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An approximate solution of fractional cable equation by homotopy analysis method

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Abstract

In this article, the homotopy analysis method (HAM) is applied to solve the fractional cable equation by the Riemann-Liouville fractional partial derivative. This method includes an auxiliary parameter h which provides a convenient way of adjusting and controlling the convergence region of the series solution. In this study, approximate solutions of the fractional cable equation are obtained by HAM. We also give a convergence theorem for this equation. A suitable value for the auxiliary parameter h is determined and results obtained are presented by tables and figures.

Keywords: cable equation; fractional differential equations; fractional cable equation; homotopy analysis method

1 Introduction

Fractional calculus has a very long history. However, this field lagged behind classic analysis. In fact, the basis of fractional calculus depended on classic analysis. Especially, in recent years fractional differential equations were used in fluid mechanics, viscoelasticity, biology, pharmacy, physics, chemistry and biochemistry, hydrology, medicine, finance, and engineering. The fractional-order models are more useful than integer-order models in many cases. Structures having fractional order are more useful in the studies that have been done by developing technology.

However, the analytic solutions of most fractional differential equations generally cannot be obtained. Thus, fractional differential equations have been solved by many approximate methods. Examples are the homotopy perturbation method [1, 2], the method of separating variables [3], the iteration method [4], the decomposition method [5], and the homotopy analysis method [6].

In this study, we will consider the cable equation that has been used in modeling the ion electro diffusion at the neurons. The cable equation occurred due to anomalous diffusion and this equation is one of the most fundamental equations for modeling neuronal dynamics [7]. The cable equation can be derived from the Nernst-Planck equation for electrodiffusion in smooth homogeneous cylinders [8]. In recent years, studies were conducted on various biological and physical systems. In this equation, the diffusion rate of species cannot be characterized by the single parameter of the diffusion constant [7]. The anomalous diffusion is characterized by a scaling parameter γ as well as the diffusion constant D and the mean square displacement of diffusing species $\langle r^2(t)\rangle$ scales as a nonlinear power law in time, *i.e.*, $\langle r^2(t)\rangle \sim t^\gamma$ [7–9]. Henry *et al.* derived a fractional cable equation from the fractional Nernst-Planck equations to model anomalous electrodiffusion of ions in spiny dentrites [9]. They subsequently found a fractional cable equation by treating the neuron and its membrane as two separate materials governed by separate fractional Nernst-Planck equations. As a result, the fractional cable equation includes two Riemann-Liouville fractional derivatives.

Consider the following fractional cable equation:

$$\frac{\partial u(x,t)}{\partial t} = {}_0D_t^{1-\gamma_1}\left(K\frac{\partial^2 u(x,t)}{\partial x^2}\right) - \mu_0^2\,{}_0D_t^{1-\gamma_2}u(x,t) + f(x,t), \tag{1.1}$$

$$u(x,0) = g(x), \quad 0 \le x \le L, \tag{1.2}$$

$$u(0,t) = \varphi(t), \qquad u(L,t) = \psi(t), \quad 0 \le t \le T, \tag{1.3}$$

where $0 < \gamma_1, \gamma_2 < 1$, $K > 0$ and μ_0^2 are constants, and ${}_0D_t^{1-\gamma_1}u(x,t)$ is the Riemann-Liouville fractional partial derivative of order $1-\gamma$ [9].

In the literature, there are few treatments of approximate solutions of the fractional cable equation in terms of (1.1). Equation (1.1) has been solved by implicit numerical methods (INM) [9], the implicit compact difference scheme (ICFDS) [10], and explicit numerical methods [11].

Here, we will use the HAM, which is an approximate solution to solve this equation. The HAM method was developed in 1992 by Liao in [12]. This method has been successfully applied by many authors [13–17]. The HAM contains the auxiliary parameter h which provides us with a simple way to adjust and control the convergence region of solution series for large or small values of x and t.

2 Preliminaries and notations

We give some basic definitions and properties of the fractional calculus theory, which are used further in this paper.

Definition 2.1 The Euler Gamma function $\Gamma(z)$ is defined by the so-called Euler integral of the second kind,

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,dt \quad \bigl(R(z) > 0\bigr), \tag{2.1}$$

where $t^{z-1} = e^{(z-1)\log t}$. This integral is convergent for all complex $z \notin \mathbb{C}$ [18].

Definition 2.2 The Riemann-Liouville fractional integral operator of order $\alpha \ge 0$ of a function $f \in C_\mu$, $\mu \ge -1$, is defined as

$$D_t^{-\alpha}u(x,t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}u(x,\tau)\,d\tau, \quad \alpha > 0, t > 0, \tag{2.2}$$

and properties of the operator $D^{-\alpha}$ can be found in [19, 20]. Also, some of properties of operator $D^{-\alpha}$ are as follows:

(i) $D_t^{-\alpha}f(t) = f(t)$,

(ii) $D_x^{-\alpha}\bigl(x^\gamma\bigr) = \dfrac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}x^{\alpha+\gamma}$,

$$D_t^{-\alpha}\left(t^{\gamma}\right) = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}t^{\alpha+\gamma},$$

(iii) $D_t^{-\alpha}D_t^{-\beta}f(t) = D_t^{-(\alpha+\beta)}f(t)$,

(iv) $D_t^{-\alpha}D_t^{-\beta}f(t) = D_t^{-\beta}D_t^{-\alpha}f(t)$.

3 Homotopy analysis method

We consider the following differential equation:

$$N\left[u(x,t)\right] = 0, \tag{3.1}$$

where N is a nonlinear differential operator, x and t denote independent variable; $u(x,t)$ is an unknown function. By means of the HAM, one first constructs a zeroth-order deformation equation

$$(1-q)L\left[\phi(x,t;q) - u_0(x,t)\right] = qhH(t)N\left[\phi(x,t;q)\right], \tag{3.2}$$

where $q \in [0,1]$ is the embedding parameter, $h \neq 0$ is a non-zero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, L is an auxiliary linear operator, $u_0(x,t)$ is an initial guess of $u(x,t)$, and $\phi(x,t;q)$ is an unknown function. It is important that one has great freedom to choose auxiliary things in the HAM. Obviously, when $q = 0$ and $q = 1$, we have

$$\phi(x,t;0) = u_0(x,t), \qquad \phi(x,t;1) = u(x,t), \tag{3.3}$$

respectively. The solution $\phi(x,t;q)$ varies from the initial guess $u_0(x,t)$ to the solution $u(x,t)$. Expanding $\phi(x,t;q)$ in a Taylor series about the embedding parameter, we have

$$\phi(x,t;q) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)q^m, \tag{3.4}$$

where

$$u_m(x,t) = \frac{1}{m!}\frac{\partial^m \phi(x,t;q)}{\partial q^m}\bigg|_{q=0}, \qquad m = 1,2,3,\ldots. \tag{3.5}$$

The convergence of the series (3.4) depends upon the auxiliary parameter h. If it is convergent at $q = 1$, one has

$$u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t). \tag{3.6}$$

According to (3.6), the governing equation can be deduced from the zeroth-order deformation equation (3.2). Define the vector

$$\vec{u}_n = \left\{u_0(x,t), u_1(x,t), u_2(x,t), \ldots, u_n(x,t)\right\}.$$

Differentiating (3.2) m times with respect to the embedding parameter q and then setting $q = 0$ and finally dividing by $m!$, we have the so-called mth-order deformation equation

$$L\left[u_m(x,t) - \chi_m u_{m-1}(x,t)\right] = hH(t)R_m(\vec{u}_{m-1},x;t), \tag{3.7}$$

where

$$R_m(\vec{u}_{m-1},x;t) = \frac{1}{(m-1)!}\frac{\partial^{m-1}N[\phi(x,t;q)]}{\partial q^{m-1}}\Big|_{q=0}, \tag{3.8}$$

and

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases} \tag{3.9}$$

It should be emphasized that $u_m(x,t)$ for $m \geq 1$ is governed by the nonlinear equation (3.7) with the linear boundary conditions that come from the original problem, which can easily be solved by symbolic computation software such as Maple and Mathematica.

4 Numerical applications and comparison

Consider the following initial and boundary problem of the fractional cable equation:

$$\frac{\partial u(x,t)}{\partial t} = {}_0D_t^{1-\gamma_1}\frac{\partial^2 u(x,t)}{\partial x^2} - {}_0D_t^{1-\gamma_2}u(x,t) + f(x,t), \tag{4.1}$$

$$u(x,0) = 0, \quad 0 \leq x \leq 1, \tag{4.2}$$

$$u(0,t) = 0, \qquad u(1,t) = 0, \quad 0 \leq t \leq T, \tag{4.3}$$

where $f(x,t) = 2(t + \frac{\pi^2 t^{1+\gamma_1}}{\Gamma(2+\gamma_1)} + \frac{t^{1+\gamma_2}}{\Gamma(2+\gamma_2)})\sin \pi x$. The exact solution of (4.1)-(4.3) is $u(x,t) = t^2 \sin \pi x$ [9].

We choose the linear operator

$$L\left[\phi(x,t;q)\right] = \frac{\partial}{\partial t}\phi(x,t;q), \tag{4.4}$$

with the property $L[C] = 0$ where C is a constant. We define a nonlinear operator by

$$N\left[\phi(x,t,q)\right] = D_t\phi(x,t,q) - {}_0D_t^{1-\gamma_1}\frac{\partial^2\phi(x,t,q)}{\partial x^2} + {}_0D_t^{1-\gamma_2}\phi(x,t,q) - f(x,t). \tag{4.5}$$

Therefore we establish the zeroth-order deformation equation

$$(1-q)L\left[\phi(x,t,q) - u_0(x,t)\right] = qhH(t)N\left[\phi(x,t,q)\right]. \tag{4.6}$$

In (4.6), $q = 0$ and $q = 1$, we can write

$$\phi(x,t,0) = u_0(x,t), \qquad \phi(x,t,1) = u(x,t). \tag{4.7}$$

So we obtain the mth-order deformation equation

$$L\left[u_m(x,t) - \chi_m u_{m-1}(x,t)\right] = hH(t)R_m\left(\vec{u}_{m-1}(x,t)\right), \tag{4.8}$$

where

$$R_m\big(\vec{u}_{m-1}(x,t)\big) = \frac{\partial u_{m-1}(x,t)}{\partial t} - {}_0D_t^{1-\gamma_1}\frac{\partial^2 u_{m-1}(x,t)}{\partial x^2} + {}_0D_t^{1-\gamma_2}u_{m-1}(x,t) - f(x,t) \tag{4.9}$$

and

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases} \tag{4.10}$$

Now the solution of the mth-order deformation equation (4.8) for $m \ge 1$ becomes

$$u_m(x,t) = \chi_m u_{m-1}(x,t) + hH(t)L^{-1}\big[R_m\big(\vec{u}_{m-1}(x,t)\big)\big]. \tag{4.11}$$

Instead of $R_m(\vec{u}_{m-1})$,

$$\begin{aligned} u_m(x,t) = {}& \chi_m u_{m-1}(x,t) + hH(t)\int_0^t \bigg(\frac{\partial u_{m-1}(x,t)}{\partial t} - {}_0D_t^{1-\gamma_1}\frac{\partial^2 u_{m-1}(x,t)}{\partial x^2} \\ & + {}_0D_t^{1-\gamma_2}u_{m-1}(x,t) - f(x,t)\bigg)\,dt \end{aligned} \tag{4.12}$$

can be written. The auxiliary function $H(t)$ can be chosen in the form $H(t) = 1$.

Rearrangement of (4.12) gives the mth-order deformation equation

$$u_m(x,t) = \chi_m u_{m-1}(x,t) + h\int_0^t \big(R_m\big(\overrightarrow{u}_{m-1}(x,t)\big)\big)\,dt. \tag{4.13}$$

Therefore, some of the symbolically computed components are found as

$$\begin{aligned}
& u_0(x,t) = 0, \\
& u_1(x,t) = ht^2\sin\pi x\bigg(-1 - \frac{2\pi^2 t^{\gamma_1}}{\Gamma(3+\gamma_1)} - \frac{2t^{\gamma_2}}{\Gamma(3+\gamma_2)}\bigg), \\
& u_2(x,t) = u_1(x,t) + h\big(-\big(t^2\sin\pi x\big(2\big(2(1+2h)\pi^2 t^{\gamma_1}\Gamma(3+2\gamma_1)\Gamma(3+\gamma_2) \\
& \qquad \times \Gamma(2+\gamma_1+\gamma_2)\Gamma(3+2\gamma_2) + \Gamma(3+\gamma_1)\big(2h\pi^4 t^{2\gamma_1}\Gamma(3+\gamma_2) \\
& \qquad \times \Gamma(2+\gamma_1+\gamma_2)\Gamma(3+2\gamma_2) + \cdots, \\
& u_3(x,t) = u_2(x,t) + h\bigg(-t^2\sin\pi x - \frac{1}{3}ht^3\sin\pi x - \frac{2t^{2+\gamma_2}\sin\pi x}{\Gamma(3+\gamma_2)} - \frac{2ht^{3+\gamma_2}\sin\pi x}{\Gamma(4+\gamma_2)} \\
& \qquad - \frac{4h^2\pi^2 t^{3+\gamma_1+\gamma_2}\sin\pi x}{\Gamma(4+\gamma_1+\gamma_2)} - \frac{h\pi^4 t^{2+2\gamma_1}\gamma_1}{\Gamma(3+2\gamma_1)(1+\gamma_1)} + \frac{(2+\gamma_1)}{(1+\gamma_1)}\frac{\pi^2 t^{2+2\gamma_1}\Gamma(\gamma_1)}{\Gamma(3+2\gamma_1)}\sin\pi x \\
& \qquad + \big(3 + 2ht + 2h^2 t + \gamma_1\big)\frac{2\pi^2 t^{2+\gamma_1}\sin\pi x}{\Gamma(4+\gamma_1)} + \frac{2t^{2+\gamma_1}\sin\pi x}{\gamma_1(2+3\gamma_1+\gamma_1^2)} - \cdots, \\
& \vdots
\end{aligned}$$

and so on.

As a result, the mth-order approximation of $u(x,t)$ is given by

$$\sum_{m=0}^{\infty} u_m(x,t). \tag{4.14}$$

Theorem 4.1 (Convergence Theorem) *As long as the series* $u(x,t) = u_0(x,t) + \sum_{m=1}^{\infty} u_m(x,t)$ *converges, where* $u_m(x,t)$ *is governed by* (4.13) *under the definitions* (4.9) *and* (4.10), *it must be a solution of the fractional cable equation* (4.1).

Proof If the series

$$\sum_{m=0}^{+\infty} u_m(x,t)$$

converges, then we can write

$$S(x,t) = \sum_{m=0}^{+\infty} u_m(x,t)$$

and we have

$$\lim_{n\to\infty} u_n(x,t) = 0. \tag{4.15}$$

Using definition (4.13), we get

$$\begin{aligned}
h\sum_{m=1}^{\infty} R_m\bigl(\vec{u}_m(x,t)\bigr) &= \sum_{m=1}^{\infty} L\bigl[u_m(x,t) - \chi_m u_{m-1}(x,t)\bigr] \\
&= \lim_{n\to\infty}\sum_{m=1}^{\infty} L\bigl[u_m(x,t) - \chi_m u_{m-1}(x,t)\bigr] \\
&= L\left[\lim_{n\to\infty}\sum_{m=1}^{\infty}\bigl(u_m(x,t) - \chi_m u_{m-1}(x,t)\bigr)\right] \\
&= L\Bigl[\lim_{n\to\infty} u_n(x,t)\Bigr] = 0.
\end{aligned}$$

Since $h \neq 0$, $\sum_{m=1}^{\infty} R_m(\vec{u}_m(x,t)) = 0$.

From (4.9), we have

$$\begin{aligned}
\sum_{m=1}^{\infty} R_m\bigl(\vec{u}_m(x,t)\bigr) &= \sum_{m=1}^{\infty}\bigl[D_t u_{m-1}(x,t) - {}_0D_t^{1-\gamma_1} u_{m-1}(x,t)_{xx} + {}_0D_t^{1-\gamma_2} u_{m-1}(x,t) - F(x,t)\bigr] \\
&= \sum_{m=0}^{\infty} D_t u_m(x,t) - \sum_{m=0}^{\infty} {}_0D_t^{1-\gamma_1} u_m(x,t)_{xx} + \sum_{m=0}^{\infty} {}_0D_t^{1-\gamma_2} u_m(x,t) - F(x,t) \\
&= D_t \sum_{m=0}^{\infty} u_m(x,t) - {}_0D_t^{1-\gamma_1} \sum_{m=0}^{\infty} u_m(x,t)_{xx} + {}_0D_t^{1-\gamma_2} \sum_{m=0}^{\infty} u_m(x,t) - F(x,t) \\
&= D_t S(x,t) - {}_0D_t^{1-\gamma_1} S(x,t)_{xx} + {}_0D_t^{1-\gamma_2} S(x,t) - F(x,t) \\
&= 0.
\end{aligned}$$

Table 1 Absolute errors obtained when $\gamma_1 = \gamma_2 = 0.5$, $x = 10^{-4}$, and $h = 1/10^8$

t	INM [9]	ICFDS [10]	HAM
0.1	4.7796×10^{-5}	3.4436×10^{-6}	3.14159×10^{-8}
0.2	2.1914×10^{-4}	6.8604×10^{-6}	6.28319×10^{-8}
0.3	5.2286×10^{-4}	9.8036×10^{-6}	9.42478×10^{-8}
0.4	9.6227×10^{-4}	1.2163×10^{-5}	1.25664×10^{-7}
0.5	1.5392×10^{-3}	1.3893×10^{-5}	1.5708×10^{-7}
0.6	2.2552×10^{-3}	1.4974×10^{-5}	1.88496×10^{-7}
0.7	3.1110×10^{-3}	1.5394×10^{-5}	2.19911×10^{-7}
0.8	4.1015×10^{-3}	1.5141×10^{-5}	2.51327×10^{-7}
0.9	5.2452×10^{-3}	1.4211×10^{-5}	2.82743×10^{-7}
1.0	6.5246×10^{-3}	1.2596×10^{-5}	3.14159×10^{-7}

Table 2 Comparison of the HPM, HAM, exact solution (ES) and absolute errors results of $u(x,t)$ when $\gamma_1 = \gamma_2 = 0.5$, $t = 0.1$, and $h = -0.0395$ for 5th-order approximation

x	HPM	HAM	ES	Error (HPM)	Error (HAM)
0.1	−0.340367	0.00309062	0.00309017	0.343458	4.53322×10^{-7}
0.2	−0.647417	0.00587871	0.00587785	0.653295	8.6227×10^{-7}
0.3	−0.891093	0.00809136	0.00809017	0.899184	1.18681×10^{-6}
0.4	−1.04754	0.00951196	0.00951057	1.05705	1.39518×10^{-6}
0.5	−1.10145	0.0100015	0.01	1.11145	1.46698×10^{-6}
0.6	−1.04754	0.00951196	0.00951057	1.05705	1.39518×10^{-6}
0.7	−0.891093	0.00809136	0.00809017	0.899184	1.18681×10^{-6}
0.8	−0.647417	0.00587871	0.00587785	0.653295	8.6227×10^{-7}
0.9	−0.340367	0.00309062	0.00309017	0.343458	4.53322×10^{-7}

Table 3 Comparison of the HPM, HAM, exact solution (ES) and absolute errors results of $u(x,t)$ when $\gamma_1 = \gamma_2 = 0.25$, $t = 0.1$, and $h = -0.004$ for 10th-order approximation

x	HPM	HAM	ES	Error (HPM)	Error (HAM)
0.1	−8614.19	0.00302986	0.00309017	8614.2	6.0306×10^{-5}
0.2	−16385.2	0.00576314	0.00587785	16385.2	1.14709×10^{-5}
0.3	−22552.2	0.00793229	0.00809017	22552.3	1.57883×10^{-4}
0.4	−26511.8	0.00932496	0.00951057	26511.8	1.85603×10^{-4}
0.5	−27876.1	0.00980485	0.01	27876.1	1.95154×10^{-4}
0.6	−26511.8	0.00932496	0.00951057	26511.8	1.85603×10^{-4}
0.7	−22552.2	0.00793229	0.00809017	22552.3	1.57883×10^{-4}
0.8	−16385.2	0.00576314	0.00587785	16385.2	1.14709×10^{-5}
0.9	−8614.19	0.00302986	0.00309017	8614.2	6.0306×10^{-5}

From the initial $u(x,0) = 0$ and $u_m(x,0) = 0$, we have

$$S(x,0) = \sum_{m=0}^{\infty} u_m(x,0) = u_0(x,t) + \sum_{m=1}^{\infty} u_{m-1}(x,0) = 0. \tag{4.16}$$

Therefore, according to the above expressions, $S(x,t)$ must be the exact solution of (4.1) and (4.2). □

We get the following tables and figures by using a series solution obtained with HAM of (4.1).

5 Conclusion

In this paper, we have achieved approximate solutions of the fractional cable equation that involve two Riemann-Liouville fractional derivatives by means of the homotopy analysis

Table 4 Comparison of the HPM, HAM, exact solution (ES) and absolute errors results of $u(x,t)$ when $\gamma_1 = 0.25$, $\gamma_2 = 0.75$, $t = 0.1$, and $h = -0.0043$ for 10th-order approximation

x	HPM	HAM	ES	Error (HPM)	Error (HAM)
0.1	−5022.62	0.00305436	0.00309017	5022.62	3.58117×10^{-5}
0.2	−9553.59	0.00580973	0.00587785	9553.6	6.8118×10^{-5}
0.3	−13149.4	0.00799641	0.00809017	13149.4	9.37564×10^{-5}
0.4	−15458.0	0.00940035	0.00951057	15458.0	1.10217×10^{-4}
0.5	−16253.5	0.00988411	0.01	16253.6	1.15889×10^{-4}
0.6	−15458.0	0.00940035	0.00951057	15458.0	1.10217×10^{-4}
0.7	−13149.4	0.00799641	0.00809017	13149.4	9.37564×10^{-5}
0.8	−9553.59	0.00580973	0.00587785	9553.6	6.8118×10^{-5}
0.9	−5022.62	0.00305436	0.00309017	5022.62	3.58117×10^{-5}

Figure 1 The *h* curves of 5th-order and 10th-order approximate solutions obtained by the HAM for $\gamma_1 = \gamma_2 = 0.5$, respectively.

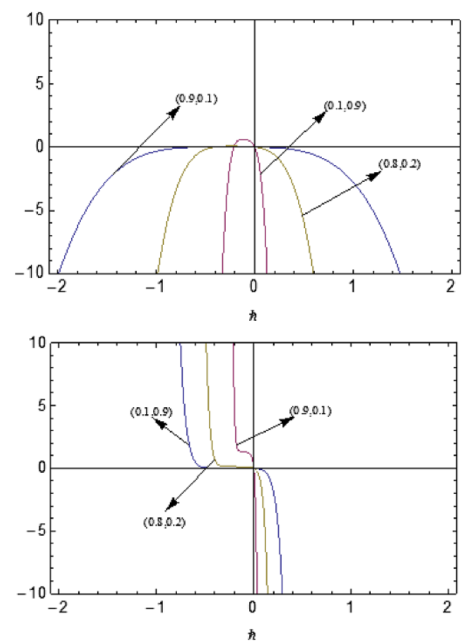
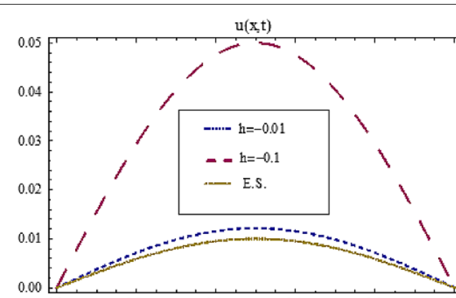


Figure 2 The 10th-order approximate solution of $u(x,t)$ with different values of *h* for $\gamma_1 = \gamma_2 = 0.5$ and $t = 0.1$.



method. We tried to find an approximate solution of this equation by HAM, which is a semi-analytical method. It is not possible to find the analytical solutions of fractional partial differential equations in most cases. In addition, there is an approximate solution of the fractional cable equation that we have considered just with the finite difference method. The HAM results were given by Tables 1-4 and Figures 1-5.

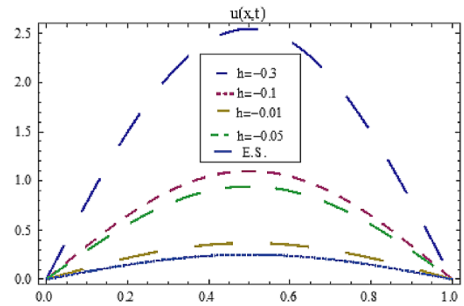


Figure 3 The 10th-order approximate solution of $u(x,t)$ with different values of h for $\gamma_1 = 0.8$, $\gamma_2 = 0.3$ and $t = 0.5$.

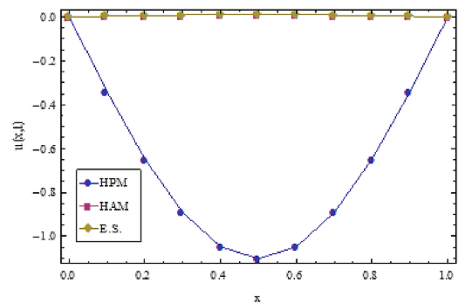


Figure 4 Comparison of the HPM, HAM and Exact solution for 5th-order approximate when $t = 0.1$, $h = -0.0395$ and $\gamma_1 = \gamma_2 = 0.5$.

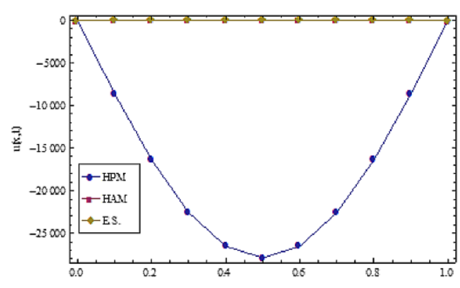


Figure 5 Comparison of the HPM, HAM and Exact solution for 10th-order approximate when $t = 0.1$, $h = -0.004$ and $\gamma_1 = \gamma_2 = 0.25$.

The range of convergence control parameter h was determined by taking a different number of terms of the series solution in Figure 1. We showed that convergent results can be obtained by selecting the appropriate values of x and t of the convergence parameter $h \neq 0$.

An approximate solution that was obtained for different values of the parameter h, the fractional-order derivatives γ_1, γ_2 of the analytical solution and some comparisons for some values of t were presented in Figures 2-3.

A comparison between HPM, HAM, and the analytical solution, when $t = 0.1$ for some values of the auxiliary parameter $h \neq 0$ and partial-order derivatives $0 < \gamma_1, \gamma_2 \leq 1$, was made in Figures 4-5. As can be seen from the figures, HAM and the analytical solution coincided and the HPM solution diverged from the analytical solution.

The absolute errors that were obtained by the implicit numerical method [9], implicit compact finite difference method [10], and HAM can be seen in Table 1. In this table $\gamma_1 = \gamma_2 = 0.5$ and $0.1 \leq t \leq 1.0$. As can be seen from this table when the convergent control parameter h takes a value close to zero, this method gave better results than the other two methods.

A comparison between HPM, HAM, and the analytical solution for γ_1, γ_2 and some values of the auxiliary parameter $h \neq 0$ were presented in Tables 2-4. As can be seen from the tables, the HPM solution diverged from the analytical solution but the HAM solution approached the analytical solution.

Although convergent results for almost every value of the independent variables and convergent control parameter h have been obtained in HAM; the approximate solution diverged at some small and large values of independent variables in HPM. Namely, it is possible to find results that converge rapidly to the analytical solution by HAM.

Consequently HAM is a recommended method for obtaining an approximate solution of the fractional cable equation with γ_1 and γ_2 Riemann-Liouville derivatives.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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