation algorithms

In this section, we characterize the LS and MMSE channel estimation algorithms by their complexity of implementation and associated MSE performance. This requires to

formulate a signal model that describes the communication of the preamble and to determine the arithmetic operations that each algorithm performs.

It is to be noted that the problem of channel estimation in a MIMO system with N_t transmit and N_r receive antennas is, in practice, a set of N_t independent single-input multiple-output problems, one per transmitter branch. This follows from observing that it cannot be assumed *a priori* that the $N_r \times N_t$ channel coefficients that compose the MIMO channel are correlated in any particular fashion. In the most unfavorable case, observations from one transmitter branch contain no information about channel parameters of another one, and transmitting information simultaneously from more than one branch provides no benefit and increases complexity. This implies that an $N_r \times N_t$ channel matrix $\mathbf{H}$ can be estimated sequentially by columns using a preamble in which only one antenna simultaneously transmits a training sequence, as shown in [7] (Figure 2). In this case, the condition $\sum_{t=1}^{N_p} s_{(n,t)} = N_p/N_t$ must hold for each transmitter branch so that column of $\mathbf{H}$ is estimated using the same number of pilot symbols.

The estimation of the jth column of $\mathbf{H}$ only requires the information of the received signal that was generated during the time when the jth branch transmitted its preamble. The received signal can be expressed as

$$\mathbf{Y}_j = \frac{1}{\sqrt{Ad^\alpha}}\mathbf{h}^j\mathbf{p}_j + \mathbf{V}_j, \tag{11}$$

where Ad^α represents the path loss, with d the link distance, α the path loss exponent, and A a parameter that depends on the transmitter and receiver antenna gains and the transmission wavelength (A may include shadow fading) [8]. $\mathbf{p}_j$ is the preamble sequence transmitted by the jth branch. $\mathbf{h}^j$ is the jth column of $\mathbf{H}$ and its elements represent the small scale fading of the MIMO channel. We assume that the wireless channel is static and flat fading. $\mathbf{V}_j$ is a matrix of independent complex Gaussian random

variables with zero mean and variance σ_n^2, representing additive white Gaussian noise. The variance σ_n^2 depends on the transmission bandwidth W, on the receiver noise figure, N_f, and on the link margin M_l [1].

The receiver compensates the path loss using an LNA. Thus, the signal observed by the channel estimator is

$$\mathbf{S}_j = \mathbf{h}^j\mathbf{p}_j + \sqrt{Ad^\alpha}\mathbf{V}_j. \tag{12}$$

In the following, we analyze the energy consumption and MSE of the LS and MMSE algorithms.

3.1 LS algorithm

Given $\mathbf{S}_j$ and knowledge of $\mathbf{p}_j$, the LS estimator for the jth column of $\mathbf{H}$ is [9]

$$\hat{\mathbf{h}}_{LS}^j = \mathbf{S}_j\frac{\mathbf{p}_j^H}{\|\mathbf{p}_j\|^2}. \tag{13}$$

As $\mathbf{p}_j^H/\|\mathbf{p}_j\|^2$ is known *a priori*, the estimation only requires the product between the $N_r \times \frac{N_p}{N_t}$ matrix $\mathbf{S}_j$ and an $\frac{N_p}{N_t} \times 1$ vector. This takes $\frac{N_r N_p}{N_t}$ complex products and $N_r\left(\frac{N_p}{N_t} - 1\right)$ complex sums each time a column of H is estimated. Standard implementations of these complex operations require four real products and two real sums for each complex product, and two real sums for each complex sum [10]. Then, performing the estimation (13) for N_t columns of $\mathbf{H}$ requires $2N_r(2N_p - N_t)$ real additions and $4N_rN_p$ real multiplications.

Finally, assuming that every pilot symbol is transmitted with power P_{tx}, the energy consumption model (10) for the case of the LS algorithm becomes

$$\mathcal{E}_{LS} = 2P_{syn}T_{tr} + \left(\check{P}_{el,tx} + N_r\check{P}_{el,rx} + 2P_{syn}\right)T_sN_p$$
$$+ \frac{\xi}{\eta}T_sN_pP_{tx} + \frac{2V_{dd}I_oN_r}{f_{APU}}\left[\left(2N_p - N_t\right)c_{sum} + 2N_pc_{prod}\right]. \tag{14}$$
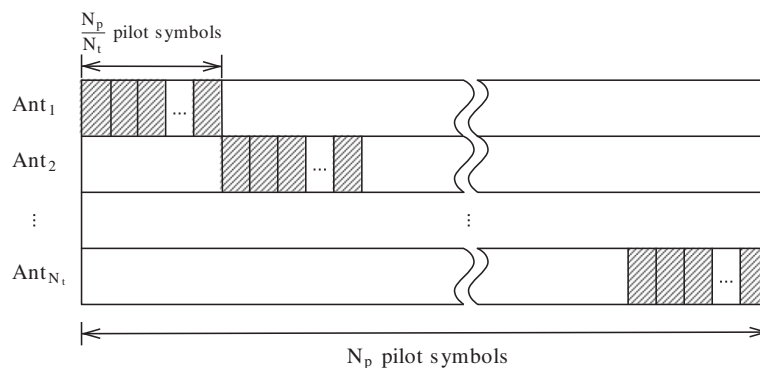
Figure 2 Distribution of pilot symbols per antenna, with a time-orthogonal preamble.

The MSE of the LS algorithm is [9]

$$\text{MSE}_{\text{LS}} = \frac{Ad^{\alpha}\sigma_n^2 N_r N_t^2}{N_p P_{\text{tx}}}, \tag{15}$$

which uses the fact that each column of **H** is estimated with $\frac{N_p}{N_t}$ equal power pilot symbols.

3.2 MMSE algorithm
The MMSE estimator for the jth column of **H** is [9]

$$\hat{\mathbf{h}}_{\text{MMSE}}^j = \mathbf{S}_j \mathbf{p}_j^H \frac{\mathbf{R}_H^{(j)}}{N_r Ad^{\alpha} \cdot \sigma_n^2 + \mathbf{R}_H^{(j)} \cdot \|\mathbf{p}_j\|^2}, \tag{16}$$

where $\mathbf{R}_H^{(j)}$ is the jth element of the diagonal of the correlation matrix $\mathbf{R}_H = \mathbb{E}\{\mathbf{H}^H\mathbf{H}\}$. We assume that $\mathbf{R}_H$ and σ_n^2 are perfectly known but can change from one transmission to the next. Therefore, the quotient in (16) has to be calculated with each new estimation of $\hat{\mathbf{h}}_{\text{MMSE}}^j$. This requires to perform two products, shown with '·' in (16), the sum in the denominator and the division plus the evaluation of $\mathbf{S}_j \mathbf{p}_j^H$, as in the LS case. The total number of real arithmetic operations required by this algorithm are shown in Table 1.

Replacing the number of arithmetic operations of Table 1 in (10), the energy consumption model for MMSE algorithm is

$$\begin{aligned}
\mathcal{E}_{\text{MMSE}} =\ & 2P_{\text{syn}}T_{\text{tr}} + \left(\check{P}_{\text{el,tx}} + N_r\check{P}_{\text{el,rx}} + 2P_{\text{syn}}\right)T_s N_p \\
& + \frac{\xi}{\eta}T_s N_p P_{\text{tx}} + \frac{V_{\text{dd}}I_o}{f_{\text{APU}}}\left[\left(4N_r N_p + 9N_t\right)c_{\text{sum}}\right. \\
& \left. + \left(4N_r N_p + 4N_r N_t + 14N_t\right)c_{\text{prod}} + 2N_t c_{\text{div}}\right].
\end{aligned} \tag{17}$$

where c_{sum}, c_{prod}, and c_{div} describe the number of cycles required for performing a sum, product, and division, respectively.

The MSE of the MMSE estimation algorithm is [9]

$$\text{MSE}_{\text{MMSE}} = Ad^{\alpha}\sigma_n^2 N_r N_t \sum_{j=1}^{N_t} \frac{\mathbf{R}_H^{(j)}}{Ad^{\alpha}\sigma_n^2 N_r N_t + \mathbf{R}_H^{(j)} N_p P_{\text{tx}}}. \tag{18}$$

It is to be noted that (14) and (17) share a common structure, namely

$$\mathcal{E} = k_1 + k_2 N_p + k_3 N_p P_{\text{tx}}, \tag{19}$$

Table 1 Number of instructions required by the MMSE estimator

Operation	Number of instructions
Sum	$n_{\text{sum}} = 4N_r N_p + 9N_t$
Product	$n_{\text{prod}} = 4N_r N_p + 4N_r N_t + 14N_t$
Division	$n_{\text{div}} = 2N_t$

with k_1, k_2, and k_3 given in Table 2. The constant k_1 is a base energy consumption independent of the number of pilot symbols transmitted. This constant is larger in the case of MMSE algorithm. The term $k_2 N_p$ represents the energy consumption due to electronics and baseband processing and depends linearly on the number on pilot symbols transmitted. $k_3 N_p P_{\text{tx}}$ represents the energy consumption due to electromagnetic radiation. The energy consumption increases as a function of N_p and P_{tx} has the same slope for MMSE and LS estimation because k_2 and k_3 are equal in both cases.

4 Minimization of the channel estimation energy consumption
In this section, we formulate and solve the optimization problem of minimizing the total energy consumption required for carrying out the LS and MMSE channel estimation algorithms as a function of the number of pilot symbols N_p and of the transmission power P_{tx}. Expression (19) is the objective function of the minimization problem.

We assume the following

A1: The transceivers have $N_r = N_t = N$ antennas and $P_{\text{syn}}, T_{\text{tr}}, \check{P}_{\text{el,tx}}, \check{P}_{\text{el,rx}}, V_{\text{dd}}, I_o, f_{\text{APU}}, c_{\text{sum}}, c_{\text{prod}}, c_{\text{div}}, \eta,$ and W are known parameters.
A2: The path loss parameter A, link distance d, and path loss exponent α are given and the matrix H is an uncorrelated flat fading MIMO Rayleigh channel with $\mathbf{R}_H = \mathbb{E}\{\mathbf{H}^H\mathbf{H}\} = N\mathbf{I}_N$.

In addition, we consider the following restrictions:

R1: In order to ensure a given estimation error, expressions (15) and (18) are upper-bounded by ϵ_{max}.
R2: The number of pilot symbols N_p should be equal or greater than N, so that at least one pilot symbol is transmitted by each antenna.
R3: Transmission power is constant P_{tx} (thus, $\xi = 1$) and limited to P_{max}.

Considering these assumptions and restrictions along with the expressions (15), (18), and (19), our optimization problem can be written in general as

$$\underset{N_p,P_{\text{tx}}}{\text{minimize}} \quad k_1 + k_2 N_p + k_3 N_p P_{\text{tx}} \tag{20a}$$

$$\text{subject to} \quad -N_p P_{\text{tx}} + k_4 \leq 0 \tag{20b}$$

$$-N_p + N \leq 0 \tag{20c}$$

$$P_{\text{tx}} - P_{\text{max}} \leq 0, \tag{20d}$$

Table 2 Constants of the energy model (19)

	LS	MMSE
k_1	$2P_{syn}T_{tr} - 2\frac{V_{dd}I_o}{f_{APU}}N_rN_tc_{sum}$	$2P_{syn}T_{tr} + \frac{V_{dd}I_o}{f_{APU}}N_t\left[9c_{sum} + (4N_r + 14)c_{prod} + 2c_{div}\right]$
k_2	$T_s(\check{P}_{el,tx} + N_r\check{P}_{el,rx} + 2P_{syn}) + \frac{4V_{dd}I_oN_r}{f_{APU}}(c_{sum} + c_{prod})$	$T_s(\check{P}_{el,tx} + N_r\check{P}_{el,rx} + 2P_{syn}) + \frac{4V_{dd}I_oN_r}{f_{APU}}(c_{sum} + c_{prod})$
k_3	$T_s\frac{\xi}{\eta}$	$T_s\frac{\xi}{\eta}$
k_4	$\frac{Ad^\alpha\sigma_n^2N_rN_t^2}{\epsilon_{max}}$	$\frac{Ad^\alpha\sigma_n^2N_r}{\epsilon_{max}}(N_t^2 - \epsilon_{max})$

where k_1 through k_4 are given in Table 2. This optimization problem has a quadratic objective function with restrictions forming a convex domain. It is to be noted that (20a) is to be solved as an integer optimization problem, because $\frac{N_p}{N} \in \mathbb{N}$ must be satisfied. We do this by first solving (20a) by means of Lagrange multipliers [11] in its continuous variable form (see Appendix) and then analyze the integer solution requirement.

The optimal values of the number of pilot symbols N_p^* and transmission power P_{tx}^* depend on constant k_4:

- If $NP_{max} \geq k_4$, then the constraints (20b) and (20c) are active. Therefore,

$$N_p^* = N \tag{21a}$$

$$P_{tx}^* = \frac{k_4}{N} \tag{21b}$$

- If $NP_{max} \leq k_4$, then the constraints (20b) and (20d) are active for the non-integer optimization problem, so that $P_{tx} = P_{max}$ and $N_p = \frac{k_4}{P_{max}}$. By imposing the integer constraint over N_p, we find

$$N_p^* = N\left\lceil \frac{k_4}{NP_{max}} \right\rceil \tag{22a}$$

$$P_{tx}^* = \frac{k_4}{N_p^*}, \tag{22b}$$

where $\lceil x \rceil$ denotes the smallest integer larger than x.

The MSE constraint (20b) is active in both cases above because it locks the trade-off between the optimal transmission power P_{tx}^* and the optimal number of pilot symbols N_p^*.

The condition $k_4 = NP_{max}$ separates the optimal solution into the two regions described above. As shown in Table 2, k_4 depends on several parameters and the dependence of the optimality of N_p and P_{tx} on them may be analyzed for each one. For illustration, we provide the analysis for how N_p^* and P_{tx}^* vary with distance. Considering d as a variable, condition $k_4 = NP_{max}$ defines a critical distance d_c. For $d \leq d_c$ the optimal solution is to transmit the minimum number of pilot symbols ($N_p^* = N$), and for $d > d_c$, the solution is to use $P_{tx} \approx P_{max}$ (this relationship

is approximated because of the ceiling function in (22a)). For the LS algorithm, the critical distance is

$$d_{c,LS} = \left(\frac{P_{max}\epsilon_{max}}{A\sigma_n^2N^2}\right)^{\frac{1}{\alpha}} \tag{23}$$

and for the MMSE algorithm it is

$$d_{c,MMSE} = \left(\frac{P_{max}\epsilon_{max}}{A\sigma_n^2(N^2 - \epsilon_{max})}\right)^{\frac{1}{\alpha}}. \tag{24}$$

5 Numerical evaluation

In order to provide numerical examples that allow for comparing the minimum energy consumption of each estimator, we use the parameters given in [5] for a generic low-power transceiver (summarized in Table 3) and the parameters given in [12,13] for a standard APU (Table 4). Finally, we choose $N = 4$ antennas for both transceivers.

Figures 3 and 4 show the optimal number of pilot symbols and transmitted power for a maximum estimation MSE of -10 dB for each algorithm. For this estimation error, the critical distance for the LS and the MMSE algorithms is about 42 m (dashed line). In both cases, the number of pilot symbols remains constant (Figure 3) while

Table 3 Generic low-power device parameters

Parameter	Description	Value
W	Bandwidth	10 kHz
T_s	Symbol period	100 μs
P_{syn}	Frequency synthesizer power consumption	50 mW
T_{tr}	Frequency synthesizer settling time	5 μs
$\check{P}_{el,tx}$	Tx electric power consumption	98.2 mW
$\check{P}_{el,rx}$	Rx electric power consumption	112.5 mW
α	Path loss coefficient	3.5
A	Channel path loss	30 dB
N_0	Noise power density	-174 dBm/Hz
N_f	Receiver noise figure	10 dB
M_l	Link margin	40 dB
η	PA drain efficiency	0.35

Table 4 Technical parameters of an arithmetic and logic unit (ALU)

Parameter	Description	Value
f_{ALU}	ALU frequency	20 MHz
V_{dd}	ALU voltage	3 V
$I_o(V_{dd}, f_{ALU})$	Average current	6.37 mA
c_{sum}	Adding cost	6 cycles
c_{prod}	Product cost	13 cycles
c_{div}	Division cost	21 cycles

the transmitted power increases (Figure 4). At the critical distance, N_p^* steps up to the next integer multiple of N, as described by (22a). At that point, the optimal transmission power steps back as shown by the sawtooth curves in Figure 4. As the distance increases further, the optimal transmitted power tends to P_{max}.

In order to compare the energy required by both algorithms, we evaluate (14) and (17) at N_p^* and P_{tx}^* (Figure 5). The energy consumption of both algorithms is identical for any practical purpose. However, the LS algorithm is simpler to implement because it does not require knowledge of the channel statistics and of the power of the noise. Therefore, it is the preferred choice for attaining a given estimation quality (MSE) at minimum energy consumption.

For a standard low-power device, the maximum operation distance is about 50 m [14]. Figure 5 shows that at this distance each antenna must send two pilot symbols in order to estimate the channel with an MSE of −10 dB with

minimum energy consumption. On the other hand, given that each antenna transmits two preamble symbols, the maximum link distance (which is achieved maximizing the transmission power) is about 53 m.

If we consider a minimum detectable signal-to-noise ratio (SNR) of 7 dB, the maximum link distance allowed by the generic system used here as example is about 123 m (Figure 6). At this distance, the channel estimators require 128 pilot symbols for an estimation MSE of −10 dB, and they consume 10 times more energy than in the 53-m link case. This suggests that under certain conditions, it may be more efficient in terms of energy consumption to use multi-hop MIMO communications. However, that analysis must incorporate the energy consumption required for transmitting correctly a message [2] and lies beyond the scope of this study.

Figure 7 shows the minimum energy consumption of each algorithm as a function of the estimation MSE. As expected, the minimum energy drops in steps as the error tolerance grows. It is to be noted that near each energy step, slight estimation performance degradation—and hence slight error rate degradation—can be traded for significant reduction in energy consumption for channel estimation.

6 Conclusions

In this study, we present a methodology for determining the length and transmission power of training signals that allow for producing estimates of MIMO channels with a given estimation error and minimal joint energy
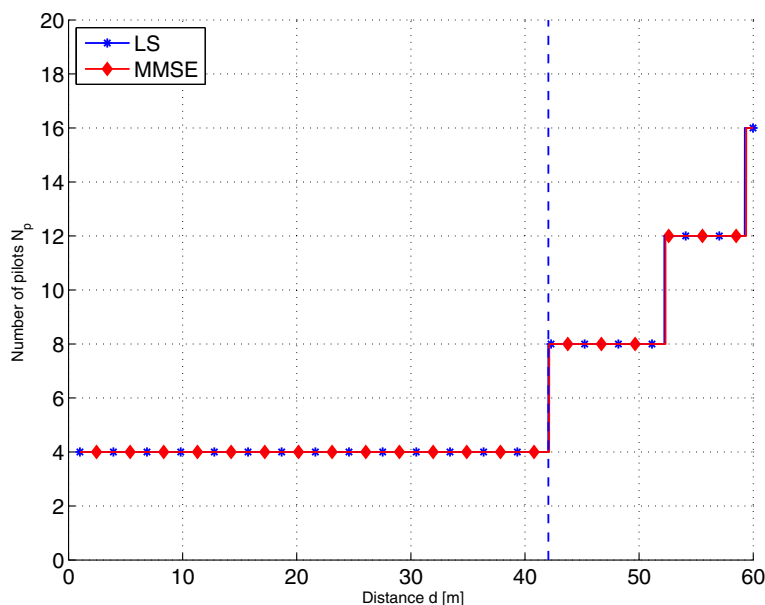

Figure 3 Optimal number of pilot symbols versus distance for the LS and MMSE algorithms with estimation MSE of −10 dB.
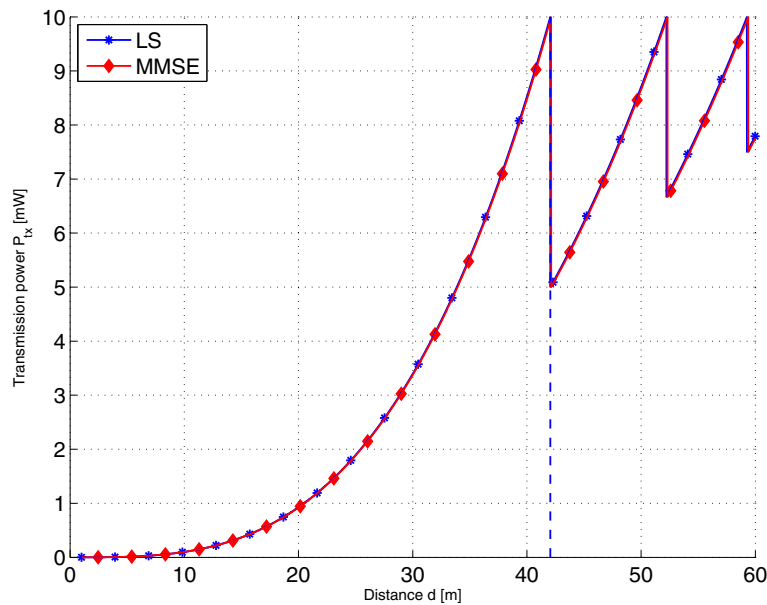
Figure 4 Optimal transmission power versus distance for the LS and MMSE algorithms with estimation MSE of −10 dB.

consumption among transmitter and receiver. We develop a general energy consumption model for the complete process of channel estimation. The model includes energy consumption due to transmission and reception of the training signals and due to the processing required to obtain the channel estimates.

The model was used for studying and optimizing the energy consumption of the LS and MMSE channel estimation algorithms. Both algorithms consume virtually the same energy when operated at their respective optimal training signal configurations of length and transmission power. However, the LS algorithm does not require the knowledge of the channel statistics and of the power of the noise, which makes the LS algorithm the preferred choice.

For link distances of about 50 m, our results show that the channel estimation with minimum energy is achieved using two preamble symbols per transmit antenna when
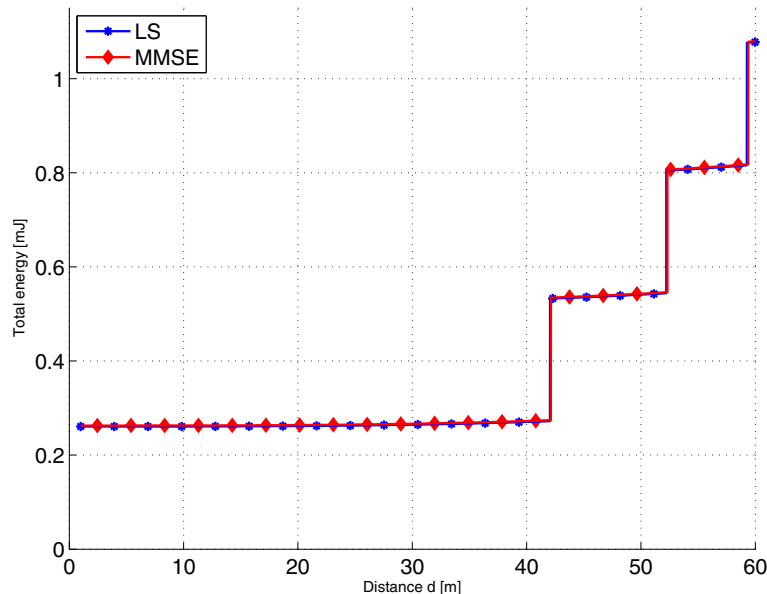

Figure 5 Minimum energy consumed by the LS and MMSE algorithms, with estimation MSE of −10 dB.
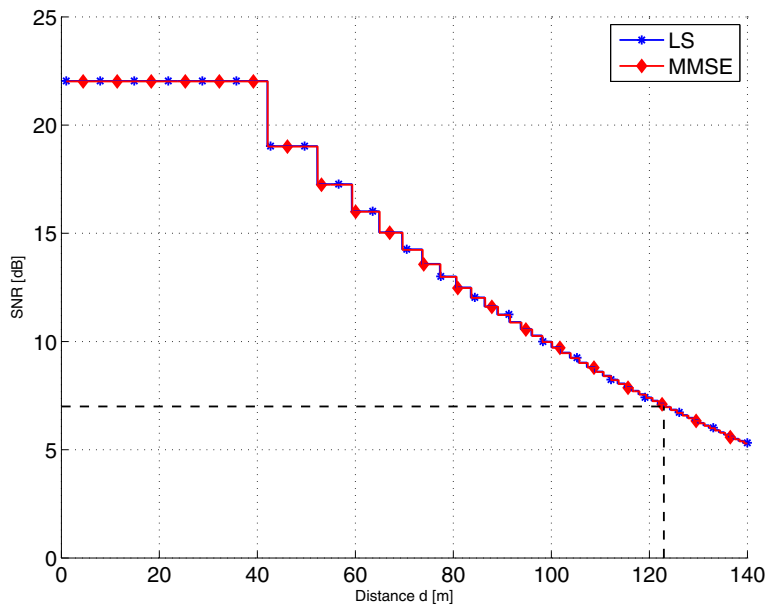
Figure 6 SNR at the input of the channel estimator. The vertical dashed line shows the maximum distance for an SNR of 7 dB.

the target estimation MSE is -10 dB. For distances of approximately 120 m, the minimum energy consumption required to achieve the same estimation quality increases tenfold for both algorithms due to path loss. This indicates that longer range MIMO communications can be performed more energy-efficiently by multi-hop routes than over single-hop links.

7 Appendix
7.1 Optimization problem
We define an auxiliary variable $N_u = \frac{N_p}{N} \in \mathbb{N}$. The Lagrangian of the optimization problem (20a) is

$$
\begin{aligned}
\mathcal{L} = {} & k_1 + k_2 N N_u + k_3 N N_u P_{\text{tx}} + \lambda_1 \left(-N N_u P_{\text{tx}} + k_4\right) \\
& + \lambda_2 \left(-N_u + 1\right) + \lambda_3 \left(P_{\text{tx}} - P_{\text{max}}\right)
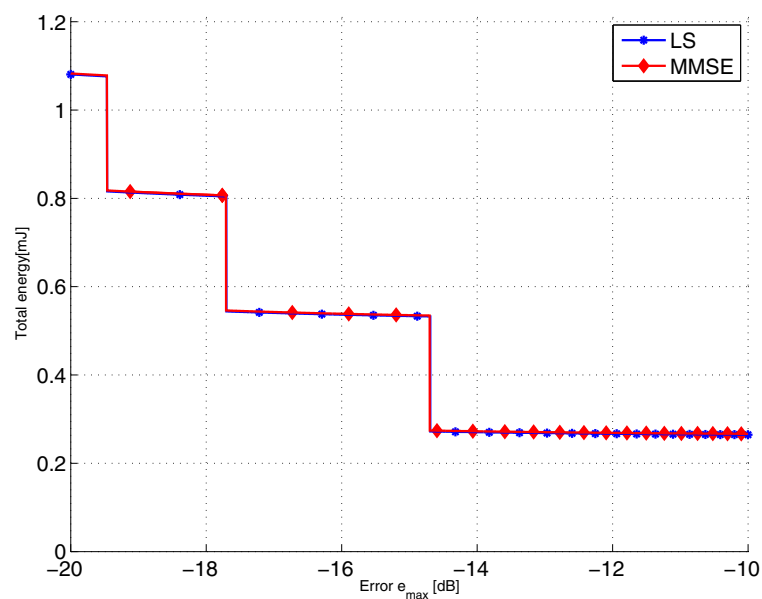\end{aligned}
\tag{25}
$$

Figure 7 Minimum energy consumed by the LS and MMSE algorithms versus mean square estimation error for a link distance of 30 m.

with multipliers λ_1, λ_2, and λ_3. After taking derivative of $\mathcal{L}$ with respect to λ_1, λ_2, λ_3, N_u, and P_{tx} we find two feasible solutions:

- Case I, $\lambda_1 \neq 0, \lambda_2 \neq 0, \lambda_3 = 0$
  The R1 (20b) and R2 (20c) constraints are active, therefore

$$N_u^* = 1 \tag{26a}$$

$$P_{\text{tx}}^* = \frac{k_4}{N}. \tag{26b}$$

  This occurs when $k_4 \leq NP_{\max}$. In this case, $N_p^* = N$ is a feasible integer solution.

- Case II, $\lambda_1 \neq 0, \lambda_2 = 0, \lambda_3 \neq 0$
  The R1 (20b) and R3 (20d) constraints are active, therefore, $P_{\text{tx}} = P_{\max}$ and $N_u = \frac{k_4}{NP_{\max}}$. This occurs when $k_4 \geq NP_{\max}$, but now N_u is not necessarily a natural number. Incorporating this constraint, we obtain $N_u^* = \left\lceil \frac{k_4}{NP_{\max}} \right\rceil$. Therefore,

$$N_p^* = N \left\lceil \frac{k_4}{NP_{\max}} \right\rceil \tag{27a}$$

$$P_{\text{tx}}^* = \frac{k_4}{N_p^*}. \tag{27b}$$

Competing interests
The authors declare that they have no competing interests.

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