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# Outage-optimal opportunistic scheduling with analog network coding in multiuser two-way relay networks

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## Abstract

This paper investigates the performance of an outage-optimal opportunistic scheduling scheme for a multiuser two-way relay network, wherein an analog network coding-based relay serves multiple pairs of users. Under a Rayleigh flat-fading environment, we derive an exact expression for cumulative distribution function (CDF) of the minimum of the two end-to-end instantaneous signal-to-noise ratios (SNRs) and utilize this to obtain an exact expression for the outage probability of such a greedy scheduling scheme. We then develop a modified scheduler that ensures fairness among user pairs of the considered system. By using a high SNR approximation of derived CDF, we present a simple closed-form expression for outage probability of the overall system and establish that a multiuser diversity of order equal to the number of user pairs is harnessed by the scheme. We also present an efficient power allocation strategy between sources and relay, subject to a total power constraint, that minimizes the outage probability of the overall system. Further, by deriving both upper and lower bound expressions for the average sum-rate of the proposed scheme, we demonstrate that an average sum-rate gain can also be achieved by increasing the number of user pairs in the system. Numerical and simulation results are presented to validate the performance of the proposed scheme.

**Keywords:** Analog network coding, Multiuser scheduling, Outage probability, Rayleigh fading, Two-way (bidirectional) relaying

## 1 Introduction

Cooperative relaying techniques have recently gained great research interest because of their potential in enhancing the throughput or reliability of wireless networks. Several schemes have been extensively studied in literature to achieve cooperative diversity utilizing the one-way relaying protocol [1]. However, the half-duplex constraint at the relays incurs a spectral efficiency loss in such schemes. Recent research has shown that such a loss can be effectively mitigated by exploiting the idea of network coding [2] in bidirectional communication scenarios [3-7]. Bidirectional cooperative relaying strategies facilitate information exchange between two users in either four, three, or two time phases via a half-duplex relay. The four-phase protocol follows the conventional

approach by requiring two separate time phases for data flow in each direction and hence is spectrally inefficient. However, the bidirectional communication has been shown to be accomplished in even three phases in [3-7]. In the three-phase protocol (called physical layer network coding (PNC) [6] or time division broadcast (TDBC) [7]), the two users transmit successively in first and second phases, the relay then decodes both the data, applies network coding, and forwards the combined data to both users in the third phase. After canceling the self-interferences (as they are known by the respective users), the intended message can be received at each of the user terminals. Recently, a two-way relaying protocol [8,9] has emerged as a promising technique to mitigate the spectral efficiency loss of conventional half-duplex relaying systems. In this scheme, the two users communicate bidirectionally (in the absence of a reliable direct link) in just two time phases, namely the multiple access channel (MAC) phase and the broadcast

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**Table 1 Selected values of channel variances over the two hops for i.n.i.d. user pairs**

$\{\sigma_{a,k}^2\}_{k=1}^K$	$\{\sigma_{b,k}^2\}_{k=1}^K$
0.2	0.5
0.8	0.3
0.4	0.1
0.9	0.7
0.4	0.8
0.2	0.3
0.9	0.5
0.1	0.7
0.7	0.2
0.2	0.9
0.3	0.4
0.6	0.5
0.8	0.2
0.5	0.3
0.2	0.6
0.4	0.7

channel (BC) phase. In the first phase (MAC), both users transmit their data simultaneously to the relay, and the relay broadcasts the processed signal to both users in the second phase (BC). When an amplify-and-forward (AF) processing is applied on the superimposed signal received during the MAC phase at the relay, such a scheme is usually termed as analog network coding (ANC) [10-13].

The two-phase two-way relaying protocol has also been generalized to a multiuser scenario in which multiple pairs of users communicate bidirectionally via one or more relays [8,14-18]. The authors in [8,14] considered several relays or antennas that orthogonalize multiple pairs by a distributed zero-forcing technique. A spread spectrum based interference management scheme wherein each pair shares a common spreading signature, and the relay uses a jointly demodulate-and-XOR forward strategy is proposed in [15]. The information theoretic capacity for such a scheme is studied in [16] and [17] by considering a deterministic channel model and a Gaussian two-pair two-way full-duplex relay network, respectively. To combat interference at each user of such a system, the authors in [18] proposed different beamforming schemes with amplify-and-forward (AF) and quantize-and-forward (QF) strategies at the relay. However, to the best of our knowledge, a performance analysis exploiting multiuser diversity for this system has not been reported so far. Although the two-phase two-way relaying protocol is spectrally efficient, it incurs a penalty in diversity as compared to conventional one-way relaying [1] due to the absence of the direct path. In traditional wireless communications under a multiuser downlink scenario, it has been shown

in many publications [19,20] that opportunistic scheduling of users can provide diversity gains. Therefore, the use of scheduling to harness multiuser diversity is well motivated in the two-way relaying context. Note that such opportunistic access avoids the difficult synchronization issues associated with simultaneous transmission of multiple user pairs, as well as the requirement of large number of antennas at the relay. However, scheduling strategies for such two-way systems are different from the commonly studied downlink scenarios. For this reason, development of good scheduling strategies in the two-way context is of considerable interest.

With the above motivation, we propose in this paper an opportunistic scheduling scheme for a multi-pair ANC-based two-way relay system and evaluate its performance over Rayleigh fading channels. We first consider a greedy scheduler based on minimizing the overall outage probability of the system. Considering channel state information (CSI)- and noise statistics-assisted gain at the relay, we derive an exact expression for the CDF of the minimum of SNRs of the two end-to-end transmissions, which is applicable for the whole SNR region. This facilitates an exact outage performance analysis for the considered greedy scheduling scheme. We then propose a scheduler that ensures fairness among user pairs of the system. By using a high SNR approximation of derived CDF, we obtain a simple closed-form expression for the outage probability of the overall system. Further, we provide an efficient power allocation scheme based on the derived expression that minimizes the system outage probability. In addition, we derive expressions for upper and lower bounds on the average sum-rate of the considered system. Numerical and simulation results illustrate the effectiveness of our analytically derived results and show that the considered scheme achieves performance gain by attaining an order of multiuser diversity equal to the number of user pairs.

The rest of the paper is organized as follows: Section 2 describes the system model. The opportunistic scheduling criteria are formulated in Section 3. The overall system is analyzed in terms of access probability, outage probability, and average sum-rate under Rayleigh fading in Section 4. Section 5 presents numerical and simulation results, and finally, Section 6 concludes the paper.

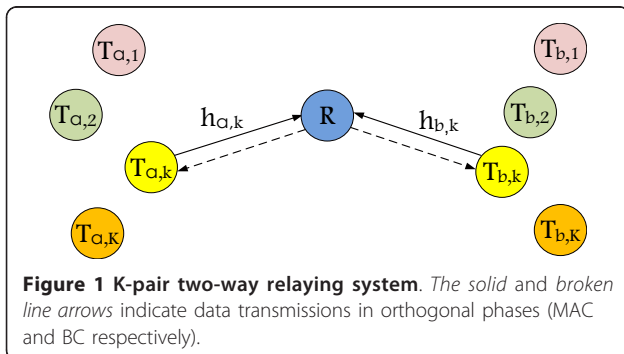
#### Notation

We use  $z \sim \mathcal{CN}(\mu, \sigma^2)$  to denote a complex circular Gaussian random variable  $z$  with mean  $\mu$  and variance  $\sigma^2$ . The operators  $\mathbb{E}[\cdot]$ ,  $\Pr[\cdot]$ , and  $|\cdot|$  represent expectation, probability, and absolute value, respectively.  $\binom{\cdot}{\cdot}$  denotes the binomial coefficient,  $\ln(\cdot)$  denotes the natural logarithm, and  $\log(\cdot)$  refers to  $\log_2(\cdot)$ .

## 2 System descriptions

We consider a multiuser two-way relay network consisting of  $2K + 1$  single-antenna nodes ( $K$  pairs of users and one relay), as depicted in Figure 1. Nodes  $T_{a,k}$  and  $T_{b,k}$  denote members of  $k$ th pair,  $k \in \{1, 2, \dots, K\}$ , that want to communicate bidirectionally via a single relay  $R$ . All nodes operate in a half-duplex fashion. The communication takes place slot-wise where one time-slot represents the end-to-end transmission duration. In what follows, we consider one time-slot to be comprised of two phases of equal duration, viz., the MAC phase and the BC phase. We assume that the channels for all links are subject to independent, but not necessarily identical frequency-flat Rayleigh fading. Let  $h_{l,k}(t)$  denote the channel fading coefficient between node  $T_{l,k}$  and relay  $R$  during the  $t$ th time-slot, where  $l \in \{a, b\}$ . We adopt the quasi-static fading-channel model such that  $h_{l,k}(t)$  remains constant during a time-slot but independently changes in different time-slots. Since the scheduling policy is determined in each slot, we drop the time index  $t$  for notational simplicity. Therefore,  $h_{l,k}$  can be modeled as  $\mathcal{CN}(0, \sigma_{l,k}^2)$  where  $\sigma_{l,k}^2$  is the average fading power of the link between  $T_{l,k}$  and  $R$ . We further assume that the relay  $R$  has perfect global channel state information (CSI) of the network. To facilitate CSI estimation at the users, the relay periodically broadcasts a common pilot signal to all users. Then, each user feeds back that CSI to the relay and assuming channel reciprocity, a scheduling strategy for opportunistic transmission by user pairs can be employed. The user pairs are informed about the scheduling decision through a certain control signal by the relay. The specific scheduling policy will be elaborated upon in Section 3.

Let us consider that the  $k$ th user pair is scheduled for transmission and has access to relay channel resources for a given time-slot. We focus on the two-step ANC protocol whereby the information exchange for the  $k$ th pair takes place in two phases of equal time duration. In the first phase (MAC), both users of  $k$ th pair simultaneously transmit their data to the relay with equal power  $P$ . Note that equal transmit power assumption at



the users does not lose generality in diversity performance analysis, as its effect can be included in the average SNR of each channel. With this, the received signal at the relay is given by

$$\gamma_{r,k} = \sqrt{P}h_{a,k}x_{a,k} + \sqrt{P}h_{b,k}x_{b,k} + n_{r,k}, \quad (1)$$

where  $x_{a,k}$  and  $x_{b,k}$  are the transmit symbols having unit energy from the sources  $T_{a,k}$  and  $T_{b,k}$ , respectively, and  $n_{r,k}$  is an additive white Gaussian noise (AWGN) at the relay. The AWGN at all nodes is assumed as independent and identically distributed (i.i.d.)  $\mathcal{CN}(0, N_0)$  with the noise variance per dimension is  $N_0/2$ . Therefore, we define  $P/N_0$  as simply the SNR.

During the second phase (BC), the signals received at the destinations  $T_{a,k}$  and  $T_{b,k}$  via the AF relay can be expressed, respectively, as

$$\gamma_{a,k} = \beta_k \sqrt{P}h_{a,k}h_{a,k}x_{a,k} + \beta_k \sqrt{P}h_{a,k}h_{b,k}x_{b,k} + \beta_k h_{a,k}n_{r,k} + n_{a,k} \quad (2)$$

and

$$\gamma_{b,k} = \beta_k \sqrt{P}h_{a,k}h_{b,k}x_{a,k} + \beta_k \sqrt{P}h_{b,k}h_{b,k}x_{b,k} + \beta_k h_{b,k}n_{r,k} + n_{b,k}, \quad (3)$$

where  $n_{a,k}$  and  $n_{b,k}$  denote AWGN at the nodes  $T_{a,k}$  and  $T_{b,k}$ , respectively, and  $\beta_k$  represents the power-constrained amplifying gain at the relay given by

$$\beta_k = \sqrt{\frac{P_r}{P|h_{a,k}|^2 + P|h_{b,k}|^2 + N_0}}, \quad (4)$$

where  $P_r$  is the transmit power at the relay. The intra-pair interferences in (2) and (3) can be canceled out as they are known at the respective terminals, and hence, the received signals can be re-expressed as

$$\tilde{\gamma}_{a,k} = \beta_k \sqrt{P}h_{a,k}h_{b,k}x_{b,k} + \beta_k h_{a,k}n_{r,k} + n_{a,k} \quad (5)$$

and

$$\tilde{\gamma}_{b,k} = \beta_k \sqrt{P}h_{a,k}h_{b,k}x_{a,k} + \beta_k h_{b,k}n_{r,k} + n_{b,k}. \quad (6)$$

The resultant end-to-end instantaneous SNRs at nodes  $T_{a,k}$  and  $T_{b,k}$  are given, respectively, by

$$\gamma_{a,k} = \frac{P_r P}{N_0} \left[ \frac{|h_{a,k}|^2 |h_{b,k}|^2}{(P_r + P)|h_{a,k}|^2 + P|h_{b,k}|^2 + N_0} \right] \quad (7)$$

and

$$\gamma_{b,k} = \frac{P_r P}{N_0} \left[ \frac{|h_{a,k}|^2 |h_{b,k}|^2}{(P_r + P)|h_{b,k}|^2 + P|h_{a,k}|^2 + N_0} \right]. \quad (8)$$

The corresponding one-sided data-rates are thus given as  $\mathcal{R}_{a,k} = \frac{1}{2} \log(1 + \gamma_{a,k})$  and  $\mathcal{R}_{b,k} = \frac{1}{2} \log(1 + \gamma_{b,k})$ . Finally, the sum-rate of  $k$ th pair opportunistic transmission is given by

$$\begin{aligned} \mathcal{R}_k^{\text{sum}} &= \mathcal{R}_{a,k} + \mathcal{R}_{b,k} \\ &= \frac{1}{2} \log[(1 + \gamma_{a,k})(1 + \gamma_{b,k})]. \end{aligned} \quad (9)$$

Note that the pre-log factor  $\frac{1}{2}$  accounts for the fact that intra-pair information exchange takes place in two time phases.

### 3 Opportunistic scheduling strategy

In this section, we explain a multi-pair scheduling strategy and suggest a selection criteria for the best user pair for the system discussed previously. With perfect global CSI knowledge, the relay determines to service a target user pair in every time-slot. The key issue is how to determine the appropriate metric for channel-aware scheduling among user pairs. We first consider a greedy scheduling policy in which the best user pair  $k^*$  is chosen among multiple pairs in each time-slot based on the following criterion:

$$k^* = \arg \max_{k \in \mathcal{K}} \{\theta_k\}, \quad (10)$$

where  $\theta_k = \min(\gamma_{a,k}, \gamma_{b,k})$  and  $\mathcal{K} = \{1, 2, \dots, K\}$  is the set of user pairs. However, by using the expressions of  $\gamma_{a,k}$  and  $\gamma_{b,k}$  given in (7) and (8), respectively, at a high SNR, one can recognize that the scheduling policy of (10) is equivalent to

$$k^* = \arg \max_{k \in \mathcal{K}} \{\phi_k\}, \quad (11)$$

where  $\phi_k = \min(|h_{a,k}|^2, |h_{b,k}|^2)$ .

The aforementioned greedy scheduling strategy thus selects the user pair for each time-slot with the largest value of the smaller end-to-end instantaneous SNRs. However, users' channels usually have different statistics due to different locations, and a user pair whose terminals are situated far away from the relay is unlikely to be ever selected by the scheduler. Hence, such a greedy scheduling scheme leads to an unfair resource allocation among user pairs, particularly for the case when the pairs are independent and non-identically distributed (i. ni.d.). To address this issue of fairness, the scheduling policy we now propose to use selects the best user pair  $k^*$  in each time-slot based on the following criterion:

$$k^* = \arg \max_{k \in \mathcal{K}} \left\{ \frac{\theta_k}{\bar{\theta}_k} \right\}, \quad (12)$$

where  $\bar{\theta}_k$  is the average value of  $\theta_k$  for  $k$ th user pair in the given time-slot. Normalization by  $\bar{\theta}_k$  in (12) is used in order to maintain long-term fairness among user pairs. To facilitate this, the relay keeps updating  $\bar{\theta}_k$  in each time-slot. It is not difficult to realize that the pairs having poor channel quality may not have to wait longer to gain access to the relay channel. Considering now

that user pairs are independent and identically distributed (i.i.d.), that is, the average values of  $\theta_k$  are identical such that  $\bar{\theta}_k = \bar{\theta}$  for all  $k$ , the scheduler policy in (12) is reduced to that stated in (10).

Next, we investigate the performance of the system discussed previously based on multi-pair scheduling policy stated in (10)-(12) in the presence of Rayleigh fading.

### 4 Performance analysis

First of all, we derive an exact expression for the cumulative distribution function (CDF) of  $\theta_k$ . Then, by approximating the derived CDF in simple closed-form at high SNR, we obtain the expressions for the probability density function (PDF) and the CDF of best user pair as defined in (12) for the fair scheduling scheme. Finally, we analyze the overall system performance in terms of access probability, outage probability, and average sum-rate.

Under Rayleigh fading,  $|h_{a,k}|^2$  and  $|h_{b,k}|^2$  for any  $k \in \mathcal{K}$  are independent but not necessarily identically distributed exponential random variables with parameters  $1/\sigma_{a,k}^2$  and  $1/\sigma_{b,k}^2$ , respectively. An exact expression for the CDF of  $\theta_k$  is provided in the following theorem.

**Theorem 1** *The CDF  $F_{\theta_k}(\theta)$  of  $\theta_k$  for any  $k \in \mathcal{K}$  is given by*

$$\begin{aligned} F_{\theta_k}(\theta) &= \Pr[\min(\gamma_{a,k}, \gamma_{b,k}) < \theta] \\ &= P_{1,k} + P_{2,k}, \end{aligned} \quad (13)$$

where  $P_{1,k}$  and  $P_{2,k}$  are given, respectively, by

$$\begin{aligned} P_{1,k} &= 1 - \frac{\eta \lambda_{\gamma_{1,k}}}{\lambda_{x_{1,k}} + \eta \lambda_{\gamma_{1,k}}} \left[ 1 - e^{-\frac{\delta}{\eta} (\lambda_{x_{1,k}} + \eta \lambda_{\gamma_{1,k}})} \right] \\ &\quad - \lambda_{\gamma_{1,k}} e^{-\gamma (\lambda_{x_{1,k}} + \lambda_{\gamma_{1,k}})} \sum_{n=0}^{\infty} \frac{(-1)^n [\lambda_{x_{1,k}} \gamma (\gamma + 1)]^n}{n! (\delta - \gamma)^{n-1}} E_n[\lambda_{\gamma_{1,k}} (\delta - \gamma)] \end{aligned} \quad (14)$$

and

$$\begin{aligned} P_{2,k} &= 1 - \frac{\eta \lambda_{x_{2,k}}}{\lambda_{\gamma_{2,k}} + \eta \lambda_{x_{2,k}}} \left[ 1 - e^{-\frac{\delta}{\eta} (\lambda_{\gamma_{2,k}} + \eta \lambda_{x_{2,k}})} \right] \\ &\quad - \lambda_{x_{2,k}} e^{-\gamma (\lambda_{\gamma_{2,k}} + \lambda_{x_{2,k}})} \sum_{n=0}^{\infty} \frac{(-1)^n [\lambda_{\gamma_{2,k}} \gamma (\gamma + 1)]^n}{n! (\delta - \gamma)^{n-1}} E_n[\lambda_{x_{2,k}} (\delta - \gamma)], \end{aligned} \quad (15)$$

with

$$\lambda_{x_{1,k}} = N_0 / (P \sigma_{a,k}^2), \lambda_{\gamma_{1,k}} = N_0 / ((P+P_r) \sigma_{a,k}^2), \lambda_{x_{2,k}} = N_0 / ((P+P_r) \sigma_{b,k}^2), \lambda_{\gamma_{2,k}} = N_0 / (P \sigma_{b,k}^2), \theta = \theta(P+P_r) / P, \eta = (P+P_r) / P,$$

and  $\delta = \frac{1}{2} \left[ \gamma (\eta + 1) + \sqrt{\gamma^2 (\eta + 1)^2 + 4 \gamma \eta} \right]$ .  $E_n[z]$  denotes the exponential integral of order  $n$  defined in [21, eq.

$$5.1.4] \text{ as } E_n[z] = \int_1^{\infty} t^{-n} e^{-zt} dt.$$

See "Appendix I" for the proof of Theorem 1. It is worth noting that the expressions in (14) and (15) involve only exponentials and exponential integral functions. These can be numerically evaluated with sufficient accuracy using symbolic software packages such as

MATHEMATICA and MATLAB. Further, the single infinite series expansion in (14) or (15) can be represented as  $\sum_{n=0}^{\infty} \Lambda_n$  where  $\Lambda_n = \frac{(-1)^n [u\gamma(\gamma+1)]^n}{n! [\delta-\gamma]^{n-1}} E_n[v(\delta-\gamma)] \times v e^{-\gamma(u+v)}$  denoting the  $n$ -th term with  $u, v \in \{\lambda_{x_{1,k}}, \lambda_{x_{2,k}}, \lambda_{y_{1,k}}, \lambda_{y_{2,k}}\}$  and  $u \neq v$ , for and  $k$ . Since  $E_n[z]$  decreases monotonically with  $n$ , it can be shown that  $\lim_{n \rightarrow \infty} \left| \frac{\Lambda_{n+1}}{\Lambda_n} \right| = \lim_{n \rightarrow \infty} \frac{u\gamma(\gamma+1)}{(\delta-\eta)(n+1)} \frac{E_{n+1}[v(\delta-\gamma)]}{E_n[v(\delta-\gamma)]} < 1$ , satisfying the convergence criteria as per the ratio test [22].

Although the expression given by (13) is exact and valid for all values of SNR, it is difficult to facilitate in particular the analysis for the case of fairness in scheduling scheme. We hence focus on deriving a simple closed-form expression of  $F_{\theta_k}(\theta)$  at high SNR ( $P \gg N_0$ ) in the following Lemma.

**Lemma 1** *The CDF  $F_{\theta_k}(\theta)$  of  $\theta_k$  can be approximated at high SNR as*

$$F_{\theta_k}(\theta) \approx 1 - e^{-\frac{\eta N_0 \theta}{P_r} \left( \frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2} \right)} \quad (16)$$

See ‘‘Appendix II’’ for the proof of Lemma 1.

Note that such an approximate expression yields very tight results in the whole SNR region and therefore can be used to make analysis feasible for the case of fairness in scheduling. We make here an interesting remark that  $\theta_k$  in (16) follows an exponential distribution with its mean value  $\bar{\theta}_k = \frac{P_r \sigma_{a,k}^2 \sigma_{b,k}^2}{\eta N_0 (\sigma_{a,k}^2 + \sigma_{b,k}^2)}$ . Moreover,  $\phi_k$  in (11) is also exponentially distributed with CDF given by

$$F_{\phi_k}(\phi) = 1 - e^{-\phi \left( \frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2} \right)}, \quad (17)$$

where the mean value of  $\phi_k$  is given by  $\bar{\phi}_k = \frac{\sigma_{a,k}^2 \sigma_{b,k}^2}{\sigma_{a,k}^2 + \sigma_{b,k}^2}$ .

Now applying a similar method as in [20], developed for downlink multiuser systems, we can express the PDF and the CDF of the best user pair (with  $\theta_{k^*}$ ) for the considered fair scheduling system, respectively, as

$$f_{\theta_{k^*}}(\theta) = \sum_{k=1}^K \frac{1}{\bar{\theta}_k} f_k \left( \frac{\theta}{\bar{\theta}_k} \right) \prod_{\substack{j=1 \\ j \neq k}}^K F_j \left( \frac{\theta}{\bar{\theta}_k} \right) \quad (18)$$

and

$$F_{\theta_{k^*}}(\theta) = \sum_{k=1}^K \int_0^{\frac{\theta}{\bar{\theta}_k}} f_k(x) \prod_{\substack{j=1 \\ j \neq k}}^K F_j(x) dx, \quad (19)$$

where  $f_k(\cdot)$  and  $F_k(\cdot)$  are the PDF and the CDF of the normalized variable  $\theta_k/\bar{\theta}_k$  for the  $k$ th pair, respectively. Using (16), we can express (18) and (19), respectively, as

$$f_{\theta_{k^*}}(\theta) \simeq \sum_{k=1}^K \frac{1}{\bar{\theta}_k} \sum_{n=0}^{K-1} \binom{K-1}{n} (-1)^n e^{-(n+1)\theta/\bar{\theta}_k} \quad (20)$$

and

$$F_{\theta_{k^*}} \simeq \frac{1}{K} \sum_{k=1}^K \left( 1 - e^{-\theta/\bar{\theta}_k} \right)^K, \quad (21)$$

where the  $\simeq$  sign denotes the equality in the region of high SNR.

#### 4.1 Access probability

The access probability can be defined as the probability that user pair  $k \in \mathcal{K}$  accesses the relay channel in the long run. It can be expressed by [20]

$$P_k^{\text{acc}} = \Pr \left[ \frac{\theta_k}{\bar{\theta}_k} \geq \frac{\theta_j}{\bar{\theta}_j} \quad \forall j \neq k \right] \\ = \int_0^{\infty} \frac{1}{\bar{\theta}_k} f_k \left( \frac{\theta}{\bar{\theta}_k} \right) \prod_{\substack{j=1 \\ j \neq k}}^K F_j \left( \frac{\theta}{\bar{\theta}_k} \right) d\theta. \quad (22)$$

We can evaluate (22) by using (16) as

$$P_k^{\text{acc}} \simeq \sum_{m=0}^{K-1} \binom{K-1}{m} \frac{(-1)^m}{m+1} \\ = \frac{1}{K} \sum_{m=0}^{K-1} \binom{K}{m+1} (-1)^m = \frac{1}{K}. \quad (23)$$

Thus, as expected, the scheduling policy in (12) is fair in the sense that each pair  $k$  can have equal access probability of  $1/K$ .

#### 4.2 Outage probability

For each user pair, an end-to-end transmission is in outage when either user of the pair is in outage, that is, when either  $\mathcal{R}_{a,k}$  or  $\mathcal{R}_{b,k}$  is smaller than the target rate  $\mathcal{R}$ . Hence, the outage probability for the best pair  $k^*$  is given by

$$P_{k^*}^{\text{out}} = \Pr[\mathcal{R}_{a,k^*} < \mathcal{R} \text{ or } \mathcal{R}_{b,k^*} < \mathcal{R}] \\ = \Pr[\min(\gamma_{a,k^*}, \gamma_{b,k^*}) < \gamma_{\text{th}}], \quad (24)$$

where  $\gamma_{\text{th}} = 2^{2\mathcal{R}} - 1$  is a threshold required for successful decoding at the receiver(s). As such, it is obvious that the considered greedy scheduling scheme minimizes the system outage probability.

Using the definition of best user pair for greedy scheduling scheme in (10) and applying the theory of order statistics [23] with  $K$  i.n.i.d. random variables, the outage probability in (24) is given as

$$P_{k^*, \text{greedy}}^{\text{out}} = \prod_{k=1}^K F_{\theta_k}(\gamma_{\text{th}}), \quad (25)$$

which can be calculated exactly by using the CDF of  $\theta_k$  as given in (13).

For the fair scheduling scheme stated in (12), we can express the outage probability in (24) by using (21) as

$$P_{k^*}^{\text{out}} = F_{\theta_{k^*}}(\gamma_{\text{th}}) \simeq \frac{1}{K} \sum_{k=1}^K \left[ 1 - e^{-\frac{\eta N_0 \gamma_{\text{th}}}{P_r} \left( \frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2} \right)} \right]^K. \quad (26)$$

We now address an efficient power allocation problem to the relay subject to a total power constraint. Specifically for the total end-to-end transmission power  $P_t = 2P + P_r$ , we consider  $P_r = \alpha P_t$  and  $P = \left( \frac{1-\alpha}{2} \right) P_t$  where  $\alpha \in (0,1)$  denotes the fraction of total power  $P_t$  allocated to the relay. Henceforth, we define  $\varrho = P_t/N_0$  as the total SNR. With such power distribution, we can rewrite (26) as

$$P_{k^*}^{\text{out}} \simeq \frac{1}{K} \sum_{k=1}^K \left[ 1 - e^{-\frac{(1+\alpha)\gamma_{\text{th}}}{\alpha(1-\alpha)\varrho} \left( \frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2} \right)} \right]^K. \quad (27)$$

Now, we can show that the expression of outage probability in (27) is minimized when  $\alpha = \sqrt{2} - 1 \approx 0.414$ . It is important to emphasize that this power allocation is independent of  $\sigma_{a,k}^2$  and  $\sigma_{b,k}^2$  for all  $k$ .

To investigate the asymptotic outage behavior, we can re-express (27) at high total SNR ( $\varrho \rightarrow \infty$ ) as

$$P_{k^*}^{\text{out}} \simeq \frac{1}{K} \left[ \frac{(1+\alpha)\gamma_{\text{th}}}{\alpha(1-\alpha)\varrho} \right]^K \sum_{k=1}^K \left( \frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2} \right), \quad (28)$$

which follows from the approximation  $e^{-z} \underset{z \rightarrow 0}{\approx} 1 - z$ . By using the definition of diversity order as  $d = -\lim_{\varrho \rightarrow \infty} \frac{\log[P_{k^*}^{\text{out}}(\varrho)]}{\log \varrho}$ , we can verify that the proposed scheduling scheme can achieve a multiuser diversity of order  $K$ .

### 4.3 Average sum-rate

For a more tractable sum-rate analysis, the expression in (9) can be approximated at high SNR as

$$\begin{aligned} \mathcal{R}_k^{\text{sum}} &\approx \frac{1}{2} \log(\gamma_{a,k} \gamma_{b,k}) \\ &\approx \log(\omega_k) + \log\left(\frac{P_r \sqrt{\Lambda_k}}{N_0}\right), \end{aligned} \quad (29)$$

where  $\omega_k = \frac{|h_{a,k}|^2 |h_{b,k}|^2}{|h_{a,k}|^2 + |h_{b,k}|^2}$  is half of the harmonic mean of channel strengths  $|h_{a,k}|^2$  and  $|h_{b,k}|^2$ , and  $\Lambda_k = \frac{(|h_{a,k}|^2 + |h_{b,k}|^2)^2}{(\eta|h_{a,k}|^2 + |h_{b,k}|^2)(\eta|h_{b,k}|^2 + |h_{a,k}|^2)}$ .

By applying the bounds  $\frac{2}{\eta^2 + 1} < \Lambda_k < \frac{1}{\eta}$  for all  $k$  [9, Theorem 2], and the well-known bounds for the harmonic mean as  $\frac{1}{2} \min(x, y) \leq \frac{xy}{x+y} < \min(x, y)$  [24], we can bound the sum-rate for best pair bidirectional transmission at high SNR as

$$\log(\phi_{k^*}) + \log\left(\frac{P_r}{N_0 \sqrt{2(\eta^2 + 1)}}\right) < \mathcal{R}_{k^*}^{\text{sum}} < \log(\phi_{k^*}) + \log\left(\frac{P_r}{N_0 \sqrt{\eta}}\right). \quad (30)$$

Therefore, the average sum-rate in (30) is bounded by

$$\mathbb{E}[\log(\phi_{k^*})] + \log\left(\frac{P_r}{N_0 \sqrt{2(\eta^2 + 1)}}\right) < \bar{\mathcal{R}}_{k^*}^{\text{sum}} < \mathbb{E}[\log(\phi_{k^*})] + \log\left(\frac{P_r}{N_0 \sqrt{\eta}}\right). \quad (31)$$

The expectation term in (31) can be evaluated as follows:

$$\mathbb{E}[\log(\phi_{k^*})] = \int_0^{\infty} \log(\phi) f_{\phi_{k^*}}(\phi) d\phi, \quad (32)$$

where  $f_{\phi_{k^*}}(\phi)$  is the PDF of  $\phi_{k^*}$ , which can be evaluated using (17) in (18) with  $\theta$  replaced by  $\varphi$  as

$$f_{\phi_{k^*}}(\phi) = \sum_{k=1}^K \frac{1}{\phi_k} \sum_{n=0}^{K-1} \binom{K-1}{n} (-1)^n e^{-(n+1)\phi/\phi_k}. \quad (33)$$

Substituting (33) into (32), we get

$$\mathbb{E}[\log(\phi_{k^*})] = \sum_{k=1}^K \frac{1}{\phi_k} \sum_{n=0}^{K-1} \binom{K-1}{n} (-1)^n \int_0^{\infty} \log(\phi) e^{-(n+1)\phi/\phi_k} d\phi. \quad (34)$$

We can evaluate the integral term in (34) using [25, eq. 4.331.1] to obtain

$$\mathbb{E}[\log(\phi_{k^*})] = \log(e) \sum_{k=1}^K \sum_{n=0}^{K-1} \binom{K-1}{n} \frac{(-1)^{n+1}}{n+1} \left[ \mathbf{C} + \ln\left(\frac{n+1}{\phi_k}\right) \right], \quad (35)$$

where  $\mathbf{C} = 0.577215664\dots$  is Euler's constant defined by [25, eq. 8.367.1].

Knowing that  $\sum_{n=0}^{K-1} \binom{K}{n+1} (-1)^{n+1} = -1$ , we can express (35) as

$$\mathbb{E}[\log(\phi_{k^*})] = \frac{1}{K} \sum_{k=1}^K \log(\bar{\phi}_k) + \zeta(K) - C \log(e), \quad (36)$$

where  $\zeta(K)$  is a constant not related with SNR or link quality given by

$$\zeta(K) = \sum_{n=0}^{K-1} \binom{K}{n+1} (-1)^{n+1} \log(n+1). \quad (37)$$

Inserting (36) into (31) and after invoking power assignment as considered previously, we obtain

$$\begin{aligned} \log \left[ \frac{\rho\alpha(1-\alpha)}{2(1+\alpha)} + \frac{1}{K} \sum_{k=1}^K \log(\bar{\phi}_k) + \zeta(K) - \log(e^C) < \bar{\mathcal{R}}_{k^*}^{\text{sum}} \right] \\ < \log \left[ \rho\alpha\sqrt{\frac{1-\alpha}{1+\alpha}} + \frac{1}{K} \sum_{k=1}^K \log(\bar{\phi}_k) + \zeta(K) - \log(e^C) \right] \end{aligned} \quad (38)$$

As  $\zeta(K) > \log(e^C)$  for  $K \geq 2$ , the average sum-rate increases with  $K$ . It can now be shown easily that the lower bound of average sum-rate is maximized when  $\alpha \approx 0.414$  (as for outage minimization), whereas the upper bound is maximized when  $\alpha \approx 0.618$  (as in [9]).

### 5 Numerical and simulation results

In this section, we present numerical and simulation results to demonstrate the performance of the considered scheme. For numerical evaluations, we use selected channel variances as listed in Table 1 to reflect randomness in  $K$  user pairs with nonidentical distributions. This is owing to the fact that different users may be placed at different distances from the relay, and hence, they may have different average SNR values. In the following numerical studies, we assume  $\gamma_{\text{th}} = 1$  (for outage probability).

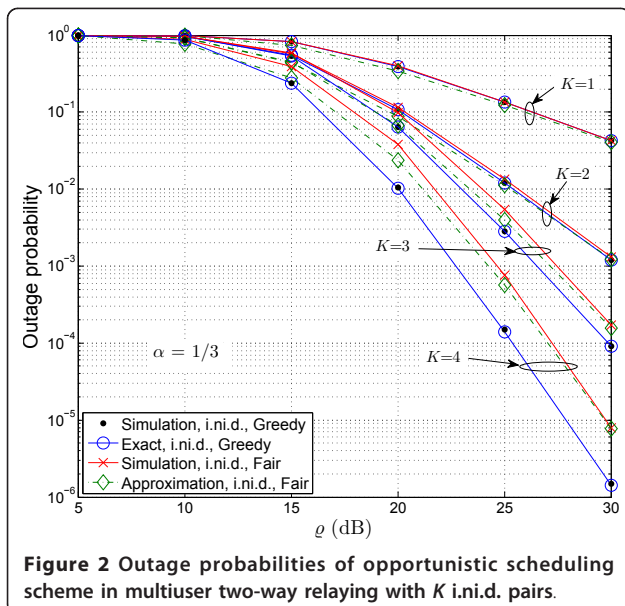
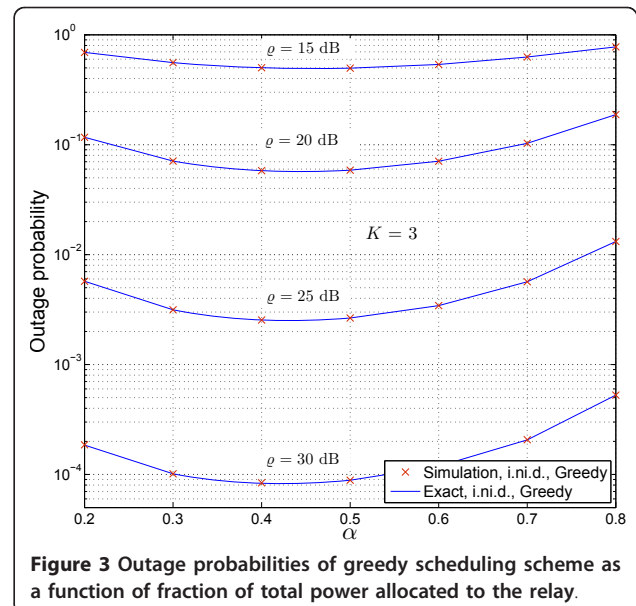
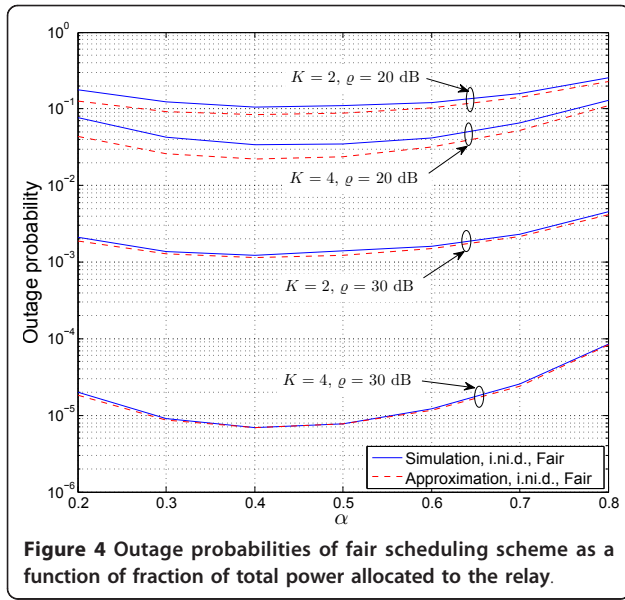


Figure 2 shows the outage probability curves for various user pairs with nonidentical  $\bar{\theta}_k$  versus total SNR ( $\rho$ ) under uniform power distribution among the selected users and the relay ( $\alpha = 1/3$ ). The exact curves corresponding to the evaluation of (25) for greedy scheduler were obtained by truncating the infinite series over index  $n$  in (14) and (15) to first few terms beyond which there is no change in the first seven decimal places of the results. As can be seen from Figure 2, the exact curves match perfectly with the results generated through Monte Carlo simulations, validating our analytical expression. Further, it can be seen that the analytical outage curves for fair scheduling scheme corresponding to the computation of (26) closely approximate the simulated values when the SNR is large. This implies that our approximated expression in (26) can provide good predictions of outage probabilities for fair scheduling scheme in the high SNR regime. The curves in Figure 2, however, illustrate that the greedy scheduling scheme outperforms the fair one. This is expected since fairness in the scheduling scheme results in some performance loss. Moreover, it is obvious from this figure that the overall system attains a multiuser diversity of order  $K$ .

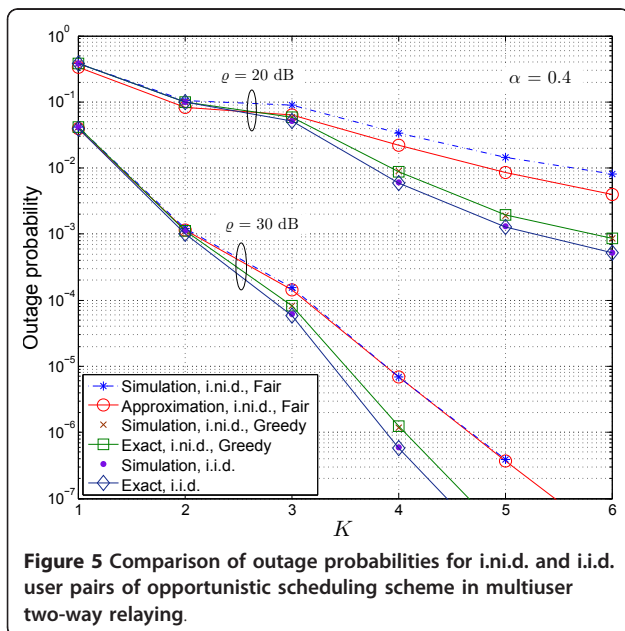
Figures 3 and 4 demonstrate the outage performance of greedy and fair scheduling schemes, respectively, with different power assignments to the selected relay (by varying  $\alpha$ ) subject to a total power constraint. We can see that the minimum of the outage probability under both the schemes lies in the range of  $\alpha$  between 0.4 and 0.5, regardless of the values of  $\rho$  and channel parameters. Also, as the outage probability is not very sensitive to  $\alpha$  in this range,  $\alpha \approx 0.4$  provides the near-





optimal performance for both the schemes. This is in good agreement with the result as derived analytically using the high SNR approximation in Section 4. It is an important result from a practical point of view since such power allocation scheme does not depend on the system/channel parameters.

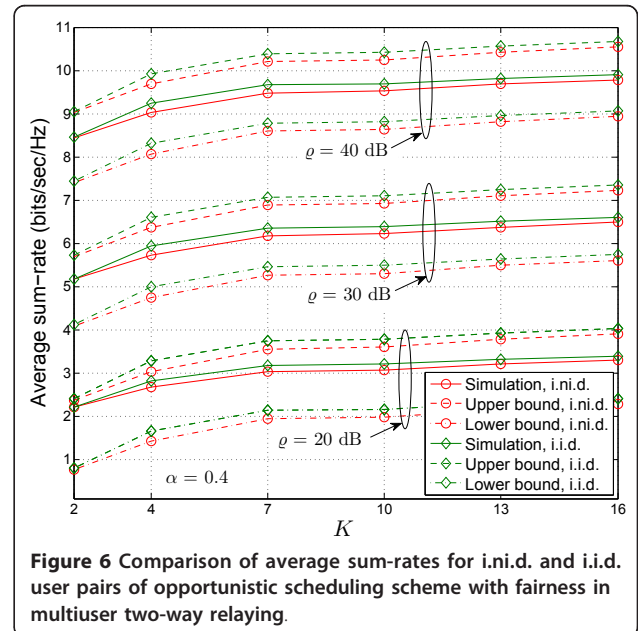
Figure 5 provides a comparison between user pairs with nonidentical and identical distributions in terms of outage probability as a function of  $K$ . For a fair comparison, we set up the simulation for i.i.d. case by considering  $\bar{\theta} = \frac{1}{K} \sum_{k=1}^K \bar{\theta}_k$  for all  $k$  (as all the user pairs are required



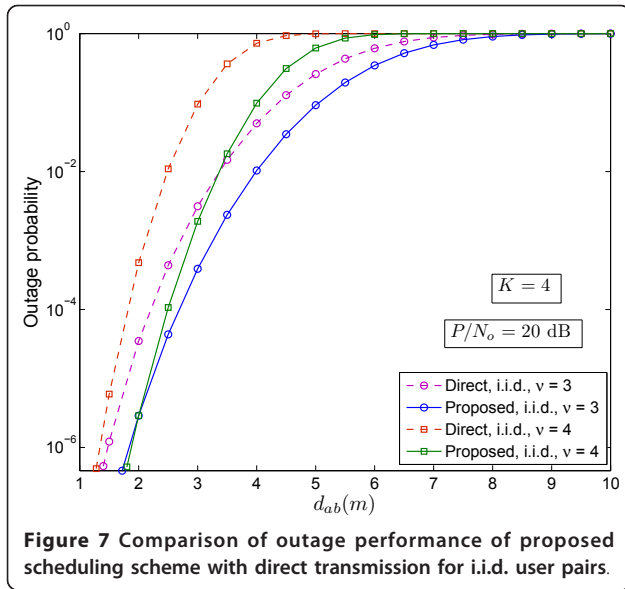
to have same statistics), so that the sum of  $\bar{\theta}_k$ s is equal to  $K\bar{\theta}$  under both cases. Note that for i.i.d. user pairs, both greedy and fair scheduling schemes have the same performance, as stated earlier. It can be observed that i.i.d. pairs achieve better performance than i.n.i.d. ones.

With the same set of parameters, we plot the average sum-rate curves versus the number of pairs  $K$  in Figure 6. This figure shows that the average sum-rate performance for two-way relaying can also be improved by including more user pairs in the considered scheme. There is a gap between the bounds and simulation curves throughout the region of high SNR. This is because at high SNR, both  $\gamma_{a,k}$  and  $\gamma_{b,k}$  have a high probability of having values close to each other, and hence, their harmonic mean does not approximate its upper or lower bound very well. Figure 6 also illustrates that the average sum-rate performance of i.i.d. user pairs is better than that of i.n.i.d. pairs.

Figures 7 and 8 present a comparison of the performance of the proposed scheme with that of the direct transmission scheme using same scheduling procedure in terms of outage probability and average sum-rate, respectively, as a function of the distance  $d_{ab}$  between two users of the best selected pair. We set up the simulation by considering an i.i.d. case with relay location lies midway between the two users so that  $d_{ab} = 2d_a = 2d_b$ , where  $d_a$  and  $d_b$  represent the distances of  $T_{a,k}$  and  $T_{b,k}$  from  $R$ , respectively, for all  $k$ . We incorporate the large-scale path loss in the signal propagation with a path loss exponent  $\nu$ . As such, we can have  $\sigma_{a,k}^2 = d_a^{-\nu}$ ,  $\sigma_{b,k}^2 = d_b^{-\nu}$ , and  $\sigma_{ab,k}^2 = d_{ab}^{-\nu}$  for all  $k$ , where  $\sigma_{ab,k}^2$  denotes the channel variance of direct link between  $T_{a,k}$  and  $T_{b,k}$ . We assume equal power  $P$  at all nodes in the network. Further, we consider radio



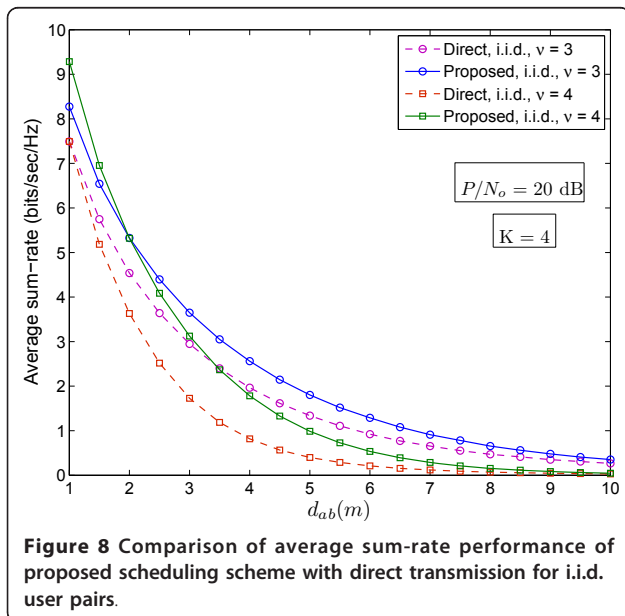




**Figure 7** Comparison of outage performance of proposed scheduling scheme with direct transmission for i.i.d. user pairs.

## 6 Conclusion and future work

We have investigated the performance of an outage-optimal opportunistic scheduling scheme with fairness for a multi-pair ANC-based two-way relay network over a



**Figure 8** Comparison of average sum-rate performance of proposed scheduling scheme with direct transmission for i.i.d. user pairs.

Rayleigh flat-fading scenario. For the greedy scheduling scheme of user pairs, we derived an exact expression for the outage probability that is valid over entire SNR region. We then proposed a scheduling strategy that ensures fairness among user pairs of the considered system. Based on a high SNR assumption, we derived an approximate expression for the outage probability and the bounds on the average sum-rate of the overall system. It was shown that the proposed scheme achieves performance gain by attaining an order of multiuser diversity equal to the number of user pairs. It is further demonstrated that near-optimal performance can be achieved when about 40% of the available power is assigned to the relay, irrespective of the system parameters.

In the present work, we have analyzed the considered scheduling scheme by assuming perfect channel estimation and no delay between the instants of estimation and best pair transmission. However, estimation errors and scheduling delays do exist in practical systems, and analyzing their effects on the performance is a subject for future work.

## Appendix I

### Proof of Theorem 1

We can express

$$\Pr[\min(\gamma_{a,k}, \gamma_{b,k}) < \theta] = P_{1,k} + P_{2,k}, \quad (39)$$

where  $P_{1,k} = \Pr[\gamma_{b,k} < \theta, \gamma_{b,k} < \gamma_{a,k}]$  and  $P_{2,k} = \Pr[\gamma_{a,k} < \theta, \gamma_{a,k} < \gamma_{b,k}]$ . Using (7), and (8) and after some straightforward manipulations, we can re-express

$$P_{1,k} = \Pr \left[ \frac{x_{1,k} \gamma_{1,k}}{x_{1,k} + \gamma_{1,k} + 1} < \gamma, x_{1,k} < \frac{\gamma_{1,k}}{\eta} \right], \quad (40)$$

where  $x_{1,k} \triangleq \frac{P}{N_0} |h_{a,k}|^2$  and  $\gamma_{1,k} \triangleq \frac{(P + P_r)}{N_0} |h_{b,k}|^2$ . For Rayleigh fading,  $x_{1,k}$  and  $\gamma_{1,k}$  are exponentially distributed random variables with parameters  $\lambda_{x_{1,k}} = N_0/(P\sigma_{a,k}^2)$  and  $\lambda_{\gamma_{1,k}} = N_0/((P + P_r)\sigma_{b,k}^2)$ , and probability density functions  $f_{x_{1,k}}(x) = \lambda_{x_{1,k}} e^{-\lambda_{x_{1,k}} x}, x \geq 0$  and  $f_{\gamma_{1,k}}(\gamma) = \lambda_{\gamma_{1,k}} e^{-\lambda_{\gamma_{1,k}} \gamma}, \gamma \geq 0$ , respectively. Now (40) can be evaluated as follows:

$$\begin{aligned} P_{1,k} &= \Pr[x_{1,k}(\gamma_{1,k} - \gamma) < \gamma(\gamma_{1,k} + 1), \eta x_{1,k} < \gamma_{1,k}] \\ &= \int_0^\gamma f_{\gamma_{1,k}}(\gamma) \int_0^{\frac{\gamma}{\eta}} f_{x_{1,k}}(x) dx d\gamma + \int_\gamma^\infty f_{\gamma_{1,k}}(\gamma) \int_0^{\min\left(\frac{\gamma}{\eta}, \frac{\gamma(\gamma+1)}{\gamma-\gamma}\right)} f_{x_{1,k}}(x) dx d\gamma. \end{aligned} \quad (41)$$

It can be shown that  $\frac{\gamma}{\eta} < \frac{\gamma(\gamma+1)}{\gamma-\gamma}$  for  $\gamma$  lying in the range  $\gamma < \gamma < \delta$ , where  $\delta$  can be obtained by solving  $\gamma^2 - \gamma(\eta + 1)\gamma - \gamma\eta < 0$  for  $\gamma$  with  $\eta > 1$  as the possible root. Hence the second term in (41) can be separated into two parts to yield

$$P_{1,k} = \int_0^\gamma f_{\gamma_{1,k}}(y) \int_0^{\frac{y}{\eta}} f_{x_{1,k}}(x) dx dy + \int_\gamma^\delta f_{\gamma_{1,k}}(y) \int_0^{\frac{y}{\eta}} f_{x_{1,k}}(x) dx dy \quad (42)$$

$$+ \int_\delta^\infty f_{\gamma_{1,k}}(y) \int_0^{\frac{y-\gamma}{\eta}} f_{x_{1,k}}(x) dx dy,$$

which can be simplified further as

$$P_{1,k} = \int_0^\delta f_{\gamma_{1,k}}(y) \int_0^{\frac{y}{\eta}} f_{x_{1,k}}(x) dx dy + \int_\delta^\infty f_{\gamma_{1,k}}(y) \int_0^{\frac{y-\gamma}{\eta}} f_{x_{1,k}}(x) dx dy \quad (43)$$

$$= I_1 + I_2,$$

where

$$I_1 \triangleq \int_0^\delta \lambda_{\gamma_{1,k}} e^{-\lambda_{\gamma_{1,k}} y} \int_0^{\frac{y}{\eta}} \lambda_{x_{1,k}} e^{-\lambda_{x_{1,k}} x} dx dy \quad (44)$$

$$= \int_0^\delta \lambda_{\gamma_{1,k}} e^{-\lambda_{\gamma_{1,k}} y} \left( 1 - e^{-\frac{\lambda_{x_{1,k}}}{\eta} y} \right) dy$$

$$= 1 - e^{-\lambda_{\gamma_{1,k}} \delta} - \frac{\eta \lambda_{\gamma_{1,k}} [1 - e^{-\frac{\delta}{\eta} (\lambda_{x_{1,k}} + \eta \lambda_{\gamma_{1,k}})}]}{\lambda_{x_{1,k}} + \eta \lambda_{\gamma_{1,k}}}$$

and

$$I_2 \triangleq \int_\delta^\infty \lambda_{\gamma_{1,k}} e^{-\lambda_{\gamma_{1,k}} y} \int_0^{\frac{y-\gamma}{\eta}} \lambda_{x_{1,k}} e^{-\lambda_{x_{1,k}} x} dx dy \quad (45)$$

$$= \int_\delta^\infty \lambda_{\gamma_{1,k}} e^{-\lambda_{\gamma_{1,k}} y} \left( 1 - e^{-\lambda_{x_{1,k}} y \left( \frac{y+1}{y-1} \right)} \right) dy$$

$$= e^{-\lambda_{\gamma_{1,k}} \delta} - \int_\delta^\infty \lambda_{\gamma_{1,k}} e^{-\lambda_{\gamma_{1,k}} y} e^{-\lambda_{x_{1,k}} y \left( \frac{y+1}{y-1} \right)} dy.$$

Operating the change of variable  $t = y - \gamma$  within the integral in (45) and some simplifications, we get

$$I_2 = e^{-\lambda_{\gamma_{1,k}} \delta} - \lambda_{\gamma_{1,k}} e^{-\gamma (\lambda_{x_{1,k}} + \lambda_{\gamma_{1,k}})} \int_{\delta-\gamma}^\infty e^{-\lambda_{\gamma_{1,k}} t} e^{-\lambda_{x_{1,k}} \gamma \left( \frac{\gamma+1}{t} \right)} dt \quad (46)$$

Since the integral in (46) has no closed-form solution, it can be evaluated by using Taylor series expansion [25, eq. 1.211.1] for the second exponential term and

interchanging the order of integration and summation as

$$I_2 = e^{-\lambda_{\gamma_{1,k}} \delta} - \lambda_{\gamma_{1,k}} e^{-\gamma (\lambda_{x_{1,k}} + \lambda_{\gamma_{1,k}})} \sum_{n=0}^\infty \frac{[-\lambda_{x_{1,k}} \gamma (\gamma+1)]^n}{n!} \int_{\delta-\gamma}^\infty \frac{e^{-\lambda_{\gamma_{1,k}} t}}{t^n} dt \quad (47)$$

$$= e^{-\lambda_{\gamma_{1,k}} \delta} - \lambda_{\gamma_{1,k}} (\delta - \gamma) e^{-\gamma (\lambda_{x_{1,k}} + \lambda_{\gamma_{1,k}})} \sum_{n=0}^\infty \frac{(-1)^n}{n!} \left[ \frac{\lambda_{x_{1,k}} \gamma (\gamma+1)}{\delta - \gamma} \right]^n$$

$$\times E_n[\lambda_{\gamma_{1,k}} (\delta - \gamma)],$$

where the last equality follows from [21, eq. 5.1.4] after a simple transformation of the integration variable. Substituting (44) and (47) into (43) yields the expression of  $P_{1,k}$  as provided in (14). Following the similar steps as above, we obtain  $P_{2,k}$  as presented in (15). And the proof of Theorem 1 is completed.

## Appendix II

### Proof of Lemma 1

The end-to-end instantaneous SNR expressions in (7) and (8) can be approximated at high SNR ( $P \gg N_0$ ), respectively, by

$$\gamma_{a,k} \approx \frac{P_r}{N_0} \left[ \frac{|h_{a,k}|^2 |h_{b,k}|^2}{\eta |h_{a,k}|^2 + |h_{b,k}|^2} \right] \quad (48)$$

and

$$\gamma_{b,k} \approx \frac{P_r}{N_0} \left[ \frac{|h_{a,k}|^2 |h_{b,k}|^2}{\eta |h_{b,k}|^2 + |h_{a,k}|^2} \right], \quad (49)$$

where the approximation is made by ignoring the noise power in the gain at the relay. Despite this, such an approximation has been shown to be very tight in the entire SNR region [9,13,24]. Using these SNR expressions and following the similar approach as in "Appendix I," we can express  $P_{1,k}$  in (14) as

$$P_{1,k} \approx 1 - \frac{\sigma_{a,k}^2}{\sigma_{a,k}^2 + \sigma_{b,k}^2} \left[ 1 - e^{-\frac{\theta \eta N_0}{P_r} \left( 1 + \frac{1}{\eta} \left( \frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2} \right) \right)} \right] \quad (50)$$

$$- \frac{\theta \eta N_0}{P_r \sigma_{b,k}^2} e^{-\frac{\theta \eta N_0}{P_r} \left( \frac{1}{\sigma_{a,k}^2} + \frac{1}{\eta \sigma_{b,k}^2} \right)} \sum_{n=0}^\infty \frac{(-1)^n}{n!} \left( \frac{\theta N_0}{P_r \sigma_{b,k}^2} \right)^n E_n \left[ \frac{\theta \eta N_0}{P_r \sigma_{b,k}^2} \right].$$

Now for  $n \geq 1$ ,  $E_n[z]$  can be expanded in series as [21, eq. 5.1.12]

$$E_n[z] = \frac{(-z)^{n-1}}{(n-1)!} (\psi(n) - \ln(z)) - \sum_{\substack{q=0 \\ q \neq n-1}}^\infty \frac{(-z)^q}{(q-n+1)q!}, \quad (51)$$

where  $\psi(n) = -C + \sum_{s=1}^{n-1} \frac{1}{s}$ ,  $n > 1$  and  $\psi(1) = -C$ , where  $C = 0.577215664\dots$  is Euler's constant. By using (51), one can verify that for high SNR ( $P/N_0 \rightarrow \infty$ ) and the fact that  $\lim_{z \rightarrow \infty} \frac{\ln(1/z)}{z^2} = 0$ , the summation term for  $n$

$\geq 1$  in (50) becomes infinitesimal of order  $2n$ . Therefore, we can write

$$P_{1,k} \approx 1 - \frac{\sigma_{a,k}^2}{\sigma_{a,k}^2 + \sigma_{b,k}^2} \left[ 1 - e^{-\frac{\theta\eta N_0}{P_r} \left(1 + \frac{1}{\eta}\right) \left(\frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2}\right)} \right] - \frac{\theta\eta N_0}{P_r \sigma_{b,k}^2} e^{-\frac{\theta\eta N_0}{P_r} \left(\frac{1}{\sigma_{a,k}^2} + \frac{1}{\eta\sigma_{b,k}^2}\right)} E_0 \left[ \frac{\theta\eta N_0}{P_r \sigma_{b,k}^2} \right], \quad (52)$$

where we have omitted the higher-order terms. Further, with  $E_0[z] = e^{-z}/z$ , we can express (52) for large SNR as

$$P_{1,k} \approx 1 - \frac{\sigma_{a,k}^2}{\sigma_{a,k}^2 + \sigma_{b,k}^2} \left[ 1 - e^{-\frac{\theta\eta N_0}{P_r} \left(1 + \frac{1}{\eta}\right) \left(\frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2}\right)} \right] - e^{-\frac{\theta\eta N_0}{P_r} \left(\frac{1}{\sigma_{a,k}^2} + \frac{1}{\eta\sigma_{b,k}^2}\right)} \frac{\theta\eta N_0}{P_r \sigma_{b,k}^2}. \quad (53)$$

Similarly, we approximate  $P_{2,k}$  in (15) for large SNR as

$$P_{2,k} \approx 1 - \frac{\sigma_{b,k}^2}{\sigma_{a,k}^2 + \sigma_{b,k}^2} \left[ 1 - e^{-\frac{\theta\eta N_0}{P_r} \left(1 + \frac{1}{\eta}\right) \left(\frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2}\right)} \right] - e^{-\frac{\theta\eta N_0}{P_r} \left(\frac{1}{\sigma_{b,k}^2} + \frac{1}{\eta\sigma_{a,k}^2}\right)} \frac{\theta\eta N_0}{P_r \sigma_{a,k}^2}. \quad (54)$$

Substituting (53) and (54) into (13), the approximate expression for  $F_{\theta_k}(\theta)$  at the high SNR region is given by

$$F_{\theta_k}(\theta) \approx 1 - e^{-\frac{\theta\eta N_0}{P_r} \left(\frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2}\right)} \left[ e^{-\frac{\theta N_0}{P_r \sigma_{a,k}^2}} + e^{-\frac{\theta N_0}{P_r \sigma_{b,k}^2}} - e^{-\frac{\theta N_0}{P_r} \left(\frac{1}{\sigma_{a,k}^2} + \frac{1}{\sigma_{b,k}^2}\right)} \right]. \quad (55)$$

Note that the above expression is dominated by the exponential term present outside the square brackets due to the factor  $\eta > 1$ . Therefore, by making use of the fact that  $e^{-z} \approx 1 - z$  for the terms within square brackets, we can express the result as given in (16).

## Additional material

**Additional file 1: 1597702927618362\_LE[1].pdf, 310K** <http://jwcn.eurasipjournals.com/imedia/1896637463640433/supp1.pdf>

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## Competing interests

The authors declare that they have no competing interests.

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