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Estimating probabilities of default of different firms and the statistical tests

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Abstract: The probability of default (PD) is the essential credit risks in the finance world. It provides an estimate of the likelihood that a borrower will be unable to meet its debt obligations.

Purpose: This paper computes the probability of default (PD) of utilizing market-based data which outlines their convenience for monetary reconnaissance. There are numerous models that provide assistance to analyze credit risks, for example, the probability of default, migration risk, and loss gain default. Every one of these models is vital for estimating credit risk, however, the most imperative model is PD, i.e., employed in this paper.

Design/methodology/approach: In this paper, the Black-Scholes Model for European Call Option (BSM-CO) is utilized to gauge the PD of the Jammu and Kashmir Bank, Bank of Baroda, Indian Overseas Bank, and Canara Bank. The information has been taken from a term of 5 years on a yearly premise from 2012 to 2016. This paper demonstrates how d_2 in Black Scholes displayed help in assessing the PD of the various firms.

Findings: The fundamental findings of this paper are whether there are any mean contrasts between the mean differences of PD between the organizations utilizing ANOVA and the Tukey strategy.

Keywords: PD, BSM-CO, Merton model, ANOVA, Tukey method

Introduction

Credit risk (default), estimation, and administration have turned out to be a standout among the most critical parts in budgetary financial matters. A default is a risk that is a neglect to pay money-related obligations, as an outline when a bank cannot return the standard sum (bank crumple). The term default basically implies an account holder who has not paid an obligation. In legitimate terms, it is indebtedness suggesting that an account holder cannot pay. The probability of default (PD) is a credit risk which gives a gauge of the probability of a borrower's will and identity unfitness to meet its obligation commitments (Bandyopadhyay 2006).

Evaluating the PD of a firm is the initial step while surveying the credit exposure and potential misfortunes faced by a firm. For corporate securities, the securities are issued by a firm and there is a plausibility that they will default eventually amid the term of the agreement. In this paper, the Black-Scholes Model for European Call Option (BSM-CO) is utilized to quantify the probability of default. The Black-Scholes Model (BSM) is the technique for displaying derivatives costs that have been first presented in

1973. The recipe of BSM demonstrates to us proper methodologies to discover the cost of an option contract (call and put option). It tends to be dictated by utilizing basic equation; anyway, in this examination, just the European call option has been utilized (Hull 2009). In 1974, Merton has shown the model, specifically the Merton model. It is utilized to gauge the default for the organizations. It is the first basic model since it gives a connection between the default risk and the advantage (capital) structure of the firm (Saunders and Allen 2002). Since the firm will default just when the obligation of the firm is over the estimation of the firm, in this condition, the proprietor will put the firm to the obligation holder. Merton has contemplated the firm's value E as a call option on its assets. The inceptions of popular credit risk auxiliary models have developed generally from the theoretical domain. Merton's works turned out to be theoretically broadened for all intents and purposes actualized by the KMV Corporation. Through this paper, it is expected that (Bohn et al. 2005; Saunders and Allen 2002; Merton 1974):

- The underlying asset (St) is replaced by the value of the firm (V)
- The strike price (K) in a call option is replaced by the debt (D)
- The risk-free rate of interest is replaced by the expected growth of the firm.

In such manner, the significance of d_2 has been clarified. It is the interior part of the Black Scholes equation. In any case, one of the intriguing and helpful methodologies $N(-\text{d\_2})$ characterizes the likelihood that an option will be practiced in a risk-neutral way instead of a real-world probability. It is realized that d_2 is the thought behind the Merton model. This paper utilizes d_2 of BSM-CO to appraise the PD of various firms. ANOVA and the Tukey strategy have been utilized to demonstrate whether there are contrasts between the mean changes of PD between the organizations.

Literature review

The distance to default was clarified by Crosbie and Bohn in Crosbie and Bohn 2003, according to the definition by Moody's KMV model. Over the most recent couple of years, the distance to default turned into the renowned measure, among checked base measures. This measure depends on Merton's model (Merton 1974). It depicts the estimation of the equity as a European call option. The strike cost of the option is equivalent to the value of obligation/debt according to Merton. The firm will be defaulted just when the estimation of the firm will fall on the face value estimation of the obligation (Kollar and Cisko 2014). The distance to default estimates the rate or probability that the value of the firm/asset falls beneath the value of the debt. The PD cannot be a zero or very less. On the off chance that it will happen, it causes vast monetary misfortunes (Kollar 2014). To appraise the PD, there are two fundamental models, structural and reduced model. Both are utilized to get the market-constructed data with respect to the premise of the diverse suppositions (Lehutova 2011). In the year 2013, a number of researchers support the Merton model to rank the organizations based on the degree to which it is distant from the default (Jessen and Lando 2015). Bohn et al. (2005) contended that the KMV Merton demonstrated gets all the data in bookkeeping factors and the traditional agency rating. In the year 2004, researchers demonstrate the KMV

Merton likelihood as a striking and prescient capacity model of the PD over the time. They trust that it can cause the structure of PD (Wang and Duffie 2004). In the year 2004, Farmen et al. investigated the PD and their qualified statics in Merton's model utilizing the real probability measure. It encourages the utilization of the BSM system for risk administration purposes and gives a hypothetical premise to experimental examinations of default probability (Farmen et al. 2004). In the year 1999, a number of researchers utilized just value costs to evaluate the default parameter for the structural model (Delianedis and Geske 1999). Bharath and Shumway (2008) evaluated the default utilizing the Merton models' distance to default for figuring defaults for non-money-related open firms recorded on the US stock trade. The Merton model approach can be utilized to create proportions of the likelihood of disappointment of individual cited UK associations (Tudela and Young 2003). In the year 2000, a researcher examined a portion of the critical theoretical models of the risky debt estimation that were based on the Merton model. It depicts both the structural and reduced forms of unforeseen case models and condenses both the hypothetical and observational research around there. Two researchers in 2004 evaluated the PD utilizing Cohort and Intensity Method (Schuermann and Hanson 2004). In the year 2003, Cangemi et al. evaluated the PD dependent on the MEU measure. Gonçalves et al. (2014) utilized the logit relapse strategy to demonstrate the conduct of credit default in a start-up firm. In the year 2017, a number of researchers clarify the components influencing the PD (Dar and Anuradha 2017). This paper depends on the structural model, and the basic model depends on the Black and Scholes Model (Black and Scholes 1973) and Merton model (Merton 1974).

Methodology

In order to estimate the distance to default and the PD of Jammu and Kashmir Bank, Bank of Baroda, Indian Overseas Bank, and Canara bank, the researchers are using the BSM-CO, the internal part of the BSM-CO (d_2). To measure the PD, the annual reports of the above-mentioned firms from the year 2012 to 2016 have been used. Finally, the ANOVA and Tukey method were applied to test whether there is any difference in mean variances of the PD between the firms.

Merton model

Black Scholes formula

The Black Scholes model, otherwise called the Black-Scholes-Merton approach, is a model of price variation over time of financial instruments, for example, stocks that can, in addition to other things, be utilized to decide the cost of a European call option. The model assumes the cost of intensely exchanged resources pursuing a geometric Brownian motion with constant drift and volatility. The Robert Merton and Myron Scholes have been awarded the Nobel Prize for economics in the year 1997 because of the derivative pricing model.

Essentially, BSM formula shows us how the price of an option contract (call and put option) can be determined by using a simple formula, but in this paper only a European call option is used. The formula for European call option is

$$C(S, T) = SN(d_1) - Ke^{-rT}N(d_2) \tag{1}$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{2}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{3}$$

S is the present price of the stock, K is the strike price, r is the free risk rate interest, σ is the volatility of the stock and N is the CDF for a standard normal distribution, and T is the time period (Hull 2009; Merton 1974; Black and Scholes 1973).

Merton model—a simple concept

The Merton model was the primary structural model which estimates the PD for firms. It accepts that the firm will issue the two debts D and an additional equity E too, giving us a chance to accept that the value of the firm is V at time t. It will shift over the time because of activities by the firm, which does not pay any sort of dividend on the equity or coupon.

The zero coupon bonds are a piece of an association's debt with an ensured reimbursement of amount D at time T. The rest of the firm V at time T will be issued to the investors, and the firm will be twisted up. On account of the firm being breezed up, the investors' rank is beneath the debt holders.

In the event that the firm will produce the great reserve in such a path, thus, to the point that it can pay the debt, at that point, the investors will get the result of:

$$V - D$$

The firm will default at time T when $V < D$.

In the above case, the bondholders will receive V instead of D and the shareholders will receive nothing.

Consider the both the conditions we get:

The shareholders will receive the payoff of:

$$max(V - D, 0)$$

This is same as the payoff of the European call option, with V as an underlying asset price and D as a strike price (Hull 2009; Valverde 2015; Dar and Anuradha 2017)

Estimate distance to default and probability of default

In order to estimate the PD of firms by using the BSM-CO, we need to estimate the total value of the firm V and its volatility σ_V. The total value of the firm V is equal to the sum of the two components that is the firm's debt (D) and its equity (E).The value of the firm V follows a Geometric Brownian Motion that is the value of the firm price changes continuously through time according to the stochastic differential equation.

$$dV = V\mu dt + V\sigma_V dZ_t \tag{4}$$

where Z_t is the standard Brownian motion, μ is the expected continuously compounded return on V, and σ_V is the volatility of the firms value (Hull 2009)

The solution of the Eq. (4) is:

$$V_t = V_0 e^{\sigma Z_t + (\mu - 0.5\sigma^2)t}$$

where V_t is the value of the firm at time t, Z_t is the standard Brownian motion, μ is the expected continuously compounded return on V_t, and σ is the volatility of the firms value.

Proof:

We know that

$$dV_t = V_t \mu dt + V_t \sigma dZ_t$$

The equation is of the form $dX_t = A_t dt + Y_t dZ_t$

where $Y_t = V_t \sigma$ and $A_t = V_t \mu$

Let us suppose that $f(t, V_t) = \log(V_t)$

Then

$\frac{d(f(t,V_t))}{dt} = 0, \frac{d(f(t,V_t))}{dV_t} = \frac{1}{V_t}, \frac{d^2(f(t,V_t))}{dV_t{}^2} = -\frac{1}{V_t{}^2}$

By Ito lemma, we have

Ito lemma: $\frac{df(.)}{dx} Y_t dZ_t + [\frac{df(.)}{dt} + A_t \frac{df(.)}{dx} + 0.5 \frac{d^2 f(.)}{dx^2} Y_t{}^2]dt$

$$\frac{df(t,V_t)}{dV_t} Y_t dZ_t + \left[\frac{df(t,V_t)}{dt} + A_t \frac{df(t,V_t)}{dV_t} + 0.5 \frac{d^2 f(t,V_t)}{dV_t{}^2} Y_t{}^2\right]dt$$

substituting A_t and Y_t in the above equation we get

$$d\log V_t = \frac{df(t,V_t)}{dV_t} V_t \sigma dZ_t + [\frac{df(t,V_t)}{dt} + \frac{df(t,c)}{dV_t} S_t \mu + 0.5 \frac{d^2 f(t,V_t)}{dV_t{}^2} (V_t \sigma)^2]dt$$

$$d\log V_t = \frac{1}{V_t} V_t \sigma dZ_t + [0 + \frac{1}{V_t} V_t \mu + 0.5(-\frac{1}{V_t{}^2})(V_t \sigma_t)^2]dt$$

$$d\log V_t = \sigma dZ_t + [0 + \mu - 0.5\sigma^2]dt$$

$$d\log V_t = \sigma dZ_t + (\mu - 0.5\sigma^2)dt$$

Integrating both sides from 0 to t

$$\int_0^t \mathrm{dlog} V_u = \sigma \int_0^T dZ_u + \int_0^t (\mu - 0.5\sigma^2)du$$

$$[\log V_u]_0^t = \sigma[Z_u]_0^t + (\mu - 0.5s^2)[u]_0^t$$

$$[\log V_t - \log V_0] = \sigma[Z_t - Z_0] + (\mu - 0.5\sigma^2)[t - 0]$$

$$\log(\frac{V_t}{V_0}) = \sigma Z_t + (\mu - 0.5\sigma^2)t$$

Therefore,

$$V_t = V_0 e^{\sigma Z_t + (\mu - 0.5\sigma^2)t} \text{ (Hence proved)}$$

The Z_t is the normally distributed and V_t is log-normally distributed with all the of the usual properties of the distribution.

So,

$$\log V_t = \log V_0 + \left(\mu - 0.5\sigma^2\right)t + \sigma dZ_t$$

It means that

$$\log V_t \sim N\left(\log V_0 + \left(\mu - 0.5\sigma^2\right)t + \sigma^2 t\right)$$
$$V_t \sim \log N\left[\log V_0 + \left(\mu - 0.5\sigma^2\right)t, \sigma^2 t\right]$$

This process is sometimes called a log-normal process, geometric Brownian motion or exponential Brownian motion.

We can estimate the value of the equity E by using the Black Scholes Formula for a call option that is:

$$E = V * N(d_1) - D * e^{-rT} * N(d_2) \tag{5}$$

where:

$$d_1 = \frac{\ln\left(\frac{V}{D}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\ln\left(\frac{V}{D}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

r is the free risk rate interest, T is the time period, σ is the volatility of the firms value, and N is the cumulative standard function (CDF) for a standard normal distribution.

Following assumptions would undermine the model efficiency:

1. The firm can default only at time T and not before
2. Assets of the firm's follow lognormal distribution
3. On the basis of accounting data, the PD for private firms can be estimated. The model does distinguish between the types of default according to their seniority, convertibility, and collaterals.

To estimate the distance to default, we need the Back Scholes Formula for a European call option which is given by:

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{6}$$

Replace:

- The risk-free rate of interest r by the expected continuously compounded return on value of the firm μ_V.
- The value of the underlying asset S_t at time t by the value of the firm at time t is V.
- The strike price K by the face value of the debt D.
- The volatility σ by the volatility of the firms value σ_V

The distance to default is given by:

$$d_2 = \frac{\ln\left(\frac{V}{D}\right) + \left(\mu_V - \frac{{\sigma_V}^2}{2}\right)T}{\sigma_V\sqrt{T}} \tag{7}$$

where μ_V is expected rate of return of the firm's asset and D is the face value of a debt and expected growth of assets is equal to $(\mu_V - \frac{\sigma_V{}^2}{2})$.

The numerator of Eq. (7) is really the distance to default; it demonstrates the distance between expected assets and D as appeared in Fig. 1. It tends to be figured as a whole of initial distance and growth of that distance within the period T. Equation (7) is the distance to default in wording as a multiplier of standard deviation. The distance to default is characterized as how much a firm is far off from the default point. Subsequent to evaluating the distance to default, we can create and gauge the probability of default.

The PD under the risk neutral measure as per the Black Scholes Merton model is given by:

$$\text{Probability default} = N(-d_2) = N\left(-\frac{\ln\left(\frac{V}{D}\right) + \left(\mu_V - \frac{\sigma_V{}^2}{2}\right)T}{\sigma_V\sqrt{T}}\right) \tag{8}$$

or

Probability of default = 1 – $N(d_2)$, see references (Hull 2009; Valverde 2015; Dar and Anuradha 2017; Merton 1974)

Equation (8) is the probability of default that is it is distance between the value of the firm and the value of the debt (V/D) adjusted for the expected growth related to asset volatility $(\mu_V - \frac{\sigma_V{}^2}{2})$.

Equation (8) is the PD with three unknowns V, σ_V, and μ_V. To appraise the probability as per Eq. (7), the authors have to locate the above three unknowns. The value of the firm V is equal to the sum of the debt and the equity of the firm, so the debt D is known and we need to find equity E.

The equity value E is a continuous time stochastic process that is Weiner process.

$$E = \mu_E E dt + \sigma_E E dZ \tag{9}$$

where dZ is the continuous time stochastic process, μ_E is the expected continuously compounded return on E, and σ_E is the volatility of the equity value.

By Ito's lemma,

$$\frac{df(.)}{dx} Y_t dZ_t + \left[\frac{df(.)}{dt} + A_t \frac{df(.)}{dx} + 0.5 \frac{d^2 f(.)}{dx^2} Y_t{}^2\right] dt$$

We can represent the process for equity E as:

$$dE = \sigma_V V \frac{dE}{dV} dZ + \left[\frac{dE}{dt} + \mu_V V \frac{dE}{dV} + 0.5(\sigma_V V)^2 \frac{d^2 E}{dV^2}\right] dt \tag{10}$$

As per the diffusion terms Eqs. (9) and (10) is equal, so we can write the relation between the two equations as:

$$\sigma_E E = \sigma_V V \frac{dE}{dV} = \sigma_V N(d_1) \tag{11}$$

Equations (5) and (11) having two unknowns one is σ_V and another is V. We can easily estimate the two parameters by solving Eqs. (5) and (11). After finding the V and σ_V, we need to find the return of the asset that is μ_V.

As per the general definition of return, it is defined as:

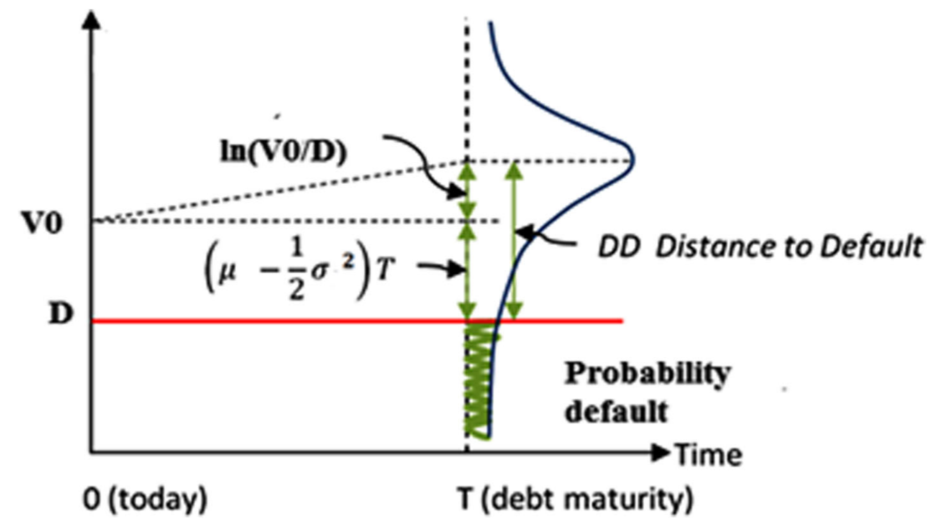

Fig. 1 PD

$$\text{Return} = \frac{\text{today's price} - \text{yestarday's price}}{\text{yestarday's price}}$$

So,

$$\mu_V = \frac{V_t - V_{t-1}}{V_{t-1}} \tag{12}$$

In most cases, the return is negative, as was mentioned in Hillegeist et al. (2004). Based on the accounting data, there are several problems when modeling probability of default, he argues. The expected return cannot be a negative. Here, assume that the expected growth is equal to the risk-free rate of interest.

Use all the known parameters that is E, r, D, T and σ_E to estimate the three unknown parameters V, σ_V, and μ_V.

As per the above calculation, draw a model for probability of default as shown in Fig. 2:

Result and discussion

This study has taken the secondary data of all the firms for 5 years starting from March 2012 to March 2016. The BSM-CO has been used to estimate the PD of all firms using BSM models:

To estimate the PD, this study used parameters like:

1. Firm value of assets, V (total equity + debt)
2. Value of a debt, D
3. Volatility of an asset, σ
4. Rate of return, μ_V.
5. Time period, T

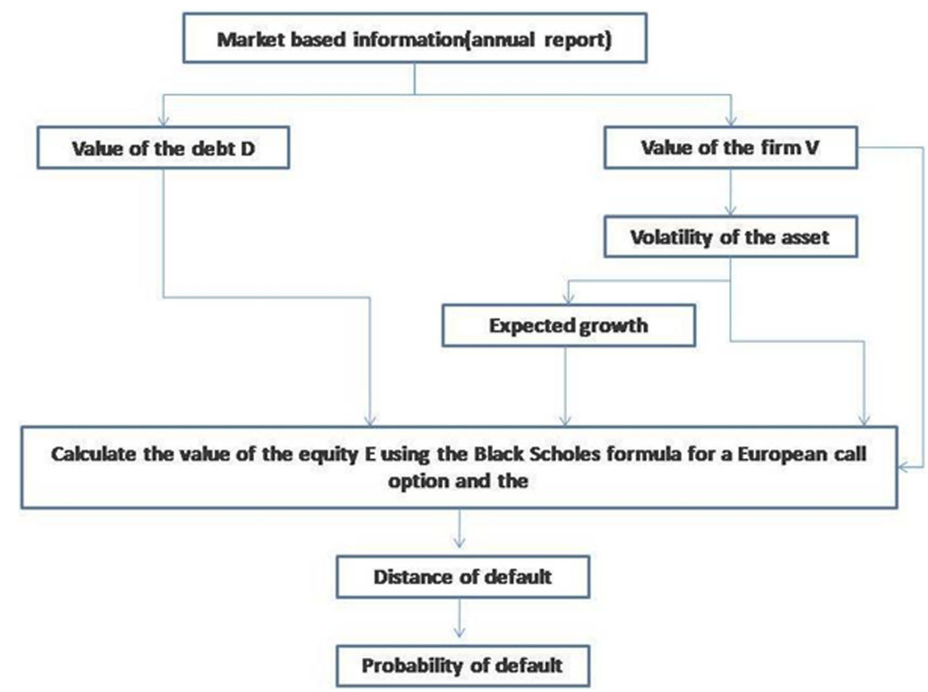

Fig. 2 Flow chart

Result

The PD of Jammu and Kashmir bank is 26.88% in year 2012 as shown in Table 1, which suggests that the likelihood of Jammu and Kashmir (JK) Bank to be a default in the year 2012 is 26.88% that is 26.88% obligations that Jammu and Kashmir Bank has not paid.

Figure 3 demonstrates the PD of the considerable number of firms. Each firm is having the distinctive PD at various day and age. The inquiry is which organization is great that a speculator will give an advance to the firm? The appropriate response is straightforward; the firm which is having the less PD, e.g., a financial specialist ABC needs to give the advance to one firm in 2013. The speculator ABC is having four firms where he can put; however, he will put in just one firm. As shown in Table 1, the PD of Jammu and Kashmir Bank, Indian Overseas Bank, Bank of Baroda, and Canara Bank is 26.88%, 20.148%, 45.55%, and 42.38%, respectively, in year 2012. Based on past data with respect to the four firms, the financial specialist will give advance to Indian Overseas Bank since it has less PD as shown in Table 1.

Table 1 PD of different firms using equation (8)

Year	Jammu and Kashmir Bank	Indian overseas Bank	Bank of Baroda	Canara Bank
2012	0.268888	0.20148	0.4555564	0.423839
2013	0.2687425	0.2016335	0.4553912	0.4237237
2014	0.268681897	0.202425	0.4552581	0.435759
2015	0.268711978	0.2022066	0.4552265	0.4234976
2016	0.268676859	0.2046978	0.4553076	0.4236178

Note:
• Taking total equity and value of debt from historical data of JK Bank
• Firm value of assets = total equity + debt
• For simplicity taking volatility as 20%

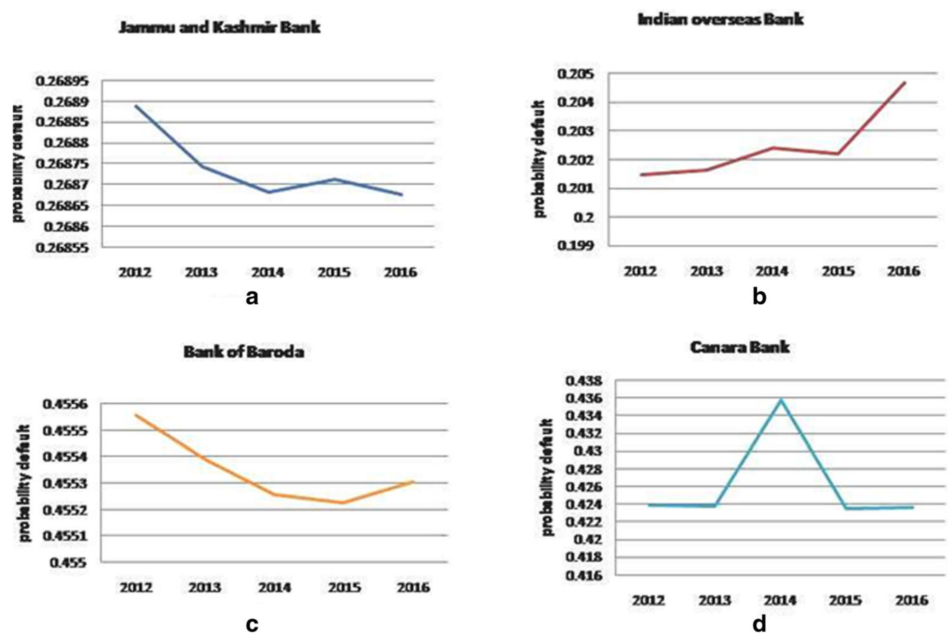

Fig. 3 PD of all firms **a** Jammu and Kashmir Bank **b** Indian overseas Bank **c** Bank of Baroda **d** Canara Bank

Take a gander at Fig. 3, where the PD in all organizations' increments or abatements is demonstrated. The Indian Overseas bank is enhancing according to our outcome. The bank of Baroda diminishes and after that it increments. The Jammu and Kashmir diminishes at all which is a decent sign, and the Canara Bank increments first and after that it begins to diminish moreover. Presently, the authors will complete a speculation test on all the four firms to observe any contrast between the methods (Table 2).

One-way ANOVA for four independent samples

In order to determine whether there is any mean difference between the PD of the firms given in Table 1. The authors compare the p values of all the mean values of parameters with the significance level ($\alpha = 5\%$). The α indicated that the risk of concluding that the parameters are significantly different (Refer Table 3).

In order to verify, we have two cases:

Table 2 Basic statistics

Data Summary	Jammu and Kashmir Bank	Indian overseas Bank	Bank of Baroda	Canara Bank	Total
N	5	5	5	5	20
Sum	1.3437	1.0124	2.2767	2.1304	6.7633
Mean	0.2687	0.2025	0.4553	0.4261	0.3382
Sumsq	0.3611	0.205	1.0367	0.9079	2.5107
SS	0	0	0	0.0001	0.2236
Variance	0	0	0	0	0.0118
St. dev.	0.0001	0.0013	0.0001	0.0054	0.1085

Variances and standard deviations are calculated with denominator = $n - 1$

Table 3 ANOVA

Source	DF	Adj SS	Adj MS	F value	*p* value
Factor	3	0.223450	0.074483	9626.24	0.000
Error	16	0.000124	0.000008		
Total	19	0.223574			

1. If p value > α (0.05), there is no mean difference (fail to reject the null hypothesis or accept the null hypothesis).
2. If p value ≤ α (0.05), there is mean difference (reject the null hypothesis).

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	$\alpha = 0.05$

Equal variances were assumed for the analysis.

Factor information

Factor	Levels	Values
Factor	4	JK Bank, IOB, BOB, Canara Bank

The ANOVA is used to explain that the mean response between the firms varies or not. If there are no differences between the means of all the firms then the F value is around 1. If the F value is large, then there are mean difference between the firms.

The standard deviation of all firms in several years gives the measurement of variance of probability of default (Fig. 4).

ANOVA summary

A hypothesis is a test whether there is any such difference between the treatments based on the F ratio. It will ask whether the F ratio for the treatments is unusually high by comparing the F ratio to a kind of a standard distribution called an F distribution. The p value for the treatments is the probability of getting such a high F ratio if all the treatments were really identical.

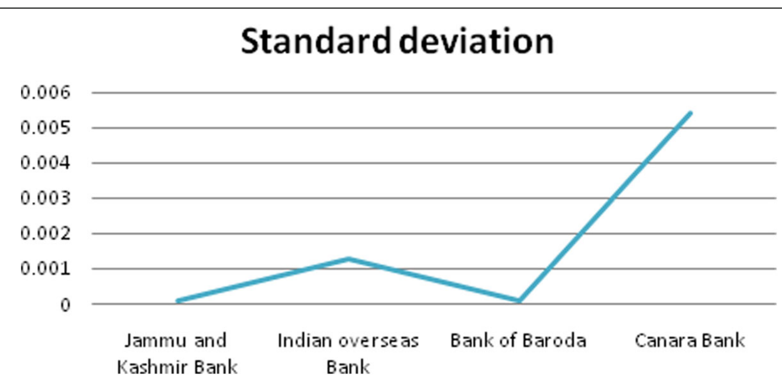

Fig. 4 Standard deviation of all firms

Table 4 Grouping information using the Tukey method and 95% confidence

Factor	*N*	Mean	Grouping			
BOB	5	0.455348	A			
Canara Bank	5	0.42609		B		
JK Bank	5	0.268740			C	
IOB	5	0.202489				D

Note: Means that do not share a letter are significantly different

The p value less that 0.005 indicates that there are differences between the treatments among the four firms. In other words, we have a strong evidence to reject the null hypothesis that the mean of PD score varies.

Tukey pairwise comparisons

It is a method that applies simultaneously to the set of pairwise comparisons

$$\{m_i - m_j\}$$

where m is a mean.

Tukey's method is used in ANOVA to create confidence intervals for all pairwise differences between factor-level means. This method is used in order to check whether there is any mean difference between the pairs of parameters. If the value zero lies in between the intervals, then there is no mean difference between the pairs.

Table 4 shows that all the factors have a different letters, group A consists Bank of Baroda (BOB), group B consists Canara Bank, group C consists JK Bank, and group D consists Indian Overseas Bank (IOB). All the firms do not share any letter, which indicates that BOB has a significantly higher mean of PD than others. But more PD means more risky. The IOB has lower PD which indicates that it is the best firm as compared to the others as shown in Fig. 5.

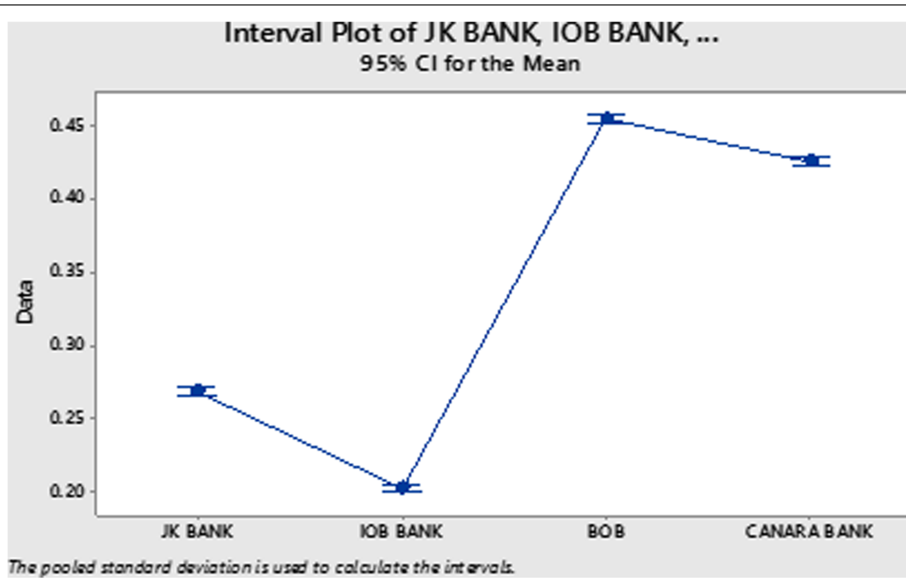

Fig. 5 Interval plot of all firms

Table 5 Tukey simultaneous tests for differences of means

Difference of levels	Difference of means	SE of Difference	95% CI	T value	Adjusted *P* value
IOB–JK Bank	− 0.06625	0.00176	(− 0.07129, − 0.06121)	− 37.66	0.000
BOB–JK Bank	0.18661	0.00176	(0.18157, 0.19165)	106.07	0.000
Canara Bank–JK Bank	0.15735	0.00176	(0.15231, 0.16239)	89.44	0.000
BOB–IOB	0.25286	0.00176	(0.24782, 0.25790)	143.73	0.000
Canara Bank–IOB	0.22360	0.00176	(0.21856, 0.22864)	127.10	0.000
Canara Bank–BOB	− 0.02926	0.00176	(− 0.03430, − 0.02422)	− 16.63	0.000

Note: Individual confidence level = 98.87%

Note: Rank the firm from D to A, D indicates the best firm because less PD means better and that is why we have to choose the ranking from D to A.

The results from Tukey simultaneous 95%CIs are (Table 5):

a. All the pairs do not contain the value zero. The pairs consist a positive or negative value. The Tukey method states that "If the pair or a group contain zero value or the interval of the pair is between positive and negatives values that indicates the groups are not significantly different." Table 5 and Fig. 6 show that all the pairs are not containing zero which means all are the firms are significantly different
b. The 95% simultaneous confidence level indicates that you can be 95% sure that all the confidence intervals have the right value
c. The 98.87% individual confidence level indicates that you can be 98.87% confident that every individual interval contains the right difference between the specific pair of group mean.

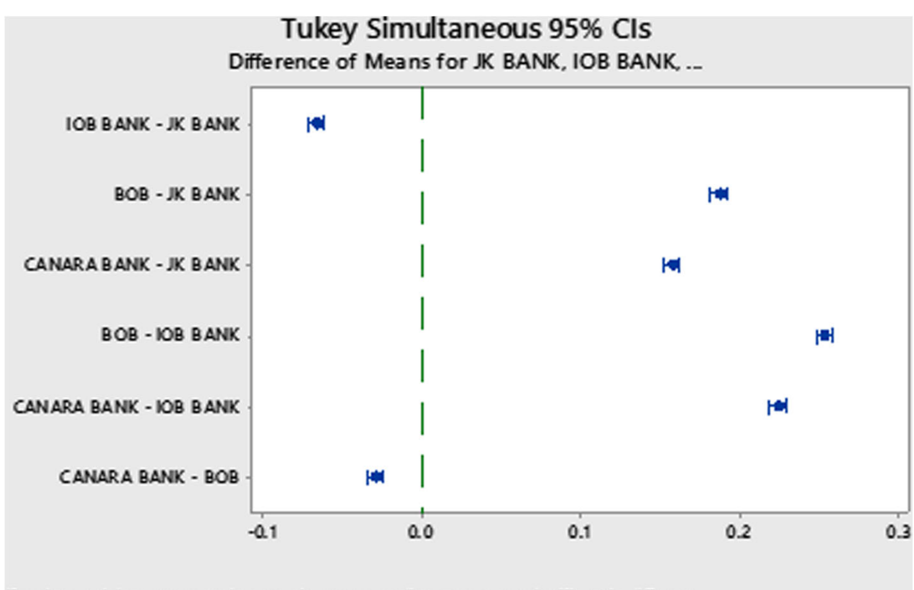

Fig. 6 Tukey simultaneous 95% CIS

Conclusion

The conclusion of this study is summarized below:

1. The solution of dV_t is $V_t = V_0 e^{\sigma Z_t + (\mu - 0.5\sigma^2)t}$
2. The researchers cannot decide the decision on the basis of overall standard deviation, better and best methods are to measure the distance to default and the PD for an investor to invest the money in any firm because the PD will give us the rate of default for a particular firm and the standard deviation of PD for several years gives the measurement of variance of the PD, and based on this, it is not helpful to calculate or decide which firm is best to invest as per the overall standard deviation of the probability of default
3. As per ANOVA, the authors reject the null hypothesis which indicates that there are differences between the mean among the four firms
4. The Tukey test clearly shows us that IOB is the better and BOB is the worst among the four firms as per rank. In other words, IOB is more significant than the other firms
5. It also indicates that the entire pairs do not contain zero values, which means that the firms are statistically significantly different from each other.

Abbreviations

ANOVA: Analysis of variance; BOB: Bank of Baroda; BSM-CO: Black-Scholes Model for European Call Option; CIs: Confidence intervals; IOB: Indian Overseas Bank; JK Bank: Jammu and Kashmir Bank; PD: Probability of default

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Availability of data and materials

The annual reports of firms from the year 2012–2016 and the data may not be correct but the procedure that I mentioned in this paper is defined well.

Authors' contributions

AAD designed the study. SQ collected the data from different firms in order to estimate the probability of default. AAD and NA analysed and interpreted the data. AAD drafted the manuscript. All the authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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