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A note on 'Modified proof of Caristi's fixed point theorem on partial metric spaces, Journal of Inequalities and Applications 2013, 2013:210'

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Abstract
In this note, we emphasize that the proofs and statements of the main results of the paper 'Modified proof of Caristi's fixed point theorem on partial metric spaces' (Journal of Inequalities and Applications 2013, 2013:210) do not have any utility to use the partial metric. Hence, it has no contribution to either partial metric theory or Caristi-type fixed point problems.

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In the following, we use the same definitions, notations and structures given in [1]. We start first with Caristi's [2] fixed point theorem.

Theorem 1.1 [2] *Let (X,d) be a complete metric space. Let $f: X \to X$ and let ϕ be a lower semi-continuous function from X into $[0,\infty)$. Assume that $d(x,f(x)) \leq \phi(x) - \phi(f(x))$ for all $x \in X$. Then f has a fixed point in X.*

Lemma 2.3 [3] *Let (X,p) be a partial metric space and let $p^s : X \times X \to [0,\infty)$ be defined by*

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{1}$$

for all $x,y \in X$. Then (X,p^s) is a metric space.

To emphasize that the function given in (1) is a metric, we use the notation d_p instead of p^s, that is,

$$d_p(x,y) = p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y) \quad \text{for all } x,y \in X. \tag{2}$$

Let (X,p) be a partial metric space. Following [1], consider $\phi : X \to [0,\infty)$ and $g : X \to X$ not necessarily a continuous function such that

$$2p\bigl(x,g(x)\bigr) - p(x,x) - p\bigl(g(x),g(x)\bigr) \leq \phi(x) - \phi\bigl(g(x)\bigr), \quad x \in X.$$

By (2), we can write

$$d_p\bigl(x,g(x)\bigr) \leq \phi(x) - \phi\bigl(g(x)\bigr).$$

The author [1] defines the class of mappings Φ and Φ_g as follows:

$$\Phi = \bigl\{f \mid f : X \to X \text{ and } 2p\bigl(x,f(x)\bigr) - p(x,x) - p\bigl(f(x),f(x)\bigr) \leq \phi(x) - \phi\bigl(f(x)\bigr)\bigr\}$$

and

$$\Phi_g = \bigl\{f \mid f \in \Phi \text{ and } \phi(f) \leq \phi(g)\bigr\}.$$

We re-write Φ as

$$\Phi = \bigl\{f \mid f : X \to X \text{ and } d_p\bigl(x,f(x)\bigr) \leq \phi(x) - \phi\bigl(f(x)\bigr)\bigr\}.$$

It is well known also that (X,p) is complete if and only if (X,d_p) is complete (see, *e.g.*, [3, 4]).

Under these observations, keeping (2) in mind, we conclude that Lemma 3.1 in [1] remains true without using any properties of a partial metric. On the other hand, in Lemma 3.2 in [1] the completeness assumption is missed. It can be re-formulated correctly as follows.

Updated Lemma 3.2 [1] *Let $\{x_n\}$ be a sequence in a complete partial metric space (X,p) such that*

$$d_p(x_{n+1},x_n) \leq \phi(x_n) - \phi(x_{n+1}) \quad \textit{for all } n \in \mathbb{N},$$

where ϕ is a lower semi-continuous function. Then $\lim_{n\to\infty} x_n = \bar{x}$ and $d_p(\bar{x},x_n) \leq \phi(x_n) - \phi(\bar{x})$ for each n.

Moreover, in Definition 2.2 in [1], the open and closed balls associated to a partial metric p are not defined correctly, because the term $p(x,x)$ is missing, that is, we should have

$$B_\varepsilon(x) = \bigl\{y \in X, p(x,y) < p(x,x) + \varepsilon\bigr\} \quad \text{and} \quad \bar{B}_\varepsilon(x) = \bigl\{y \in X, p(x,y) \leq p(x,x) + \varepsilon\bigr\}.$$

It is clear that there is nothing in this paper [1] to prove. Indeed, the main result of [1] is a consequence of Theorem 1.1.

The following definition already exists in the literature.

Definition 3.3 (*cf.* [1]) Let (X,p) be a partial metric space.

(1) For $A \subset X$, define the diameter of a subset A, written $D(A)$, by

$$D(A) = \sup_{(x_i,x_j)\in A} \{2p(x_i,x_j) - p(x_i,x_i) - p(x_j,x_j)\} = \sup_{(x_i,x_j)\in A} d_p(x_i,x_j).$$

(2) Let $r(A) = \inf_{x\in A}(\phi(x))$. Note that $B \subset A$ implies $r(B) \geq r(A)$.

(3) Let $\Phi' \subset \Phi$. For each $x \in X$, define $S_x = \{f(x) | f \in \Phi'\}$.

Keeping (2) in mind, we conclude easily.

Lemma 3.4 [1] $D(S_x) \leq 2(\phi(x) - r(S_x))$.

Consequently, we derive Theorem 3.5 in [1] without using any property of the partial metric. As a conclusion, this paper is just a repetition of usual results by using equality (2).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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