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Periodic solutions of delay differential equations with feedback control for enterprise clusters based on ecology theory

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Abstract

By utilizing a fixed point theorem of strict-set-contraction, some criteria are established for the existence of positive periodic solutions of enterprise clusters based on ecology theory with time-varying delays and feedback controls.

Keywords: feedback control; strict-set-contraction; positive periodic solution; fixed point

1 Introduction

During recent years, many authors have worked on the existence of periodic solutions of delay differential equations with feedback control [1–6]. However, few papers have been published about the existence of these solutions of the aforementioned differential equations with feedback control for enterprise clusters based on ecology theory [7–13]. Zhou divided enterprise clusters into two models: the concentration of subcontractors around the dominant firm and the concentration of simple competitors, called the center half-back model and the net model [13]. Moreover, two models from biology were given and studied from an economic point of view. Accordingly, sufficient conditions were obtained to guarantee the co-existence and stability of enterprise clusters. Then, the developing strategy of enterprises was analyzed based on the logistic model, while the suggestions of constructing cooperative relation and choosing generalization or specialization tactics for the commodity were suggested [7]. In addition, based on a theoretical model of ecological population science, Wang and Pan made a detailed analysis of the equilibrium mechanism of enterprise clusters, including a net model and a center halfback model, and they drew the conclusion that the relationship of fierce competition and beneficial cooperation among enterprise clusters was the crucial factor to keep stability [11]. In a recent study the competition and cooperation system of two enterprises based on an ecosystem have been considered [9]:

$$\begin{cases} \frac{dx_1(t)}{dt} = r_1(t)x_1(t)(1 - \frac{x_1(t)+\alpha(x_2(t)-c_2)^2}{K}), \\ \frac{dx_2(t)}{dt} = r_2(t)x_2(t)(1 - \frac{x_2(t)-\beta(x_1(t)-c_1)^2}{K}), \end{cases}$$

where $x_1(t)$, $x_2(t)$ are the output of enterprises A and B, r_1, r_2 represent the intrinsic growth rate, K is the carrying capacity of the market under natural unlimited conditions, α, β

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denote the competitive coefficients of the two enterprises, c_1, c_2 are the initial production of the two enterprises. Therefore, the equation with non-constant coefficients is assumed, which can be got as a modified previous system with variable coefficients (letting $a_1 = \frac{r_1}{K}$, $a_2 = \frac{r_2}{K}$, $b_1 = \frac{r_1\alpha}{K}$, $b_2 = \frac{r_2\beta}{K}$ in previous system):

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t)(r_1(t) - a_1(t)x_1(t) - b_1(t)(x_2(t) - c_2(t))^2), \\ \frac{dx_2(t)}{dt} = x_2(t)(r_2(t) - a_2(t)x_2(t) - b_2(t)(x_1(t) - c_1(t))^2). \end{cases}$$

In real situations there is an undeniable time delay, and a long time delay may damage the stability of a system. Also the situation of enterprises is often distributed by unforeseen forces, which can lead to changes in the enterprises' parameters, such as intrinsic growth rates, therefore it is vitally important to study models with control variables which are the so-called disturbance functions [3, 14–16]. Resuming the latest studies, the following competitive and cooperation model of n satellite enterprises and a dominant enterprise under the center halfback model with time-varying delays and feedback controls is considered to find whether there is any periodic solution for this system:

$$\begin{cases} \frac{dx_i(t)}{dt} = x_i(t)[r_i(t) - a_i(t)x_i(t) - \sum_{j=1,j\neq i}^{n} b_{ij}(t)\frac{dx_j}{dt}(t - \tau_j(t)) \\ \qquad - c(t)[y(t - \gamma(t)) - d(t)]^2 - e_i(t)u_i(t - \lambda_i(t))], \\ \frac{du_i(t)}{dt} = -\alpha_i(t)u_i(t) - \sum_{j=1}^{n} \beta_j(t)x_j(t - \eta_j(t)), \\ \frac{dy(t)}{dt} = y(t)[r(t) - a(t)y(t) - b(t)\frac{dy(t)}{dt} - \sum_{i=1}^{n} c_i(t)[x_i(t - \delta_i(t)) - d_i(t)]^2 \\ \qquad - e(t)v(t - \lambda(t))], \\ \frac{dv(t)}{dt} = -\mu(t)v(t) + v(t)y(t - \theta(t)), \end{cases}$$

where $x_i(t)$, $y(t)$ represent the output of the satellite enterprises, Ax_i, and the core enterprise, Ay, respectively, r_i, r are the intrinsic growth rates, a_i, a account for their respective self-regulations, b_{ij} accounts for the rates of inter-enterprises Ax_i competition, c represents the rate of intra-enterprise competition from Ay, c_i represents the rate of conversion of a commodity into the reproduction of enterprise Ax_i, d_i and d represent the initial production of the enterprises, respectively, u_i, v are the control variables, $i = 1, 2, \ldots, n$.

Let R and R^+ denote the set of all real numbers and nonnegative real numbers. Let h be a continuous bounded function defined on R, and we set

$$\overline{h} = \max\{h(t) : t \in [0, w]\}$$

and

$$\underline{h} = \min\{h(t) : t \in [0, w]\},$$

where h is a continuous w-periodic function.

In this paper we shall use the following hypotheses.

(H1) r_i, r, a_i, a, b_{ij}, b, c_i, c, d_i, d, e_i, e, α_i, τ_i, γ, λ_i, λ, δ_i, η_i, μ, θ are continuous w-periodic functions,

$$p_i = e^{\int_0^w \alpha_i(s)\,ds} > 1, \qquad p = e^{\int_0^w \mu(s)\,ds} > 1,$$

$$q_i = e^{-\int_0^w r_i(s)\,ds} < 1, \qquad q = e^{-\int_0^w r(s)\,ds} < 1,$$

$$G_i(t,s) = \frac{e^{\int_t^s \alpha_i(s)\,ds}}{e^{\int_0^w \alpha_i(s)\,ds} - 1}, \qquad G(t,s) = \frac{e^{\int_t^s \mu(s)\,ds}}{e^{\int_0^w \mu(s)\,ds} - 1},$$

$$H(t,s) = \frac{e^{-\int_t^s r_i(s)\,ds}}{1 - e^{-\int_0^w r_i(s)\,ds}}, \qquad H(t,s) = \frac{e^{-\int_t^s r(s)\,ds}}{1 - e^{-\int_0^w r(s)\,ds}},$$

$$B(t) = \max\{b_i(t) : i = 1, \ldots, n, b(t)\},$$

$$F_i(t) = \int_t^{t+w} G_i(t,s) \sum_{i=1}^n \beta_i(s)\,ds, \qquad F(t) = v(t) = \int_t^{t+w} G(t,s)v(s)\,ds,$$

$$M_{i1} = \int_0^w \left(q_i a_i(s) + \sum_{i=1,i\neq j}^n b_{ij}(s) + q_i c(s)\|x\|_1 + q_i e_i(s) F_i(s - \lambda_i(s)) \right) ds,$$

$$M_1 = \int_0^w \left[qa(s) + b(s) + q\sum_{i=1}^n c_i(s)\|x\|_1 + qe(s)F(s - \lambda(s)) \right] ds,$$

$$M_{i2} = \int_0^w \left(a_i(s) + \sum_{i=1,i\neq j}^n b_{ij}(s) + c(s)\|x\|_1 + e_i(s) F_i(s - \lambda_i(s)) \right) ds,$$

$$M_2 = \int_0^w \left[a(s) + b(s) + \sum_{i=1}^n c_i(s)\|x\|_1 + e(s)F(s - \lambda(s)) \right] ds.$$

(H2)

$$q_i a_i(t)\|x\|_1 + q_i \sum_{i=1,i\neq j}^n b_{ij}(t)\|x\|_1 + q_i^2 c(t)\|x\|_1^2 + q_i e_i(t) F_i(t - \lambda_i(t))\|x\|_1 \geq 0$$

and

$$qa(t)\|x\|_1 + qb(t)\|x\|_1 + q^2 \sum_{i=1}^n c_i(t)\|x\|_1^2 + qe(t)F(t - \lambda_i(t))\|x\|_1 \geq 0.$$

(H3)

$$(1 + \underline{r}_i)\frac{q_i^2}{1 - q_i}M_{i1} \geq \max\left\{ a_i(t) + \sum_{i=1,i\neq j}^n b_{ij}(t) + c(t)\|x\|_1 + e_i(t)F_i(t - \lambda_i(t)) : t \in [0,w] \right\},$$

$$(1 + \underline{r})\frac{q^2}{1 - q}M_1 \geq \max\left\{ a(t) + b(t) + \sum_{i=1}^n c_i(t)\|x\|_1 + e(t)F(t - \lambda(t)) : t \in [0,w] \right\}.$$

(H4)

$$(1 - \overline{r}_i)\frac{1}{q_i(1 - q_i)}M_{i2}$$

$$\leq \min\left\{ q_i a_i(t) + q_i \sum_{i=1,i\neq j}^n b_{ij}(t) + q_i c(t)\|x\|_1 + q_i e_i(t)F_i(t - \lambda_i(t)) : t \in [0,w] \right\},$$

$$(1 - \overline{r})\frac{1}{q(1 - q)}M_2$$

$$\leq \min\left\{ qa(t) + qb(t) + q\sum_{i=1}^n c_i(t)\|x\|_1 + qe(t)F(t - \lambda(t)) : t \in [0,w] \right\}.$$

In order to obtain our main results, we first make the following preparations.

Definition 1.1 Let X be Banach space and E be a closed, nonempty subset of X; E is said to be a cone if

  (i) $\alpha u + \beta v \in E$, for all $u, v \in E$ and all $\alpha, \beta > 0$,
  (ii) $u, -u \in E$ imply $u = 0$.

Let X be a Banach space and E be a cone in X. The semi-order induced by the cone E is denoted by '$\leq$'. That is, $x \leq y$ if and only if $y - x \in E$. For a bounded subset $A \subset X$, let $\alpha_X(A)$ denote the (Kuratowski) measure of non-compactness defined by

$$\alpha_X(A) = \inf\left\{\delta > 0 : \text{there is a finite number of subsets } A_i \subset A\right.$$

$$\left.\text{such that } A = \bigcup_i A_i \text{ and } \operatorname{diam}(A_i) = \delta\right\},$$

where $\operatorname{diam}(A_i)$ denotes the diameter of the set A_i.

Definition 1.2 Let E, F be two Banach spaces and $D \subset E$, a continuous and bounded map $T : D \to F$ is called k-set-contractive if for any bounded set $S \subset D$ we have

$$\alpha_F\big(T(S)\big) \leq k\alpha_E(S).$$

T is called strict-set-contractive if it is k-set-contractive for some $0 \leq k < 1$.

The following lemma is cited from [17, 18], which is useful for the proof of our main result of this paper.

Lemma 1.1 *Let P be a cone of the real Banach space X and*

$$\Omega_R = \big\{x \in X : \|x\| < R\big\}, \qquad \Omega_r = \big\{x \in X : \|x\| < r\big\},$$

with $R > r > 0$. Suppose that $T : P \cap \overline{\Omega}_R \backslash \Omega_r \to P$ is strict-set-contractive such that one of the following two conditions is satisfied:

  (i) *Not $Tx \geq x$ $\forall x \in P \cap \partial\Omega_r$.*
  (ii) *Not $Tx \leq x$ $\forall x \in P \cap \partial\Omega_R$.*

Then T has at least one fixed point in $P \cap \overline{\Omega}_R \backslash \Omega_r$.

2 Main results

In this section, at first based on a fixed point theorem for the strict-set-contraction, we shall study the existence of at least one positive periodic solution of the system, then we state and prove our main results.

Now, each w-periodic solution of the equation

$$\frac{du_i(t)}{dt} = -\alpha_i(t)u_i(t) - \sum_{j=1}^{n} \beta_j(t)x_j\big(t - \eta_j(t)\big)$$

and

$$\frac{dv(t)}{dt} = -\mu(t)v(t) + v(t)y\big(t - \theta(t)\big)$$

is equivalent to that of the equations

$$u_i(t)(t) = \int_t^{t+w} G_i(t,s) \sum_{j=1}^n \beta_j(s) x_j\big(s - \eta_j(s)\big)\, ds = \Phi_{x_i}(t)$$

and

$$v(t) = \int_t^{t+w} G(t,s) v(s) y\big(s - \theta(s)\big)\, ds = \Phi_y(t).$$

Therefore, the existence problem of a w-periodic solution of system is equivalent to that of a w-periodic solution of the equations:

$$x_i(t) = \int_t^{t+w} H_i(t,s) x_i(s) \Bigg(a_i(s) x_i(s) + \sum_{j=1, i \neq j}^n b_{ij}(s) \frac{dx_j}{ds}\big(s - \tau_j(s)\big)$$

$$+ c(s)\big[y\big(s - \gamma(s)\big) - d(s)\big]^2 + e_i(s) \Phi_{x_i}\big(s - \lambda_i(s)\big) \Bigg)\, ds,$$

$i = 1, 2, \ldots, n,$

$$y(t) = \int_t^{t+w} H(t,s) y(s) \Bigg[a(s) y(s) + b(s) \frac{dy(s)}{ds} + \sum_{i=1}^n c_i(s)\big[x_i\big(s - \delta_i(s)\big) - d_i(s)\big]^2$$

$$+ e(s) \Phi_y\big(s - \lambda(s)\big) \Bigg]\, ds.$$

In order to apply Lemma 1.1 to the equations, set

$$C_w^0 = \left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ y \end{bmatrix} : x_i, y \in C(R, R), x_i(t + w) = x_i(t), y(t + w) = y(t), t \in R \right\},$$

with the norm defined by

$$\|x\| = \max\big\{ \max\big\{ |x_i(t)| : t \in [0, w] \big\}, i = 1, 2, \ldots, n, \max\big\{ |y(t)| : t \in [0, w] \big\} \big\}$$

and

$$C_w^1 = \left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ y \end{bmatrix} : x_i, y \in C^1(R, R), x_i(t + w) = x_i(t), y(t + w) = y(t), t \in R \right\},$$

with the norm defined by $\|x\|_1 = \max\{\|x\|, \|x'\|\}$. Then C_w^0, C_w^1 are of course Banach spaces. Define the cone P in C_w^1 by

$$P = \left\{ x : x \in C_w^1, x_i(t) \geq q_i\|x\|_1, y(t) \geq q\|x\|_1, t \in [0,w] \right\}.$$

Also for $R > r > 0$, set

$$\Omega_R = \left\{ x \in C_w^1 : \|x\|_1 < R \right\}, \qquad \Omega_r = \left\{ x \in C_w^1 : \|x\|_1 < r \right\}.$$

Let the map T be defined by

$$Tx = \begin{bmatrix} Tx_1 \\ Tx_2 \\ \vdots \\ Tx_n \\ Ty \end{bmatrix},$$

where

$$Tx_i(t) = \int_t^{t+w} H_i(t,s)x_i(s)\left(a_i(s)x_i(s) + \sum_{j=1, i\neq j}^n b_{ij}(s)\frac{dx_j}{ds}\left(s - \tau_j(s)\right) \right.$$

$$\left. + c(s)\left[y\left(s - \gamma(s)\right) - d(s)\right]^2 + e_i(s)\Phi_{x_i}\left(s - \lambda_i(s)\right) \right)ds,$$

$i = 1, 2, \ldots, n$,

$$Ty(t) = \int_t^{t+w} H(t,s)y(s)\left[a(s)y(s) + b(s)\frac{dy(s)}{ds} + \sum_{i=1}^n c_i(s)\left[x_i\left(s - \delta_i(s)\right) - d_i(s)\right]^2 \right.$$

$$\left. + e(s)\Phi_y\left(s - \lambda(s)\right) \right]ds.$$

Lemma 2.1 *Assume that* (H1)-(H3) *hold.*
  (i) *If* $\overline{r}_i \leq 1$, $\overline{r} \leq 1$, *then* $T : P \to P$ *is well defined.*
 (ii) *If* (H4) *holds and* $\overline{r}_i > 1$, $\overline{r}_i > 1$, *then* $T : P \to P$ *is well defined.*

Proof (i) For any $x \in P$, it follows that $Tx \in C_w^1$. Under the assumption (H2), for $t \in [0,w]$, we have

$$a_i(t)x_i(t) + \sum_{j=1, i\neq j}^n b_{ij}(t)\frac{dx_j}{dt}\left(t - \tau_j(t)\right) + c(t)\left[y\left(t - \gamma(t)\right) - d(t)\right]^2 + e_i(t)\Phi_{x_i}\left(t - \lambda_i(t)\right)$$

$$\geq q_i a_i(t)\|x\|_1 + q_i \sum_{j=1, i\neq j}^n b_{ij}(t)\|x\|_1 + q_i^2 c(t)\|x\|_1^2 + q_i e_i(t)F_i\left(t - \lambda_i(t)\right)\|x\|_1 \geq 0$$

and

$$
a(t)y(t) + b(t)\frac{dy(t)}{dt} + \sum_{i=1}^{n} c_i(t)\big[x_i\big(t - \delta_i(t)\big) - d_i(t)\big]^2 + e(t)\Phi_y\big(t - \lambda(t)\big)
$$

$$
\geq qa(t)\|x\|_1 + qb(t)\|x\|_1 + q^2\sum_{i=1}^{n} c_i(t)\|x\|_1^2 + qe(t)F\big(t - \lambda_i(t)\big)\|x\|_1 \geq 0;
$$

moreover,

$$
\max\big\{\big|Tx_i(t)\big| : t \in [0, w]\big\}
$$

$$
\leq \max\Bigg\{\frac{1}{1-q_i}\int_t^{t+w} x_i(s)\Bigg(a_i(s)x_i(s) + \sum_{i=1,i\neq j}^{n} b_{ij}(s)\frac{dx_j}{ds}(s - \tau_j(s))
$$

$$
+ c(s)\big[y\big(s - \gamma(s)\big) - d(s)\big]^2 + e_i(s)\Phi_{x_i}\big(s - \lambda_i(s)\big)\Bigg)\,ds : t \in [0, w]\Bigg\}
$$

$$
= \frac{1}{1-q_i}\int_0^w x_i(s)\Bigg(a_i(s)x_i(s) + \sum_{i=1,i\neq j}^{n} b_{ij}(s)\frac{dx_j}{ds}(s - \tau_j(s))
$$

$$
+ c(s)\big[y\big(s - \gamma(s)\big) - d(s)\big]^2 + e_i(s)\Phi_{x_i}\big(s - \lambda_i(s)\big)\Bigg)\,ds
$$

and

$$
\max\big\{\big|Ty(t)\big| : t \in [0, w]\big\}
$$

$$
\leq \max\Bigg\{\frac{1}{1-q}\int_t^{t+w} y(s)\Bigg(a(s)y(s) + b(s)\frac{dy(s)}{ds}
$$

$$
+ \sum_{i=1}^{n} c_i(s)\big[x_i\big(s - \delta_i(s)\big) - d_i(s)\big]^2 + e(s)\Phi_y\big(s - \lambda(s)\big)\Bigg)\,ds : t \in [0, w]\Bigg\}
$$

$$
= \frac{1}{1-q}\int_0^w y(s)\Bigg(a(s)y(s) + b(s)\frac{dy(s)}{ds} + \sum_{i=1}^{n} c_i(s)\big[x_i\big(s - \delta_i(s)\big) - d_i(s)\big]^2
$$

$$
+ e(s)\Phi_y\big(s - \lambda(s)\big)\Bigg)\,ds.
$$

Therefore, for $t \in [0, w]$, we obtain

$$
Tx_i(t) \geq \frac{q_i}{1-q_i}\int_t^{t+w} x_i(s)\Bigg(a_i(s)x_i(s) + \sum_{j=1,i\neq j}^{n} b_{ij}(s)\frac{dx_j}{ds}(s - \tau_j(s))
$$

$$
+ c(s)\big[y\big(s - \gamma(s)\big) - d(s)\big]^2 + e_i(s)\Phi_{x_i}\big(s - \lambda_i(s)\big)\Bigg)\,ds
$$

$$
= \frac{q_i}{1-q_i}\int_0^w x_i(s)\Bigg(a_i(s)x_i(s) + \sum_{j=1,i\neq j}^{n} b_{ij}(s)\frac{dx_j}{dt}(s - \tau_j(s))
$$

$$+ c(s)\big[y\big(s - \gamma(s)\big) - d(s)\big]^2 + e_i(s)\Phi_{x_i}\big(s - \lambda_i(s)\big)\bigg) ds$$

$$\geq q_i\|Tx\|$$

and

$$Ty(t) \geq \frac{q}{1-q}\int_t^{t+w} y(s)\bigg(a(s)y(s) + b(s)\frac{dy(s)}{ds}$$

$$+ \sum_{i=1}^{n} c_i(s)\big[x_i\big(s - \delta_i(s)\big) - d_i(s)\big]^2 + e(s)\Phi_y\big(s - \lambda(s)\big)\bigg) ds$$

$$= \frac{q}{1-q}\int_0^{w} y(s)\bigg(a(s)y(s) + b(s)\frac{dy(s)}{ds}$$

$$+ \sum_{i=1}^{n} c_i(s)\big[x_i\big(s - \delta_i(s)\big) - d_i(s)\big]^2 + e(s)\Phi_y\big(s - \lambda(s)\big)\bigg) ds$$

$$\geq q\|Tx\|.$$

Now we prove $Tx_i(t) \geq q_i\|(Tx)'\|$, $Ty(t) \geq q\|(Tx)'\|$, for $t \in [0, w]$; we find

$$(Tx_i)'(t) = H_i(t, t + w)x_i(t + w)\bigg(a_i(t + w)x_i(t + w)$$

$$+ \sum_{j=1, i\neq j}^{n} b_{ij}(t + w)\frac{dx_j}{dt}\big(t + w - \tau_j(t + w)\big)$$

$$+ c(t + w)\big[y\big(t + w - \gamma(t + w)\big) - d(t + w)\big]^2$$

$$+ e_i(t + w)\Phi_{x_i}\big(t + w - \lambda_i(t + w)\big)\bigg)$$

$$- H_i(t, t)x_i(t)\bigg(a_i(t)x_i(t) + \sum_{j=1, i\neq j}^{n} b_{ij}(t)\frac{dx_j}{dt}\big(t - \tau_j(t)\big)$$

$$+ c(t)\big[y\big(t - \gamma(t)\big) - d(t)\big]^2 + e_i(t)\Phi_{x_i}\big(t - \lambda_i(t)\big)\bigg)$$

and

$$(Ty)'(t) = H(t, t + w)y(t + w)\bigg[a(t + w)y(t + w) + b(t + w)\frac{dy}{dt}(t + w)$$

$$+ \sum_{i=1}^{n} c_i(t + w)\big[x_i\big(t + w - \delta_i(t + w)\big) - d_i(t + w)\big]^2$$

$$+ e(t + w)\Phi_y\big(t + w - \lambda(t + w)\big)\bigg]$$

$$- H(t, t)y(t)\bigg[a(t)y(t) + b(t)\frac{dy}{dt}(t) + \sum_{i=1}^{n} c_i(t)\big[x_i(t - \delta_i) - d_i\big]^2$$

$$+ e(t)\Phi_y\big(t - \lambda(t)\big)\Bigg] + r(t)Ty(t)$$

$$= r(t)Ty(t) - y(t)\Bigg[a(t)y(t) + b(t)\frac{dy}{dt}(t) + \sum_{i=1}^{n} c_i(t)\big[x_i(t - \delta_i) - d_i\big]^2$$

$$+ e(t)\Phi_y\big(t - \lambda(t)\big)\Bigg].$$

We have

$$(Tx_i)'(t) \le r_i(t)Tx_i(t) \le \bar{r}_i Tx_i(t) \le Tx_i(t)$$

and

$$(Ty)'(t) \le r(t)Ty(t) \le \bar{r}Ty(t) \le Ty(t).$$

On the other hand, we find

$$-(Tx_i)'(t) = x_i(t)\Bigg(a_i(t)x_i(t) + \sum_{i=1,i\ne j}^{n} b_{ij}(t)\frac{dx_j}{dt}\big(t - \tau_j(t)\big)$$

$$+ c(t)\big[y\big(t - \gamma(t)\big) - d(t)\big]^2 + e_i(t)\Phi_{x_i}\big(t - \lambda_i(t)\big)\Bigg) - r_i(t)Tx_i(t)$$

$$\le \|x\|_1^2\Bigg(a_i(t) + \sum_{i=1,i\ne j}^{n} b_{ij}(t) + c(t)\|x\|_1 + e_i(t)F_i\big(t - \lambda_i(t)\big)\Bigg) - r_i(t)Tx_i(t)$$

$$\le (1 + \underline{r}_i)\frac{q_i^2}{1 - q_i}\|x\|_1^2 \int_0^w \Bigg(a_i(s) + \sum_{i=1,i\ne j}^{n} b_{ij}(s) + c(s)\|x\|_1 + e_i(s)F_i\big(s - \lambda_i(s)\big)\Bigg)\, ds,$$

$$-\underline{r}_i Tx_i(t)$$

$$= (1 + \underline{r}_i) \int_t^{t+w} \frac{q_i}{1 - q_i} q_i \|x\|_1^2 \Bigg(a_i(s) + \sum_{i=1,i\ne j}^{n} b_{ij}(s) + c(s)\|x\|_1 + e_i(s)F_i\big(s - \lambda_i(s)\big)\Bigg)\, ds,$$

$$-\underline{r}_i Tx_i(t) \le (1 + \underline{r}_i)Tx_i(t) - \underline{r}_i Tx_i(t) = Tx_i(t),$$

and we have

$$-(Ty)'(t) = y(t)\Bigg[a(t)y(t) + b(t)\frac{dy}{dt}(t) + \sum_{i=1}^{n} c_i(t)\big[x_i(t - \delta_i) - d_i\big]^2 + e(t)\Phi_y\big(t - \lambda(t)\big)\Bigg]$$

$$- r(t)Ty(t)$$

$$\le \|x\|_1^2\Bigg[a(t) + b(t) + \sum_{i=1}^{n} c_i(t)\|x\|_1 + e(t)F\big(t - \lambda(t)\big)\Bigg] - r(t)Ty(t)$$

$$\le (1 + \underline{r})\frac{q^2}{1 - q}\|x\|_1^2 \int_0^w \Bigg[a(s) + b(s) + \sum_{i=1}^{n} c_i(s)\|x\|_1 + e(s)F\big(s - \lambda(s)\big)\Bigg]\, ds$$

$$- r(t)Ty(t)$$

$$= (1 + \underline{r}) \int_t^{t+w} \frac{q}{1-q} q \|x\|_1^2 \left[a(s) + b(s) + \sum_{i=1}^n c_i(s) \|x\|_1 + e(s)F(s - \lambda(s)) \right] ds$$

$$- \underline{r} Ty(t)$$

$$\leq (1 + \underline{r}) Ty(t) - \underline{r} Ty(t) = Ty(t).$$

So we know $\|(Tx)'\| \leq \|Tx\|$, so $\|Tx\|_1 = \|Tx\|$. We have

$$(Tx_i)(t) \geq q_i \|Tx\|_1, \qquad (Ty)(t) \geq q \|Tx\|_1, \quad t \in [0, w].$$

Hence $Tx \in P$.

(ii) In view of the proof of (i) we need to prove that

$$(Tx_i)'(t) \leq Tx_i(t) \quad \text{and} \quad (Ty)'(t) \leq Ty(t).$$

We obtain

$$(Tx_i)'(t) \leq -q_i \|x\|_1 \left(a_i(t)x_i(t) + \sum_{j=1, i \neq j}^n b_{ij}(t) \frac{dx_j}{dt}(t - \tau_j(t)) + c(t)[y(t - \gamma(t)) - d(t)]^2 \right.$$

$$\left. + e_i(t)\Phi_{x_i}(t - \lambda_i(t)) \right) + r_i(t)Tx_i(t)$$

$$\leq r_i(t)Tx_i(t) + q_i^2 \|x\|_1^2 \left(a_i(t) + \sum_{j=1, i \neq j}^n b_{ij}(t) + qc(t)\|x\|_1 + e_i(t)F_i(t - \lambda_i(t)) \right)$$

and

$$(Ty)'(t) \leq -q \|x\|_1 \left[a(t)y(t) + b(t) \frac{dy}{dt}(t) + \sum_{i=1}^n c_i(t)[x_i(t - \delta_i) - d_i]^2 + e(t)\Phi_y(t - \lambda(t)) \right]$$

$$+ r(t)Ty(t)$$

$$\leq r(t)Ty(t) + q^2 \|x\|_1^2 \left[a(t) + b(t) + \sum_{i=1}^n c_i(t)\|x\|_1 + e(t)F(t - \lambda(t)) \right]. \qquad \Box$$

Lemma 2.2 *Assume that* (H1)-(H3) *hold and* $R_{\overline{B}} < 1$.

(i) *If* $\overline{r}_i \leq 1$, $\overline{r} \leq 1$, *then* $T : P \cap \overline{\Omega}_R \backslash \Omega_r \to P$ *is strict-set-contractive.*

(ii) *If* (H4) *holds and* $\overline{r}_i > 1$, $\overline{r} > 1$, *then* $T : P \cap \overline{\Omega}_R \backslash \Omega_r \to P$ *is strict-set-contractive.*

Proof We only need to prove (i), since the proof of (ii) is similar. It is easy to see that T is continuous and bounded. Now we prove that $\alpha_{C_w^1}(T(S)) \leq R_{\overline{B}} \alpha_{C_w^1}(S)$ for any bounded set $S \subset P \cap \overline{\Omega}_R$. Let $\eta = \alpha_{C_w^1}(S)$, then, for any positive number $\varepsilon < R_{\overline{B}}\eta$, there is a finite family of subset $S_i \subset S$ satisfying $S = \bigcup_i S_i$ with $\text{diam}(S_i) \leq \eta + \varepsilon$. Therefore

$$\|x - y\|_1 \leq \varepsilon \quad \text{for any } x, y \in S_i.$$

As S and S_i are precompact in C_w^0, it follows that there is a finite family of subsets $S_{ij} \subset S_i$ satisfying $S_i = \bigcup_j S_{ij}$ and

$$\|x - z\| \leq \varepsilon \quad \text{for any } x, z \in S_{ij}.$$

In addition, for any

$$
x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ y \end{bmatrix} \in S
$$

and $t \in [0, w]$ we have

$$
\begin{aligned}
\left|(Tx_i)(t)\right| &= \int_t^{t+w} H_i(t,s) x_i(s) \Bigg(a_i(s) x_i(s) + \sum_{j=1, i \neq j}^n b_{ij}(s) \frac{dx_j}{ds}(s - \tau_j(s)) \\
&\quad + c(s) \big[y(s - \nu(s)) - d(s) \big]^2 + e_i(s) \Phi_{x_i}(s - \lambda_i(s)) \Bigg) ds \\
&\leq \frac{R^2}{1 - q_i} \int_0^w \Bigg(a_i(s) + \sum_{j=1, i \neq j}^n b_{ij}(s) + c(s)R + e_i(s) F_i(s - \lambda_i(s)) \Bigg) ds := M_{i3}
\end{aligned}
$$

and

$$
\begin{aligned}
\left|(Tx_i)'(t)\right| &= \Bigg| r_i(t) Tx_i(t) - x_i(t) \Bigg(a_i(t) x_i(t) + \sum_{j=1, i \neq j}^n b_{ij}(t) \frac{dx_j}{dt}(t - \tau_j(t)) \\
&\quad + c(t) \big[y(t - \nu(t)) - d(t) \big]^2 + e_i(t) \Phi_{x_i}(t - \lambda_i(t)) \Bigg) \Bigg| \\
&\leq \bar{r}_i M_{i3} + R^2 \Bigg(\bar{a}_i + \sum_{j=1, i \neq j}^n \bar{b}_{ij} + \bar{c}_i R + \frac{p_i}{p_i - 1} \bar{e}_i \Bigg).
\end{aligned}
$$

Also we have

$$
\begin{aligned}
\left|(Ty)(t)\right| &= \int_t^{t+w} H(t,s) y(s) \Bigg[a(s) y(s) + b(s) \frac{dy}{ds}(s) + \sum_{i=1}^n c_i(s) \big[x_i(s - \delta_i(s)) - d_i(s) \big]^2 \\
&\quad + e(s) \Phi_y(s - \lambda(s)) \Bigg] ds \\
&\leq \frac{R^2}{1 - q} \int_0^w \Bigg[a(s) + b(s) + \sum_{i=1}^n c_i(s)R + e(s) F_y(s - \lambda(s)) \Bigg] ds := M_3
\end{aligned}
$$

and

$$
\begin{aligned}
\left|(Ty)'(t)\right| &= \Bigg| r(t) Ty(t) \\
&\quad + y(t) \Bigg[a(t) y(t) + b(t) \frac{dy}{dt}(t) + \sum_{i=1}^n c_i(t) \big[x_i(t - \delta_i) - d_i \big]^2 + e(t) \Phi_y(t - \lambda(t)) \Bigg] \Bigg| \\
&\leq \bar{r} M_3 + R^2 \Bigg(\bar{a} + \bar{b} + \sum_{i=1}^n \bar{c}_i + \frac{p}{p - 1} \bar{e} \Bigg).
\end{aligned}
$$

Applying the Arzela-Ascoli theorem, we know that $T(S)$ is precompact in C_w^0. Then there is a finite family of subset $S_{ijk} \subset S_{ij}$ satisfying $S_{ij} = \bigcup_k S_{ijk}$ and

$$\|Tx - Tz\| \le \varepsilon \quad \text{for any } x, z \in S_{ijk}$$

for any

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ y \end{bmatrix}, z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \\ n \end{bmatrix} \in S_{ijk},$$

we obtain

$$\max\left\{ \left| (Tx_i)'(t) - (Tz_i)'(t) \right| : t \in [0, w] \right\}$$

$$= \max\left\{ \left| r_i(t) T x_i(t) - r_i(t) T z_i(t) + x_i(t) \left(a_i(t) x_i(t) + \sum_{j=1, i \neq j}^{n} b_{ij}(t) \frac{dx_j}{dt}(t - \tau_j(t)) \right. \right.\right.$$

$$+ c(t) \left[y(t - \nu(t)) - d(t) \right]^2 + e_i(t) \Phi_{x_i}(t - \lambda_i(t)) \bigg) - z_i(t) \bigg(a_i(t) z_i(t)$$

$$+ \sum_{j=1, i \neq j}^{n} b_{ij}(t) \frac{dz_j}{dt}(t - \tau_j(t)) + c(t) \left[n(t - \nu(t)) - d(t) \right]^2$$

$$\left.\left.+ e_i(t) \Phi_{z_i}(t - \lambda_i(t)) \right) \right| : t \in [0, w] \right\}$$

$$\le \overline{r}_i \varepsilon + R \left(\overline{a}_i \varepsilon + \sum_{j=1, i \neq j}^{n} \overline{b}_{ij}(\varepsilon + \eta) + \overline{c}\varepsilon^2 + \frac{p_i}{p_i - 1} \overline{e}_i \varepsilon \right)$$

$$+ \varepsilon \left(\overline{a}_i R + \sum_{j=1, i \neq j}^{n} \overline{b}_{ij} R + \overline{c} R^2 + \frac{p_i}{p_i - 1} \overline{e}_i R \right) = R_{\overline{B}} \eta + M_{i4} \varepsilon,$$

where $M_{i4} = \overline{r}_i + 2R\overline{a}_i + 2R \sum_{j=1, i \neq j}^{n} \overline{b}_{ij} + 2R^2 \overline{c} + 2R \frac{p_i}{p_i - 1} \overline{e}_i$ and

$$\max\left\{ \left| (Ty)'(t) - (Tn)'(t) \right| : t \in [0, w] \right\}$$

$$= \max\left\{ \left| r(t) Ty(t) - r(t) Tn(t) + y(t) \left[a(t) y(t) + b(t) \frac{dy}{dt}(t) \right] \right.\right.$$

$$+ \sum_{i=1}^{n} c_i(t) \left[x_i(t - \delta_i) - d_i \right]^2 + e(t) \Phi_y(t - \lambda(t))$$

$$- n(t) \left[a(t) n(t) + b(t) \frac{dn}{dt}(t) + \sum_{i=1}^{n} c_i(t) \left[z_i(t - \delta_i) - d_i \right]^2 \right.$$

$$\left.\left.\left.+ e(t) \Phi_n(t - \lambda(t)) \right] \right| : t \in [0, w] \right\}$$

$$\leq \overline{r}\varepsilon + R\left(\overline{a}\varepsilon + \overline{b}(\varepsilon + \eta) + \sum_{i=1}^{n}\overline{c}_i\varepsilon^2 + \frac{p}{p-1}\overline{e}\varepsilon\right) + \varepsilon\left(\overline{a}R + \overline{b}R + \sum_{i=1}^{n}\overline{c}_iR^2 + \frac{p}{p-1}\overline{e}R\right)$$

$$= R_{\overline{B}}\eta + M_4'\varepsilon,$$

where $M_4' = \overline{r} + 2R\overline{a} + 2R\overline{b} + 2R^2\sum_{i=1}^{n}\overline{c}_i + 2R\frac{p}{p-1}\overline{e}$ and $M_4 = \max\{M_4', M_{i4}\}$.

We have

$$\|Tx - Tz\|_1 \leq R_{\overline{B}}\eta + M_4\varepsilon \quad \text{for any } x, z \in S_{ijk}.$$

As ε is arbitrarily small, it follows that

$$\alpha_{C_w^1}\big(T(S)\big) \leq R_{\overline{B}}\alpha_{C_w^1}(S).$$

Therefore, T is strict-set-contractive. $\qquad\qquad\Box$

Theorem 2.1 *Assume that* (H1)-(H3) *hold.*
  (i) *If $\overline{r}_i \leq 1$, $\overline{r} \leq 1$, then the system has at least one positive w-periodic solution.*
  (ii) *If* (H4) *holds and $\overline{r}_i > 1$, $\overline{r} > 1$, then system has at least one positive w-periodic solution.*

Proof We only need to prove (i), since the proof of (ii) is similar. Let $R = \max\{\frac{1-q_i}{q_i^2 M_{i1}}, \frac{1-q}{q^2 M_1}\}$ and $0 < r < \min\{\frac{q_i(1-q_i)}{M_{i2}}, \frac{q(1-q)}{M_2}\}$. Then we find $0 < r < R$. It follows from Lemmas 2.1 and 2.2 that $T : P \cap \overline{\Omega}_R \backslash \Omega_r \to P$ is strict-set-contractive.

If there exists $\widetilde{x} \in P \cap \partial\Omega_r$ or $\widetilde{x} \in P \cap \partial\Omega_R$ such that $T\widetilde{x} = \widetilde{x}$ then the system has at least one positive w-periodic solution.

Now, we shall show that condition (ii) of Lemma 1.1 holds.

$Tx \geq x$ so $Tx - x \in P$, which implies that

$$Tx_i(t) - Tx_i(t) \geq q_i\|Tx - x\|_1 \geq 0 \quad \text{for all } t \in [0, w].$$

We obtain

$$Tx_i(t) = \int_t^{t+w} H_i(t,s)x_i(s)\left(a_i(s)x_i(s) + \sum_{j=1,i\neq j}^{n} b_{ij}(s)\frac{dx_j}{ds}\big(s - \tau_j(s)\big)\right.$$

$$\left. + c(s)\big[y\big(s - \gamma(s)\big) - d(s)\big]^2 + e_i(s)\Phi_{x_i}\big(s - \lambda_i(s)\big)\right)ds$$

$$= \frac{1}{1-q_i}\|x\|\int_0^w\left(a_i(s) + \sum_{j=1,i\neq j}^{n} b_{ij}(s) + c(s)R + e_i(s)F_i\big(s - \lambda_i(s)\big)\right)ds < q_i\|x\|.$$

We find

$$\|x\| \leq \|Tx\| \leq q_i\|x\| \leq \|x\|.$$

This is a contradiction.

We only need prove that

$$\text{Not } Tx \leq x, \quad \forall x \in P \cap \partial\Omega_R.$$

Suppose, for the sake of contradiction, that there exists $x \in P \cap \partial\Omega_R$ such that $Tx > x$. Thus $Tx - x \in P \backslash \{0\}$. Furthermore, for any $t \in [0, w]$, we have

$$x_i(t) - Tx_i(t) \geq q_i \|x - Tx\|_1 \geq 0 \quad \text{for all } t \in [0, w],$$

and we obtain

$$Tx_i(t) = \int_t^{t+w} H_i(t,s)x_i(s)\left(a_i(s)x_i(s) + \sum_{j=1,i\neq j}^n b_{ij}(s)\frac{dx_j}{ds}\big(s - \tau_j(s)\big)\right.$$

$$\left. + c(s)\big[y\big(s - \gamma(s)\big) - d(s)\big]^2 + e_i(s)\Phi_{x_i}\big(s - \lambda_i(s)\big)\right) ds$$

$$\geq \frac{q_i}{1 - q_i}q_i\|x\|_1^2 \int_0^w \left(a_i(s) + \sum_{j=1,i\neq j}^n b_{ij}(s) + c(s)R + e_i(s)F_i\big(s - \lambda_i(s)\big)\right) ds.$$

It follows that

$$\|x\| > \|Tx\| > R,$$

which is a contradiction. Therefore, conditions (i) and (ii) hold. By Lemma 2.1, we see that T has at least one nonzero fixed point in $P \cap \overline{\Omega}_R \backslash \Omega_r$. Thus the system has at least one positive w-periodic solution. The proof of Theorem 2.1 is complete. $\qquad\square$

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