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# Multiple-level power allocation strategy for secondary users in cognitive radio networks

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**Abstract**

In this paper, we propose a new multiple-level power allocation strategy for the secondary user (SU) in cognitive radio (CR) networks. Different from the conventional strategies, where SU either stays silent or transmit with a constant/binary power depending on the busy/idle status of the primary user (PU), the proposed strategy allows SU to choose different power levels according to a carefully designed function of the receiving energy. The way of the power level selection at SU side is optimized to maximize the achievable rate of SU under the constraints of average transmit power at SU and average interference temperature to PU. Simulation results demonstrate that the proposed strategy can significantly improve the throughput of SU compared to the conventional strategies.

**Keywords:** Cognitive radio (CR); Multiple-level power allocation; Spectrum sensing; Statistical reliability; Sensing-based spectrum sharing

## Introduction

Cognitive radio (CR) has recently emerged as a promising technology to improve spectrum utilization and to solve the spectrum scarcity problem [1]. Consequently, spectrum sensing and power allocation play as two key functionalities of a CR system, which involves monitoring the spectrum usage and accessing the primary band under given interference constraints.

The earliest spectrum access approach is the *opportunistic spectrum access* where secondary user (SU) can only access the primary band when it is detected to be idle [2]. The second approach is the *underlay* where SU is allowed to transmit beneath the primary user (PU) signal, while sensing is not needed as long as the quality of service (QoS) of PU is protected [3]. The recent approach, *sensing-based spectrum sharing*, performs spectrum sensing to determine the status of PU and then accesses the primary band with a high transmit power if PU is claimed to be absent, or with a low power otherwise [4,5]. These three approaches adopt either constant or binary power allocation at SU which is too 'hard' and limits the performance of SU.

In order to make the power allocation 'softer,' we propose a multiple-level power allocation strategy for SU, where the power level used at SU varies based on its receiving energy during the sensing period. It can be easily known that the conventional constant or binary power allocations are special cases of the proposed strategy. The whole strategy is composed of two phases: (i) sensing phase, where the receiving energy is accumulated and the transmit power of SU is decided; (ii) transmission phase, where SU sends its own data with the corresponding power level.

Under the constraints of the average transmit power at SU and the average interference temperature to PU, the sensing duration, energy threshold, and power levels are optimized to maximize the average achievable rate at SU.

## System model

Consider a CR network with a pair of primary and secondary transceivers as depicted in Figure 1. Let $g_1$, $g_2$, $\gamma$, and $h$ denote the instantaneous channel power gains from the primary transmitter (PT) to the secondary transmitter (ST), from PT to the secondary receiver (SR), from ST to the primary receiver (PR), and from ST to SR, respectively. We consider the simplest case that the channel gains are assumed to be constant and known at the secondary systems and mainly focus on broadcasting the idea of multiple-level power allocation strategy. The idea

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**Figure 1 System model of the cognitive radio network.**
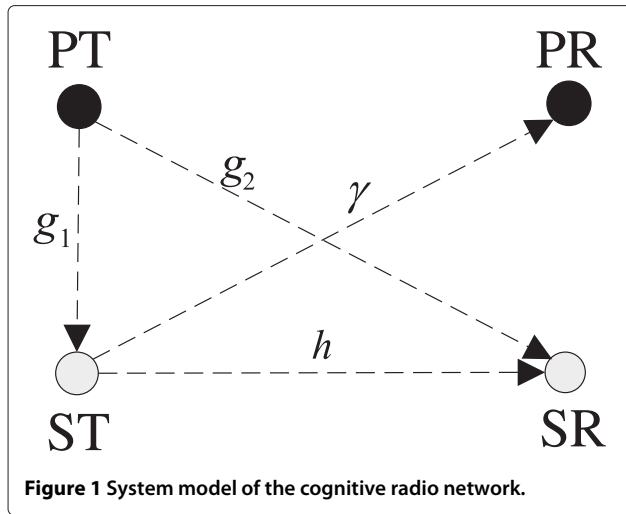
and the results can be extended to other cases of full/statistic/partial channel information in the future work.

One data frame of CR is divided into the sensing slot with duration $\tau$ and the transmission slot with duration $T - \tau$. During the sensing slot, ST listens to the primary channel and obtains its accumulated energy. In the conventional schemes, spectrum sensing is performed in this slot and the decisions on the status (active/idle) of the channels are made. When transmitting, ST accesses the primary band with the optimal power in order to maximize the throughput while at the same time keeping the interference to PR.

During the sensing slot, the $j$th received sample symbol at ST is

$$
r_j = \begin{cases} n_j, & H_0, \\ \sqrt{g_1}e^{j\phi}s_j + n_j, & H_1, \end{cases} \tag{1}
$$

where $H_0$ and $H_1$ denote the hypothesis that PT is absent and present, respectively; $\phi$ is the instant channel phase that is unknown; $n_j$ is the additive noise which is assumed to follow a circularly symmetric complex Gaussian distribution with zero mean and variance $N_0$, i.e., $n_j \backsim \mathcal{N}_c(0, N_0)$; $s_j$ is the $j$th symbol transmitted from PT. For the purpose of computing the achievable channel rate, the transmitted symbols $s_j$ from the Gaussian constellation are typically assumed [4,5], i.e., $s_j \backsim \mathcal{N}_c(0, P_p)$, where $P_p$ is the symbol power. Without loss of generality, we assume that $s_j$ and $n_j$ are independent of each other.

During the sensing period, the detection statistic $x$ using the accumulated received sample energy can be written as

$$
x = \sum_{j=1}^{\tau f_s} |r_j|^2, \tag{2}
$$

where $f_s$ is the sampling frequency at ST. Then the probability density functions (pdf), conditioned on $H_0$ and $H_1$, are given by [6]

$$
\begin{aligned}
f(x|H_0) &= \frac{x^{\tau f_s - 1}e^{-\frac{x}{N_0}}}{\Gamma(\tau f_s)N_0^{\tau f_s}}, \\
f(x|H_1) &= \frac{x^{\tau f_s - 1}e^{-\frac{x}{N_0 + g_1 P_p}}}{\Gamma(\tau f_s)(N_0 + g_1 P_p)^{\tau f_s}},
\end{aligned} \tag{3}
$$

where $\Gamma(.)$ is the gamma function defined as $\Gamma(x) = \int_0^{+\infty} t^{x-1}e^{-t}dt$. Indeed, $f(x|H_0)$ and $f(x|H_1)$ are both variants of the Gamma distribution.

In the conventional CR, ST compares $x$ with a threshold $\rho$ and makes decision according to $x \underset{H_0}{\overset{H_1}{\gtrless}} \rho$. Specifically,

- In opportunistic spectrum access approach, ST can only access the primary band when $x < \rho$ (it means $H_0$) [7,8].
- In sensing-based spectrum sharing, if $x < \rho$, ST transmits with one higher power and otherwise with a lower power (binary power) [9].
- In underly approach, ST transmits with a constant power for all $x$ according to the interference constraint at PU (constant power). No sensing time slot is needed.

## Proposed multiple-level power allocation strategy

It can be easily realized that the conventional constant or binary power of SU does not fully exploit the capability of the co-existing transmission. Motivated by this, we propose a multiple-level power allocation strategy for SU to improve the average achievable rate.

### Strategy of multiple-level power allocation

Define $\{\Re_1, \ldots, \Re_M\}$ as $M$ disjoint spaces of the receiving energy $x$, and $\{P_1, \ldots, P_M\}$ as the corresponding allocated power of SU. Then the proposed power allocation strategy can be written as

$$
P(x) = \sum_{i=1}^{M} P_i I_{x \in \Re_i}, \tag{4}
$$

where $I_A$ is the indicating function that $I_A = 1$ if $A$ is true and $I_A = 0$ otherwise. Note that the conventional power allocation rules are special cases when $M = 1$ or 2.

Using (4), the instantaneous rates of SU with receiving $x$, at the absence and the presence of PU, are given by

$$
R(x)|_{H_0} = \sum_{i=1}^{M} \log_2\left(1 + \frac{P_i h}{N_0}\right) I_{x \in \Re_i}, \tag{5}
$$

$$
R(x)|_{H_1} = \sum_{i=1}^{M} \log_2\left(1 + \frac{P_i h}{N_0 + g_2 P_p}\right) I_{x \in \Re_i}, \tag{6}
$$

respectively. Then the average throughput of SU for the proposed multiple-level power allocation strategy using the total probability formula can be formulated as

$$
R = \frac{T-\tau}{T} \sum_{i=1}^{M} \left[ q_0 \log_2 \left( 1 + \frac{P_i h}{N_0} \right) p_{i,0} \right.
$$
$$
\left. + q_1 \log_2 \left( 1 + \frac{P_i h}{N_0 + g_2 P_p} \right) p_{i,1} \right],
$$
(7)

where $q_0$ and $q_1 = 1 - q_0$ are the idle and busy probabilities of the PU respectively; $p_{i,0}$ and $p_{i,1}$ are functions of $\tau$ and can be computed from

$$
p_{i,j} = \Pr(x \in \mathfrak{R}_i | H_j) = \int_0^\infty I_{x \in \mathfrak{R}_i} f(x|H_j) dx, \ j = 0, 1.
$$
(8)

In order to keep the long-term power budget of SU, the average transmit power, denoted by $\bar{P}$, is constrained as

$$
\frac{T-\tau}{T} \sum_{i=1}^{M} P_i \left[ q_0 p_{i,0} + q_1 p_{i,1} \right] \leq \bar{P}.
$$
(9)

Moreover, to protect the QoS of PU, an interference temperature constraint should be applied as well. Under (4), the interference is caused only when PU is present. Denoting $\bar{I}$ as the maximum average allowable interference at PU, the average interference power constraint can be formulated as

$$
\frac{T-\tau}{T} \sum_{i=1}^{M} \gamma q_1 P_i p_{i,1} \leq \bar{I}.
$$
(10)

Our target is to find the optimal space division $\{\mathfrak{R}_i\}$,[a] the power allocation $\{P_i\}$, as well as the sensing time $\tau$ in order to maximize the average achievable rate of SU under the power constraints. The optimization is then formulated as

$$
\max_{\tau, P_i, \mathfrak{R}_i} R
$$
$$
\text{s.t. } (9), (10), \ 0 \leq \tau \leq T, \ P_i \geq 0, \ \forall i.
$$
(11)

The term $\frac{T-\tau}{T}$ means that the power constraints occur in the transmission slot. Note that (11) is nonlinear and non-convex over $\tau$. Hence, following [4,10], we will simply use the one-dimensional search within the interval $[0, T]$ to find the optimal $\tau$, whose complexity is generally acceptable as known from [11,12].

**The algorithm**
The Lloyd's algorithm is employed here to solve problem (11), where the local convergence has been proved for some cases in one-dimensional space. But in general, there is no guarantee that Lloyd's algorithm will converge to the global optimal [13]. Starting from a feasible solution as the initial value, e.g., subspaces $\{\mathfrak{R}_i\}$ satisfying $p_{i,0} = \frac{1}{M}$, we

repeat the following two steps until the convergence: step 1 - determine the power allocations $\{P_i\}$ given the subspaces $\{\mathfrak{R}_i\}$; step 2 - determine the subspaces $\{\mathfrak{R}_i\}$ given power allocations $\{P_i\}$.

*Subspaces design*
First, we demonstrate that the design of the optimal subspace division $\{\mathfrak{R}_i\}$ and power allocation $\{P_i\}$ is equivalent to a modified *distortion measure* design [14]. Incorporating the power constraints by the Lagrange multipliers $\lambda$ and $\mu$, we define the following *distortion measure* for optimizing the rate

$$
R(x, P_i) = q_0 \log_2 \left( 1 + \frac{P_i h}{N_0} \right) f(x|H_0) - \mu q_1 \gamma P_i f(x|H_1)
$$
$$
+ q_1 \log_2 \left( 1 + \frac{P_i h}{N_0 + g_2 P_p} \right) f(x|H_1)
$$
$$
- \lambda P_i \left[ q_0 f(x|H_0) + q_1 f(x|H_1) \right].
$$
(12)

The optimization problem in (11) is equivalent to selecting $\{\mathfrak{R}_i\}$ and $\{P_i\}$ to maximize the *average distortion* given by

$$
R = \frac{T-\tau}{T} \sum_{i=1}^{M} \int_{x \in \mathfrak{R}_i} R(x, P_i) dx.
$$
(13)

The optimal subspaces $\{\mathfrak{R}_i\}$ are then determined by the *farthest neighbor rule* [14] as

$$
\mathfrak{R}_i = \{x : R(x, P_i) \geq R(x, P_k), \ \forall k \neq i\}.
$$
(14)

The following lemma is instrumental to deriving the optimal subspaces $\{\mathfrak{R}_i\}$.

**Lemma 1.** *For $x_1 < x_2 < x_3$, if $x_1 \in \mathfrak{R}_i$, $x_2 \in \mathfrak{R}_k$ and $i \neq k$, then $x_3 \notin \mathfrak{R}_i$ must hold.*

*Proof.* Define a function of $x$ as

$$
S_{i,k}(x) = R(x, P_i) - R(x, P_k)
$$
$$
= \frac{x^{\tau f_s - 1} e^{-\frac{x}{N_0}}}{\Gamma(\tau f_s)} \left[ \frac{a_{i,k}}{(N_0 + g_2 P_p)^{\tau f_s}} e^{\frac{x g_2 P_p}{N_0(N_0 + g_2 P_p)}} + \frac{b_{i,k}}{N_0^{\tau f_s}} \right],
$$

where

$$
a_{i,k} = q_1 \left[ \log_2 \left( 1 + \frac{P_i h}{N_0 + g_2 P_p} \right) - \log_2 \left( 1 + \frac{P_k h}{N_0 + g_2 P_p} \right) \right]
$$
$$
- \lambda q_1 (P_i - P_k) - \mu q_1 \gamma (P_i - P_k),
$$
$$
b_{i,k} = q_0 \left[ \log_2 \left( 1 + \frac{P_i h}{N_0} \right) - \log_2 \left( 1 + \frac{P_k h}{N_0} \right) \right]
$$
$$
- \lambda q_0 (P_i - P_k).
$$

From $x_1 \in \mathfrak{R}_i$, $x_2 \in \mathfrak{R}_k$ and (14), we know that $S_{i,k}(x_1) > 0$ and $S_{i,k}(x_2) < 0$. In (14), the sign of $S_{i,k}(x)$ is decided by $\frac{a_{i,k}}{(N_0 + g_2 P_p)^{\tau f_s}} e^{\frac{x g_2 P_p}{N_0(N_0 + g_2 P_p)}} + \frac{b_{i,k}}{N_0^{\tau f_s}}$ which is a strictly monotonic

function. Thus, for any $x_3 > x_2$, there are $S_{i,k}(x_3) < 0$ and $x_3 \notin \Re_i$. □

**Proposition 1.** $\Re_i, i = 1, \ldots, M$ *are continuous intervals and satisfy* $\bigcup_{i=1,\ldots,M} \Re_i = [0, \infty]$.

*Proof.* The proof can be easily obtained from the law of contradiction. Assuming that $\Re_i$ has more than two non-continuous intervals, it is contradicted with Lemma 1. This proposition is instrumental to obtaining the explicit formulation of $\Re_i$. □

Define $M + 1$ thresholds $\eta_0, \eta_1, \ldots, \eta_M$ with $\eta_0 = 0$, $\eta_M = +\infty$. Thus, $\Re_i$ corresponds to one of $[\eta_{j-1}, \eta_j)$, $j = [1, \ldots, M]$. Based on Lemma 1, we can calculate $\eta_j$ sequentially and assign $\{\Re_i\}$ in Algorithm 1. The answer of $x_k$ that satisfies $S_{i,k}(x_k) = 0$ is given by

$$x_k = \frac{N_0(N_0 + g_2 P_p)}{g_2 P_p} \cdot \ln\left(\frac{-b_{i,k}(N_0 + g_2 P_p)^{\tau f_s}}{a_{i,k} N_0^{\tau f_s}}\right). \tag{15}$$

---

**Algorithm 1** Subspaces design for $x$ given $\{P_i\}$

---

▶ Initialize the set $\Theta = \{1, \ldots, M\}$;
   Set $i = \arg \max_{j \in \Theta} R(0, P_j)$, $\Theta \leftarrow \Theta \setminus i$
▶ For $l = 1 : M - 1$, do
   1) Calculate $x_k$ that satisfies $S_{i,k}(x_k) = 0, k \in \Theta$
   2) Set $\eta_l = \min_{k \in \Theta} x_k$. Assign $\Re_i = [\eta_{l-1}, \eta_l)$
   3) Set $i = \arg \min_{k \in \Theta} x_k$, $\Theta \leftarrow \Theta \setminus i$
▶ End for
▶ Set the last element in $\Theta$ as $i$, $\Re_i = [\eta_{M-1}, \eta_M)$

---

*Power allocation*
After obtaining the threshold $\eta_i$, the probabilities $p_{i,j}$ in (11) can be explicitly expressed as

$$p_{i,j} = \int_{\eta_{i-1}}^{\eta_i} f(x|H_j)dx, \ i \in [1, \ldots, M], \ j = 0, 1. \tag{16}$$

Let us first write the lagrangian $L(P_i, \lambda, \mu)$ for problem (11) under the constraints (9) and (10) as

$$L(P_i, \lambda, \mu) = R + \lambda\left(\bar{P} - \frac{T - \tau}{T}\sum_{i=1}^{M} P_i\left[q_0 p_{i,0} + q_1 p_{i,1}\right]\right)$$
$$+ \mu\left(\bar{I} - \frac{T - \tau}{T}\sum_{i=1}^{M} q_1 \gamma P_i p_{i,1}\right), \tag{17}$$

where $\lambda, \mu \geq 0$ are dual variables corresponding to (9) and (10). The Lagrange dual optimization can be formulated as

$$\min_{\lambda \geq 0, \ \mu \geq 0} g(\lambda, \mu) \triangleq \sup_{P_i \geq 0} L(P_i, \lambda, \mu). \tag{18}$$

In (11), $\frac{\partial^2 R}{\partial^2 P_i} = -\frac{T-\tau}{T}\left\{\frac{\log_2(e)q_0 p_{i,0}}{(P_i+N_0/h)^2} + \frac{\log_2(e)q_1 p_{i,1}}{(P_0+(N_0+g_2 P_p)/h)^2}\right\} < 0$, and $\frac{\partial^2 R}{\partial P_i \partial P_j} = 0, i \neq j$. Since the constraints are linear functions, problem (11) is concave over $P_i$. Thus the optimal value $P_i$ of problem (18) is equal to that of (11), and we can solve (18) instead of (11). From (18), we have to obtain the supremum of $L(P_i, \lambda, \mu)$. Taking the derivative of $L(P_i, \lambda, \mu)$ with respect to $P_i$ leads to

$$\frac{\partial L(P_i, \lambda, \mu)}{\partial P_i} = \frac{T - \tau}{T}\left\{\frac{\log_2(e)q_0 p_{i,0}}{P_i + N_0/h} + \frac{\log_2(e)q_1 p_{i,1}}{P_i + (N_0 + g_2 P_p)/h}\right.$$
$$\left. -\lambda\left[q_0 p_{i,0} + q_1 p_{i,1}\right] - \mu q_1 \gamma p_{i,1}\right\}. \tag{19}$$

By setting the above equation to 0 and applying the constraint $P_i \geq 0$, the optimal power allocation $P_i$ for given Lagrange multipliers $\lambda$ and $\mu$ is computed as

$$P_i = \left[\frac{A_i + \sqrt{\Delta_i}}{2}\right]^+, \tag{20}$$

where $[x]^+$ denotes max $(0, x)$, and

$$A_i = \frac{\log_2(e)\left[q_0 p_{i,0} + q_1 p_{i,1}\right]}{\lambda\left[q_0 p_{i,0} + q_1 p_{i,1}\right] + \mu q_1 \gamma p_{i,1}} - \frac{2N_0 + g_2 P_p}{h}, \tag{21}$$

$$\Delta_i = A_i^2 + \frac{4}{h}\left\{\frac{\log_2(e)\left[q_0 p_{i,0}(N_0 + g_2 P_p) + q_1 p_{i,1} N_0\right]}{\lambda\left[q_0 p_{i,0} + q_1 p_{i,1}\right] + \mu q_1 \gamma p_{i,1}}\right.$$
$$\left. - \frac{N_0(N_0 + g_2 P_p)}{h}\right\}. \tag{22}$$

**Proposition 2.** *The power allocation functions* $P_i$ *are non-increasing over i.*

*Proof.* First, from (3), we have

$$\frac{f(x|H_1)}{f(x|H_0)} = e^{\frac{xg_1 P_p}{N_0(N_0 + g_1 P_p)}}\left(\frac{N_0}{N_0 + g_1 P_p}\right)^{\tau f_s}, \tag{23}$$

and obviously it is an increasing function over $x$. Through some simple manipulations, the monotonicity of $A_i$ is equivalent to the monotonicity of the following term:

$$C_i = \frac{1 + \frac{q_1}{q_0}\frac{p(i,1)}{p(i,0)}}{1 + (1 + \mu\gamma/\lambda)\frac{q_1}{q_0}\frac{p(i,1)}{p(i,0)}}. \tag{24}$$

From (23), we can get that

$$\frac{p(i,1)}{p(i,0)} > \frac{p(i+1,1)}{p(i+1,0)}, \forall i. \tag{25}$$

Jointly from (24) and (25), we know that $A_i$ is a decreasing function over $i$. The monotonicity of $\frac{4}{h}\left\{\frac{\log_2(e)[q_0 p_{i,0}(N_0+g_2 P_p)+q_1 p_{i,1}N_0]}{\lambda[q_0 p_{i,0}+q_1 p_{i,1}]+\mu q_1 \gamma p_{i,1}} - \frac{N_0(N_0+g_2 P_p)}{h}\right\}$ is equivalent to the monotonicity of the following term:

$$D_i = \frac{1+\frac{N_0}{N_0+g_2 P_p}\frac{q_1}{q_0}\frac{p(i,1)}{p(i,0)}}{1+(1+\mu\gamma/\lambda)\frac{q_1}{q_0}\frac{p(i,1)}{p(i,0)}}. \tag{26}$$

Similarly, we get that $\triangle_i$ is a decreasing function over $i$. Thus, from (20), we can conclude that $P_i$ is a non-increasing function with respect to $i$. □

*Remark 1.* Proposition 2 shows that at smaller $x$, the probability of PU being busy is smaller, so SU can use higher transmit power to better exploit the primary band. On the other hand, at the larger $x$, lower transmit power should be used to prevent harmful interference to PU. Thus, the proposed multiple-level power allocation strategy can also be defined on the probability of PU being busy.

Subgradient-based methods are used here to find the optimal Lagrange multipliers $\lambda$ and $\mu$, e.g., the ellipsoid method and the Newton's method [15]. The subgradient of $g(\lambda,\mu)$ is $[C,D]^T$, where

$$C = \bar{P} - \frac{T-\tau}{T}\sum_{i=1}^{M}\bar{P}_i\left[q_0 p_{i,0}+q_1 p_{i,1}\right],$$

$$D = \bar{I} - \frac{T-\tau}{T}\sum_{i=1}^{M}q_1\gamma\bar{P}_i p_{i,1}, \tag{27}$$

while $\bar{P}_i$ is the optimal power allocation for fixed $\lambda$ and $\mu$ [16]. Finally, we summarize the algorithm that computes the sensing time and multiple-level power allocations in Algorithm 2.

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**Algorithm 2 Sensing time and multiple-level power allocations**

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▶ For each $\tau$ in $[0,T]$, do
  1) Initialize $\lambda$, $\mu$, $\eta_i$ satisfying $p_{i,0} = \frac{1}{M}$
  2) Repeat until $\{\Re_i\}$ converge:
    - Get $\{P_i\}$ using (20); Update $\lambda$ and $\mu$ using the subgradient-based method; Until $\lambda$ and $\mu$ converge
    - Update $\{\Re_i\}$ using Algorithm 1
▶ End for
▶ Optimal parameters: $\tau^* = \arg\max_{\tau} R(\tau, P_i, \Re_i)$,
$(P_i^*, \Re_i^*) = (P_i, \Re_i)|_{\tau=\tau^*}$

---

*Remark 2.* All computations are performed offline and the resulting power control rule is stored in a look-up table for real-time implementation. Thus, the computational complexity is not significant.
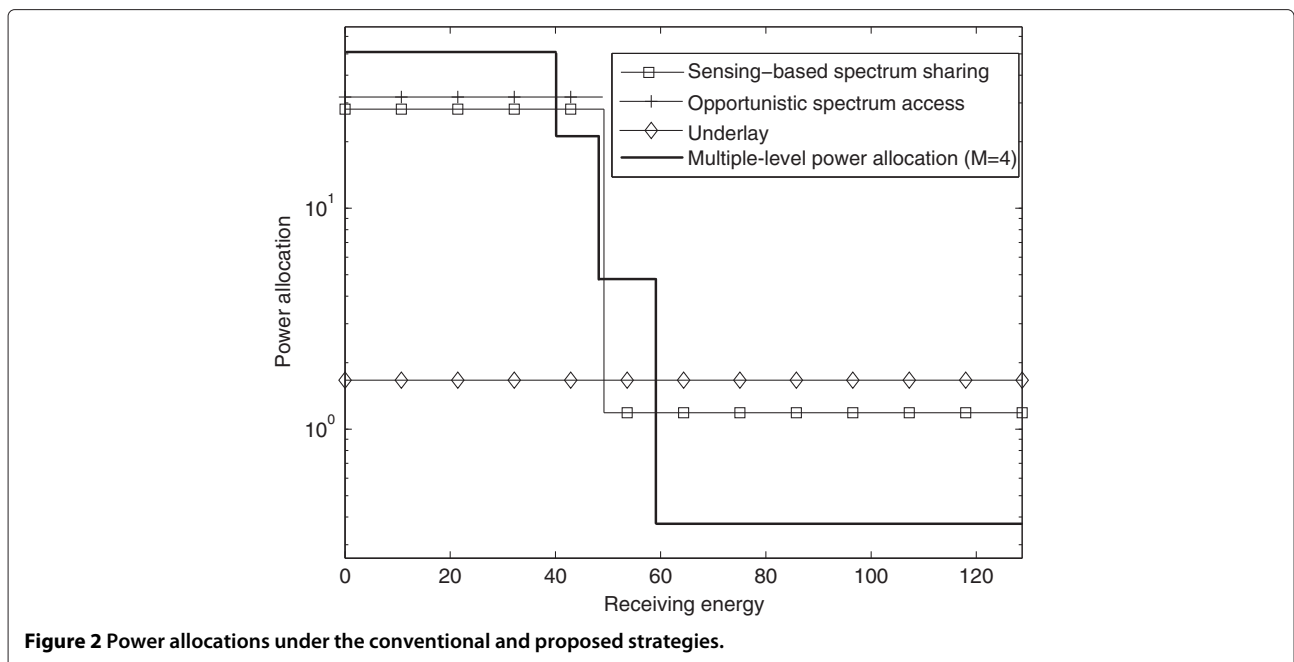


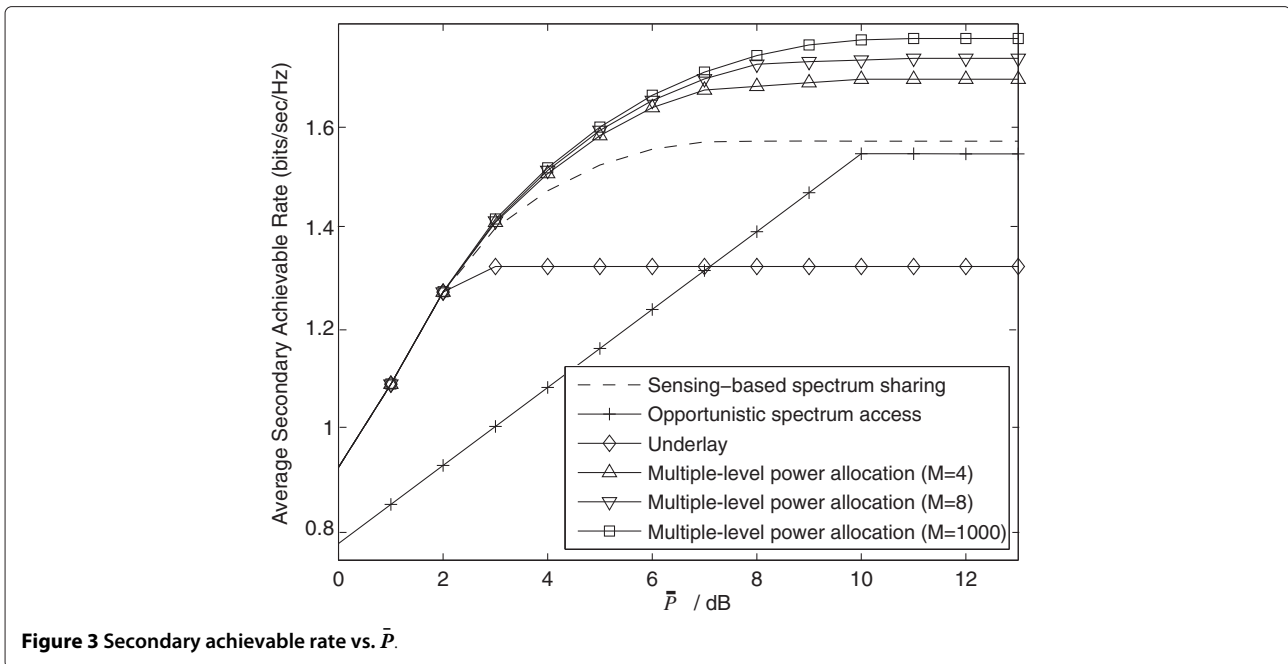**Figure 2 Power allocations under the conventional and proposed strategies.**

**Figure 3 Secondary achievable rate vs. $\bar{P}$.**

## Simulation results

In this section, simulations are performed to evaluate the proposed multiple-level power allocation strategy in a CR system where the system parameters similar to the references [4,5,10] are used. The frame duration is taken as $T = 100$ ms and the sampling frequency $f_s = 1$ MHz. The target detection probability is set as 0.9 in the opportunistic spectrum access scheme. We set $g_1 = N_0 = 0$ dB, $q_0 = 0.7$, $\bar{I} = P_p = 0.5$, $\bar{P} = 10$ dB, $\gamma = h = g_2 = 0$ dB, unless otherwise mentioned.

Figure 2 compares the power allocations under the conventional strategies as well as the proposed one. The figure shows that $P_i$ for the proposed strategy is a non-increasing function of the received signal energy. When $x$ is small, the proposed strategy allocates more power than the conventional ones, while when $x$ is large, it allocates less power, thus the average transmit powers are the same for all the strategies.

Figure 3 shows the average secondary achievable rate. In the low $\bar{P}$ region, the proposed strategy and the conventional ones have the same rates. However, when $\bar{P}$ is high, the proposed strategy achieves much higher rates. The rates of all strategies flatten out when $\bar{P}$ is sufficiently large since the rate is decided by $\bar{I}$ under this condition. When $M$ increases, the rate of the proposed strategy becomes larger, but the gain does not improve much when $M$ is large. As $M$ becomes extremely large, say $M = 1,000$ in the figure, the rate approaches an upper limit. In practice, we can choose the right $M$ to tradeoff the system complexity and performance, and in this example $M = 4$ serves as a good choice.

## Conclusions

In this paper, we propose a novel multiple-level power allocation strategy for SU in a CR system. The receiving signal energy from PU is divided into different categories and SU transmits with different power for each category. The power levels at SU are obtained by maximizing the average achievable rate under the constraints of the average transmit power at SU and the average interference temperature to PU. Compared with the conventional power allocation strategies, the proposed scheme offers significant rate improvement for SU. Furthermore, we are working on extending the idea to the cases of multiple SUs and statistic channel information and on how to decrease the computational complexity.

## Endnote

[a]Namely, we have multiple thresholds to categorize $x$ rather than only using $\rho$ as did in convention.

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