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Partial relay-based cooperative primary user detection in cognitive radio networks

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Abstract

In order to identify spectrum holes, secondary users (SUs) should detect whether the spectrum band is occupied by primary user (PU). Cooperative PU detection is the most prevalent PU detection method in cognitive radio (CR) networks. Existing cooperative PU detection usually consumes a dedicated reporting channel to share the detection results. This paper proposes a partial relay-based cooperative PU detection scheme that does not require any reporting channels. Our idea is to let the SU with higher signal-to-noise ratio (SNR) sacrifices part of its detection period acting as the relay node to help other SUs. Although the performance of this SU is inevitably impaired, other SUs may benefit from the relay, and performance of the whole CR network can be improved. Moreover, this paper investigates the relay policy of proposed scheme under two criteria. Considering the criterion of balancing detection accuracy, an interesting conclusion that the optimal relay policy merely depends on the SNR gap is derived. Both lower and upper bounds of the optimal relay policy are deduced as well. Considering the criterion of maximizing detection agility, numerical approach to obtain the optimal relay policy is introduced, and the agility gain is also analyzed. Simulation results are provided to verify all conclusions above.

Keywords: Partial relay, Cooperative PU detection, Relay policy, Detection accuracy, Detection agility

1 Introduction

The dramatic flourish of wireless technologies has aroused great concern about spectrum resources in recent years [1–3]. Coined by Mitola [4], cognitive radio (CR) is regarded as a promising solution to the so-called "spectrum scarcity" issue by enabling the users without dedicated spectrum resources, namely secondary users (SUs), to reuse the temporal and spatial "holes" on the spectrum band that has been licensed to the primary user (PU) for exclusive use. In order to identify spectrum holes, SUs should detect whether PU is occupying its band [5, 6].

Various methods have been suggested for PU detection, and cooperative PU detection increasingly becomes the most prevalent choice as it exploits the diversity of multiple SUs in CR networks [7]. Based on how multiple SUs share their detection results, cooperative PU detection can be classified into three categories [8]: centralized manner that all SUs forward the results to a fusion center (FC) [7, 9–11], distributed manner that SUs communicate with others directly [12], and relay-assisted manner that one or more SUs serve as relays to assist other SUs in forwarding the results [13, 14]. No matter in which category of cooperation, in order to avoid interfering with PU, a dedicated reporting channel (or control channel) is used to transmit detection results, not only consuming extra spectrum resource but also incurring additional transmission errors as the channel cannot be perfect [15]. In [16], a selective-relay-based scheme is proposed to upload SUs' binary detection results to FC on the licensed band without causing severe interference to PU. However, after combining SUs' detection results, FC should feed its global decision back to SUs and inform them of either using or vacating the licensed band. This feedback link (from FC to SUs) still needs extra channel resources.

This paper concentrates on cooperative PU detection and proposes a partial relay-based scheme that does not require any dedicated channels. To achieve this goal, ⟨1⟩ PU signal that can be transmitted on licensed band is relayed instead of SUs' detection results, and ⟨2⟩ decisions are individually made by SUs themselves to avoid

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feeding back the global decision. Different from existing partial relay selection techniques [17], the term "partial relay" in our scheme refers to that one SU spends part of its detection period acting as the relay node. Although the performance of relaying SU is inevitably impaired as its detection time is reduced, other SUs in the CR network may benefit from the relay mechanism, and thus, our scheme could be superior to the non-relay scheme from the entire CR network point of view, e.g., via improving the network detection agility. Moreover, this paper investigates how much of the detection period should be used for partial relay and deduces the optimal relay policy under the criteria of balancing detection accuracy and maximizing detection agility.

The rest of this paper is organized as follows. Section 2 introduces the model of partial relay. Section 3 analyzes the performance of partial relay-based cooperative PU detection. In Section 4, the impact of relay policy is studied under different criteria. Section 5 provides some simulation results, and Section 6 concludes this paper.

2 Partial relay model

Similar to [13, 18], a CR network with two SUs, namely SU_1 and SU_2, is considered for simplicity, as shown in Fig. 1. If there exist more than two SUs in the CR network, similar relay mechanism can be conducted by choosing one SU with the highest signal-to-noise ratio (SNR) as the relay according to literature [19]. Due to lack of spectrum resource, SUs periodically detect whether the licensed band is being used by PU; if a spectrum hole is found, they use different sub-channels of the band to transmit.

In case of non-cooperation, each SU individually detects PU during the detection period T_d; if the absence of PU is detected, they transmit on the licensed band during the period T_c, as depicted in Fig. 2a.

Because of various propagation conditions, the received signal-to-noise ratios (SNRs) of SU_1 and SU_2, denoted by γ_1 and γ_2, respectively, are usually different. Therefore, given the same detection period T_d, the SU with higher SNR (say SU_2) may perform detection with excessive accuracy, while the SU with lower SNR (say SU_1) may suffer from numerous missed detections and cause intolerable interference to PU. Our idea is to let SU_2 sacrifice part of its detection period $\eta T_d (0 < \eta < 1)$ to help SU_1 via relay. Then, the whole detection period of SU_2 is divided into two phases, as depicted in Fig. 2b.

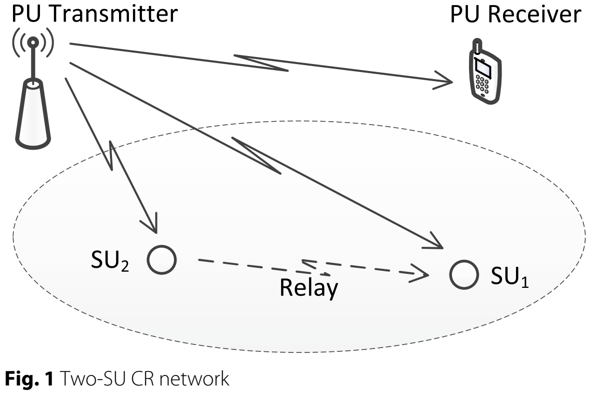

Fig. 1 Two-SU CR network

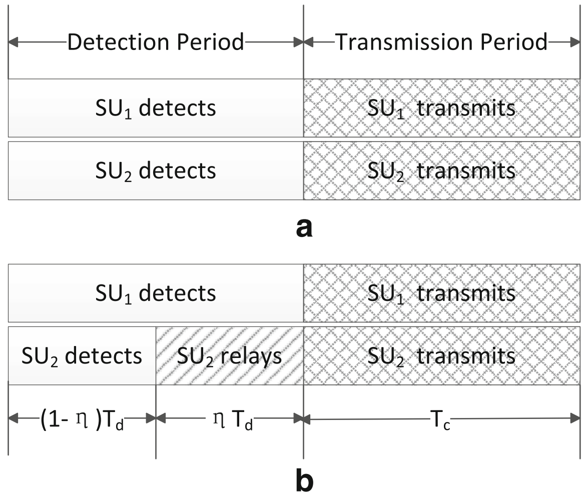

Fig. 2 a Non-cooperation, and **b** partial relay-based cooperation

In the first phase of $(1-\eta)T_d$, both SU_1 and SU_2 perform PU detection as before. The received signals of SU_1 and SU_2 are given by

$$y_1^{(1)} = h_1(k) \cdot s(k) + n_1(k), \tag{1}$$

$$y_2(k) = h_2(k) \cdot s(k) + n_2(k), \tag{2}$$

where $s(k)$ denotes the signal of PU; $s(k) = 0$ if PU is absent; $h_1(k)$ and $h_2(k)$ denote the channels from PU transmitter to SU_1 and SU_2, respectively; $n_1(k)$ and $n_2(k)$ denote the additive noises of SU_1 and SU_2, respectively.

In the second phase of $\eta T_d, SU_2$ stops detecting and begins to relay its received signal in an immediate amplify-and-forward manner. Note that the signal is forwarded on licensed band, and thus, no extra channel is required. Although this signal affects PU receiver simultaneously, it can be treated as a multipath copy of original PU signal and collected by use of Rake technique. Consequently, the received signal of SU_1 in this phase can be expressed as

$$\begin{aligned} y_1^{(2)}(k) &= h_1(k) \cdot s + \sqrt{\kappa} h_{21}(k) \cdot y_2(k) + n_1(k) \\ &= \left[h_1(k) + \sqrt{\kappa} h_{21}(k) h_2(k)\right] s(k) \\ &\quad + \left[n_1(k) + \sqrt{\kappa} h_{21}(k) n_2(k)\right], \end{aligned} \tag{3}$$

where κ denotes the relay gain, $h_{21}(k)$ denotes the channel from SU_2 to SU_1.

Consider $s(k)$ is binary phase shift keying signal; $h_1(k), h_2(k)$, and $h_{21}(k)$ are real Gaussian with zero means;

$n_1(k)$ and $n_2(k)$ are zero mean and unit variance Gaussian white noises; all these random processes are independent of each other [20]. Then, the SNR of SU_1 in the second phase is

$$\begin{aligned}\gamma_1^{(2)} &= \frac{E\left\{\left|\left[h_1(k)+\sqrt{\kappa}h_{21}(k)h_2(k)\right]s(k)\right|^2\right\}}{E\{|n_1(k)+\sqrt{\kappa}h_{21}(k)n_2(k)|^2\}} \\ &= \frac{\gamma_1+\kappa\gamma_h\gamma_2}{1+\kappa\gamma_h},\end{aligned} \tag{4}$$

where γ_h denotes the impact of the relay channel from SU_2 to SU_1,

$$\begin{aligned}\gamma_h &= E\left\{|h_{21}(k)|^2\right\}, \\ \gamma_1 &= \frac{E\left\{|h_1(k)s(k)|^2\right\}}{E\left\{|n_1(k)|^2\right\}}, \\ \gamma_2 &= \frac{E\left\{|h_2(k)s(k)|^2\right\}}{E\left\{|n_2(k)|^2\right\}}.\end{aligned}$$

Since $\gamma_2 > \gamma_1$, it is obvious that $\gamma_1^{(2)} > \gamma_1$ according to (4). In other words, the SNR of SU_1 is improved via relay, and the benefit of partial relay is proved.

3 Performance analysis

Without loss of generality, assume both SUs adopt energy detector to make decisions throughout this paper [7, 9, 10, 20]. The observed energy of SU_1 and SU_2 can be calculated as

$$\begin{aligned}v_1 &= v_1^{(1)} + v_1^{(2)} \\ &= \sum_{k=1}^{n_1}\left|y_1^{(1)}(k)\right|^2 + \sum_{k=n_1+1}^{n}\left|y_1^{(2)}(k)\right|^2,\end{aligned} \tag{5}$$

$$v_2 = \sum_{k=1}^{n_1}\left|y_2(k)\right|^2, \tag{6}$$

where $v_1^{(1)}$ and $v_1^{(2)}$ are the energy observed by SU_1 in the first and the second phases, respectively; $n = T_d f_s$ and $n_1 = (1-\eta)T_d f_s$ are the sample numbers of the whole detection period and its first phase, respectively; f_s is the sampling frequency.

Comparing v_1 and v_2 with corresponding thresholds, SU_1 and SU_2 can make decisions individually by themselves rather than obtain the global decision from FC via a dedicated channel,

$$\begin{cases} v_i \le \lambda_i \;\; SU_i \text{ accpect } H_0 \\ v_i > \lambda_i \;\; SU_i \text{ accpect } H_1 \end{cases}, i = 1, 2,$$

where λ_1 and λ_2 represent the decision thresholds of SU_1 and SU_2, respectively; H_0 and H_1 represent the hypotheses of PU's absence and presence, respectively.

Since the sample number used by energy detector is usually very large, $v_1^{(1)}$, $v_1^{(2)}$, and v_2 approximately follow Gaussian distributions according to the central limit theorem. The means and variances of these Gaussian variables can be derived as follows [18, 20],

$$E\left\{v_1^{(1)}\right\} = \begin{cases} n(1-\eta) & H_0 \\ n(1-\eta)(1+\gamma_1) & H_1 \end{cases}, \tag{7}$$

$$\mathrm{Var}\left\{v_1^{(1)}\right\} = \begin{cases} 2n(1-\eta) & H_0 \\ 2n(1-\eta)(1+\gamma_1)^2 & H_1 \end{cases}, \tag{8}$$

$$E\left\{v_1^{(2)}\right\} = \begin{cases} n\eta(1+\kappa\gamma_h) & H_0 \\ n\eta\left(1+\gamma_1+\kappa\gamma_h+\kappa\gamma_h\gamma_2\right) & H_1 \end{cases}, \tag{9}$$

$$\mathrm{Var}\left\{v_1^{(2)}\right\} = \begin{cases} 2n\eta\left[(1+\kappa\gamma_h)^2+3\kappa^2\gamma_h^2\right] & H_0 \\ 2n\eta\left[(1+\gamma_1+\kappa\gamma_h+\kappa\gamma_h\gamma_2)^2\right. & \\ \qquad \left.+3\kappa^2\gamma_h^2(1+\gamma_2)^2\right] & H_1 \end{cases}, \tag{10}$$

$$E\{v_2\} = \begin{cases} m_{20} = n(1-\eta) & H_0 \\ m_{21} = n(1-\eta)(1+\gamma_2) & H_1 \end{cases}, \tag{11}$$

$$\mathrm{Var}\{v_2\} = \begin{cases} \delta_{20}^2 = 2n(1-\eta) & H_0 \\ \delta_{21}^2 = 2n(1-\eta)(1+\gamma_2)^2 & H_1 \end{cases}. \tag{12}$$

Note that $v_1^{(1)}$ is independent of $v_1^{(2)}$, and thus $v_1 = v_1^{(1)} + v_1^{(2)}$ also follows Gaussian distribution with the mean and variance as below,

$$\begin{aligned}E\{v_1\} &= E\left\{v_1^{(1)}+v_1^{(2)}\right\} \\ &= \begin{cases} m_{10} = n(1+\eta\kappa\gamma_h) & H_0 \\ m_{11} = n\left(1+\gamma_1+\eta\kappa\gamma_h+\eta\kappa\gamma_h\gamma_2\right) & H_1 \end{cases},\end{aligned} \tag{13}$$

$$\begin{aligned}\mathrm{Var}\{v_1\} &= \mathrm{Var}\left\{v_1^{(1)}+v_1^{(2)}\right\} \\ &= \begin{cases} \delta_{10}^2 = 2n\left(1+2\eta\kappa\gamma_h+4\eta\kappa^2\gamma_h^2\right) & H_0 \\ \delta_{11}^2 = 2n\left[(1+\gamma_1)^2+2\eta\kappa\gamma_h(1+\gamma_2)\right. & \\ \qquad \left.\cdot(1+\gamma_1+2\kappa\gamma_h+2\kappa\gamma_h\gamma_2)\right] & H_1 \end{cases}.\end{aligned} \tag{14}$$

For simplicity, we assume that the relay gain is properly chosen to compensate for the path loss of relay channel, that is, $\kappa = 1/\gamma_h$. In this case, the mean and variance of v_1 can be rewritten as

$$E\{v_1\} = \begin{cases} m_{10} = n(1+\eta) & H_0 \\ m_{11} = n\left(1+\gamma_1+\eta+\eta\gamma_2\right) & H_1 \end{cases}, \tag{15}$$

$$\mathrm{Var}\{v_1\} = \begin{cases} \delta_{10}^2 = 2n(1+6\eta) & H_0 \\ \delta_{11}^2 = 2n\left[(1+\gamma_1)^2+2\eta\right. & \\ \qquad \left.\cdot(1+\gamma_2)\left(3+\gamma_1+2\gamma_2\right)\right] & H_1 \end{cases}. \tag{16}$$

Based on the distributions above, detection accuracy of SU_1 and SU_2, evaluated in term of false alarm and missed detection probabilities, can be expressed as

$$P_{f1} = Q\left(\frac{\lambda_1 - m_{10}}{\delta_{10}}\right), \tag{17}$$

$$P_{m1} = 1 - Q\left(\frac{\lambda_1 - m_{11}}{\delta_{11}}\right), \tag{18}$$

$$P_{f2} = Q\left(\frac{\lambda_2 - m_{20}}{\delta_{20}}\right), \tag{19}$$

$$P_{m2} = 1 - Q\left(\frac{\lambda_2 - m_{21}}{\delta_{21}}\right), \tag{20}$$

where $Q(\cdot)$ is the Q-function.

4 Relay policy optimization

In our scheme, SU_2 takes $T_R = \eta T_d$ time off from the entire T_d period to act as the relay node and help SU_1, while its detection time is reduced to $(1-\eta)T_d$. If adopting a conservative relay policy and choosing a small T_R, little help will be offered to SU_1. On the contrary, if adopting an aggressive relay policy and selecting a large T_R, SU_2 itself will suffer from severe performance degradation. This section deals with the issue of how to determine relay policy

$$\eta \triangleq \frac{T_R}{T_d}, (0 < \eta < 1), \tag{21}$$

where the more η approaches 0, the more conservative relay policy is; the more η approaches 1, the more aggressive relay policy is.

Many criteria can be implemented to find an optimal η, and this paper gives two examples of balancing detection accuracy and maximizing detection agility.

4.1 Balancing detection accuracy

Under the criterion of balancing detection accuracy, our aim is to let different SUs achieve the same detection accuracy to avoid either of them producing too many detection errors. Without loss of generality, consider the energy detector with constant false alarm probability α. Then, the same detection accuracy of SU_1 and SU_2 can be expressed as

$$\begin{cases} P_{f1} &= P_{f2} = \alpha \\ P_{m1} &= P_{m2} \end{cases}. \tag{22}$$

Substituting (17), (18), (19) and (20) into (22) and eliminating λ_1 and λ_2, an implicit equation for η is obtained

$$\begin{aligned} &Q^{-1}(\alpha)\sqrt{2} - \sqrt{n(1-\eta)}\gamma_2 \\ &= Q^{-1}(\alpha)\sqrt{2}\sqrt{\frac{1+6\eta}{\phi^2+\eta(4+2\phi)}} - \frac{\sqrt{n}(\gamma_1+\eta\gamma_2)}{\sqrt{\phi^2+\eta(4+2\phi)}}, \end{aligned} \tag{23}$$

where $\phi \triangleq (1+\gamma_1)/(1+\gamma_2)$. According to (23), the desired relay policy, namely η_{opt}, is determined by parameters α, n, γ_1, and γ_2.

Considering the low SNR scenarios in which the signal from PU is usually very weak [7, 8, 20], we have $(1+\gamma_1)/(1+\gamma_2) \approx 1$. Then, an interesting conclusion can be derived as

$$\eta_{opt}(\rho) = \frac{5\rho - 2 + \sqrt{53\rho^2 - 20\rho - 24}}{14\rho}, \tag{24}$$

where $\rho = \gamma_2/\gamma_1 (\rho > 1)$ is the ratio of γ_2 to γ_1. That is, the optimal relay policy η_{opt} to balance the detection accuracy of SU_1 and SU_2 merely depends on their SNR gap ρ.

Moreover, since η_{opt} is monotonically increasing in ρ, its lower and upper bounds are given by

$$\lim_{\rho \to 1^+} \eta_{opt}(\rho) = \frac{3}{7} \approx 42.9\%, \tag{25}$$

$$\lim_{\rho \to \infty} \eta_{opt}(\rho) = \frac{5+\sqrt{53}}{14} \approx 87.2\%. \tag{26}$$

Therefore, SU_2 should spend at least 42.9% and at most 87.2% of its detection period in assisting SU_1 regardless of ρ.

It should be pointed out that, if SU_1 and SU_2 have the same SNR, their detection accuracy will be originally equal and no relay is required to balance their accuracy, namely $\eta_{opt}(1) = 0$. This phenomenon is not in contradiction with our results. When deducing (24), a quadratic equation is solved and two solutions are produced,

$$\eta_{opt}(\rho) = \frac{5\rho - 2 \pm \sqrt{53\rho^2 - 20\rho - 24}}{14\rho}. \tag{27}$$

Note that another solution $(5\rho - 2 - \sqrt{53\rho^2 - 20\rho - 24})/(14\rho)$ has been discarded as the relay policy cannot be negative. Its right-hand limit is 0 as ρ approaches 1.

4.2 Maximizing detection agility

The concept of agility is developed to describe the ability of SUs to quickly vacate the licensed spectrum band after PU emerges [13, 19], and it can be evaluated in terms of the time needed to vacate the band, namely the vacating time,

$$T_\eta = k_\eta \left(T_d + T_c\right), \tag{28}$$

where k_η denotes the number of detections required to vacate the band. Obviously, larger T_η results in lower agility, and smaller T_η leads to higher agility.

Detailedly, in the two-SU CR network of this paper, since SU_1 and SU_2 usually share the licensed band by using different code or frequency sub-channels when PU is absent, the band is vacated only if they both stop transmitting. Moreover, as there exists no fusion center in the CR network, if one SU finds the presence of PU out, it can hardly inform another SU about this message and ask it to vacate the band as considered in [13]. Consequently, the licensed spectrum band can be completely vacated only in case that both SU_1 and SU_2 have detected out the presence of PU separately.

Note that, due to the inevitable missed detection errors, how many detections are required by SU_1 and SU_2 to both detect out the presence of PU after PU emerges is not fixed. This number k_η is essentially a random variable, and the probability of $k_\eta = l$ is given by

$$\begin{aligned} Pr\left(k_\eta = l\right) = P_{m1}^{l-1}(1-P_{m1}) \cdot \left(1-P_{m2}^{l}\right) \\ + \left(1-P_{m1}^{l}\right) \cdot P_{m2}^{l-1}(1-P_{m2}) \\ - P_{m1}^{l-1}(1-P_{m1})P_{m2}^{l-1}(1-P_{m2}). \end{aligned} \tag{29}$$

Based on (29), the mean of k_η can be expressed as

$$\begin{aligned} E\left\{k_\eta\right\} &= \sum_{l=1}^{\infty} l \cdot Pr\left(k_\eta = l\right) \\ &= \frac{1}{1-P_{m1}} + \frac{1}{1-P_{m2}} - \frac{1}{1-P_{m1}P_{m2}}. \end{aligned} \tag{30}$$

Consequently, the average vacating time in this paper yields to

$$\begin{aligned} E\left\{T_\eta\right\} &= E\left\{k_\eta\left(T_d + T_c\right)\right\} \\ &= \left(\frac{1}{1-P_{m1}} + \frac{1}{1-P_{m2}} - \frac{1}{1-P_{m1}P_{m2}}\right)\left(T_d + T_c\right). \end{aligned} \tag{31}$$

Given a specific relay policy η, both P_{m1} and P_{m2} can be calculated according to (18) and (20). Substituting these missed detection probabilities into (31), the average vacating time $E\left\{T_\eta\right\}$ can also be obtained theoretically. Repeating two steps above and implementing numerical search, it is easy to find out the optimal relay policy η_{opt}, with which $E\left\{T_\eta\right\}$ can be minimized. In partial scenarios, if SU_2 spends η_{opt} portion of its detection period acting as the relay node, detection agility of the entire CR network can be maximized eventually.

Furthermore, in order to highlight the benefit of partial relay-based cooperation, this paper compares its detection agility with that of non-cooperation and defines the agility gain as follows,

$$\tau_\eta \triangleq \frac{E\left\{T_0\right\}}{E\left\{T_\eta\right\}}, \tag{32}$$

where $E\left\{T_0\right\}$ is the average vacating time of non-cooperation. The detailed analysis on agility gain can be found in the next section.

5 Simulation results

This section provides some simulation results to illustrate the partial relay-based cooperative PU detection and its relay policy. The simulation settings are $\alpha = 0.1, \gamma_1 = -20dB, n = 600, T_d = T_c = nf_s$, and $f_s = 21.524476$ M samples/sec [21]. Unless otherwise specified, we assume $\kappa \cdot \gamma_h = 1$.

Figure 3 plots the missed detection probabilities of SU_1 and SU_2 versus the relay policy under different SNRs of $\gamma_2 = -8, -10, -12, -14$ dB. Obviously, in case of non-cooperation ($\eta = 0$), SU_1 causes too many detection errors as its SNR is extremely low. So no matter how well SU_2 performs, PU is interfered seriously. As η increases, P_{m1} decreases and P_{m2} increases; the larger γ_2 is, the more sharply P_{m1} and P_{m2} change. It is because SU_2 sacrifices ηT_d time to aid SU_1 by relay, and more powerful SU_2 could offer more help. The curves of P_{m1} and P_{m2} intersect eventually. This indicates the detection accuracy of two SUs has been balanced. Note that the points of (η_{opt}, P_m) calculated according to (24) and (18) and the bounds of η_{opt} derived from (25) and (26) are also plotted in this figure. Since all curve intersections agree well with these points and bounds, theoretical analysis on relay policy is verified. If η continues increasing and becomes larger than η_{opt}, the performance of SU_1 will get better while SU_2 will suffer from excessive missed detection errors. That is not what we expected either.

According to (24), the optimal relay policy for balancing detection accuracy merely depends on the SNR ratio of SU_2 to SU_1. Figure 4 depicts their relationship. As shown in this figure, SU_2 should spend at least 42.9% of its detection period on relay. When the SNR ratio ρ increases, the optimal relay policy η_{opt} grows, but its growth speed gradually slows down. If ρ is larger than 15 dB, increase in ρ causes little impact on η_{opt}, and η_{opt} approaches 82.7%. That is, SU_2 should spend at most 82.7% of its detection period acting as the relay.

Figure 5 plots the curves of average vacating time $E\left\{T_\eta\right\}$ versus relay policy η for different SNRs of SU_2 ($\gamma_2 =$

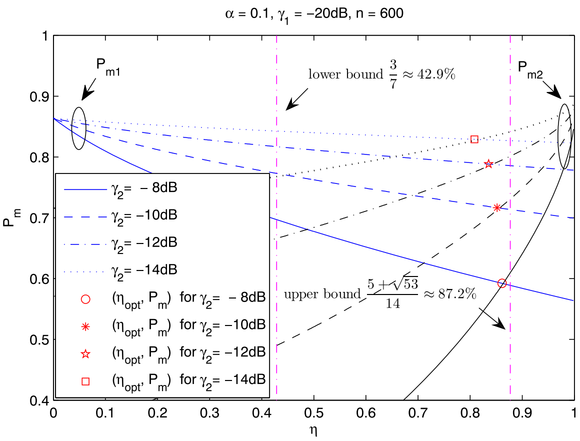

Fig. 3 Missed detection probabilities of SU_1 and SU_2 versus relay policy under different SNRs of SU_2

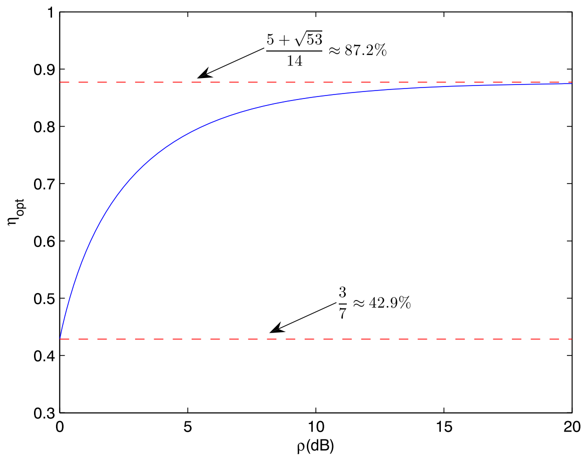

Fig. 4 Optimal relay policy versus SNR ratio under balancing detection accuracy criterion

$0, -5, -10, -15$ dB). In this figure, all curves are concave, which means that both excessively conservative η and excessively aggressive η result in high vacating time and low agility. Therefore, it is necessary to choose an appropriate η. Implementing numerical search as suggested in Section 4, we derive the optimal relay policy η_{opt} for each γ_2 and plot them with red diamond markers. As shown in the figure, all markers are located at the bottom of the corresponding vacating time curves, so our method of deducing η_{opt} is verified. In addition, η_{opt} increases as γ_2 increases. This phenomenon indicates that SU_2 should adopt a more aggressive relay policy if its SNR is higher.

Figure 6 depicts the curves of detection agility gain τ_η versus relay policy η under different SNRs of SU_2 ($\gamma_2 = 0, -5, -10, -15$ dB). The red diamond marker on each curve shows the optimal relay policy η_{opt} that has been discussed in Fig. 5 and its corresponding maximum agility gain. As shown in Fig. 6, all curves approach 1 as η approaches 0. This is because when η is very small, SU_2 spends little time acting as the relay node, and thus produces little agility improvement. Moreover, when $\eta < \eta_{opt}$, τ_η increases as η increases, and its increase speed is higher if γ_2 is larger. For $\gamma_2 = 0$ dB, τ_η can reach as much as 7.3. In other words, the detection agility can be improved by 630%. It should also be pointed out that, when $\eta > \eta_{opt}$, increase in η does not cause positive impact on τ_η any longer. Especially, when $\gamma_2 = -15$ dB and $\eta > 0.5$, τ_η is smaller than 1, which indicates that the detection agility turns worse after partial relay. As a result, it is better to avoid adopting an extremely aggressive relay policy when the SNR of SU_2 is very low.

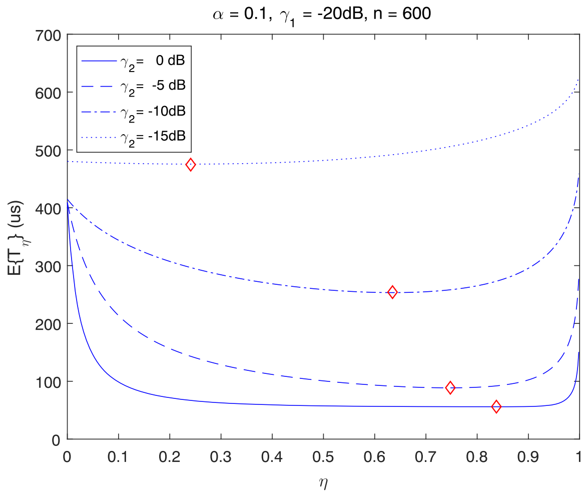

Fig. 5 Average vacating time versus relay policy under different SNRs of SU_2

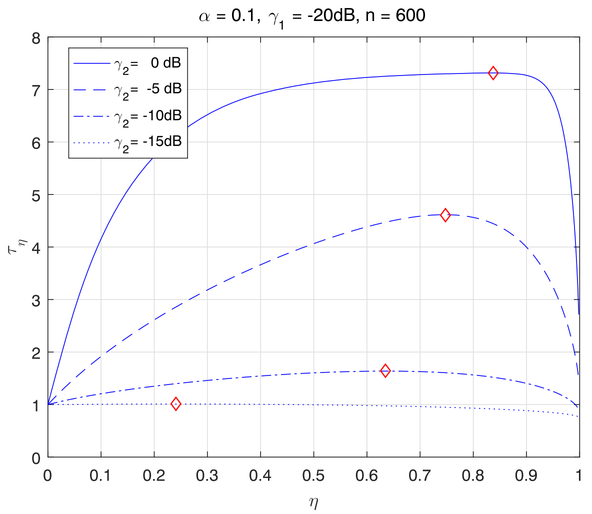

Fig. 6 Detection agility gain versus relay policy under different SNRs of SU_2

Finally, in order to investigate the impacts of relay gain κ and relay channel γ_h, Fig. 7 considers different values of $\kappa \cdot \gamma_h$ ($\kappa \cdot \gamma_h = 6, 3, 0, -3, -6$ dB) with $\gamma_2 = -5$ dB. Obviously, given the same relay policy η, higher detection agility gain is achieved if $\kappa \cdot \gamma_h$ is larger. This phenomenon can be explained as follows. A large κ incurs great amplification of relay signal and a high γ_h (low path loss of relay channel) represents little attenuation in relay signal, both of which are beneficial for our partial relay mechanism. In addition, the highest points of agility gain curves are also marked with red diamond markers in Fig. 7. These markers show that the optimal relay policy should be more conservative (smaller) if $\kappa \cdot \gamma_h$ is larger.

6 Conclusions

A partial relay-based cooperative PU detection scheme that did not require any dedicate reporting channels was

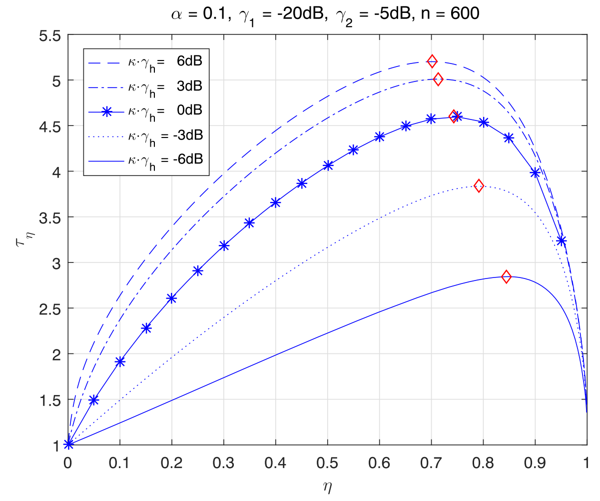

Fig. 7 Impacts of relay gain and relay channel

proposed in this paper. False alarm and missed detection probabilities of the proposed scheme were analyzed. How to determine its relay policy was also investigated. Taking the criterion of balancing detection accuracy for example, closed-form expression, lower bound, and upper bound of the optimal relay policy were deduced. We proved that, in order to avoid either of SUs causing too much interference, the SU with higher SNR should spend at least 42.9% and at most 87.2% of its detection period acting as the relay node. Considering the criterion of maximizing detection agility, we showed that, although the performance of relaying SU was inevitably impaired, detection agility of the whole CR network could be improved by as much as 630% via choosing an optimal relay policy.

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Authors' contributions
The majority of this work is done by SP. All other authors revised the manuscript. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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