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The strong deviation theorem for discrete-time and continuous-state nonhomogeneous Markov chains

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Abstract

In this paper, the notion of asymptotic average log-likelihood ratio, as a measure of the difference between the sequence of random variables and Markov chains, is introduced, and by constructing a nonnegative martingale, the strong deviation theorem for discrete-time and continuous-state nonhomogeneous Markov chains is established.

Keywords: discrete-time and continuous-state nonhomogeneous Markov chain; asymptotic average log-likelihood ratio; transition probability density; strong deviation theorem

1 Introduction

Let (Ω, \mathcal{F}, P) be the probability space, and let $\{X_n, n \geq 0\}$ be a sequence of continuous random variables taking values in R and with the joint density function $f_n(X_0, X_1 \cdots X_n)$, $n = 1, 2, \dots$. Let Q be another probability measure on (Ω, \mathcal{F}) , and $\{X_n, n \geq 0\}$ be an independent random sequence on the measure Q , with the joint density function $g_n(X_0, X_1 \cdots X_n)$, $n = 1, 2, \dots$.

Let

$$r_n(\omega) = \frac{g_n(X_0, X_1 \cdots X_n)}{f_n(X_0, X_1 \cdots X_n)},$$
$$r(\omega) = -\liminf_{n \rightarrow \infty} \frac{1}{n} \ln r_n(\omega) \quad (\ln 0 = -\infty), \quad (1)$$

where ω is a sample point. In statistical terms, $r_n(\omega)$ and $r(\omega)$ are called the likelihood ratio and the asymptotic average log-likelihood ratio, respectively [1]. Obviously, if $f_n(x_0, x_1 \cdots x_n) = g_n(x_0, x_1 \cdots x_n)$, $n \geq 1$, then $r_n(\omega) \equiv 0$, a.s. So $r(\omega)$ can be used as a measure of deviation between $f_n(x_0, x_1 \cdots x_n)$ and $g_n(x_0, x_1 \cdots x_n)$ when n tends to infinity. The smaller $r(\omega)$ is, the smaller the deviation is.

Definition 1 [2] Let $\{X_n, n \geq 0\}$ be a nonhomogeneous Markov chain with the initial distribution $u(x)$, $x \in R$, and the transition probability density $p_n = p_n(x, y)$, $x, y \in R$, $n \geq 1$. If

$$P(X_0 \in B) = \int_B u(x) dx, \quad P(X_{n+1} \in B | X_n = x) = \int_B p_n(x, y) dy,$$

this Markov chain is called a discrete-time and continuous-state nonhomogeneous Markov chain.

Let $\{X_n, n \geq 0\}$ be a discrete-time and continuous-state nonhomogeneous Markov chain on the measure Q with the initial distribution density $u(x)$, $x \in R$ and the transition probability density $p_n = p_n(x, y)$, $x, y \in R$, $n \geq 1$. Then for any Borel set B

$$Q(X_0 \in B) = \int_B u(x) dx,$$

$$Q(X_{n+1} \in B | X_n = x) = \int_B p_n(x, y) dy,$$

then

$$g_n(x_0, x_1, \dots, x_n) = u(x_0)p_0(x_0, x_1) \cdots p_{n-1}(x_{n-1}, x_n) = u(x_0) \prod_{k=1}^n p_{k-1}(x_{k-1}, x_k),$$

so

$$r_n(\omega) = \frac{u(X_0) \prod_{k=1}^n p_{k-1}(X_{k-1}, X_k)}{f_n(X_0, X_1 \cdots X_n)}. \tag{2}$$

There have been some works on deviation theorem, a kind of strong limit theorem represented by inequalities. Liu and Yang [3] have studied the limit properties of a class of averages of functions of two variables of arbitrary information sources. Liu and Yang [4] investigated the strong deviation theorems for arbitrary information source relative to Markov information source. Liu [5] discussed a class of strong deviation theorems for an arbitrary stochastic sequence with respect to the marginal distribution by using generating function method, and also studied the problem above by means of Laplace transform [6]. Liu and Wang [7] have studied a strong limit theorem expressed by inequalities for the sequences of absolutely continuous random variables. Recently, Fan [8] has studied some strong deviation theorems for dependent continuous random sequence.

In this paper, by using the notion of asymptotic log-likelihood and the martingale convergence theorem, and extending the analytic technique proposed by Liu [9], Liu and Yang [7] to the case of discrete-time and continuous-state nonhomogeneous Markov chains, we obtain the strong deviation theorem for discrete-time and continuous-state nonhomogeneous Markov chains.

2 Main result

Theorem 1 *Let $\{X_n, n \geq 0\}$ be a discrete-time and continuous-state nonhomogeneous Markov chain on the measure Q , $r(\omega)$ and $r_n(\omega)$ be defined by (1) and (2), respectively. Let $\{B_n, n \geq 1\}$ be a sequence of Borel set of the real line, and I_{B_n} be the indicative function of B_n . Let*

$$a(\omega) = \limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \leq b, \quad \forall \omega \in \Omega, \tag{3}$$

and

$$D_1 = \{\omega : r(\omega) \leq b\}, \quad D_2 = \{\omega : r(\omega) \geq b\}.$$

Then

$$(a) \quad \limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \leq 2\sqrt{br(\omega)} + r(\omega) \quad a.s.; \quad (4)$$

$$(b) \quad \liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \geq -2\sqrt{br(\omega)} \quad a.s. \text{ on } D_1, \quad (5)$$

and

$$\liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \geq -b - r(\omega) \quad a.s. \text{ on } D_2. \quad (6)$$

Proof Let λ be a nonnegative constant, and let

$$h_k(x_{k-1}, x_k) = \begin{cases} \frac{\lambda p_{k-1}(x_{k-1}, x_k)}{1 + (\lambda - 1) \int_{B_k} p_{k-1}(x_{k-1}, x_k) dx_k}, & x_k \in B_k; \\ \frac{p_{k-1}(x_{k-1}, x_k)}{1 + (\lambda - 1) \int_{B_k} p_{k-1}(x_{k-1}, x_k) dx_k}, & x_k \notin B_k. \end{cases} \quad (7)$$

It is easy to see that $u(x_0) \prod_{k=1}^n h_k(x_{k-1}, x_k)$ is a density function of $n + 1$ variables. Let

$$t_n(\lambda, \omega) = \frac{u(X_0) \prod_{k=1}^n h_k(X_{k-1}, X_k)}{f_n(X_0, X_1 \cdots X_n)}, \quad (8)$$

then $t_n(\lambda, \omega)$ is a nonnegative supermartingale that converges a.s. Hence there exists $A(\lambda) \in \mathcal{F}$, $P(A(\lambda)) = 1$ such that

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \ln t_n(\lambda, \omega) \leq 0, \quad \omega \in A(\lambda). \quad (9)$$

Letting $\lambda = 1$ in (9), we obtain

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \ln r_n(\omega) \leq 0, \quad \omega \in A(1). \quad (10)$$

This implies that

$$r(\omega) \geq 0, \quad \omega \in A(1). \quad (11)$$

We have by (7)

$$\begin{aligned} \prod_{k=1}^n h_k(X_{k-1}, X_k) &= \prod_{k=1}^n \frac{\lambda^{I_{B_k}(X_k)} p_{k-1}(X_{k-1}, X_k)}{1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k} \\ &= \lambda^{\sum_{k=1}^n I_{B_k}(X_k)} \prod_{k=1}^n \frac{p_{k-1}(X_{k-1}, X_k)}{1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k}. \end{aligned} \quad (12)$$

It follows from (2), (8), and (12) that

$$\ln t_n(\lambda, \omega) = \sum_{k=1}^n I_{B_k}(X_k) \ln \lambda - \sum_{k=1}^n \ln \left[1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] + \ln r_n(\omega). \quad (13)$$

By (9) and (13), we have

$$\begin{aligned} & \limsup_{n \rightarrow \infty} \frac{1}{n} \left(\sum_{k=1}^n I_{B_k}(X_k) \ln \lambda + \ln r_n(\omega) - \sum_{k=1}^n \ln \left[1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \right) \\ & \leq 0, \quad \omega \in A(\lambda). \end{aligned} \tag{14}$$

(a) Let $\lambda > 1$. Dividing the two sides of (14) by $\ln \lambda$, we obtain

$$\begin{aligned} & \limsup_{n \rightarrow \infty} \frac{1}{n} \left(\sum_{k=1}^n I_{B_k}(X_k) + \frac{\ln r_n(\omega)}{\ln \lambda} - \sum_{k=1}^n \frac{\ln [1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k]}{\ln \lambda} \right) \\ & \leq 0, \quad \omega \in A(\lambda). \end{aligned} \tag{15}$$

By (1) and (15), we have

$$\begin{aligned} & \limsup_{n \rightarrow \infty} \frac{1}{n} \left(\sum_{k=1}^n I_{B_k}(X_k) - \sum_{k=1}^n \frac{\ln [1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k]}{\ln \lambda} \right) \\ & \leq \frac{r(\omega)}{\ln \lambda}, \quad \omega \in A(\lambda). \end{aligned} \tag{16}$$

By (3), (16), the property of the superior limit

$$\limsup_{n \rightarrow \infty} (a_n - b_n) \leq d \quad \Rightarrow \quad \limsup_{n \rightarrow \infty} (a_n - c_n) \leq \limsup_{n \rightarrow \infty} (b_n - c_n) + d,$$

and the inequality $0 \leq \ln(1 + x) \leq x$ ($x \geq 0$), we have

$$\begin{aligned} & \limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ & \leq \limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[\frac{\ln [1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k]}{\ln \lambda} - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ & \quad + \frac{r(\omega)}{\ln \lambda} \\ & \leq \limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[\frac{(\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k}{\ln \lambda} - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] + \frac{r(\omega)}{\ln \lambda} \\ & \leq b \left(\frac{\lambda - 1}{\ln \lambda} - 1 \right) + \frac{r(\omega)}{\ln \lambda}, \quad \omega \in A(\lambda). \end{aligned} \tag{17}$$

By using the inequality $1 - \frac{1}{\lambda} < \ln \lambda$ ($\lambda > 1$), we have by (17)

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \leq b(\lambda - 1) + \frac{\lambda r(\omega)}{\lambda - 1}, \quad \omega \in A(\lambda). \tag{18}$$

Let Q^* be the set of rational numbers in the interval $(1, +\infty)$, and let

$$A^* = \bigcap_{\lambda \in Q^*} A(\lambda), \quad g(\lambda, r) = b(\lambda - 1) + \frac{\lambda r}{\lambda - 1}. \tag{19}$$

Then we have by (18),

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \leq g(\lambda, r(\omega)), \quad \omega \in A^*, \lambda \in Q^*. \quad (20)$$

Let $b > 0$. It is easy to see if $r > 0$, $g(\lambda, r)$ as a function of λ attains its smallest value $g(1 + \sqrt{\frac{r}{b}}, r) = 2\sqrt{br} + r$ on the interval $(1, +\infty)$, and $g(\lambda, 0)$ is increasing on the interval $(1, +\infty)$ and $\lim_{\lambda \rightarrow 1+0} g(\lambda, 0) = 0$. For each $\omega \in A^* \cap A(1)$ if $r(\omega) \neq \infty$, take $\lambda_n(\omega) \in Q^*$, $n = 1, 2, \dots$ such that $\lambda_n(\omega) \rightarrow 1 + \sqrt{\frac{r(\omega)}{b}}$, we have

$$\lim_{n \rightarrow \infty} g(\lambda_n(\omega), r(\omega)) = 2\sqrt{br(\omega)} + r(\omega). \quad (21)$$

By (20), we have

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \leq g(\lambda_n(\omega), r(\omega)), \quad n = 1, 2, \dots \quad (22)$$

By (21) and (22), we have

$$\begin{aligned} \limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ \leq 2\sqrt{br(\omega)} + r(\omega), \quad \omega \in A^* \cap A(1). \end{aligned} \quad (23)$$

If $r(\omega) = \infty$, (23) holds obviously. Since $P(A^* \cap A(1)) = 1$, (4) holds by (23) when $b > 0$.

When $b = 0$, letting $\lambda = e$ in (20), we have

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \leq r(\omega), \quad \omega \in A(e). \quad (24)$$

Since $P(A(e)) = 1$, (4) also holds by (24) when $b = 0$.

(b) Let $0 < \lambda < 1$. Dividing the two sides of (14) by $\ln \lambda$, we have

$$\begin{aligned} \liminf_{n \rightarrow \infty} \frac{1}{n} \left(\sum_{k=1}^n I_{B_k}(X_k) - \sum_{k=1}^n \frac{\ln[1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k]}{\ln \lambda} + \frac{\ln r_n(\omega)}{\ln \lambda} \right) \\ \geq 0, \quad \omega \in A(\lambda). \end{aligned} \quad (25)$$

By (1) and (25), we have

$$\begin{aligned} \liminf_{n \rightarrow \infty} \frac{1}{n} \left(\sum_{k=1}^n I_{B_k}(X_k) - \sum_{k=1}^n \frac{\ln[1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k]}{\ln \lambda} \right) \\ \geq \frac{r(\omega)}{\ln \lambda}, \quad \omega \in A(\lambda). \end{aligned} \quad (26)$$

By (26), (3), the property of the inferior limit

$$\liminf_{n \rightarrow \infty} (a_n - b_n) \geq d \quad \Rightarrow \quad \liminf_{n \rightarrow \infty} (a_n - c_n) \geq \liminf_{n \rightarrow \infty} (b_n - c_n) + d,$$

and the inequality $\ln(1 + x) \leq x$ ($-1 < x \leq 0$), we have

$$\begin{aligned} & \liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ & \geq \liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[\frac{\ln[1 + (\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k]}{\ln \lambda} - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ & \quad + \frac{r(\omega)}{\ln \lambda} \\ & \geq \liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[\frac{(\lambda - 1) \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k}{\ln \lambda} - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] + \frac{r(\omega)}{\ln \lambda} \\ & \geq b \left(\frac{\lambda - 1}{\ln \lambda} - 1 \right) + \frac{r(\omega)}{\ln \lambda}, \quad \omega \in A(\lambda). \end{aligned} \tag{27}$$

By using the inequality $1 - \frac{1}{\lambda} < \ln \lambda < 0$ and $\ln \lambda < \lambda - 1 < 0$ ($0 < \lambda < 1$), we have by (27)

$$\begin{aligned} & \liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ & \geq b(\lambda - 1) + \frac{r(\omega)}{\lambda - 1}, \quad \omega \in A(\lambda) \cap A(1). \end{aligned} \tag{28}$$

Let Q^* be the set of rational numbers in the interval $(0, 1)$, and let

$$A^* = \bigcap_{\lambda \in Q^*} A(\lambda), \quad h(\lambda, r) = b(\lambda - 1) + \frac{r}{\lambda - 1}. \tag{29}$$

Then we have by (28)

$$\begin{aligned} & \liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ & \geq h(\lambda, r(\omega)), \quad \omega \in A^* \cap A(1), \lambda \in Q^*. \end{aligned} \tag{30}$$

Let $b > 0$. It is easy to see that if $0 < r < b$, then $h(\lambda, r)$ as a function of λ attains its largest value $h(1 - \sqrt{\frac{r}{b}}, r) = -2\sqrt{br}$ on the interval $(0, 1)$, and $h(\lambda, 0)$ is increasing on the interval $(0, 1)$ and $\lim_{\lambda \rightarrow 1-0} h(\lambda, 0) = 0$, and $h(\lambda, b) = b(\lambda - 1 + \frac{1}{\lambda - 1})$ is decreasing on the interval $(0, 1)$ and $\lim_{\lambda \rightarrow 0^+} h(\lambda, b) = -2b$. For each $\omega \in A^* \cap A(1) \cap D_1$, take $\tau_n(\omega) \in Q^*$, $n = 1, 2, \dots$ such that $\tau_n(\omega) \rightarrow 1 - \sqrt{\frac{r(\omega)}{b}}$. Then we have

$$\lim_{n \rightarrow \infty} h(\tau_n(\omega), r(\omega)) = -2\sqrt{br(\omega)}. \tag{31}$$

By (30), we have

$$\liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \geq h(\tau_n(\omega), r(\omega)), \quad n = 1, 2, \dots \tag{32}$$

By (31) and (32),

$$\begin{aligned} & \liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ & \geq -2\sqrt{br(\omega)}, \quad \omega \in A^* \cap A(1) \cap D_1. \end{aligned} \tag{33}$$

Since $P(A^* \cap A(1)) = 1$, (5) holds by (33) when $b > 0$.

When $b = 0$, $r(\omega) = 0$ for $\omega \in D_1 \cap A(1)$, hence we have by (30)

$$\liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \geq 0, \quad \omega \in A(\lambda) \cap A(1) \cap D_1, \tag{34}$$

since $P(A(\lambda) \cap A(1)) = 1$, (5) also holds by (34) when $b = 0$.

It is easy to see that when $0 \leq b < r$, $h(\lambda, r)$ as a function of λ is decreasing on the interval $(0, 1)$ and $\lim_{\lambda \rightarrow 0^+} h(\lambda, r) = -(r + b)$. For each $\omega \in A^* \cap A(1) \cap D_2$, when $r(\omega) \neq \infty$, take $\lambda_n(\omega) \in Q^*$, $n = 1, 2, \dots$, such that $\lambda_n(\omega) \rightarrow 0$. We have

$$\lim_{n \rightarrow \infty} h(\lambda_n(\omega), r(\omega)) = -r(\omega) - b. \tag{35}$$

By (30), we have

$$\liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \geq h(\lambda_n(\omega), r(\omega)), \quad n = 1, 2, \dots \tag{36}$$

It follows from (35) and (36) that

$$\begin{aligned} & \liminf_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left[I_{B_k}(X_k) - \int_{B_k} p_{k-1}(X_{k-1}, x_k) dx_k \right] \\ & \geq -r(\omega) - b, \quad \omega \in A^* \cap A(1) \cap D_2 \end{aligned} \tag{37}$$

when $r(\omega) = \infty$, (37) also holds obviously. Since $P(A^* \cap A(1)) = 1$, (6) follows from (37) directly. \square

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

BW carried out the study of strong deviation theorem and drafted the manuscript. ZS participated in the proof of theorem. All authors read and approved the final manuscript.

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