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Positive solutions of second-order non-local boundary value problem with singularities in space variables

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Abstract
We discuss a non-local boundary value problem of second-order, where the involved nonlinearity depends on the derivative and may be singular. The boundary conditions are given by Riemann-Stieltjes integrals. We establish sufficient conditions for the existence of positive solutions of the considered problem. Our approach is based on the Krasnoselskii-Guo fixed point theorem on cone expansion and compression.

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1 Introduction

In the paper we are interested in the existence of positive solutions for the following singular non-local boundary value problem (BVP):

$$\begin{cases} u''(t) + f(t, u(t), u'(t)) = 0, & t \in [0,1], \\ au(0) - bu'(0) = \alpha[u], \\ u'(1) = \beta[u]. \end{cases} \tag{1}$$

Throughout the paper we assume that:

(H1) $a > 0$ and $b > 0$,

(H2) f is continuous and nonnegative on $[0,1] \times (0,\infty) \times (0,\infty)$,

and we consider f to be singular at the value 0 of its space variables, that is, f may be singular in its second and third variable. The boundary conditions (BCs) involve linear functionals given by Riemann-Stieltjes integrals

$$\alpha[u] = \int_0^1 u(s)\,dA(s) \quad \text{and} \quad \beta[u] = \int_0^1 u(s)\,dB(s),$$

such that:

(H3) A and B are of bounded variation, and dA and dB are *positive* measures.

Many interesting results on the existence of solutions for the BVPs singular in the independent and/or the dependent variables can be found in the monographs [1] and [2] and in the recent papers; see for example [3–11] and [12]. Some of the techniques applied to the singular BVPs are based on the fixed point theorems in cones (see [3–6] and [12]). For other methods, including Leray-Schauder alternative and *a priori* bounds, see for example [2, 7, 8, 10, 13] and the references therein.

We point out that both regular and singular BVPs under the BCs involving Riemann-Stieltjes integrals are extensively discussed objects. We refer the reader to [9, 11, 14, 15] and [16] for some recent results on this topic.

A direct inspiration for studying (1) in the present paper were the problems considered in [4] and [5]. In [4], Yan, O'Regan and Agarwal dealt with the following *local* singular BVP:

$$\begin{cases} u''(t) + f(t, u(t), u'(t)) = 0, \quad t \in (0,1), \\ au(0) - bu'(0) = 0, \\ u'(1) = 0. \end{cases} \tag{2}$$

They established the existence of multiple positive solutions using the fixed point index technique combined with the approximation of the singular BVP (2) by an appropriate sequence of regular BVPs. The nonlinearity f in (2) allowed to be singular in its second and third variable. In [5], Infante studied the following *non-local* singular BVP:

$$\begin{cases} -u''(t) = g(t)f(t, u(t)), \quad t \in (0,1), \\ u'(0) + \alpha[u] = 0, \\ \sigma u'(1) + u(\eta) = \beta[u], \quad \eta \in [0,1], \end{cases}$$

with f singular in its space variable. This time the fixed point index technique was employed together with the truncation method, that is, the singular nonlinear term f was extended to all of $[0,1] \times [0,\infty)$ (see also [17, 18]).

The aim of our paper is to establish sufficient conditions for the existence of positive solutions for (1), that is, for the singular BVP with the *derivative dependence* and *non-local* boundary conditions. The main idea of our method is to restrict the singular nonlinear term f to an appropriately chosen subset $[0,1] \times [\rho_1,\infty) \times [\rho_2,\infty)$ of $[0,1] \times (0,\infty) \times (0,\infty)$. Then, following to some extent the approach developed by Webb and Infante in [19], we study the existence of fixed points of a perturbed Hammerstein integral operator of the form

$$Fu(t) = \alpha[u]\gamma(t) + \beta[u]\delta(t) + \int_0^1 G(t,s)f\bigl(s, u(s), u'(s)\bigr)\,ds, \tag{3}$$

where $G(t,s)$ is the Green's function of the problem

$$\begin{cases} u''(t) = 0, \quad t \in [0,1], \\ au(0) - bu'(0) = 0, \\ u'(1) = 0, \end{cases} \tag{4}$$

and γ and δ are the unique solutions of

$$\begin{cases} u''(t) = 0, & t \in [0,1], \\ au(0) - bu'(0) = 1, \\ u'(1) = 0, \end{cases} \quad \text{and} \quad \begin{cases} u''(t) = 0, & t \in [0,1], \\ au(0) - bu'(0) = 0, \\ u'(1) = 1, \end{cases}$$

respectively. Clearly, $\gamma(t) = \frac{1}{a}$ and $\delta(t) = t + \frac{b}{a}$. Throughout the paper we work under assumption (see for example [19])

(H4) $(1 - \alpha[\gamma])(1 - \beta[\delta]) - \alpha[\delta]\beta[\gamma] \neq 0$.

This implies that (1) is non-resonant, that is, the following BVP:

$$\begin{cases} u''(t) = 0, & t \in [0,1], \\ au(0) - bu'(0) = \alpha[u], \\ u'(1) = \beta[u], \end{cases}$$

has only the trivial solution. In order to prove the existence of a fixed point of (3) we make use of the Krasnoselskii-Guo fixed point theorem on cone expansion and compression (see [20]). It is well known that the key step when one applies the Krasnoselskii-Guo result is to find a suitable cone. We would like to point out here that in our case the choice of a cone is determined not only by the properties of the Green's function of (4) as it can be frequently found in the literature. The technique we use essentially takes into account the upper bound of the term f on $[0,1] \times [\rho_1, R] \times [\rho_2, R]$ with R being a suitable chosen positive constant. In this way we can deal with f singular in both its space variables.

2 Preliminaries

Let $\rho_1, \rho_2 > 0$. Denote by $\widetilde{f}$ the restriction of f to $[0,1] \times [\rho_1, \infty) \times [\rho_2, \infty)$. Clearly, $\widetilde{f}$ is continuous and nonnegative on $[0,1] \times [\rho_1, \infty) \times [\rho_2, \infty)$ and if u_0 is a positive solution of the following regular BVP:

$$\begin{cases} u''(t) + \widetilde{f}(t, u(t), u'(t)) = 0, & t \in [0,1], \\ au(0) - bu'(0) = \alpha[u], \\ u'(1) = \beta[u], \end{cases} \tag{5}$$

then $u_0(t) \geq \rho_1 > 0$ and $u_0'(t) \geq \rho_2 > 0$, so u_0 is a positive solution of (1).

In what follows we will employ the Green's function G of the homogeneous BVP (4) corresponding to (5). It is easy to check that, under (H1), G is given by the formula (see [4])

$$G(t,s) = \frac{1}{a} \begin{cases} at + b, & 0 \leq t \leq s \leq 1, \\ as + b, & 0 \leq s \leq t \leq 1. \end{cases}$$

Then

$$G_t(t,s) = \begin{cases} 1, & 0 \leq t < s \leq 1, \\ 0, & 0 \leq s < t \leq 1, \end{cases}$$

and

$$G(s,s) \geq G(t,s) \geq \frac{b}{a+b} G(s,s) > 0 \tag{6}$$

for $t, s \in [0,1]$. Now we recall some standard facts on cone theory in Banach spaces.

Definition 1 A nonempty subset P, $P \neq \{0\}$, of a real Banach space E is called a cone if P is closed, convex and

(i) $\lambda u \in P$ for all $u \in P$ and $\lambda \geq 0$,
(ii) if $u, -u \in P$, then $u = 0$.

Our existence result on positive solutions for (5) is based on the following Krasnoselskii-Guo fixed point theorem on cone expansion and compression.

Theorem 1 [20] *Let P be a cone in a Banach space E and let Ω_1, Ω_2 be open bounded subsets of E with $0 \in \Omega_1$ and $\overline{\Omega}_1 \subset \Omega_2$. If $F : P \cap (\overline{\Omega}_2 \setminus \Omega_1) \to P$ is a completely continuous operator such that either*

1° $\|Fu\| \leq \|u\|$ *for every* $u \in P \cap \partial\Omega_1$ *and* $\|Fu\| \geq \|u\|$ *for every* $u \in P \cap \partial\Omega_2$ *or*
2° $\|Fu\| \leq \|u\|$ *for every* $u \in P \cap \partial\Omega_2$ *and* $\|Fu\| \geq \|u\|$ *for every* $u \in P \cap \partial\Omega_1$,

then F has a fixed point in $P \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

3 Existence result for the regular BVP

In this section we state a result for the existence of a positive solution of (5). For positive numbers r and R we set

$$M_R := \max\bigl\{\widetilde{f}(t,u,v) : (t,u,v) \in [0,1] \times [\rho_1, R] \times [\rho_2, R]\bigr\} \tag{7}$$

and

$$c := \frac{\frac{b}{a+b} \min\{1, \frac{b}{a}\} r \int_0^1 dB(s)}{\frac{1}{a} R \int_0^1 dA(s) + (\frac{1}{2} + \frac{b}{a}) M_R + \frac{b}{a} \min\{1, \frac{b}{a}\} r \int_0^1 dB(s)}. \tag{8}$$

Observe that (H1), (H2), and (H3) imply

$$0 < c < \frac{a}{a+b}. \tag{9}$$

In addition to (H1)-(H4), we make the following assumptions on the function $\widetilde{f}$, the functionals α and β, and the coefficients a and b that appear in (5).

We assume there exist $0 < r < R$ and $M, m > 0$ such that:

(H5) $\frac{b}{a+b} \min\{1, \frac{b}{a}\} r \geq \rho_1$ and $c \min\{1, \frac{b}{a}\} r \geq \rho_2$.
(H6) $\widetilde{f}(t,u,v) \leq RM$ for $(t,u,v) \in [0,1] \times [R\frac{b}{a+b}, R] \times [Rc \min\{1, \frac{b}{a}\}, R]$.
(H7) $\frac{1}{a} \int_0^1 dA(s) + (1 + \frac{b}{a}) \int_0^1 dB(s) + M \max\{\frac{1}{2} + \frac{b}{a}, 1\} \leq 1$.
(H8) $\widetilde{f}(t,u,v) \geq rm$ for $(t,u,v) \in [0,1] \times [r\frac{b}{a+b}, r] \times [rc \min\{1, \frac{b}{a}\}, r]$.
(H9) $\frac{b}{a+b} \min\{1, \frac{b}{a}\} [\frac{1}{a} \int_0^1 dA(s) + (1 + \frac{b}{a}) \int_0^1 dB(s)] + \frac{3}{2} m \geq 1$.

Theorem 2 *Under the assumptions* (H1)-(H9), *the regular BVP* (5) *has a solution* u, *positive on* $[0,1]$, *with*

$$\frac{b}{a+b}\min\left\{1,\frac{b}{a}\right\}r \le u(t) \le R$$

and

$$c\min\left\{1,\frac{b}{a}\right\}r \le u'(t) \le R.$$

Proof Let $C^1[0,1]$ denote a Banach space of continuously differentiable functions with the norm

$$\|u\| = \max\{\|u\|_\infty, \|u'\|_\infty\},$$

where

$$\|u\|_\infty = \max\{|u(t)| : t \in [0,1]\}.$$

Let

$$P = \left\{u \in C^1[0,1] : u(t) \ge \frac{b}{a+b}\|u\|_\infty, u(0) \ge \frac{b}{a}\|u'\|_\infty, \text{and } u'(t) \ge c\|u\|_\infty \text{ on } [0,1]\right\}.$$

Then P is a cone in $C^1[0,1]$. Observe that the constant c that appears in P involves the maximum M_R of $\widetilde{f}$ on the set $[0,1] \times [\rho_1, R] \times [\rho_2, R]$ (see (7) and (8)). Moreover, if $u \in P$, then u is increasing on $[0,1]$ and

$$u(t) \ge \frac{b}{a+b}\|u\|_\infty \ge \frac{b}{a+b}u(0) \ge \frac{b^2}{a(a+b)}\|u'\|_\infty.$$

Hence

$$\begin{aligned} u(t) &\ge \max\left\{\frac{b}{a+b}\|u\|_\infty, \frac{b^2}{a(a+b)}\|u'\|_\infty\right\} \ge \frac{b}{a+b}\min\left\{1,\frac{b}{a}\right\}\max\{\|u\|_\infty, \|u'\|_\infty\} \\ &= \frac{b}{a+b}\min\left\{1,\frac{b}{a}\right\}\|u\|. \end{aligned} \tag{10}$$

We also have

$$u'(t) \ge c\|u\|_\infty \ge c\frac{b}{a}\|u'\|_\infty.$$

Hence

$$u'(t) \ge c\max\left\{\|u\|_\infty, \frac{b}{a}\|u'\|_\infty\right\} \ge c\min\left\{1,\frac{b}{a}\right\}\|u\|. \tag{11}$$

Let

$$\Omega_1 = \{u \in C^1[0,1] : \|u\| < r\} \quad \text{and} \quad \Omega_2 = \{u \in C^1[0,1] : \|u\| < R\}.$$

For $u \in P \cap (\overline{\Omega}_2 \setminus \Omega_1)$ consider the operator (3)

$$Fu(t) = \alpha[u]\gamma(t) + \beta[u]\delta(t) + \int_0^1 G(t,s)f\bigl(s,u(s),u'(s)\bigr)\,ds,$$

that is,

$$Fu(t) = \alpha[u]\frac{1}{a} + \beta[u]\left(t + \frac{b}{a}\right) + \int_0^1 G(t,s)\widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds.$$

It is clear that every fixed point of F is a solution of (5) (see for example [5] and [19]). We will show that F fulfills the assumptions of Theorem 1. First we prove that $F : P \cap (\overline{\Omega}_2 \setminus \Omega_1) \to P$. If $u \in P \cap (\overline{\Omega}_2 \setminus \Omega_1)$ then $Fu \in C^1[0,1]$ and by (6) we have

$$\begin{aligned}
\frac{b}{a+b}\|Fu\|_\infty &\le \frac{b}{a+b}\left(\alpha[u]\frac{1}{a} + \beta[u]\left(1 + \frac{b}{a}\right) + \int_0^1 G(s,s)\widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds\right) \\
&\le \alpha[u]\frac{1}{a} + \beta[u]\left(t + \frac{b}{a}\right) + \frac{b}{a+b}\int_0^1 G(s,s)\widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds \\
&\le \alpha[u]\frac{1}{a} + \beta[u]\left(t + \frac{b}{a}\right) + \int_0^1 G(t,s)\widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds = Fu(t).
\end{aligned}$$

Since

$$(Fu)'(t) = \beta[u] + \int_t^1 \widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds \le \beta[u] + \int_0^1 \widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds,$$

we get

$$\begin{aligned}
Fu(0) &= \alpha[u]\frac{1}{a} + \beta[u]\frac{b}{a} + \frac{b}{a}\int_0^1 \widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds \\
&\ge \frac{b}{a}\left(\beta[u] + \int_0^1 \widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds\right) \ge \frac{b}{a}\bigl\|(Fu)'\bigr\|_\infty.
\end{aligned}$$

To show that $(Fu)'(t) \ge c\|Fu\|_\infty$, we observe first that for $t \in [0,1]$ we have

$$(Fu)'(t) = \beta[u] + \int_t^1 \widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds \ge \beta[u].$$

On the other hand, (6) and (7) give

$$\begin{aligned}
\|Fu\|_\infty &\le \alpha[u]\frac{1}{a} + \beta[u]\left(1 + \frac{b}{a}\right) + \int_0^1 G(s,s)\widetilde{f}\bigl(s,u(s),u'(s)\bigr)\,ds \\
&\le \alpha[u]\frac{1}{a} + \beta[u]\left(1 + \frac{b}{a}\right) + M_R\int_0^1 G(s,s)\,ds \\
&= \alpha[u]\frac{1}{a} + \beta[u]\left(1 + \frac{b}{a}\right) + \left(\frac{1}{2} + \frac{b}{a}\right)M_R.
\end{aligned}$$

By (H3) and (10) we obtain

$$\begin{aligned}\beta[u] &= \int_0^1 u(s)\,dB(s) \geq \frac{b}{a+b}\min\left\{1,\frac{b}{a}\right\}\|u\|\int_0^1 dB(s)\\ &\geq \frac{b}{a+b}\min\left\{1,\frac{b}{a}\right\}r\int_0^1 dB(s),\end{aligned}\tag{12}$$

and

$$\alpha[u] = \int_0^1 u(s)\,dA(s) \leq R\int_0^1 dA(s).\tag{13}$$

Moreover, we can rewrite (8) as

$$\left(1-c\left(1+\frac{b}{a}\right)\right)\frac{b}{a+b}\min\left\{1,\frac{b}{a}\right\}r\int_0^1 dB(s) = c\left(\frac{1}{a}R\int_0^1 dA(s)+\left(\frac{1}{2}+\frac{b}{a}\right)M_R\right).\tag{14}$$

Then (9) combined with (12), (13), and (14) implies

$$\left(1-c\left(1+\frac{b}{a}\right)\right)\beta[u] \geq c\left(\alpha[u]\frac{1}{a}+\left(\frac{1}{2}+\frac{b}{a}\right)M_R\right),$$

and therefore

$$\beta[u] \geq c\left(\alpha[u]\frac{1}{a}+\beta[u]\left(1+\frac{b}{a}\right)+\left(\frac{1}{2}+\frac{b}{a}\right)M_R\right),$$

which gives $(Fu)'(t) \geq c\|Fu\|_\infty$. Thus F maps $P\cap(\overline{\Omega}_2\setminus\Omega_1)$ to P. By standard arguments we can show that F is completely continuous on $P\cap(\overline{\Omega}_2\setminus\Omega_1)$. Let $u\in P\cap\partial\Omega_2$. Then, in particular, $\|u\|=R$, $R\frac{b}{a+b}\leq u(t)\leq R$ and $Rc\min\{1,\frac{b}{a}\}\leq u'(t)\leq R$. Since dA and dB are positive measures, we get by (H6)

$$\begin{aligned}\|Fu\|_\infty = (Fu)(1) &= \alpha[u]\frac{1}{a}+\beta[u]\left(1+\frac{b}{a}\right)+\int_0^1 G(s,s)\widetilde{f}\big(s,u(s),u'(s)\big)\,ds\\ &\leq \frac{1}{a}\|u\|_\infty\int_0^1 dA(s)+\left(1+\frac{b}{a}\right)\|u\|_\infty\int_0^1 dB(s)+\frac{1}{a}RM\int_0^1(as+b)\,ds\\ &\leq \left[\frac{1}{a}\int_0^1 dA(s)+\left(1+\frac{b}{a}\right)\int_0^1 dB(s)+\frac{1}{a}M\left(\frac{a}{2}+b\right)\right]R\end{aligned}\tag{15}$$

and

$$\big\|(Fu)'\big\|_\infty \leq \|u\|_\infty\int_0^1 dB(s)+\int_0^1\widetilde{f}\big(s,u(s),u'(s)\big)\,ds \leq \left(\int_0^1 dB(s)+M\right)R.\tag{16}$$

Thus, (H7), (15), and (16) imply

$$\begin{aligned}\|Fu\| &= \max\left\{\|Fu\|_\infty,\big\|(Fu)'\big\|_\infty\right\}\\ &\leq \left[\frac{1}{a}\int_0^1 dA(s)+\left(1+\frac{b}{a}\right)\int_0^1 dB(s)+M\max\left\{1,\frac{1}{2}+\frac{b}{a}\right\}\right]R \leq R = \|u\|.\end{aligned}$$

For $u \in P \cap \partial\Omega_1$ we have $\|u\| = r$, $r\frac{b}{a+b} \le u(t) \le r$ and $rc\min\{1, \frac{b}{a}\} \le u'(t) \le r$. Hence, from (H8) and (H9), we obtain

$$\begin{aligned}
\|Fu\| \ge \|Fu\|_\infty = Fu(1) &= \alpha[u]\frac{1}{a} + \beta[u]\left(1 + \frac{b}{a}\right) + \int_0^1 G(s,s)\widetilde{f}\big(s, u(s), u'(s)\big)\,ds \\
&\ge \frac{b}{a(a+b)}\min\left\{1, \frac{b}{a}\right\} r \int_0^1 dA(s) \\
&\quad + \left(1 + \frac{b}{a}\right)\frac{b}{a+b}\min\left\{1, \frac{b}{a}\right\} r \int_0^1 dB(s) + rm\int_0^1 G(s,s)\,ds \\
&= \left[\frac{b}{a(a+b)}\min\left\{1, \frac{b}{a}\right\}\int_0^1 dA(s)\right. \\
&\quad \left. + \left(1 + \frac{b}{a}\right)\frac{b}{a+b}\min\left\{1, \frac{b}{a}\right\}\int_0^1 dB(s) + \frac{3}{2}m\right] r \ge r = \|u\|.
\end{aligned}$$

Application of Theorem 1 yields the result. □

Remark 1 In [4] the authors used the cone

$$P_1 = \left\{u \in C^1[0,1] : u(t) \ge \frac{b}{a+b}\|u\|_\infty \text{ on } [0,1], u(0) \ge \frac{b}{a}\|u'\|_\infty\right\}.$$

The cone we consider in the proof of Theorem 2 is of the form

$$P = \left\{u \in C^1[0,1] : u(t) \ge \frac{b}{a+b}\|u\|_\infty, u(0) \ge \frac{b}{a}\|u'\|_\infty, \text{and } u'(t) \ge c\|u\|_\infty \text{ on } [0,1]\right\},$$

which provides the lower bound not only for $u(t)$ but for $u'(t)$ as well (see (10) and (11)). Since it is sufficient for our method to work that F defined in (3) maps $P \cap (\overline{\Omega}_2 \setminus \Omega_1)$ to P, we would like to emphasize here that we do not need F to be positive on P.

Remark 2 Observe that Theorem 2 implies the existence of a positive solution of (1). Indeed, by (H5), a solution u of (5) evidently satisfies $u(t) \ge \frac{b}{a+b}\min\{1, \frac{b}{a}\}r \ge \rho_1$ and $u'(t) \ge c\min\{1, \frac{b}{a}\}r \ge \rho_2$ on $[0,1]$.

We conclude this section with one numerical example illustrating Theorem 2. Some calculations have been made here with MAPLE.

Example 1 Consider the following four-point boundary value problem:

$$\begin{cases} u''(t) + h(t)(\frac{0.1}{u(t)} + \frac{0.0001}{u'(t)}) = 0, & t \in [0,1], \\ u(0) - u'(0) = \frac{1}{10}u(\frac{1}{4}), \\ u'(1) = \frac{2}{5}u(\frac{1}{2}), \end{cases} \tag{17}$$

where the function h is continuous on $[0,1]$ and $1 \le h(t) \le 2$ on $[0,1]$. In this case, $f(t,u,v) = h(t)(\frac{0.1}{u} + \frac{0.0001}{v})$, $a = b = 1$, $\alpha[u] = \frac{1}{10}u(\frac{1}{4})$ and $\beta[u] = \frac{2}{5}u(\frac{1}{2})$. Fix $\rho_1 = 0.1$ and $\rho_2 = 0.0001$. For $r = \frac{1}{5}$ and $R = 3$ we have $M_R = 4$ and $c = \frac{2}{319} \approx 0.0063$ and we can take $M = \frac{1}{20}$ and $m = 2$. By Theorem 2, the BVP (17) has a solution u such that $\frac{1}{5} \le \|u\| \le 3$, $\frac{1}{10} \le u(t) \le 3$ and $\frac{6}{319} \le u'(t) \le 3$ on $[0,1]$.

Competing interests
The author declares that she has no competing interests.

Author's contributions
The sole author personally prepared the manuscript.

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