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Optimal solutions for nonlinear proximal C_N-contraction mapping in metric space

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Abstract
The objective of this paper is to solve an optimal solutions for new nonlinear mapping in the setting of metric spaces. Our results extend, generalize, and improve some known results from best proximity point theory and fixed point theory. Examples are given to support our main results.
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1 Introduction

Fixed point theory is one of the famous and traditional theories in mathematics and has a large number of applications in various fields of pure and applied mathematics, as well as in physical, chemical, life, and social sciences. It is an important tool for solving equations of the form $Tx = x$, where T is a self-mapping on a subset of a metric space. On the other hand, if T is not a self-mapping, say $T : A \to B$ where A and B are nonempty subsets of a metric space, then T does not necessarily have a fixed point. Consequently, the equation $Tx = x$ could have no solutions, and in this case, it is of a certain interest to determine an element x that is in some sense closest to Tx. Here best approximation theorems explore the existence of an approximate solution whereas best proximity point theorems analyze the existence of an approximate solution that is optimal. Thus, we can say that the aim of the best proximity point theorems is to provide sufficient conditions to solve a minimization problem. In view of the fact that $d(x, Tx)$ is at least $d(A, B) := \inf\{d(x, y) : x \in A \text{ and } y \in B\}$, a best proximity point theorem concerns the global minimum of the real valued function $x \to d(x, Tx)$, that is, an indicator of the error involved for an approximate solution of the equation $Tx = x$, by complying with the condition $d(x, Tx) = d(A, B)$. A classical best approximation theorem was introduced by Fan [1], that is, if A is a nonempty compact convex subset of a Hausdorff locally convex topological vector space B and $T : A \to B$ is a continuous mapping, then there exists an element $x \in A$ such that $d(x, Tx) = d(Tx, A)$. Afterward, several generalizations and extensions of this theorem appeared in the literature (see *e.g.* [2–4], and references cited therein). It turns out that many of the contractive conditions which are investigated for fixed points ensure the existence of best proximity points, and many results of this kind are obtained in [5–25].

In this paper, we introduce a new class of non-self-mappings, called proximal C_N-contractions of the first and second kinds, which contains the proximal contractions as a subclass. In this class, we will consider the following nonlinear problem: Find

$$\min_{x\in A} d(x, Tx),$$

where $T : A \to B$ belongs a to new class of non-self-mappings, and A and B are nonempty subsets of a metric space (X, d). Also, we give some illustrative examples to support our results.

2 Preliminaries

In the sequel, $\mathbb{R}$ and $\mathbb{N}$ denote the set of real numbers and the set of positive integers, respectively. Let (X, d) be a metric space, A and B be two nonempty subsets of X and $T : A \to B$ be a non-self-mapping. The following notations will be used in the sequel:

$$\begin{aligned}
&d(A,B) := \inf\{d(x,y) : x \in A \text{ and } y \in B\},\\
&A_0 := \{x \in A : d(x,y) = d(A,B) \text{ for some y } \in B\},\\
&B_0 := \{y \in B : d(x,y) = d(A,B) \text{ for some } x \in A\}.
\end{aligned}$$

Kirk *et al.* [9] gave sufficient conditions to ensure that A_0 and B_0 are nonempty. Also, we find that if A and B are closed subsets of a normed linear space such that $d(A, B) > 0$, then A_0 and B_0 are contained in the boundaries of A and B, respectively (see [12]).

In 1961, Efimov and Stechkin [26] introduced the concept of an approximatively compact set. The properties of approximatively compact sets have been largely studied. It is well known that the concept of approximative compactness plays an important role in the theory of approximation [27]. Borodin [28] showed that in every infinite-dimensional separable Banach space there exists a bounded approximatively compact set which is not compact.

Remark 2.1 For a metric space (X, d), the bounded compactness of a set is equivalent to its closure and the possibility of selecting from any bounded sequence contained in it a converging subsequence.

Definition 2.1 Let A and B be two nonempty subsets of metric space (X, d). Then B is said to be approximatively compact with respect to A if every sequence $\{y_n\}$ of B, satisfying the condition $d(x, y_n) \to d(x, B)$ as $n \to \infty$ for some $x \in A$, has a convergent subsequence.

We see that any set is approximatively compact with respect to itself.

3 Main results

In this section, we give sequentially two new classes of non-self-mappings that are essential to state and prove the existence of best proximity point theorems.

Definition 3.1 Let (X, d) be a metric space and A and B are two nonempty subsets of X. A mapping $T : A \to B$ is said to be a *proximal C_N-contraction of the first kind* if there exist nonnegative real numbers α, β, and L with $\alpha + \beta < 1$, such that the conditions

$$d(u, Tx) = d(A,B) \quad \text{and} \quad d(v, Ty) = d(A,B)$$

imply that

$$d(u,v) \le \alpha d(x,y) + \beta\left[\frac{1+d(x,u)}{1+d(x,y)}d(y,v)\right] + L\min\{d(y,u),d(x,v),d(x,u),d(y,v)\} \tag{3.1}$$

for all $u,v,x,y \in A$.

Remark 3.1 If T is a self-mapping on A, then the requirement in the above definition reduces to the following generalized contractive condition which is useful in establishing a fixed point theorem:

$$d(Tx,Ty) \le \alpha d(x,y) + \beta\left[\frac{1+d(x,Tx)}{1+d(x,y)}d(y,Ty)\right] + L\min\{d(y,Tx),d(x,Ty),d(x,Tx),d(y,Ty)\}$$

for all $x,y \in X$.

Definition 3.2 Let (X,d) be a metric space and A and B be two nonempty subsets of X. A mapping $T: A \to B$ is said to be a *proximal C_N-contraction of the second kind* if there exist nonnegative real numbers α, β, and L with $\alpha + \beta < 1$, such that the conditions

$$d(u,Tx) = d(A,B) \quad \text{and} \quad d(v,Ty) = d(A,B)$$

imply that

$$d(Tu,Tv) \le \alpha d(Tx,Ty) + \beta\left[\frac{1+d(Tx,Tu)}{1+d(Tx,Ty)}d(Ty,Tv)\right] + L\min\{d(Ty,Tu),d(Tx,Tv),d(Tx,Tu),d(Ty,Tv)\} \tag{3.2}$$

for all $u,v,x,y \in A$.

Here, we give our first main result which is the best proximity point theorem for a proximal C_N-contraction of the first kind.

Theorem 3.1 *Let (X,d) be a complete metric space and A and B be two nonempty, closed subsets of X such that B is approximatively compact with respect to A. Assume that A_0 and B_0 are nonempty and $T: A \to B$ is a non-self-mapping such that*:

(a) *T is a proximal C_N-contraction of the first kind*;

(b) $T(A_0) \subseteq B_0$.

Then there exists a unique element $x \in A$ such that $d(x,Tx) = d(A,B)$. Moreover, for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$, defined by $d(x_{n+1},Tx_n) = d(A,B)$, converges to x.

Proof Let $x_0 \in A_0$. Since $T(A_0) \subseteq B_0$, then by the definition of B_0, there exists $x_1 \in A_0$ such that $d(x_1,Tx_0) = d(A,B)$. Again, since $Tx_1 \in B_0$, it follows that there is $x_2 \in A_0$ such that $d(x_2,Tx_1) = d(A,B)$. Continuing this process, we can construct a sequence $\{x_n\}$ in A_0, such that

$$d(x_{n+1},Tx_n) = d(A,B)$$

for every nonnegative integer n. Since T is a proximal C_N-contraction, we have

$$\begin{aligned} d(x_n, x_{n+1}) &\le \alpha d(x_{n-1}, x_n) + \beta\left[\frac{1 + d(x_{n-1}, x_n)}{1 + d(x_{n-1}, x_n)} d(x_n, x_{n+1})\right] \\ &\quad + L\min\{d(x_n, x_n), d(x_{n-1}, x_{n+1}), d(x_{n-1}, x_n), d(x_n, x_{n+1})\} \\ &= \alpha d(x_{n-1}, x_n) + \beta d(x_n, x_{n+1}). \end{aligned}$$

It follows that $d(x_n, x_{n+1}) \le \frac{\alpha}{1-\beta} d(x_{n-1}, x_n)$, where $\frac{\alpha}{1-\beta} < 1$. Therefore, $\{x_n\}$ is a Cauchy sequence. Now, since the space is complete and A is closed, the sequence $\{x_n\}$ converges to some $x \in A$. Further, we have

$$\begin{aligned} d(x, B) &\le d(x, Tx_n) \\ &\le d(x, x_{n+1}) + d(x_{n+1}, Tx_n) \\ &= d(x, x_{n+1}) + d(A, B) \\ &\le d(x, x_{n+1}) + d(x, B). \end{aligned}$$

Therefore, $d(x, Tx_n) \to d(x, B)$. Since B is approximatively compact with respect to A, the sequence $\{Tx_n\}$ has a subsequence $\{Tx_{n_k}\}$ converging to some element $y \in B$. Therefore,

$$d(x, y) = \lim_{k\to\infty} d(x_{n_k+1}, Tx_{n_k}) = d(A, B),$$

and hence x must be a member of A_0. Because of the fact that $T(A_0)$ is contained in B_0, $d(u, Tx) = d(A, B)$ for some element u in A. Since T is a proximal C_N-contraction of the first kind, we get

$$\begin{aligned} d(u, x_{n+1}) &\le \alpha d(x, x_n) + \beta\left[\frac{1 + d(x, u)}{1 + d(x, x_n)} d(x_n, x_{n+1})\right] \\ &\quad + L\min\{d(x_n, u), d(x, x_{n+1}), d(x, u), d(x_n, x_{n+1})\}. \end{aligned}$$

Taking the limit $n \to \infty$, we have $x = u$. Thus, it follows that $d(x, Tx) = d(u, Tx) = d(A, B)$.

Now, to prove the uniqueness of the best proximity point, assume that z is another best proximity point of T so that $d(z, Tz) = d(A, B)$. Since T is a proximal C_N-contraction of the first kind, we have

$$\begin{aligned} d(x, z) &\le \alpha d(x, z) + \beta\left[\frac{1 + d(x, x)}{1 + d(x, z)} d(z, z)\right] \\ &\quad + L\min\{d(z, x), d(x, z), d(x, x), d(z, z)\}, \end{aligned}$$

which implies that $x = z$. Hence T has a unique best proximity point. □

It is easy to see that the preceding result yields the following corollaries.

Corollary 3.2 *Let (X, d) be a complete metric space and A and B be two nonempty, closed subsets of X such that B is approximatively compact with respect to A. Assume that A_0 and B_0 are nonempty and $T : A \to B$ is a non-self-mapping such that:*

(a) *there exist nonnegative real numbers* α *and* L *with* $\alpha < 1$, *such that, for all* $u, v, x, y \in A$, *the conditions* $d(u, Tx) = d(A, B)$ *and* $d(v, Ty) = d(A, B)$ *imply that*

$$d(u, v) \leq \alpha d(x, y) + L \min\{d(y, u), d(x, v), d(x, u), d(y, v)\};$$

(b) $T(A_0) \subseteq B_0$.

Then there exists a unique element $x \in A$ *such that* $d(x, Tx) = d(A, B)$. *Moreover, for any fixed* $x_0 \in A_0$, *the sequence* $\{x_n\}$, *defined by* $d(x_{n+1}, Tx_n) = d(A, B)$, *converges to* x.

Corollary 3.3 *Let* (X, d) *be a complete metric space and* A *and* B *be two nonempty, closed subsets of* X *such that* B *is approximatively compact with respect to* A. *Assume that* A_0 *and* B_0 *are nonempty and* $T : A \to B$ *is a non-self-mapping such that*:

(a) *there exist nonnegative real numbers* $\alpha < 1$ *such that, for all* $u, v, x, y \in A$, *the conditions* $d(u, Tx) = d(A, B)$ *and* $d(v, Ty) = d(A, B)$ *imply that* $d(u, v) \leq \alpha d(x, y)$;

(b) $T(A_0) \subseteq B_0$.

Then there exists a unique element $x \in A$ *such that* $d(x, Tx) = d(A, B)$. *Moreover, for any fixed* $x_0 \in A_0$, *the sequence* $\{x_n\}$, *defined by* $d(x_{n+1}, Tx_n) = d(A, B)$, *converges to* x.

In Theorem 3.1, if T is a self-mapping, then we get the following fixed point theorem.

Corollary 3.4 *Let* T *be a self-mapping of a complete metric space* (X, d). *Assume that there exist nonnegative real numbers* α, β, *and* L *with* $\alpha + \beta < 1$, *such that*

$$\begin{aligned} d(Tx, Ty) \leq{}& \alpha d(x, y) + \beta \left[\frac{1 + d(x, Tx)}{1 + d(x, y)} d(y, Ty) \right] \\ &+ L \min\{d(y, Tx), d(x, Ty), d(x, Tx), d(y, Ty)\} \end{aligned}$$

for all $x, y \in X$. *Then* T *has a unique fixed point.*

Remark 3.2 It is well known that a contraction mapping must be continuous. Therefore, Corollary 3.4 is a real proper extension of the Banach contraction mapping principle of Banach [29] because the continuity of the mapping T is not required.

Next, we give the existence of best proximity point theorem for proximal C_N-contraction of the second kind.

Theorem 3.5 *Let* (X, d) *be a complete metric space and* A *and* B *be two nonempty, closed subsets of* X *such that* A *is approximatively compact with respect to* B. *Assume that* A_0 *and* B_0 *are nonempty and* $T : A \to B$ *is a non-self-mapping such that*:

(a) T *is a continuous proximal* C_N*-contraction of the second kind*;

(b) $T(A_0) \subseteq B_0$.

Then there exists $x \in A$ *such that* $d(x, Tx) = d(A, B)$. *Moreover, for any fixed* $x_0 \in A_0$, *the sequence* $\{x_n\}$, *defined by* $d(x_{n+1}, Tx_n) = d(A, B)$, *converges to* x. *Further,* $Tx = Tz$ *for all* x *and* z *belong to best proximity of* T.

Proof Following the arguments in Theorem 3.1, we can construct a sequence $\{x_n\}$ in A_0, such that

$$d(x_{n+1}, Tx_n) = d(A, B)$$

for every nonnegative integer n. Since T is a proximal C_N-contraction of the second kind, we have

$$\begin{aligned} d(Tx_n, Tx_{n+1}) &\le \alpha d(Tx_{n-1}, Tx_n) + \beta\left[\frac{1 + d(Tx_{n-1}, Tx_n)}{1 + d(Tx_{n-1}, Tx_n)} d(Tx_n, Tx_{n+1})\right] \\ &\quad + L\min\bigl\{d(Tx_n, Tx_n), d(Tx_{n-1}, Tx_{n+1}), d(Tx_{n-1}, Tx_n), d(Tx_n, Tx_{n+1})\bigr\} \\ &= \alpha d(Tx_{n-1}, Tx_n) + \beta d(Tx_n, Tx_{n+1}). \end{aligned}$$

It follows that $d(Tx_n, Tx_{n+1}) \le \frac{\alpha}{1-\beta} d(Tx_{n-1}, Tx_n)$, where $\frac{\alpha}{1-\beta} < 1$. Therefore, $\{Tx_n\}$ is a Cauchy sequence. Now, since the space is complete and B is closed, the sequence $\{Tx_n\}$ converges to some $y \in B$. Further, we have

$$\begin{aligned} d(y, A) &\le d(y, x_{n+1}) \\ &\le d(y, Tx_n) + d(Tx_n, x_{n+1}) \\ &= d(y, Tx_n) + d(A, B) \\ &\le d(y, Tx_n) + d(y, A). \end{aligned}$$

Therefore, $d(y, x_n) \to d(y, A)$. Since A is approximatively compact with respect to B, then the sequence $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ converging to some element $x \in A$. Now, using the continuity of T, we obtain $d(x, Tx) = \lim_{k\to\infty} d(x_{n_k+1}, Tx_{n_k}) = d(A, B)$.

Now, further assume that z is another best proximity point of T so that $d(z, Tz) = d(A, B)$. Since T is a proximal C_N-contraction of the second kind, we get

$$\begin{aligned} d(Tx, Tz) &\le \alpha d(Tx, Tz) + \beta\left[\frac{1 + d(Tx, Tx)}{1 + d(Tx, Tz)} d(Tz, Tz)\right] \\ &\quad + L\min\bigl\{d(Tz, Tx), d(Tx, Tz), d(Tx, Tx), d(Tz, Tz)\bigr\}, \end{aligned}$$

which implies that $Tx = Tz$. □

As consequences of Theorem 3.5, we state the following corollaries.

Corollary 3.6 *Let (X, d) be a complete metric space and A and B be two nonempty, closed subsets of X such that A is approximatively compact with respect to B. Assume that A_0 and B_0 are nonempty and $T : A \to B$ is a non-self-mapping such that:*

(a) *there exist nonnegative real numbers α and L with $\alpha < 1$ such that, for all $u, v, x, y \in A$, the conditions $d(u, Tx) = d(A, B)$ and $d(v, Ty) = d(A, B)$ imply that*

$$d(Tu, Tv) \le \alpha d(Tx, Ty) + L\min\bigl\{d(Ty, Tu), d(Tx, Tv), d(Tx, Tu), d(Ty, Tv)\bigr\};$$

(b) *T is a continuous;*

(c) *$T(A_0) \subseteq B_0$.*

Then there exists $x \in A$ such that $d(x, Tx) = d(A, B)$. Moreover, for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$, defined by $d(x_{n+1}, Tx_n) = d(A, B)$, converges to x. Further, $Tx = Tz$ for all x and z belong to best proximity of T.

Corollary 3.7 *Let (X,d) be a complete metric space and A and B be two nonempty, closed subsets of X such that A is approximatively compact with respect to B. Assume that A_0 and B_0 are nonempty and $T : A \to B$ is a non-self-mapping such that*:

(a) *there exists a nonnegative real number $\alpha < 1$ such that, for all $u,v,x,y \in A$, the conditions $d(u,Tx) = d(A,B)$ and $d(v,Ty) = d(A,B)$ imply that $d(Tu,Tv) \leq \alpha d(Tx,Ty)$*;

(b) *T is a continuous*;

(c) *$T(A_0) \subseteq B_0$*.

Then there exists $x \in A$ such that $d(x,Tx) = d(A,B)$. Moreover, for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$, defined by $d(x_{n+1},Tx_n) = d(A,B)$, converges to x. Further, $Tx = Tz$ for all x and z belong to the best proximity of T.

In Theorem 3.5, if T is a self-mapping, then we get the following fixed point theorem.

Corollary 3.8 *Let T be a continuous self-mapping of a complete metric space (X,d). Assume that there exist nonnegative real numbers α, β, and L with $\alpha + \beta < 1$, such that*

$$\begin{aligned} d(TTx,TTy) \leq{} & \alpha d(Tx,Ty) + \beta\left[\frac{1 + d(Tx,TTx)}{1 + d(Tx,Ty)} d(Ty,TTy)\right] \\ & + L\min\bigl\{d(Ty,TTx), d(Tx,TTy), d(Tx,TTx), d(Ty,TTy)\bigr\} \end{aligned}$$

for $x,y \in X$. Then T has a unique fixed point.

Remark 3.3 It is easy to see that Corollary 3.8 is a special case of Corollary 3.4.

The next theorem, we give conditions for the existence of best proximity point for a non-self-mapping that is a proximal C_N-contraction of the first and second kind. In this theorem, we consider only a completeness hypothesis without assuming the continuity of the non-self-mapping and the approximatively compactness of subspace.

Theorem 3.9 *Let (X,d) be a complete metric space and A and B be two nonempty, closed subsets of X. Assume that A_0 and B_0 are nonempty and $T : A \to B$ is a non-self-mapping such that*:

(a) *T is a proximal C_N-contraction of the first and second kind*;

(b) *$T(A_0) \subseteq B_0$*.

Then there exists a unique $x \in A$ such that $d(x,Tx) = d(A,B)$. Moreover, for any fixed $x_0 \in A_0$, the sequence $\{x_n\}$, defined by $d(x_{n+1},Tx_n) = d(A,B)$, converges to x.

Proof Following the arguments in Theorem 3.1, we can construct a sequence $\{x_n\}$ in A_0, such that

$$d(x_{n+1},Tx_n) = d(A,B)$$

for every nonnegative integer n. Also, using similar arguments as in the proof of Theorem 3.1, we deduce that the sequence $\{x_n\}$ is a Cauchy sequence, and hence converges to some $x \in A$. Moreover, on the lines of Theorem 3.5, we find that the sequence $\{Tx_n\}$ is a Cauchy sequence and hence converges to some $y \in B$. Therefore, we have $d(x,y) =$

$\lim_{n\to\infty} d(x_{n+1}, Tx_n) = d(A,B)$, hence x must be in A_0. Since $T(A_0) \subseteq B_0$, then $d(u, Tx) = d(A,B)$ for some $u \in A$. Using the fact that T is proximal C_N-contraction of first kind, we have

$$d(u,x_{n+1}) \leq \alpha d(x,x_n) + \beta\left[\frac{1+d(x,u)}{1+d(x,x_n)} d(x_n,x_{n+1})\right] + L\min\bigl\{d(x_n,u), d(x,x_{n+1}), d(x,u), d(x_n,x_{n+1})\bigr\}.$$

Taking the limit $n \to \infty$, we have $x = u$. Thus, it follows that $d(x, Tx) = d(u, Tx) = d(A,B)$.

Now, to prove the uniqueness of the best proximity point, assume that z is another best proximity point of T so that $d(z, Tz) = d(A,B)$. Since T is a proximal C_N-contraction of the first kind, we have

$$d(x,z) \leq \alpha d(x,z) + \beta\left[\frac{1+d(x,x)}{1+d(x,z)} d(z,z)\right] + L\min\bigl\{d(z,x), d(x,z), d(x,x), d(z,z)\bigr\},$$

which implies that $x = z$. Hence T has a unique best proximity point. □

Finally, we give some illustrative example which demonstrate the validity of the hypotheses and degree of utility of our results.

Example 3.10 Let $X = \mathbb{R}$ endowed with the usual metric $d(x,y) = |x-y|$, for all $x,y \in X$. Define two closed subsets A, B of X by $A = [0,1]$ and $B = (-\infty,-2] \cup [3,\infty)$. Then $d(A,B) = 2$, $A_0 = \{0,1\}$ and $B_0 = \{-2,3\}$. Also define $T : A \to B$ by

$$Tx = \begin{cases} 3 & \text{if } x \text{ is rational}, \\ 5 & \text{if } x \text{ is irrational}. \end{cases}$$

It is easy to see that $T(A_0) \subseteq B_0$ and T is a proximal C_N-contraction of the first and second kind. Now all the hypotheses of Theorem 3.9 are satisfied and $d(1, T(1)) = d(A,B)$. Obviously, Theorem 3.5 is not applicable in this case since T is not continuous.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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