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Approximation of homomorphisms and derivations on Lie C^*-algebras via fixed point method

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Abstract
In this paper, using fixed point methods, we prove the generalized Hyers-Ulam stability of homomorphisms in C^*-algebras and Lie C^*-algebras and of derivations on C^*-algebras and Lie C^*-algebras for an m-variable additive functional equation.

MSC: Primary 39A10; 39B52; 39B72; 46L05; 47H10; 46B03

Keywords: additive functional equation; fixed point; homomorphism in C^*-algebras and Lie C^*-algebras; generalized Hyers-Ulam stability; derivation on C^*-algebras and Lie C^*-algebras

1 Introduction and preliminaries

The stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms:

Let $(G_1,*)$ be a group and let $(G_2,\diamond,d)$ be a metric group with the metric $d(\cdot,\cdot)$. Given $\epsilon>0$, does there exist a $\delta(\epsilon)>0$ such that if a mapping $h:G_1\to G_2$ satisfies the inequality $d(h(x*y),h(x)\diamond h(y))<\delta$ for all $x,y\in G_1$, then there is a homomorphism $H:G_1\to G_2$ with $d(h(x),H(x))<\epsilon$ for all $x\in G_1$?

If the answer is affirmative, we say that the equation of homomorphism $H(x*y)=H(x)\diamond H(y)$ is *stable*.

Since Ulam's question, recently, many authors have given many answers and proved many kinds of functional equations in various spaces, for example, Banach algebras [2], random normed spaces [3–8], fuzzy normed spaces [9, 10], non-Archimedean Banach spaces [11], non-Archimedean lattice random spaces [12], inner product spaces [13–15] and others [16–21].

In this paper, using the fixed point method, we prove the generalized Hyers-Ulam stability of homomorphisms and derivations in Lie C^*-algebras for the following additive functional equation (see [22]):

$$\sum_{i=1}^{m} f\left(mx_i+\sum_{j=1,j\neq i}^{m} x_j\right)+f\left(\sum_{i=1}^{m} x_i\right)=2f\left(\sum_{i=1}^{m} mx_i\right) \tag{1.1}$$

for all $m\in\mathbb{N}$ with $m\geq 2$.

2 Stability of homomorphisms and derivations in C^*-algebras

Throughout this section, assume that A is a C^*-algebra with a norm $\|\cdot\|_A$ and B is a C^*-algebra with a norm $\|\cdot\|_B$.

For any mapping $f: A \to B$, we define

$$D_\mu f(x_1,\ldots,x_m) := \sum_{i=1}^{m} \mu f\left(mx_i + \sum_{j=1, j\neq i}^{m} x_j\right) + f\left(\mu \sum_{i=1}^{m} x_i\right) - 2f\left(\mu \sum_{i=1}^{m} mx_i\right)$$

for all $\mu \in \mathbb{T}^1 := \{\nu \in \mathbb{C} : |\nu| = 1\}$ and $x_1,\ldots,x_m \in A$.

Recall that a $\mathbb{C}$-linear mapping $H: A \to B$ is called a *homomorphism* in C^*-algebras if H satisfies $H(xy) = H(x)H(y)$ and $H(x^*) = H(x)^*$ for all $x, y \in A$.

Recently, in [23], O'Regan *et al.* proved the generalized Hyers-Ulam stability of homomorphisms in C^*-algebras for the functional equation $D_\mu f(x_1,\ldots,x_m) = 0$.

Theorem 2.1 [23] *Let $f: A \to B$ be a mapping for which there are functions $\varphi: A^m \to [0,\infty)$, $\psi: A^2 \to [0,\infty)$ and $\eta: A \to [0,\infty)$ such that*

$$\lim_{j\to\infty} m^{-j}\varphi\left(m^j x_1,\ldots,m^j x_m\right) = 0,$$

$$\left\|D_\mu f(x_1,\ldots,x_m)\right\|_B \le \varphi(x_1,\ldots,x_m),$$

$$\left\|f(xy) - f(x)f(y)\right\|_B \le \psi(x,y),$$

$$\lim_{j\to\infty} m^{-2j}\psi\left(m^j x, m^j y\right) = 0,$$

$$\left\|f\left(x^*\right) - f(x)^*\right\|_B \le \eta(x),$$

$$\lim_{j\to\infty} m^{-j}\eta\left(m^j x\right) = 0$$

for all $\mu \in \mathbb{T}^1$ and $x_1,\ldots,x_m, x, y \in A$. If there exists $0 < L < 1$ such that

$$\varphi(mx, 0,\ldots,0) \le mL\varphi(x,0,\ldots,0)$$

for all $x \in A$, then there exists a unique homomorphism $H: A \to B$ such that

$$\left\|f(x) - H(x)\right\|_B \le \frac{1}{m - mL}\varphi(x,0,\ldots,0)$$

for all $x \in A$.

Theorem 2.2 [23] *Let $f: A \to B$ be a mapping for which there are functions $\varphi: A^m \to [0,\infty)$, $\psi: A^2 \to [0,\infty)$ and $\eta: A \to [0,\infty)$ such that*

$$\lim_{j\to\infty} m^{j}\varphi\left(m^{-j} x_1,\ldots,m^{-j} x_m\right) = 0,$$

$$\left\|D_\mu f(x_1,\ldots,x_m)\right\|_B \le \varphi(x_1,\ldots,x_m),$$

$$\left\|f(xy) - f(x)f(y)\right\|_B \le \psi(x,y),$$

$$\lim_{j\to\infty} m^{2j}\psi\left(m^{-j} x, m^{-j} y\right) = 0,$$

$$\|f(x^*) - f(x)^*\|_B \le \eta(x),$$
$$\lim_{j\to\infty} m^j \eta(m^{-j}x) = 0$$

for all $\mu \in \mathbb{T}^1$ *and* $x_1, \dots, x_m, x, y \in A$. *If there exists* $0 < L < 1$ *such that*

$$\varphi(x, 0, \dots, 0) \le \frac{L}{m}\varphi(mx, 0, \dots, 0)$$

for all $x \in A$, *then there exists a unique homomorphism* $H : A \to B$ *such that*

$$\|f(x) - H(x)\|_B \le \frac{L}{m - mL}\varphi(x, 0, \dots, 0)$$

for all $x \in A$.

Recall that a $\mathbb{C}$-linear mapping $\delta : A \to A$ is called a *derivation* on A if δ satisfies $\delta(xy) = \delta(x)y + x\delta(y)$ for all $x, y \in A$.

In [23], also, O'Regan *et al.* proved the generalized Hyers-Ulam stability of derivations on C^*-algebras for the functional equation $D_\mu f(x_1, \dots, x_m) = 0$.

Theorem 2.3 [23] *Let* $f : A \to B$ *be a mapping for which there are functions* $\varphi : A^m \to [0, \infty)$ *and* $\psi : A^2 \to [0, \infty)$ *such that*

$$\lim_{j\to\infty} m^{-j}\varphi(m^j x_1, \dots, m^j x_m) = 0,$$
$$\|D_\mu f(x_1, \dots, x_m)\|_B \le \varphi(x_1, \dots, x_m),$$
$$\|f(xy) - f(x)y - xf(y)\|_B \le \psi(x, y),$$
$$\lim_{j\to\infty} m^{-2j}\psi(m^j x, m^j y) = 0$$

for all $\mu \in \mathbb{T}^1$ *and* $x_1, \dots, x_m, x, y \in A$. *If there exists* $0 < L < 1$ *such that*

$$\varphi(mx, 0, \dots, 0) \le mL\varphi(x, 0, \dots, 0)$$

for all $x \in A$, *then there exists a unique derivation* $\delta : A \to A$ *such that*

$$\|f(x) - \delta(x)\|_B \le \frac{1}{m - mL}\varphi(x, 0, \dots, 0)$$

for all $x \in A$.

Theorem 2.4 [23] *Let* $f : A \to B$ *be a mapping for which there are functions* $\varphi : A^m \to [0, \infty)$ *and* $\psi : A^2 \to [0, \infty)$ *such that*

$$\lim_{j\to\infty} m^j\varphi(m^{-j}x_1, \dots, m^{-j}x_m) = 0,$$
$$\|D_\mu f(x_1, \dots, x_m)\|_B \le \varphi(x_1, \dots, x_m),$$
$$\|f(xy) - f(x)y - xf(y)\|_B \le \psi(x, y),$$
$$\lim_{j\to\infty} m^{2j}\psi(m^{-j}x, m^{-j}y) = 0$$

for all $\mu \in \mathbb{T}^1$ *and* $x_1, \ldots, x_m, x, y \in A$. *If there exists* $0 < L < 1$ *such that*

$$\varphi(mx, 0, \ldots, 0) \le \frac{L}{m}\varphi(x, 0, \ldots, 0)$$

for all $x \in A$, *then there exists a unique derivation* $\delta : A \to A$ *such that*

$$\|f(x) - \delta(x)\|_B \le \frac{L}{m - mL}\varphi(x, 0, \ldots, 0)$$

for all $x \in A$.

3 Stability of homomorphisms in Lie C^*-algebras

A C^*-algebra $\mathcal{C}$, endowed with the Lie product

$$[x, y] := \frac{xy - yx}{2}$$

on $\mathcal{C}$, is called a *Lie* C^**-algebra* (see [16, 24–26]).

Definition 3.1 Let A and B be Lie C^*-algebras. A $\mathbb{C}$-linear mapping $H : A \to B$ is called a *Lie* C^**-algebra homomorphism* if $H([x, y]) = [H(x), H(y)]$ for all $x, y \in A$.

Throughout this section, assume that A is a Lie C^*-algebra with a norm $\|\cdot\|_A$ and B is a Lie C^*-algebra with a norm $\|\cdot\|_B$.

Now, we prove the generalized Hyers-Ulam stability of homomorphisms in Lie C^*-algebras for the functional equation $D_\mu f(x_1, \ldots, x_m) = 0$.

Theorem 3.2 *Let* $f : A \to B$ *be a mapping for which there are functions* $\varphi : A^m \to [0, \infty)$ *and* $\psi : A^2 \to [0, \infty)$ *such that*

$$\lim_{j\to\infty} m^{-j}\varphi\left(m^j x_1, \ldots, m^j x_m\right) = 0, \tag{3.1}$$

$$\left\|D_\mu f(x_1, \ldots, x_m)\right\|_B \le \varphi(x_1, \ldots, x_m), \tag{3.2}$$

$$\left\|f\left([x, y]\right) - \left[f(x), f(y)\right]\right\|_B \le \psi(x, y), \tag{3.3}$$

$$\lim_{j\to\infty} m^{-2j}\psi\left(m^j x, m^j y\right) = 0 \tag{3.4}$$

for all $\mu \in \mathbb{T}^1$ *and* $x_1, \ldots, x_m, x, y \in A$. *If there exists* $0 < L < 1$ *such that*

$$\varphi(mx, 0, \ldots, 0) \le mL\varphi(x, 0, \ldots, 0)$$

for all $x \in A$, *then there exists a unique Lie* C^**-algebra homomorphism* $H : A \to B$ *such that*

$$\|f(x) - H(x)\|_B \le \frac{1}{m - mL}\varphi(x, 0, \ldots, 0) \tag{3.5}$$

for all $x \in A$.

Proof By the same method as in the proof of Theorem 2.1, we can find the mapping $H : A \to B$ given by

$$H(x) = \lim_{n\to\infty} \frac{f(m^n x)}{m^n}$$

for all $x \in A$. Thus it follows from (3.3) that

$$\begin{aligned}\|H([x,y]) - [H(x),H(y)]\|_B &= \lim_{n\to\infty} \frac{1}{m^{2n}} \|f(m^{2n}[x,y]) - [f(m^n x), f(m^n y)]\|_B \\ &\le \lim_{n\to\infty} \frac{1}{m^{2n}} \psi(m^n x, m^n y) = 0\end{aligned}$$

for all $x, y \in A$, and so

$$H([x,y]) = [H(x),H(y)]$$

for all $x, y \in A$. Therefore, $H : A \to B$ is a Lie C^*-algebra homomorphism satisfying (3.5). This completes the proof. □

Corollary 3.3 *Let* $0 < r < 1$ *and* θ *be nonnegative real numbers. If* $f : A \to B$ *is a mapping such that*

$$\begin{aligned}&\|D_\mu f(x_1,\ldots,x_m)\|_B \le \theta(\|x_1\|_A^r + \|x_2\|_A^r + \cdots + \|x_m\|_A^r), \\ &\|f([x,y]) - [f(x),f(y)]\|_B \le \theta \cdot \|x\|_A^r \cdot \|y\|_A^r\end{aligned}$$

for all $\mu \in \mathbb{T}^1$ *and* $x_1,\ldots,x_m,x,y \in A$, *then there exists a unique Lie* C^**-algebra homomorphism* $H : A \to B$ *such that*

$$\|f(x) - H(x)\|_B \le \frac{\theta}{m - m^r} \|x\|_A^r$$

for all $x \in A$.

Proof The proof follows from Theorem 3.2 by taking

$$\varphi(x_1,\ldots,x_m) = \theta(\|x_1\|_A^r + \|x_2\|_A^r + \cdots + \|x_m\|_A^r), \qquad \psi(x,y) := \theta \cdot \|x\|_A^r \cdot \|y\|_A^r$$

for all $x_1,\ldots,x_m,x,y \in A$ and putting $L = m^{r-1}$. □

Theorem 3.4 *Let* $f : A \to B$ *be a mapping for which there are functions* $\varphi : A^m \to [0,\infty)$ *and* $\psi : A^2 \to [0,\infty)$ *satisfying* (3.1)-(3.4) *for all* $\mu \in \mathbb{T}^1$ *and* $x_1,\ldots,x_m,x,y \in A$. *If there exists* $0 < L < 1$ *such that*

$$\varphi(x,0,\ldots,0) \le \frac{L}{m}\varphi(x,0,\ldots,0)$$

for all $x \in A$, *then there exists a unique Lie* C^**-algebra homomorphism* $H : A \to B$ *such that*

$$\|f(x) - H(x)\|_B \le \frac{L}{m - mL}\varphi(x,0,\ldots,0)$$

for all $x \in A$.

Corollary 3.5 *Let $r > 1$ and θ be nonnegative real numbers. If $f : A \to B$ is a mapping such that*

$$\begin{aligned} &\left\|D_\mu f(x_1, \dots, x_m)\right\|_B \le \theta \cdot \left(\|x_1\|_A^r + \|x_2\|_A^r + \cdots + \|x_m\|_A^r\right), \\ &\left\|f\left([x, y]\right) - \left[f(x), f(y)\right]\right\|_B \le \theta \cdot \|x\|_A^r \cdot \|y\|_A^r \end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and $x_1, \dots, x_m, x, y \in A$, then there exists a unique Lie C^*-algebra homomorphism $H : A \to B$ such that

$$\left\|f(x) - H(x)\right\|_B \le \frac{\theta}{m^r - m} \|x\|_A^r$$

for all $x \in A$.

Proof The proof follows from Theorem 3.4 by taking

$$\begin{aligned} &\varphi(x_1, \dots, x_m) = \theta \cdot \left(\|x_1\|_A^r + \|x_2\|_A^r + \cdots + \|x_m\|_A^r\right), \\ &\psi(x, y) := \theta \cdot \|x\|_A^r \cdot \|y\|_A^r \end{aligned}$$

for all $x_1, \dots, x_m, x, y \in A$ and putting $L = m^{1-r}$. □

4 Stability of derivations in Lie C^*-algebras

Definition 4.1 Let A be a Lie C^*-algebra. A $\mathbb{C}$-linear mapping $\delta : A \to A$ is called a *Lie derivation* if $\delta([x, y]) = [\delta(x), y] + [x, \delta(y)]$ for all $x, y \in A$.

Throughout this section, assume that A is a Lie C^*-algebra with a norm $\| \cdot \|_A$.

Finally, we prove the generalized Hyers-Ulam stability of derivations on Lie C^*-algebras for the functional equation $D_\mu f(x_1, \dots, x_m) = 0$.

Theorem 4.2 *Let $f : A \to A$ be a mapping for which there are functions $\varphi : A^m \to [0, \infty)$ and $\psi : A^2 \to [0, \infty)$ such that*

$$\lim_{j \to \infty} m^{-j} \varphi\left(m^j x_1, \dots, m^j x_m\right) = 0, \tag{4.1}$$

$$\left\|D_\mu f(x_1, \dots, x_m)\right\|_B \le \varphi(x_1, \dots, x_m), \tag{4.2}$$

$$\left\|f\left([x, y]\right) - \left[f(x), y\right] - \left[x, f(y)\right]\right\|_B \le \psi(x, y), \tag{4.3}$$

$$\lim_{j \to \infty} m^{-2j} \psi\left(m^j x, m^j y\right) = 0 \tag{4.4}$$

for all $\mu \in \mathbb{T}^1$ and $x_1, \dots, x_m, x, y \in A$. If there exists $0 < L < 1$ such that

$$\varphi(mx, 0, \dots, 0) \le mL\varphi(x, 0, \dots, 0)$$

for all $x \in A$, then there exists a unique Lie derivation $\delta : A \to A$ such that

$$\left\|f(x) - \delta(x)\right\|_B \le \frac{1}{m - mL} \varphi(x, 0, \dots, 0) \tag{4.5}$$

for all $x \in A$.

Proof By the same method as in the proof of Theorem 2.3, there exists a unique $\mathbb{C}$-linear mapping $\delta : A \to A$ satisfying (3.5). Also, we can find the mapping $\delta : A \to A$ given by

$$\delta(x) = \lim_{n\to\infty} \frac{f(m^n x)}{m^n} \tag{4.6}$$

for all $x \in A$. Thus it follows from (4.3), (4.4) and (4.6) that

$$\begin{aligned} &\big\|\delta\big([x,y]\big) - \big[\delta(x),y\big] - \big[x,\delta(y)\big]\big\|_A \\ &\quad = \lim_{n\to\infty} \frac{1}{m^{2n}} \big\|f\big(m^{2n}[x,y]\big) - \big[f\big(m^n x\big), \cdot m^n y\big] - \big[m^n x, f\big(m^n y\big)\big]\big\|_A \\ &\quad \le \lim_{n\to\infty} \frac{1}{m^{2n}} \psi\big(m^n x, m^n y\big) = 0 \end{aligned}$$

for all $x, y \in A$, and so

$$\delta\big([x,y]\big) = \big[\delta(x),y\big] + \big[x,\delta(y)\big]$$

for all $x, y \in A$. Thus $\delta : A \to A$ is a Lie derivation satisfying (4.5). □

Corollary 4.3 *Let $0 < r < 1$ and θ be nonnegative real numbers. If $f : A \to A$ is a mapping such that*

$$\begin{aligned} &\big\|D_\mu f(x_1, \dots, x_m)\big\|_B \le \theta \cdot \big(\|x_1\|_A^r + \cdots \|x_m\|_A^r\big), \\ &\big\|f\big([x,y]\big) - \big[f(x),y\big] - \big[x,f(y)\big]\big\|_A \le \theta \cdot \|x\|_A^r \cdot \|y\|_A^r \end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and $x_1, \dots, x_m, x, y \in A$, then there exists a unique derivation $\delta : A \to A$ such that

$$\big\|f(x) - \delta(x)\big\|_A \le \frac{\theta}{m - m^r} \|x\|_A^r$$

for all $x \in A$.

Proof The proof follows from Theorem 4.2 by taking

$$\varphi(x_1, \dots, x_m) := \theta \cdot \big(\|x_1\|_A^r + \cdots + \|x_m\|_A^r\big)$$

and

$$\psi(x,y) := \theta \cdot \|x\|_A^r \cdot \|y\|_A^r$$

for all $x_1, \dots, x_m, x, y \in A$ and putting $L = m^{r-1}$. □

Theorem 4.4 *Let $f : A \to A$ be a mapping for which there are functions $\varphi : A^m \to [0,\infty)$ and $\psi : A^2 \to [0,\infty)$ such that*

$$\begin{aligned} &\lim_{j\to\infty} m^j \varphi\big(m^{-j}x_1, \dots, m^{-j}x_m\big) = 0, \\ &\big\|D_\mu f(x_1, \dots, x_m)\big\|_B \le \varphi(x_1, \dots, x_m), \end{aligned}$$

$$\|f([x,y]) - [f(x),y] - [x,f(y)]\|_B \le \psi(x,y),$$
$$\lim_{j\to\infty} m^{2j}\psi(m^{-j}x, m^{-j}y) = 0$$

for all $\mu \in \mathbb{T}^1$ *and* $x_1,\ldots,x_m,x,y \in A$. *If there exists* $0 < L < 1$ *such that*

$$\varphi(mx,0,\ldots,0) \le \frac{L}{m}\varphi(x,0,\ldots,0)$$

for all $x \in A$, *then there exists a unique Lie derivation* $\delta : A \to A$ *such that*

$$\|f(x) - \delta(x)\|_B \le \frac{L}{m-mL}\varphi(x,0,\ldots,0)$$

for all $x \in A$.

Proof The proof is similar to the proof of Theorem 4.2. □

Corollary 4.5 *Let* $r > 1$ *and* θ *be nonnegative real numbers. If* $f : A \to A$ *is a mapping such that*

$$\|D_\mu f(x_1,\ldots,x_m)\|_B \le \theta \cdot (\|x_1\|_A^r + \cdots \|x_m\|_A^r),$$
$$\|f([x,y]) - [f(x),y] - [x,f(y)]\|_A \le \theta \cdot \|x\|_A^r \cdot \|y\|_A^r$$

for all $\mu \in \mathbb{T}^1$ *and* $x_1,\ldots,x_m,x,y \in A$, *then there exists a unique Lie derivation* $\delta : A \to A$ *such that*

$$\|f(x) - \delta(x)\|_A \le \frac{\theta}{m^r - m}\|x\|_A^r$$

for all $x \in A$.

Proof The proof follows from Theorem 4.4 by taking

$$\varphi(x_1,\ldots,x_m) := \theta \cdot (\|x_1\|_A^r + \cdots \|x_m\|_A^r)$$

and

$$\psi(x,y) := \theta \cdot \|x\|_A^r \cdot \|y\|_A^r$$

for all $x_1,\ldots,x_m,x,y \in A$ and putting $L = m^{1-r}$. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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References

1. Ulam, SM: A Collection of the Mathematical Problems. Interscience, New York (1960)
2. Cho, YJ, Kang, JI, Saadati, R: Fixed points and stability of additive functional equations on the Banach algebras. J. Comput. Anal. Appl. **14**, 1103-1111 (2012)
3. Baktash, E, Cho, YJ, Jalili, M, Saadati, R, Vaezpour, SM: On the stability of cubic mappings and quadratic mappings in random normed spaces. J. Inequal. Appl. **2008**, Article ID 902187 (2008)
4. Cho, YJ, Eshaghi Gordji, M, Zolfaghari, S: Solutions and stability of generalized mixed type QC functional equations in random normed spaces. J. Inequal. Appl. **2010**, Article ID 403101 (2010)
5. Cho, YJ, Kang, SM, Sadaati, R: Nonlinear random stability via fixed-point method. J. Appl. Math. **2012**, Article ID 902931 (2012)
6. Kenary, HA, Cho, YJ: Stability of mixed additive-quadratic Jensen type functional equation in various spaces. Comput. Math. Appl. **61**, 2704-2724 (2011)
7. Mohammadi, M, Cho, YJ, Park, C, Vetro, P, Saadati, R: Random stability of an additive-quadratic-quartic functional equation. J. Inequal. Appl. **2010**, Article ID 754210 (2010)
8. Saadati, R, Cho, YJ, Vahidi, J: The stability of the quartic functional equation in various spaces. Comput. Math. Appl. **60**, 1994-2002 (2010)
9. Agarwal, RP, Cho, YJ, Saadati, R, Wang, S: Nonlinear *L*-fuzzy stability of cubic functional equations. J. Inequal. Appl. **2012**, Article ID 77 (2012)
10. Najati, A, Kang, JI, Cho, YJ: Local stability of the pexiderized Cauchy and Jensen's equations in fuzzy spaces. J. Inequal. Appl. **2011**, Article ID 78 (2011)
11. Cho, YJ, Park, C, Saadati, R: Functional inequalities in non-Archimedean Banach spaces. Appl. Math. Lett. **60**, 1994-2002 (2010)
12. Cho, YJ, Saadati, R: Lattice non-Archimedean random stability of ACQ functional equation. Adv. Differ. Equ. **2011**, Article ID 31 (2011)
13. Cho, YJ, Park, C, Rassias, TM, Saadati, R: Inner product spaces and functional equations. J. Comput. Anal. Appl. **13**, 296-304 (2011)
14. Moslehian, MS: On the orthogonal stability of the Pexiderized quadratic equation. J. Differ. Equ. Appl. **11**, 999-1004 (2005)
15. Park, C, Cho, YJ, Kenary, HA: Orthogonal stability of a generalized quadratic functional equation in non-Archimedean spaces. J. Comput. Anal. Appl. **14**, 526-535 (2012)
16. Cho, YJ, Saadati, R, Vahidi, J: Approximation of homomorphisms and derivations on non-Archimedean Lie C^*-algebras via fixed point method. Discrete Dyn. Nat. Soc. **2012**, Article ID 373904 (2012)
17. Eshaghi Gordji, M, Cho, YJ, Ghaemi, MB, Majani, H: Approximately quintic and sextic mappings from *r*-divisible groups into Šerstnev probabilistic Banach spaces: fixed point method. Discrete Dyn. Nat. Soc. **2011**, Article ID 572062 (2011)
18. Eshaghi Gordji, M, Ghaemi, MB, Cho, YJ, Majani, M: A general system of Euler-Lagrange-type quadratic functional equations in Menger probabilistic non-Archimedean 2-normed spaces. Abstr. Appl. Anal. **2011**, Article ID 208163 (2011)
19. Khodaei, H, Eshaghi Gordji, M, Kim, SS, Cho, YJ: Approximation of radical functional equations related to quadratic and quartic mappings. J. Math. Anal. Appl. **397**, 284-297 (2012)
20. Najati, A, Cho, YJ: Generalized Hyers-Ulam stability of the Pexiderized Cauchy functional equation in non-Archimedean spaces. Fixed Point Theory Appl. **2011**, Article ID 309026 (2011)
21. Park, C, Eshaghi Gordji, M, Cho, YJ: Stability and superstability of generalized quadratic ternary derivations on non-Archimedean ternary Banach algebras: a fixed point approach. Fixed Point Theory Appl. **2012**, Article ID 97 (2012)
22. Eskandani, GZ: On the Hyers-Ulam-Rassias stability of an additive functional equation in quasi-Banach spaces. J. Math. Anal. Appl. **345**, 405-409 (2008)
23. O'Regan, D, Rassias, JM, Saadati, R: Approximations of ternary Jordan homomorphisms and derivations in multi-C^* ternary algebras. Acta Math. Hung. **134**(1-2), 99-114 (2012)
24. Park, C: Lie $*$-homomorphisms between Lie C^*-algebras and Lie $*$-derivations on Lie C^*-algebras. J. Math. Anal. Appl. **293**, 419-434 (2004)
25. Park, C: Homomorphisms between Lie JC^*-algebras and Cauchy-Rassias stability of Lie JC^*-algebra derivations. J. Lie Theory **15**, 393-414 (2005)
26. Park, C: Homomorphisms between Poisson JC^*-algebras. Bull. Braz. Math. Soc. **36**, 79-97 (2005)

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