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G-β-ψ contractive-type mappings and related fixed point theorems

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Abstract
In this paper, we introduce the notion of generalized G-β-ψ contractive mappings which is inspired by the concept of α-ψ contractive mappings. We showed the existence and uniqueness of a fixed point for such mappings in the setting of complete G-metric spaces. The main results of this paper extend, generalize and improve some well-known results on the topic in the literature. We state some examples to illustrate our results. We consider also some applications to show the validity of our results.

1 Introduction and preliminaries

In nonlinear functional analysis, the importance of fixed point theory has been increasing rapidly as an interesting research field. One of the most important reasons for this development is the potential of application of fixed point theory not only in various branches of applied and pure mathematics, but also in many other disciplines such as chemistry, biology, physics, economics, computer science, engineering *etc.* We also emphasize the crucial role of celebrated results of Banach [1], known as a Banach contraction mapping principle or a Banach fixed point theorem, in the growth of this theory. In 1922, Banach proved that every contraction in a complete metric space has a unique fixed point. After this remarkable paper, a number of authors have extended/generalized/improved the Banach contraction mapping principle in various ways in different abstract spaces (see, *e.g.*, [2–22]). One of the interesting and recent results in this direction was given by Samet *et al.* [23]. They defined the notion of α-ψ contractive mappings and proved that including the Banach fixed point theorems, some well-known fixed point results turn into corollaries of their results. Another interesting result was given in 2004 by Mustafa and Sims [24] by introducing the notion of a G-metric space as a generalization of the concept of a metric space. The authors characterized the Banach fixed point theorem in the context of a G-metric space. After this result, many authors have paid attention to this space and proved the existence and uniqueness of a fixed point in the context of a G-metric space (see, *e.g.*, [11, 17–20, 24–48]). In this paper, we combine these two notions by introducing a G-β-ψ contractive mapping which is a characterization α-ψ contractive mappings in the context of G-metric spaces. Our main results generalize, extend and improve the existence results on the topic in the literature.

Let Ψ be a family of functions $\psi : [0,\infty) \to [0,\infty)$ satisfying the following conditions:

(i) ψ is nondecreasing;

(ii) there exist $k_0 \in \mathbb{N}$ and $a \in (0,1)$ and a convergent series of nonnegative terms $\sum_{k=1}^{\infty} v_k$ such that

$$\psi^{k+1}(t) \leq a\psi^k(t) + v_k$$

for $k \geq k_0$ and any $t \in \mathbb{R}^+$, where $\mathbb{R}^+ = [0,\infty)$.

These functions are known in the literature as Bianchini-Grandolfi gauge functions in some sources (see, *e.g.*, [21, 22, 49]) and as (c)-comparison functions in some other sources (see, *e.g.*, [50]).

Lemma 1 (See [50]) *If $\psi \in \Psi$, then the following hold*:

(i) $(\psi^n(t))_{n\in\mathbb{N}}$ *converges to* 0 *as* $n \to \infty$ *for all* $t \in \mathbb{R}^+$;
(ii) $\psi(t) < t$ *for any* $t \in \mathbb{R}^+$;
(iii) ψ *is continuous at* 0;
(iv) *the series* $\sum_{k=1}^{\infty} \psi^k(t)$ *converges for any* $t \in \mathbb{R}^+$.

Very recently, Karapınar and Samet [32] introduced the following concepts.

Definition 2 Let (X,d) be a metric space and $T: X \to X$ be a given mapping. We say that T is a generalized α-ψ contractive mapping if there exist two functions $\alpha: X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that

$$\alpha(x,y)d(Tx,Ty) \leq \psi\bigl(M(x,y)\bigr)$$

for all $x,y \in X$, where

$$M(x,y) = \max\bigl\{d(x,y), \bigl(d(x,Tx) + d(y,Ty)\bigr)/2, \bigl(d(x,Ty) + d(y,Tx)\bigr)/2\bigr\}.$$

Clearly, since ψ is nondecreasing, every α-ψ contractive mapping, presented in [23], is a generalized α-ψ contractive mapping.

Definition 3 Let $T: X \to X$ and $\alpha: X \times X \to [0,\infty)$. We say that T is α-admissible if for all $x,y \in X$, we have

$$\alpha(x,y) \geq 1 \quad \Longrightarrow \quad \alpha(Tx,Ty) \geq 1.$$

Various examples of such mappings are presented in [23]. The main results in [32] are the following fixed point theorems.

Theorem 4 *Let (X,d) be a complete metric space and $T: X \to X$ be a generalized α-ψ contractive mapping. Suppose that*

(i) T *is* α*-admissible*;
(ii) *there exists* $x_0 \in X$ *such that* $\alpha(x_0, Tx_0) \geq 1$;
(iii) T *is continuous*.

Then there exists $u \in X$ such that $Tu = u$.

Theorem 5 *Let (X,d) be a complete metric space and $T: X \to X$ be a generalized α-ψ contractive mapping. Suppose that*

(i) *T is α-admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;
(iii) *if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then $\alpha(x_n, x) \geq 1$ for all n.*

Then there exists $u \in X$ such that $Tu = u$.

Theorem 6 *Adding to the hypotheses of Theorem* 4 (*resp. Theorem* 5) *the condition*: *For all $x, y \in \mathrm{Fix}(T)$, there exists $z \in X$ such that $\alpha(x,z) \geq 1$ and $\alpha(y,z) \geq 1$, we obtain the uniqueness of the fixed point of T.*

Mustafa and Sims [24] introduced the concept of G-metric spaces as follows.

Definition 7 [24] Let X be a nonempty set and $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following properties:
(G1) $G(x,y,z) = 0$ if $x = y = z$;
(G2) $0 < G(x,x,y)$ for all $x, y \in X$ with $x \neq y$;
(G3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $y \neq z$;
(G4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$ (symmetry in all three variables);
(G5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$ (rectangle inequality).

Then the function G is called a generalized metric or, more specifically, a G-metric on X, and the pair (X,G) is called a G-metric space.

Every G-metric on X defines a metric d_G on X by

$$d_G(x,y) = G(x,y,y) + G(y,x,x) \quad \text{for all } x,y \in X.$$

Example 8 Let (X,d) be a metric space. The function $G : X \times X \times X \to \mathbb{R}^+$, defined as

$$G(x,y,z) = \max\{d(x,y), d(y,z), d(z,x)\}$$

or

$$G(x,y,z) = d(x,y) + d(y,z) + d(z,x)$$

for all $x,y,z \in X$, is a G-metric on X.

Definition 9 [24] Let (X,G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of X. We say that $\{x_n\}$ is G-convergent to $x \in X$ if

$$\lim_{n,m\to\infty} G(x, x_n, x_m) = 0,$$

that is, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x, x_n, x_m) < \varepsilon$ for all $n, m \geq N$. We call x the limit of the sequence and write $x_n \to x$ or $\lim_{n\to\infty} x_n = x$.

Proposition 10 [24] *Let (X,G) be a G-metric space. The following are equivalent*:
(1) *$\{x_n\}$ is G-convergent to x*;
(2) *$G(x_n, x_n, x) \to 0$ as $n \to \infty$*;

(3) $G(x_n, x, x) \to 0$ *as* $n \to \infty$;
(4) $G(x_n, x_m, x) \to 0$ *as* $n, m \to \infty$.

Definition 11 [24] Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called a G-Cauchy sequence if for any $\varepsilon > 0$, there is $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$ for all $n, m, l \geq N$, that is, $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.

Proposition 12 [24] *Let* (X, G) *be a* G*-metric space. Then the following are equivalent*:
(1) *the sequence* $\{x_n\}$ *is* G*-Cauchy*;
(2) *for any* $\varepsilon > 0$, *there exists* $N \in \mathbb{N}$ *such that* $G(x_n, x_m, x_m) < \varepsilon$ *for all* $n, m \geq N$.

Definition 13 [24] A G-metric space (X, G) is called G-complete if every G-Cauchy sequence is G-convergent in (X, G).

Lemma 14 [24] *Let* (X, G) *be a* G*-metric space. Then, for any* $x, y, z, a \in X$, *it follows that*
(i) *if* $G(x, y, z) = 0$, *then* $x = y = z$;
(ii) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$;
(iii) $G(x, y, y) \leq 2G(y, x, x)$;
(iv) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$;
(v) $G(x, y, z) \leq \frac{2}{3}[G(x, y, a) + G(x, a, z) + G(a, y, z)]$;
(vi) $G(x, y, z) \leq G(x, a, a) + G(y, a, a) + G(z, a, a)$.

Definition 15 (See [24]) Let (X, G) be a G-metric space. A mapping $T : X \to X$ is said to be G-continuous if $\{T(x_n)\}$ is G-convergent to $T(x)$, where $\{x_n\}$ is any G-convergent sequence converging to x.

In [36], Mustafa characterized the well-known Banach contraction principle mapping in the context of G-metric spaces in the following way.

Theorem 16 (See [36]) *Let* (X, G) *be a complete* G*-metric space and* $T : X \to X$ *be a mapping satisfying the following condition for all* $x, y, z \in X$:

$$G(Tx, Ty, Tz) \leq kG(x, y, z), \tag{1}$$

where $k \in [0, 1)$. *Then* T *has a unique fixed point*.

Theorem 17 (See [36]) *Let* (X, G) *be a complete* G*-metric space and* $T : X \to X$ *be a mapping satisfying the following condition for all* $x, y \in X$:

$$G(Tx, Ty, Ty) \leq kG(x, y, y), \tag{2}$$

where $k \in [0, 1)$. *Then* T *has a unique fixed point*.

Remark 18 The condition (1) implies the condition (2). The converse is true only if $k \in [0, \frac{1}{2})$. For details, see [36].

From [24, 36], each G-metric G on X generates a topology τ_G on X whose base is a family of open G-balls $\{B_G(x, \varepsilon) : x \in X, \varepsilon > 0\}$, where $B_G(x, \varepsilon) = \{y \in X : G(x, y, y) < \varepsilon\}$ for all $x \in X$

and $\varepsilon > 0$. Moreover,

$$x \in \overline{A} \quad \Leftrightarrow \quad B_G(x,\varepsilon) \cap A \neq \emptyset, \quad \text{for all } \varepsilon > 0.$$

Proposition 19 *Let (X, G) be a G-metric space and A be a nonempty subset of X. Then A is G-closed if for any G-convergent sequence $\{x_n\}$ in A with limit x, one has $x \in A$.*

2 Main results

We introduce the concept of generalized α-ψ contractive mappings as follows.

Definition 20 Let (X, G) be a G-metric space and $T : X \to X$ be a given mapping. We say that T is a generalized G-β-ψ contractive mapping of type I if there exist two functions $\beta : X \times X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that for all $x, y, z \in X$, we have

$$\beta(x,y,z)G(Tx,Ty,Tz) \le \psi\big(M(x,y,z)\big), \tag{3}$$

where

$$M(x,y,z) = \max \left\{ \begin{array}{c} G(x,y,z), G(x,Tx,Tx), G(y,Ty,Ty), G(z,Tz,Tz), \\ \frac{1}{3}(G(x,Ty,Ty) + G(y,Tz,Tz) + G(z,Tx,Tx)) \end{array} \right\}.$$

Definition 21 Let (X, G) be a G-metric space and $T : X \to X$ be a given mapping. We say that T is a generalized G-β-ψ contractive mapping of type II if there exist two functions $\beta : X \times X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that for all $x, y \in X$, we have

$$\beta(x,y,y)G(Tx,Ty,Ty) \le \psi\big(M(x,y,y)\big), \tag{4}$$

where

$$M(x,y,y) = \max \left\{ \begin{array}{c} G(x,y,y), G(x,Tx,Tx), G(y,Ty,Ty), \\ \frac{1}{3}(G(x,Ty,Ty) + G(y,Ty,Ty) + G(y,Tx,Tx)) \end{array} \right\}.$$

Remark 22 Clearly, any contractive mapping, that is, a mapping satisfying (1), is a generalized G-β-ψ contractive mapping of type I with $\beta(x,y,z) = 1$ for all $x, y, z \in X$ and $\psi(t) = kt$, $k \in (0,1)$. Analogously, a mapping satisfying (2) is a generalized G-β-ψ contractive mapping of type II with $\beta(x,y,y) = 1$ for all $x, y \in X$ and $\psi(t) = kt$, where $k \in (0,1)$.

Definition 23 Let $T : X \to X$ and $\beta : X \times X \times X \to [0, \infty)$. We say that T is β-admissible if for all $x, y, z \in X$, we have

$$\beta(x,y,z) \ge 1 \quad \Longrightarrow \quad \beta(Tx,Ty,Tz) \ge 1.$$

Example 24 Let $X = [0, \infty)$ and $T : X \to X$. Define $\beta(x,y,z) : X \times X \times X \to [0, \infty)$ by $Tx = \ln(1 + x)$ and

$$\beta(x,y,z) = \begin{cases} e & \text{if } x \ge y \ge z, \\ 0 & \text{if otherwise.} \end{cases}$$

Then T is β-admissible.

Our first result is the following.

Theorem 25 *Let (X,G) be a complete G-metric space. Suppose that $T : X \to X$ is a generalized G-β-ψ contractive mapping of type I and satisfies the following conditions*:

(i)$_a$ *T is β-admissible*;
(ii)$_a$ *there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, Tx_0) \geq 1$*;
(iii)$_b$ *T is G-continuous.*

Then there exists $u \in X$ such that $Tu = u$.

Proof Let $x_0 \in X$ be such that $\beta(x_0, Tx_0, Tx_0) \geq 1$ (such a point exists from the condition (ii)$_a$). Define the sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for all $n \geq 0$. If $x_{n_0} = x_{n_0+1}$ for some n_0, then $u = x_{n_0}$ is a fixed point of T. So, we can assume that $x_n \neq x_{n+1}$ for all n. Since T is β-admissible, we have

$$\beta(x_0,x_1,x_1) = \beta(x_0, Tx_0, Tx_0) \geq 1 \quad\Longrightarrow\quad \beta(Tx_0, Tx_1, Tx_1) = \beta(x_1,x_2,x_2) \geq 1.$$

Inductively, we have

$$\beta(x_n,x_{n+1},x_{n+1}) \geq 1, \quad \text{for all } n = 0,1,\ldots. \tag{5}$$

From (3) and (5), it follows that for all $n \geq 1$, we have

$$\begin{aligned} G(x_n,x_{n+1},x_{n+1}) &= G(Tx_{n-1}, Tx_n, Tx_n) \\ &\leq \beta(x_{n-1},x_n,x_n)G(Tx_{n-1}, Tx_n, Tx_n) \\ &\leq \psi\bigl(M(x_{n-1},x_n,x_n)\bigr). \end{aligned}$$

On the other hand, we have

$$\begin{aligned} M(x_{n-1},x_n,x_n) &= \max\left\{\begin{array}{c} G(x_{n-1},x_n,x_n), G(x_{n-1}, Tx_{n-1}, Tx_{n-1}), G(x_n, Tx_n, Tx_n), \\ \frac{1}{3}(G(x_{n-1}, Tx_n, Tx_n) + G(x_n, Tx_n, Tx_n) + G(x_n, Tx_{n-1}, Tx_{n-1})) \end{array}\right\} \\ &= \max\left\{\begin{array}{c} G(x_{n-1},x_n,x_n), G(x_{n-1},x_n,x_n), G(x_n,x_{n+1},x_{n+1}), \\ \frac{1}{3}(G(x_{n-1},x_{n+1},x_{n+1}) + G(x_n,x_{n+1},x_{n+1}) + G(x_n,x_n,x_n)) \end{array}\right\} \\ &\leq \max\left\{\begin{array}{c} G(x_{n-1},x_n,x_n), G(x_n,x_{n+1},x_{n+1}), \\ \frac{1}{3}(G(x_{n-1},x_n,x_n) + G(x_n,x_{n+1},x_{n+1}) + G(x_n,x_{n+1},x_{n+1})) \end{array}\right\} \\ &= \max\left\{\begin{array}{c} G(x_{n-1},x_n,x_n), G(x_n,x_{n+1},x_{n+1}), \\ \frac{1}{3}(G(x_{n-1},x_n,x_n) + 2G(x_n,x_{n+1},x_{n+1})) \end{array}\right\} \\ &= \max\bigl\{G(x_{n-1},x_n,x_n), G(x_n,x_{n+1},x_{n+1})\bigr\}. \end{aligned}$$

Thus, we have

$$G(x_n,x_{n+1},x_{n+1}) \leq \psi\bigl(\max\bigl\{G(x_{n-1},x_n,x_n), G(x_n,x_{n+1},x_{n+1})\bigr\}\bigr).$$

We consider the following two cases:

Case 1: If $\max\{G(x_{n-1},x_n,x_n),G(x_n,x_{n+1},x_{n+1})\} = G(x_n,x_{n+1},x_{n+1})$ for some n, then

$$G(x_n,x_{n+1},x_{n+1}) \le \psi\big(G(x_n,x_{n+1},x_{n+1})\big) < G(x_n,x_{n+1},x_{n+1}),$$

which is a contradiction.

Case 2: If $\max\{G(x_{n-1},x_n,x_n),G(x_n,x_{n+1},x_{n+1})\} = G(x_{n-1},x_n,x_n)$, then

$$G(x_n,x_{n+1},x_{n+1}) \le \psi\big(G(x_{n-1},x_n,x_n)\big)$$

for all $n \ge 1$. Since ψ is nondecreasing, by induction, we get

$$G(x_n,x_{n+1},x_{n+1}) \le \psi^n\big(G(x_0,x_1,x_1)\big) \quad \text{for all } n \ge 1. \tag{6}$$

Using (G5) and (6), we have

$$\begin{aligned} G(x_n,x_m,x_m) &\le G(x_n,x_{n+1},x_{n+1}) + G(x_{n+1},x_{n+2},x_{n+2}) \\ &\quad + G(x_{n+2},x_{n+3},x_{n+3}) + \cdots + G(x_{m-1},x_m,x_m) \\ &= \sum_{k=n}^{m-1} G(x_k,x_{k+1},x_{k+1}) \\ &\le \sum_{k=n}^{m-1} \psi^k\big(G(x_0,x_1,x_1)\big). \end{aligned}$$

Since $\psi \in \Psi$ and $G(x_0,x_1,x_1) > 0$, by Lemma 1, we get that

$$\sum_{k=0}^{\infty} \psi^k\big(G(x_0,x_1,x_1)\big) < \infty.$$

Thus, we have

$$\lim_{n,m\to 0} G(x_n,x_m,x_m) = 0.$$

By Proposition 12, this implies that $\{x_n\}$ is a G-Cauchy sequence in the G-metric space (X,G). Since (X,G) is complete, there exists $u \in X$ such that $\{x_n\}$ is G-convergent to u. Since T is G-continuous, it follows that $\{Tx_n\}$ is G-convergent to Tu. By the uniqueness of the limit, we get $u = Tu$, that is, u is a fixed point of T. □

Definition 26 (See [51]) Let (X,G) be a G-metric space and $T : X \to X$ be a given mapping. We say that T is a G-β-ψ contractive mapping of type I if there exist two functions $\beta : X \times X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that for all $x,y,z \in X$, we have

$$\beta(x,y,z)G(Tx,Ty,Tz) \le \psi\big(G(x,y,z)\big) \tag{7}$$

by following the lines of the proof of Theorem 25.

Corollary 27 *Let (X,G) be a complete G-metric space. Suppose that $T : X \to X$ is a G-β-ψ contractive mapping of type I and satisfies the following conditions*:

(i)$_a$ *T is β-admissible*;

$(\mathrm{ii})_a$ *there exists* $x_0 \in X$ *such that* $\beta(x_0, Tx_0, Tx_0) \geq 1$;

$(\mathrm{iii})_b$ T *is G-continuous.*

Then there exists $u \in X$ *such that* $Tu = u$.

Example 28 Let $X = [0, \infty)$ be endowed with the G-metric

$$G(x,y,z) = |x-y| + |y-z| + |z-x| \quad \text{for all } x,y,z \in X.$$

Define $T : X \to X$ by $Tx = 3x$ for all $x \in X$. We define $\beta : X \times X \times X \to [0, \infty)$ in the following way:

$$\beta(x,y,z) = \begin{cases} \frac{1}{9} & \text{if } (x,y,z) \neq (0,0,0), \\ 1 & \text{otherwise.} \end{cases}$$

One can easily show that

$$\beta(x,y,z)G(Tx,Ty,Tz) \leq \frac{1}{9}G(x,y,z) \quad \text{for all } x,y,z \in X.$$

Then T is a G-β-ψ contractive mapping of type I with $\psi(t) = \frac{1}{9}t$ for all $t \in [0, \infty)$. Take $x,y,z \in X$ such that $\beta(x,y,z) \geq 1$. By the definition of T, this implies that $x = y = z = 0$. Then we have $\beta(Tx,Ty,Tz) = \beta(0,0,0) = 1$, and so T is β-admissible. All the conditions of Corollary 27 are satisfied. Here, 0 is the fixed point of T. Notice also that the Banach contraction mapping principle is not applicable. Indeed, $d(x,y) = |x-y|$ for all $x,y \in X$. Then we have $x \neq y$ $d(Tx,Ty) = 3|x-y| > k|x-y|$ for all $k \in [0,1)$.

It is clear that Theorem 16 is not applicable.

The following result can be easily concluded from Theorem 25.

Corollary 29 *Let* (X,G) *be a complete G-metric space. Suppose that* $T : X \to X$ *is a generalized G-*β*-*ψ *contractive mapping of type II and satisfies the following conditions*:

$(\mathrm{i})_a$ T *is* β*-admissible*;

$(\mathrm{ii})_a$ *there exists* $x_0 \in X$ *such that* $\beta(x_0, Tx_0, Tx_0) \geq 1$;

$(\mathrm{iii})_b$ T *is G-continuous.*

Then there exists $u \in X$ *such that* $Tu = u$.

The next theorem does not require the continuity of T.

Theorem 30 *Let* (X,G) *be a complete G-metric space. Suppose that* $T : X \to X$ *is a generalized G-*β*-*ψ *contractive mapping of type I such that* ψ *is continuous and satisfies the following conditions*:

$(\mathrm{i})_b$ T *is* β*-admissible*;

$(\mathrm{ii})_b$ *there exists* $x_0 \in X$ *such that* $\beta(x_0, Tx_0, Tx_0) \geq 1$;

$(\mathrm{iii})_b$ *if* $\{x_n\}$ *is a sequence in* X *such that* $\beta(x_n, x_{n+1}, x_{n+1}) \geq 1$ *for all* n *and* $\{x_n\}$ *is a G-convergent to* $x \in X$, *then* $\beta(x_n, x, x) \geq 1$ *for all* n.

Then there exists $u \in X$ *such that* $Tu = u$.

Proof Following the proof of Theorem 25, we know that the sequence $\{x_n\}$ defined by $x_{n+1} = Tx_n$ for all $n \geq 0$, is a G-Cauchy sequence in the complete G-metric space (X, G), that is, G-convergent to $u \in X$. From (5) and $(\mathrm{iii})_b$, we have

$$\beta(x_n, u, u) \geq 1 \quad \text{for all } n \geq 0. \tag{8}$$

Using (8), we have

$$\begin{aligned} G(x_{n+1}, Tu, Tu) &= G(Tx_n, Tu, Tu) \\ &\leq \beta(x_n, u, u) G(Tx_n, Tu, Tu) \\ &\leq \psi\big(M(x_n, u, u)\big), \end{aligned}$$

where

$$\begin{aligned} M(x_n, u, u) &= \max \left\{ \begin{array}{c} G(x_n, u, u), G(x_n, Tx_n, Tx_n), G(u, Tu, Tu), \\ \frac{1}{3}(G(x_n, Tu, Tu) + G(u, Tu, Tu) + G(u, Tx_n, Tx_n)) \end{array} \right\} \\ &= \max \left\{ \begin{array}{c} G(x_n, u, u), G(x_n, x_{n+1}, x_{n+1}), G(u, Tu, Tu), \\ \frac{1}{3}(G(x_n, Tu, Tu) + G(u, Tu, Tu) + G(u, x_{n+1}, x_{n+1})) \end{array} \right\}. \end{aligned}$$

Letting $n \to \infty$ in the following inequality:

$$G(x_{n+1}, Tu, Tu) \leq \psi\big(M(x_n, u, u)\big),$$

it follows that

$$G(u, Tu, Tu) \leq \psi\big(G(u, Tu, Tu)\big),$$

which is a contradiction. Thus, we obtain $G(u, Tu, Tu) = 0$, that is, by Lemma 14, $u = Tu$. □

The following corollary can be easily derived from Theorem 30.

Corollary 31 *Let (X, G) be a complete G-metric space. Suppose that $T : X \to X$ is a generalized G-β-ψ contractive mapping of type II such that ψ is continuous and satisfies the following conditions*:

- $(\mathrm{i})_b$ *T is β-admissible*;
- $(\mathrm{ii})_b$ *there exists $x_0 \in X$ such that $\beta(x_0, Tx_0, Tx_0) \geq 1$*;
- $(\mathrm{iii})_b$ *if $\{x_n\}$ is a sequence in X such that $\beta(x_n, x_{n+1}, x_{n+1}) \geq 1$ for all n and $\{x_n\}$ is a G-convergent to $x \in X$, then $\beta(x_n, x, x) \geq 1$ for all n.*

Then there exists $u \in X$ such that $Tu = u$.

With the following example, we will show that the hypotheses in Theorems 25 and 30 do not guarantee uniqueness.

Example 32 Let $X = \{(1,0),(0,1)\} \subset \mathbb{R}^2$ be endowed with the following G-metric:

$$G\big((x,y),(u,v),(z,w)\big) = |x-u| + |u-z| + |z-x| + |y-v| + |v-w| + |w-y|$$

for all $(x,y),(u,v),(z,w) \in X$. Obviously, (X,G) is a complete metric space. The mapping $T(x,y) = (x,y)$ is trivially continuous and satisfies, for any $\psi \in \Psi$,

$$\beta\big((x,y),(u,v),(z,w)\big)G\big(T(x,y),T(u,v),T(z,w)\big) \le \psi\big(M\big((x,y),(u,v),(z,w)\big)\big)$$

for all $(x,y),(u,v),(z,w) \in X$, where

$$\beta\big((x,y),(u,v),(z,w)\big) = \begin{cases} 1 & \text{if } (x,y) = (u,v) = (z,w), \\ 0 & \text{otherwise.} \end{cases}$$

Thus T is a generalized G-β-ψ contractive mapping. On the other hand, for all (x,y), $(u,v),(z,w) \in X$, we have

$$\beta\big((x,y),(u,v),(z,w)\big) \ge 1 \to (x,y) = (u,v) = (z,w),$$

which yields that

$$T(x,y) = T(u,v) = T(z,w) \to \beta\big(T(x,y),T(u,v),T(z,w)\big) \ge 1.$$

Hence T is β-admissible. Moreover, for all $(x,y) \in X$, we have $\beta((x,y),T(x,y),T(x,y)) \ge 1$. So, the assumptions of Theorem 25 are satisfied. Note that the assumptions of Theorem 30 are also satisfied, indeed, if $\{(x_n,y_n)\}$ is a sequence in X that converges to some point $(x,y) \in X$ with $\beta((x_n,y_n),(x_{n+1},y_{n+1}),(x_{n+1},y_{n+1})) \ge 1$ for all n, then from the definition of β, we have $(x_n,y_n) = (x,y)$ for all n, which implies that $\beta((x_n,y_n),(x,y),(x,y)) = 1$ for all n. However, in this case, T has two fixed points in X.

Let X be a set and T be a self-mapping on X. The set of all fixed points of T will be denoted by $\mathrm{Fix}(T)$.

Theorem 33 *Adding the following condition to the hypotheses of Theorem* 25 (*resp. Theorem* 30, *Corollary* 29, *Corollary* 31), *we obtain the uniqueness of the fixed point of* T.
(iv) *For* $x \in \mathrm{Fix}(T)$, $\beta(x,z,z) \ge 1$ *for all* $z \in X$.

Proof Let $u,v \in \mathrm{Fix}(T)$ be two fixed points of T. By (iv), we derive

$$\beta(u,v,v) \ge 1.$$

Notice that $\beta(Tu,Tv,Tv) = \beta(u,v,v)$ since u and v are fixed points of T. Consequently, we have

$$\begin{aligned} G(u,v,v) &= G(Tu,Tv,Tv) \\ &\le \beta(u,v,v)G(Tu,Tv,Tv) \le \psi\big(M(u,v,v)\big), \end{aligned}$$

where

$$\begin{aligned} M(u,v,v) &= \max\left\{\begin{array}{c} G(u,v,v), G(u,Tu,Tu), G(v,Tv,Tv), \\ \frac{1}{3}(G(u,Tv,Tv)+G(v,Tv,Tv)+G(v,Tu,Tu)) \end{array}\right\} \\ &= \max\left\{G(u,v,v), \frac{1}{3}\big(G(u,v,v)+G(v,u,u)\big)\right\} \\ &\leq \max\left\{G(u,v,v), \frac{1}{3}\big(G(u,v,v)+2G(u,v,v)\big)\right\} \\ &= G(u,v,v). \end{aligned}$$

Thus, we get that

$$G(u,v,v) \leq \psi\big(M(u,v,v)\big) \leq \psi\big(G(u,v,v)\big) < G(u,v,v),$$

which is a contradiction. Therefore, $u = v$, *i.e.*, the fixed point of T is unique. □

Corollary 34 *Let (X,G) be a complete G-metric space and let $T : X \to X$ be a given mapping. Suppose that there exists a continuous function $\psi \in \Psi$ such that*

$$G(Tx,Ty,Tz) \leq \psi\big(M(x,y,z)\big)$$

for all $x,y,z \in X$. Then T has a unique fixed point.

Corollary 35 *Let (X,G) be a complete G-metric space and let $T : X \to X$ be a given mapping. Suppose that there exists a function $\psi \in \Psi$ such that*

$$G(Tx,Ty,Tz) \leq \psi\big(G(x,y,z)\big)$$

for all $x,y,z \in X$. Then T has a unique fixed point.

Corollary 36 *Let (X,G) be a complete G-metric space and let $T : X \to X$ be a given mapping. Suppose that there exists $\lambda \in [0,1)$ such that*

$$G(Tx,Ty,Tz) \leq \lambda \max\left\{\begin{array}{c} G(x,y,z), G(x,Tx,Tx), G(y,Ty,Ty), G(z,Tz,Tz), \\ \frac{1}{3}(G(x,Ty,Ty)+G(y,Tz,Tz)+G(z,Tx,Tx)) \end{array}\right\}$$

for all $x,y,z \in X$. Then T has a unique fixed point.

Corollary 37 *Let (X,G) be a complete G-metric space and let $T : X \to X$ be a given mapping. Suppose that there exist nonnegative real numbers a, b, c, d and e with $a+b+c+d+e < 1$ such that*

$$\begin{aligned} G(Tx,Ty,Tz) \leq{}& aG(x,y,z) + bG(x,Tx,Tx) + cG(y,Ty,Ty) + dG(z,Tz,Tz) \\ &+ \frac{e}{3}\big(G(x,Ty,Ty)+G(y,Tz,Tz)+G(z,Tx,Tx)\big) \end{aligned}$$

for all $x,y,z \in X$. Then T has a unique fixed point.

Corollary 38 (See [40]) *Let (X,G) be a complete G-metric space and let $T:X\to X$ be a given mapping. Suppose that there exists $\lambda\in[0,1)$ such that*

$$G(Tx,Ty,Tz)\le\lambda G(x,y,z)$$

for all $x,y,z\in X$. Then T has a unique fixed point.

3 Consequences

3.1 Fixed point theorems on metric spaces endowed with a partial order

Definition 39 Let $(X,\preceq)$ be a partially ordered set and $T:X\to X$ be a given mapping. We say that T is nondecreasing with respect to $\preceq$ if

$$x,y\in X,\quad x\preceq y\quad\Longrightarrow\quad Tx\preceq Ty.$$

Definition 40 Let $(X,\preceq)$ be a partially ordered set. A sequence $\{x_n\}\subset X$ is said to be nondecreasing with respect to $\preceq$ if

$$x_n\preceq x_{n+1}\quad\text{for all } n.$$

Definition 41 Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X. We say that $(X,\preceq,G)$ is G-regular if for every nondecreasing sequence $\{x_n\}\subset X$ such that $x_n\to x\in X$ as $n\to\infty$, $x_n\preceq x$ for all n.

Theorem 42 *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Let $T:X\to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a function $\psi\in\Psi$ such that*

$$G(Tx,Ty,Ty)\le\psi\big(M(x,y,y)\big)\tag{9}$$

for all $x,y\in X$ with $x\preceq y$. Suppose also that the following conditions hold:

(i) *there exists $x_0\in X$ such that $x_0\preceq Tx_0$;*

(ii) *T is G-continuous or $(X,\preceq,G)$ is G-regular and ψ is continuous.*

Then there exists $u\in X$ such that $Tu=u$. Moreover, if for $x\in\mathrm{Fix}(T)$, $x\preceq z$ for all $z\in X$, one has the uniqueness of the fixed point.

Proof Define the mapping $\beta:X\times X\times X\to[0,\infty)$ by

$$\beta(x,y,y)=\begin{cases}1 & \text{if } x\preceq y,\\ 0 & \text{otherwise.}\end{cases}\tag{10}$$

From (9), for all $x,y\in X$, we have

$$\beta(x,y,y)G(Tx,Ty,Ty)\le\psi\big(M(x,y,y)\big).$$

It follows that T is a generalized G-β-ψ contractive mapping of type II. From the condition (i), we have

$$\beta(x_0,Tx_0,Tx_0)\ge1.$$

By the definition of β and since T is a nondecreasing mapping with respect to $\preceq$, we have

$$\beta(x,y,y) \geq 1 \quad \Longrightarrow \quad x \preceq y \quad \Longrightarrow \quad Tx \preceq Ty \quad \Longrightarrow \quad \beta(Tx,Ty,Ty) \geq 1.$$

Thus T is β-admissible. Moreover, if T is G-continuous, by Theorem 25, T has a fixed point.

Now, suppose that $(X,\preceq,G)$ is G-regular. Let $\{x_n\}$ be a sequence in X such that $\beta(x_n,x_{n+1},x_{n+1}) \geq 1$ for all n and x_n is G-convergent to $x \in X$. By Definition 41, $x_n \preceq x$ for all n, which gives us $\beta(x_n,x,x) \geq 1$ for all k. Thus, all the hypotheses of Theorem 30 are satisfied and there exists $u \in X$ such that $Tu = u$. To prove the uniqueness, since $u \in \mathrm{Fix}(T)$, we have, $u \preceq z$ for all $z \in X$. By the definition of β, we get that $\beta(u,z,z) \geq 1$ for all $z \in X$. Therefore, the hypothesis (iv) of Theorem 33 is satisfied and we deduce the uniqueness of the fixed point. □

Corollary 43 *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Let $T: X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a function $\psi \in \Psi$ such that*

$$G(Tx,Ty,Ty) \leq \psi\bigl(G(x,y,y)\bigr)$$

for all $x,y \in X$ with $x \preceq y$. Suppose also that the following conditions hold:

(i) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*

(ii) *T is G-continuous or $(X,\preceq,G)$ is G-regular.*

Then there exists $u \in X$ such that $Tu = u$. Moreover, if for $x \in \mathrm{Fix}(T)$, $x \preceq z$ for all $z \in X$, one has the uniqueness of the fixed point.

Corollary 44 *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Let $T: X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists $\lambda \in [0,1)$ such that*

$$G(Tx,Ty,Ty) \leq \lambda \max \left\{ \begin{matrix} G(x,y,y), G(x,Tx,Tx), G(y,Ty,Ty), \\ \frac{1}{3}(G(x,Ty,Ty) + G(y,Ty,Ty) + G(y,Tx,Tx)) \end{matrix} \right\}$$

for all $x,y \in X$ with $x \preceq y$. Suppose also that the following conditions hold:

(i) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*

(ii) *T is G-continuous or $(X,\preceq,G)$ is G-regular.*

Then there exists $u \in X$ such that $Tu = u$. Moreover, if for $x \in \mathrm{Fix}(T)$, $x \preceq z$ for all $z \in X$, one has the uniqueness of the fixed point.

Corollary 45 *Let $(X,\preceq)$ be a partially ordered set and G be a G-metric on X such that (X,G) is a complete G-metric space. Let $T: X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exist nonnegative real numbers a, b, c and d with $a+b+c+d < 1$ such that*

$$\begin{aligned} G(Tx,Ty,Ty) \leq{} & aG(x,y,y) + bG(x,Tx,Tx) + cG(y,Ty,Ty) \\ & + \frac{d}{3}\bigl(G(x,Ty,Ty) + G(y,Ty,Ty) + G(y,Tx,Tx)\bigr) \end{aligned}$$

for all $x, y \in X$ with $x \preceq y$. Suppose also that the following conditions hold:

(i) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$*;

(ii) *T is G-continuous or $(X, \preceq, G)$ is G-regular.*

Then there exists $u \in X$ such that $Tu = u$. Moreover, if for $x \in \mathrm{Fix}(T)$, $x \preceq z$ for all $z \in X$, one has the uniqueness of the fixed point.

Corollary 46 *Let $(X, \preceq)$ be a partially ordered set and G be a G-metric on X such that (X, G) is a complete G-metric space. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exists a constant $\lambda \in [0, 1)$ such that*

$$G(Tx, Ty, Ty) \leq \lambda G(x, y, y)$$

for all $x, y \in X$ with $x \preceq y$. Suppose also that the following conditions hold:

(i) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$*;

(ii) *T is G-continuous or $(X, \preceq, G)$ is G-regular.*

Then there exists $u \in X$ such that $Tu = u$. Moreover, if for $x \in \mathrm{Fix}(T)$, $x \preceq z$ for all $z \in X$, one has the uniqueness of the fixed point.

3.2 Cyclic contraction

Now, we will prove our results for cyclic contractive mappings in a G-metric space.

Theorem 47 (See [30, 33]) *Let A, B be a nonempty G-closed subset of a complete G-metric space (X, G). Suppose also that $Y = A \cup B$ and $T : Y \to Y$ is a given self-mapping satisfying*

$$T(A) \subseteq B \quad \textit{and} \quad T(B) \subseteq A. \tag{11}$$

If there exists a continuous function $\psi \in \Psi$ such that

$$G(Tx, Ty, Ty) \leq \psi\big(M(x, y, y)\big), \quad \forall x \in A, y \in B, \tag{12}$$

then T has a unique fixed point $u \in A \cap B$, that is, $Tu = u$.

Proof Notice that (Y, G) is a complete G-metric space since A, B is a closed subset of a complete G-metric space (X, G). We define $\beta : X \times X \times X \to [0, \infty)$ in the following way:

$$\beta(x, y, y) = \begin{cases} 1 & \text{if } (x, y) \in (A \times B) \cup (B \times A), \\ 0 & \text{otherwise.} \end{cases}$$

Due to the definition of β and the assumption (12), we have

$$\beta(x, y, y) G(Tx, Ty, Ty) \leq \psi\big(M(x, y, y)\big), \quad \forall x, y \in Y. \tag{13}$$

Hence, T is a generalized G-β-ψ contractive mapping.

Let $(x, y) \in Y \times Y$ be such that $\beta(x, y, y) \geq 1$. If $(x, y) \in A \times B$ then by the assumption (11), $(Tx, Ty) \in B \times A$, which yields that $\beta(Tx, Ty, Ty) \geq 1$. If $(x, y) \in B \times A$, we get again $\beta(Tx, Ty, Ty) \geq 1$ by analogy. Thus, in any case, we have $\beta(Tx, Ty, Ty) \geq 1$, that is,

T is β-admissible. Notice also that for any $z \in A$, we have $(z, Tz) \in A \times B$, which yields $\beta(z, Tz, Tz) \geq 1$.

Take a sequence $\{x_n\}$ in X such that $\beta(x_n, x_{n+1}, x_{n+1}) \geq 1$ for all n and $x_n \to u \in X$ as $n \to \infty$. Regarding the definition of β, we derive that

$$(x_n, x_{n+1}) \in (A \times B) \cup (B \times A) \quad \text{for all } n. \tag{14}$$

By assumption, A, B and hence $(A \times B) \cup (B \times A)$ is a G-closed set. Hence, we get that $(u, u) \in (A \times B) \cup (B \times A)$, which implies that $u \in A \cap B$. We conclude, by the definition of β, that $\beta(x_n, u, u) \geq 1$ for all n.

Now, all hypotheses of Theorem 30 are satisfied and we conclude that T has a fixed point. Next, we show the uniqueness of a fixed point u of T. Since $u \in \mathrm{Fix}(T)$ and $u \in A \cap B$, we get $\beta(u, a, a) \geq 1$ for all $a \in Y$. Thus, the condition (iv) of Theorem 33 is satisfied. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
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