Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2010, Article ID 495014, 15 pages
doi:10.1155/2010/495014

Research Article

Channel Capacity Bounds in the Presence of Quantized Channel State Information

Behrooz Makki, Lotfollah Beygi, and Thomas Eriksson

Department of Signals and Systems, Chalmers University of Technology, 41296, Gothenburg, Sweden

Correspondence should be addressed to Behrooz Makki, behrooz.makki@chalmers.se

Received 13 April 2010; Accepted 1 December 2010

Academic Editor: Tongtong Li

The goal of this paper is to investigate the effect of channel side information on increasing the achievable rates of continuous power-limited non-Gaussian channels. We focus on the case where (1) there is imperfect channel quality information available to the transmitter and the receiver and (2) while the channel gain is continuously varying, there are few cross-region changes, and the noise characteristics remain in each detection region for a long time. The results are presented for two scenarios, namely, reliable and unreliable region detection. Considering short- and long-term power constraints, the capacity bounds are found for log-normal and two different Nakagami-based channel distributions, and for both Max-Lloyd and equal probability quantization approaches. Then, the optimal gain partitioning approach, maximizing the achievable rates, is determined. Finally, general equations for the channel capacity bounds and optimal channel partitioning in the case of unreliable region detection are presented. Interestingly, the results show that, for high SNR's, it is possible to determine a power-independent optimal gain partitioning approach maximizing the capacity lower bound which, in both scenarios, is identical for both short- and long-term power constraints.

1. Introduction

As shown by Shannon [1], having perfect knowledge about the channel, the Shannon capacity is achieved via updating the transmission power and rate relative to channel quality. However, assuming perfect channel information at the transmitter and receiver is an overly optimistic assumption, which does not match with reality [2–6]. Often, the receiver channel side information (CSI) is limited to knowledge of what SNR interval the channel quality belongs to, that is, the CSI is quantized to the best modulation and coding scheme (MCS) (see [3–6]). Then, implementing predefined coding and modulation selection tables, the transmitter is informed about the acceptable transmission rates via a limited number of feedback bits. This is a suboptimal but practical approach, and the considerable throughput improvement has led adaptive modulation with imperfect channel state information to be a major issue in, for example, the 3rd Generation Partnership Project (3GPP) [7] and some standards like UMTS/WCDMA [8].

Started by Shannon [1], Dobrushin [9], and Wolfowits [10], there have been several attempts during the last decades to deal with channel capacity in the presence of side information; Goldsmith and Varaiya [11] presented a coding strategy for achieving the channel capacity in the presence of perfect channel knowledge at the transmitter and receiver. Caire and Shamai [12], and Das and Narayan [13] investigated different classes of channels with memory and side information at the transmitter and receiver. Further, Tatikonda [14] presented a new model of feedback channels which generalized the Verdu and Han [15] capacity formula for channels without feedback. Lapidoth and Shamai [16] examined the effect of side information imprecision in the capacity of fading channels. Also, Skoglund and Jöngren [17] presented numerical results verifying the effect of channel quality information quantization on the capacity of multiple-antenna systems. Lau et al. [18] worked on the capacity of memoryless and block-fading channels with partial channel state feedback and proposed a Max-Lloyd-like algorithm to find the capacity. Considering memoryless feedback channel, Yüksel and Tatikonda [19] obtained the capacity of Markov channel with causal deterministic quantized state feedback. Moreover, there have been some other works such as [20–22] dealing with feedback capacity in Markov channels.

Research in the field of feedback channel capacity has received more attention in the recent years: Jafar [23] provided a common form of capacity expression for the cases where causal or noncausal side information is available to the transmitter. Kim et al. [24] found the upper and lower bounds on the reliability function of the additive white Gaussian noise channels in noisy feedback channels. Moreover, Martins and Weissman [25] proposed some coding strategies for reliable communication over additive white Gaussian channel in the presence of feedback corrupted by quantization or additive bounded noise and analyzed the channel capacity in this case. Dabora and Goldsmith [26] found the capacity region of the discrete-time, time-varying broadcast channel with memory in two different cases of feedback knowledge. Permuter et al. [27] determined the capacity region of finite-state multiple-access channel in the presence or absence of feedback information. Finally, Tatikonda and Mitter [28] developed a general framework for channels with memory and feedback.

Particularly, in presence of noise-free quantized channel SNR information at the transmitter and perfect knowledge at the receiver, Liu et al. [29] have recently obtained the achievable rates of slow fading channels using optimal channel SNR quantization and power allocation. In their work, in order to have error-free data transmission, the channel SNR is assumed to be constant, equal to its *worst case* (consequently, we denote it "the worst case approach.") within each quantization interval. This is a simple practical approach which results in Gaussian input distributions reaching the maximum reliable rates if the channel remains fixed for a long time so that the ergodic capacity is achieved. Later, Kim and Skoglund [30] showed that the worst case is, in fact, the best value which can be considered in each quantization region maximizing the total achievable rates. Also, the same approach, but with noisy feedback channel, has been investigated in [31]. However, it is obvious that the proposed rates reduce to a lower bound of channel capacity in the cases where the channel SNR is not fixed but changes within each quantization region for a long time. Moreover, the results are based on perfect CSI at the receiver which is not a valid assumption in practice. Finally, as illustrated in the following, in the above approach no data is transmitted in one of the quantization regions which deteriorates the transmission performance.

Most analyses of digital communication systems are based on the additive Gaussian noise channel. However, experimental measurements have confirmed the presence of non-Gaussian noise in many communication channels such as underwater acoustic channels [32], narrowband cellular networks, for example, GSM [33], or urban/indoor mobile and portable radio channels [34]. This effect, which is mainly due to crosstalk or other types of additive interference, has been studied from different practical and theoretical points of view; Pinsker et al. [35] analyzed the sensitivity of channel capacity to additive non-Gaussian noise. Further, The authors in [36] studied the higher order asymptotics of mutual information for nonlinear channels experiencing non-Gaussian noise. The authors in [37, 38] provided some upper bounds on the capacity of non-Gaussian channels. Then, considering Gaussian input distributions, The authors in [39] investigated the effect of nearest neighbor decoding on the achievable rates of non-Gaussian channels. Moreover, Das [40], Tchamkerten [41], and Smith [42] obtained some sufficient conditions on the noise characteristics which guarantee the presence and some properties of the capacity-achieving input distributions. Finally, among more practical works, we can mention [33, 34, 43–46] in which different topics, such as channel estimation [33], multiuser detection [34], equalizer design [43, 44], sequence detection [45] and decoder design [46], have been studied in non-Gaussian channels.

Based on this perspective, this work attempts to analyze the maximum achievable rates of a continuous power-limited non-Gaussian channel in the presence of channel side information. Here, motivated by practical adaptive modulation systems, we focus on the case where (1) there is imperfect channel quality information (CQI) available to the transmitter and receiver, and (2) while the channel gain is continuously varying, there are rare abrupt cross-region changes in the channel conditions, and the noise (and interference) characteristics remain in each detection region for some time so that it is meaningful to send CQI feedback. This case is motivated by the aforementioned practical MCS transmission schemes, and it will also serve as a lower bound of the performance of a system with perfect receiver CSI and quantized transmitter CSI.

As it will be seen, our proposed bounds lead to higher achievable rates in comparison to the *worst case* approach discussed above. Furthermore, in contrast to the *worst case* approach, data transmission is done in all states, improving the transmission performance. The results are presented in two scenarios, namely, reliable and unreliable region detection. In the first scenario, considering short- and long-term power constraints, the channel capacity bounds are obtained for log-normal and two ($\mu = 2$ and $\mu = 3$) Nakagami-based distributions, and for two (equal probability and Max-Lloyd [47]) quantization approaches. Further, we propose a method for determining the optimal gain partitioning approach maximizing the proposed achievable rates which is tested for the three mentioned distributions. Moreover, the general equations for the capacity bounds and optimal channel partitioning maximizing the achievable rates in the case of unreliable region detection are presented in the last part and are illustrated in more details via some examples. The results show that, for high SNR's, it is possible to determine a power-independent optimal gain partitioning approach maximizing the capacity lower bound which, in both scenarios, is identical for both short- and long-term power constraints.

The rest of the paper is organized as follows. In Section 2, the problem statement is presented. Based on our channel model, the bounds of channel capacity with no side

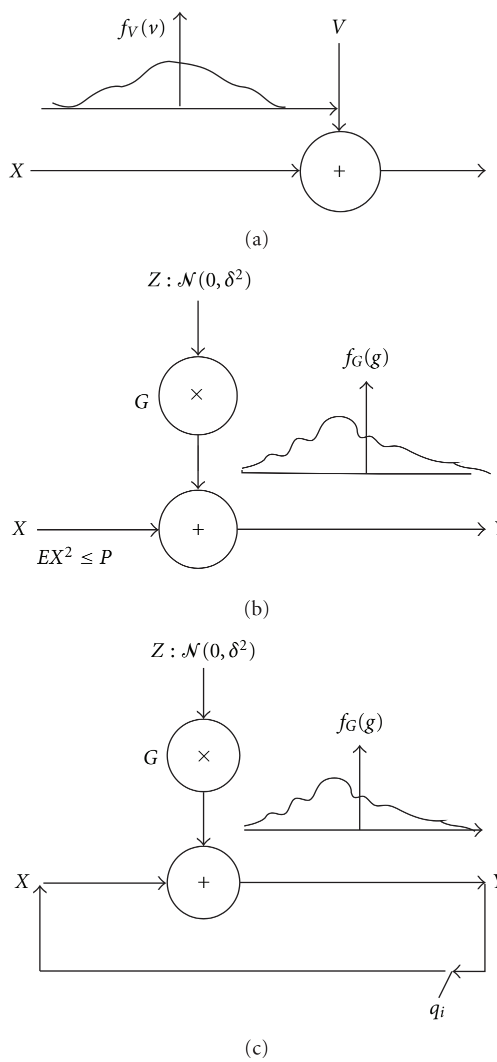


FIGURE 1: Channel model: (a) original model, (b) new representation of channel model, (c) channel with feedback.

information and in the presence of imperfect channel state information at the transmitter and receiver are presented in Sections 3 and 4, respectively. Then, considering error-free region detection, the optimal partitioning approach maximizing the achievable rates is obtained in Section 5. In Section 6, the channel capacity bounds, optimal channel partitioning, and power allocation are obtained in the presence of unreliable channel state information available at the transmitter and the receiver. Finally, Section 7 concludes the paper.

2. Problem Statement

We consider the general non-Gaussian communication channel, depicted in Figure 1(a), in which V is a zero-mean non-Gaussian colored noise. To be able to verify the effect of different fading conditions, we assume that the noise can be modeled as $V = ZG$ in which $Z : \mathcal{N}(0, \delta^2)$ is a white Gaussian random variable, and $G = 1/U$ is an independent fading random variable modeling different fading conditions (we denote G as channel gain (CG) in the following). This is because, as illustrated in [11], log-normal and Nakagami distributions of the variable U can model severe and nonsevere fading channels, respectively. In this way, as illustrated in Figure 1(b), the channel can be represented as

$$Y = X + ZG, \tag{1}$$

where Y denotes the channel output in response to the communicating message X. It should be mentioned that for the log-normal and two ($\mu = 2$ and $\mu = 3$) Nakagami distributions of U, considered here, the properties of the random variable G is given by

log-normal (all results are presented in natural logarithm basis):

$$\begin{aligned}
f_U(u) &= \frac{1}{\sqrt{2\pi\gamma^2}u} e^{-(\log(u))^2/2\gamma^2}, \quad u > 0, \\
f_G(g) &= \frac{1}{g^2} f_U\left(\frac{1}{g}\right) = \frac{1}{\sqrt{2\pi\gamma^2}g} e^{-(\log(g))^2/2\gamma^2}, \quad g > 0, \\
F_G(g) &= \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{\log(g)}{\sqrt{2}\gamma}\right)\right) \\
\int_a^b g^2 f_G(g)\,dg &= \frac{e^{2\gamma^2}}{2}\left\{\operatorname{erf}\left(\frac{\log(b)}{\sqrt{2}\gamma} - \sqrt{2}\gamma\right)\right. \\
&\quad \left. - \operatorname{erf}\left(\frac{\log(a)}{\sqrt{2}\gamma} - \sqrt{2}\gamma\right)\right\} \\
EG^2 &= \int_0^\infty g^2 f_G(g)\,dg = e^{2\gamma^2}.
\end{aligned} \tag{2}$$

Nakagami $\mu = 2$:

$$\begin{aligned}
f_U(u) &= \frac{8}{\Gamma(2)w^2} u^3 e^{-2u^2/w}, \quad u \ge 0, \\
f_G(g) &= \frac{1}{g^2} f_U\left(\frac{1}{g}\right) = \frac{8}{w^2}\frac{1}{g^5} e^{-2/wg^2}, \quad g \ge 0, \\
F_G(g) &= \left(\frac{2}{wg^2} + 1\right) e^{-2/wg^2} \\
\int_a^b g^2 f_G(g)\,dg &= \frac{2}{w}\left\{e^{-2/wb^2} - e^{-2/wa^2}\right\} \\
EG^2 &= \int_0^\infty g^2 f_G(g)\,dg = \frac{2}{w}.
\end{aligned} \tag{3}$$

Nakagami $\mu = 3$:

$$\begin{aligned} f_U(u) &= \frac{54}{\Gamma(3)w^3}u^5 e^{-3u^2/w}, \quad u \geq 0, \\ f_G(g) &= \frac{1}{g^2} f_U\left(\frac{1}{g}\right) = \frac{27}{w^3}\frac{1}{g^7}e^{-3/wg^2}, \quad g \geq 0, \\ F_G(g) &= \frac{(9 + 6wg^2 + 2w^2g^4)}{2w^2g^4}e^{-3/wg^2} \\ \int_a^b g^2 f_G(g)\,\mathrm{d}g &= \frac{3}{2w^2}\left\{\left(\frac{3}{b^2} + w\right)e^{-3/wb^2} - \left(\frac{3}{a^2} + w\right)e^{-3/wa^2}\right\} \\ EG^2 &= \int_0^\infty g^2 f_G(g)\,dg = \frac{3}{2w}, \end{aligned} \tag{4}$$

where $f_G(g)$, $F_G(g)$, and EG^2 are the probability density function (pdf), cumulative distribution function (cdf), and power of the random variable G, w is the second parameter of Nakagami distributions, γ is the log-normal distribution parameter, and $\Gamma(\cdot)$ and $\mathrm{erf}(\cdot)$ denote the standard Gamma and error functions, respectively. Moreover, $EX^2 \leq P$ represents the input power constraint which can be interpreted in two different ways, namely, short and long term. Let $X_l, l = 1, \ldots, L$ be a collection of different random variables each of which is selected at different conditions with probability p_l. The short-term power constraint requires that, for every single X_l, the power allocated cannot exceed P, that is,

$$P_l \leq P, \quad l = 1, \ldots, L, \tag{5}$$

where $P_l = EX_l^2$ is the power of input variable X_l. This constraint is normally considered when the transmission powers are limited by the hardware, for example, amplifiers, properties. Under the more relaxed long-term (battery-limited) power constraint, the transmitter can adapt the power based on channel conditions such that the average transmission power does not exceed P, that is,

$$\sum_{l=1}^{L} p_l P_l \leq P. \tag{6}$$

In order to verify the effect of imperfect channel quality information, we have considered the model of Figure 1(c). In this model, the receiver first finds the quantization region in which the gain realizations fall. Then, considering 2^N quantization regions, the quantization indices are sent back to the transmitter via N bits feedback. In the following, first the lower and upper bounds of channel capacity with no side information are presented, and then they will be determined for two different quantizers (equal probability and Max-Lloyd), assuming that the quantization regions are correctly detected and fed back to the transmitter. Further, note that all results are based on the assumption that, while there are in-region variations, the channel gain G stays at each region for a long enough time so that it is meaningful to send channel quality feedback.

3. Channel Capacity Bounds without Side Information at Transmitter and Receiver

With no side information about the channel quality, the capacity of a power-limited channel can be written as

$$\begin{aligned} C &= \max_{f_X(x);\, EX^2 \leq P} I(X;Y) = \max_{f_X(x);\, EX^2 \leq P} \{h(Y) - h(Y \mid X)\} \\ &= \max_{f_X(x);\, EX^2 \leq P} \{h(Y) - h(ZG)\}, \end{aligned} \tag{7}$$

where maximization is done over the input pdf $f_X(x)$ and $EX^2 \leq P$ represents the input power constraint. However, since in general (due to the gain pdf) it is difficult to determine the channel capacity in this case, we find a lower and an upper bound to the maximum achievable mutual information $I(X;Y)$.

Similar to the method presented in [2], the lower bound of the mutual information $I(X;Y) = h(X) - h(X \mid Y)$, is found by fixing the input distribution entropy and upper bounding the second term. Hence, the input distribution is selected to be zero-mean Gaussian of power P (which is not necessarily the optimal one maximizing the mutual information) and then, an upper bound of the term $h(X \mid Y)$ is determined. Therefore, for any known value of α, one can write

$$\begin{aligned} I(X;Y) &= h(X) - h(X \mid Y) \\ &= \frac{1}{2}\log(2\pi eP) - h(X \mid Y) \\ &\overset{(a)}{=} \frac{1}{2}\log(2\pi eP) - h(X - \alpha Y \mid Y) \\ &\overset{(b)}{\geq} \frac{1}{2}\log(2\pi eP) - h(X - \alpha Y) \\ &\overset{(c)}{\geq} \frac{1}{2}\log(2\pi eP) - \frac{1}{2}\log(2\pi e \delta^2_{X-\alpha Y}), \end{aligned} \tag{8}$$

where (a)–(c) follow from the facts that

(a) adding a known random variable does not change the mutual information,

(b) conditioning reduces the differential entropy, and

(c) for a fixed power, Gaussian distribution maximizes the differential entropy.

Since (8) is valid for any known value of α, it is selected such that αY becomes the linear minimum mean square error estimate of X in terms of Y. Therefore, since X, G, and Z are independent and X and Z are zero mean, we have

$$\alpha = \frac{EXY}{EY^2} = \frac{E\{X(X+ZG)\}}{E\left\{(X+ZG)^2\right\}} = \frac{P}{P + \delta^2 EG^2} \tag{9}$$

and so,

$$\begin{aligned} \delta^2_{X-\alpha Y} &= E\left((X - \alpha Y)^2\right) = EX^2 + \alpha^2 EY^2 - 2\alpha EXY \\ &\overset{\alpha = EXY/EY^2}{=} P - \frac{(EXY)^2}{EY^2} = \frac{P\delta^2 EG^2}{P + \delta^2 EG^2}. \end{aligned} \tag{10}$$

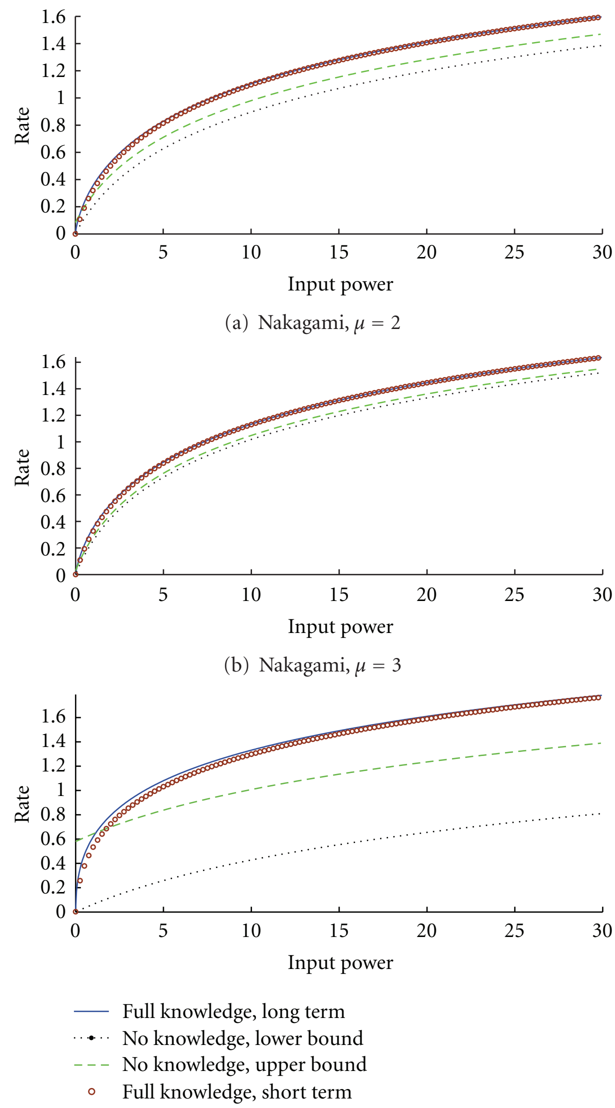
According to (8) and (10), the channel capacity is lower bounded by

$$I(X;Y) \geq \frac{1}{2}\log(2\pi eP) - \frac{1}{2}\log\left(2\pi e\frac{P\delta^2 EG^2}{P+\delta^2 EG^2}\right) = \frac{1}{2}\log\left(1+\frac{P}{\delta^2 EG^2}\right). \tag{11}$$

On the other hand, the upper bound of achievable rates is simply found based on the following consequences:

$$I(X;Y) = h(Y) - h(Y \mid X) \leq \frac{1}{2}\log\left(2\pi e\left(P+\delta^2 EG^2\right)\right) - h(ZG), \tag{12}$$

which again follows from the fact that for a fixed power, $EY^2 = P + \delta^2 EG^2$, the Gaussian distribution maximizes the differential entropy and $h(ZG)$ can be determined numerically. Figure 2 shows the lower and upper bounds of channel capacity for the three mentioned channel gain distributions ($w = 1$, $\gamma = 1$) along with the case where there is full knowledge about the channel gain at the transmitter and receiver (as we know, full knowledge channel capacity is obtained by $C = (1/2)E_G\{\ln(1 + P(g)/\delta^2 g^2\}$ in which $P(g)$ is power allocation function optimally determined according to the considered power constraint). As it can be seen, having full knowledge, the channel capacity under the short- and long-term power constraints are so close that they are hardly distinguishable from each other. While focusing on lower bound of achievable rates, the proposed bounds are applied in the next section to verify the effect of quantized channel gain information on the channel capacity.



(a) Nakagami, $\mu = 2$

(b) Nakagami, $\mu = 3$

(c) Log−normal

Figure 2: Channel capacity for the three distributions in two full and no knowledge cases. $w = 1$, $\gamma = 1$.

4. Channel Capacity Bounds: Predefined Quantization and Reliable Region Detection

Considering the case of error-free quantization region detection, the channel capacity in the presence of a predefined N-bits quantizer can be formulated as

$$C = \max_{f_{X|Q}(x)} I(X;Y \mid Q) = \max_{f_{X|Q}(x)} \sum_{i=1}^{2^N} p_i I(X;Y \mid q_i) = \sum_{i=1}^{2^N} p_i \max_{f_{X_i}(x)} I(X_i;Y_i \mid q_i), \tag{13}$$

where q_i and p_i, $i = 1,\ldots,2^N$, are the quantization outputs and their selection probabilities, respectively, and maximization is again done over $f_{X_i}(x)$, $i = 1,\ldots,2^N$ which are the 2^N input distributions considered for different quantization regions. Note that the last equality is a consequence of the quantizer predefinition (which is the case considered in this section) while, in general, the optimization should be done over both input distributions and the quantizer.

For a given quantization region i, the channel model will change to Figure 3 in which the channel gain will change within the ith quantization interval S_i according to the pdf of

$$f_{G_i}(g) = \begin{cases} \dfrac{1}{p_i} f_G(g), & g \in S_i, \\ 0 & \text{otherwise}, \end{cases} \qquad p_i = \int_{S_i} f_G(g)\,dg, \tag{14}$$

with no knowledge about the in-region variations at the transmitter and the receiver. Therefore, based on Figure 3

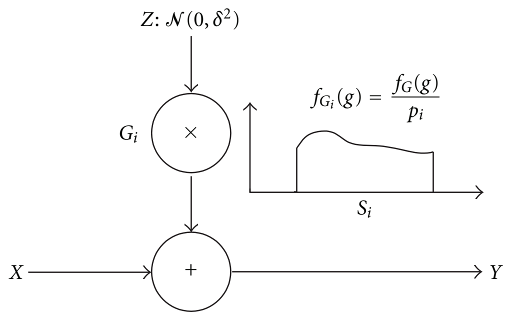


Figure 3: Channel model for a given q_i.

and the bounds obtained in the previous section, the following equations follow:

$$I(X_i; Y_i \mid q_i) = I(X_i; Y_i \mid G \in S_i, \text{ no knowledge about } G)$$

$$\frac{1}{2}\log\left(1 + \frac{T_i}{\delta^2 E_i}\right) \leq I(X_i; Y_i \mid q_i) \leq \frac{1}{2}\log(2\pi e(T_i + \delta^2 E_i)) - h(ZG_i) \quad (15)$$

and so, the total maximum achievable mutual information C will be bounded to

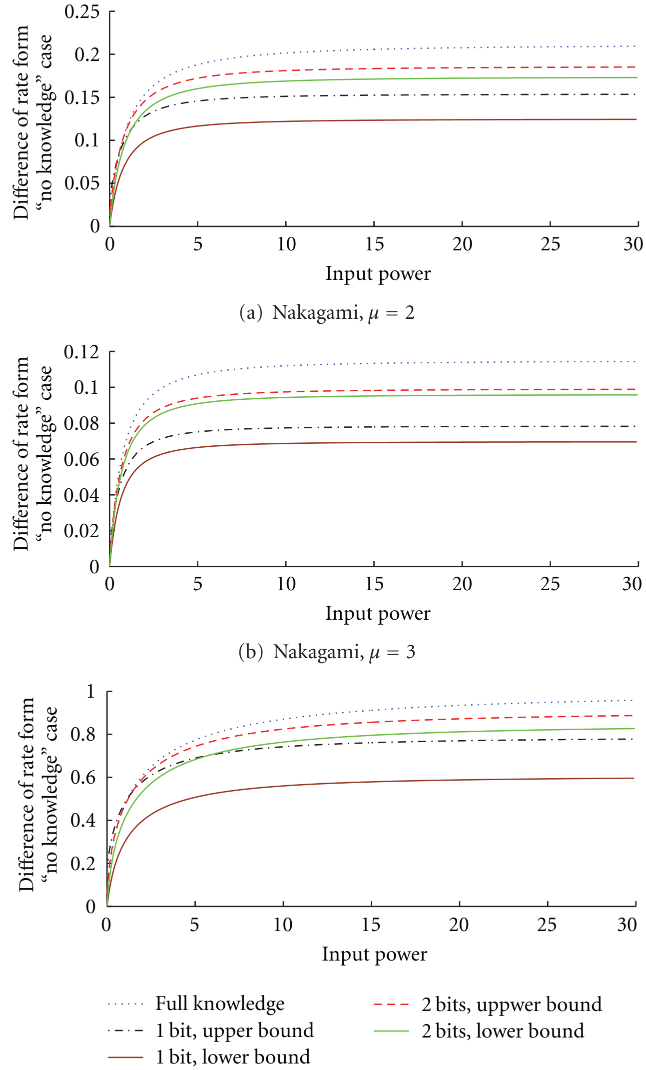
$$\frac{1}{2}\sum_{i=1}^{2^N} p_i \log\left(1 + \frac{T_i}{\delta^2 E_i}\right) \leq C \leq \sum_{i=1}^{2^N} p_i \left\{\frac{1}{2}\log(2\pi e(T_i + \delta^2 E_i)) - h(ZG_i)\right\}, \quad (16)$$

where $E_i = EG_i^2 = (1/p_i)\int_{S_i} g^2 f_G(g) \mathrm{d}g$ denotes the channel gain power for the ith quantization interval, and $EX_i^2 = T_i$ is the input power allocated to the ith input distribution. Considering $N = 1$ and 2 bits equal probability quantization and short-term power constraint, that is, $T_i = P$, $i = 1, \ldots, 2^N$, the bounds of channel capacity relative the lower bound of no knowledge case have been obtained for the three mentioned gain distributions, as illustrated in Figure 4. Note that in N-bits equal probability quantization the distribution is partitioned into 2^N subspaces that have the same probabilities. The parameters of the distributions (2)–(4) are set to

$$w = 1, \qquad \delta^2 = 1, \qquad \gamma = 1. \quad (17)$$

While the short-term power constraint leads to uniform power distribution, the achievable rates can be increased by optimal power allocation under long-term power constraint condition. In this way, there are 2^N input distributions selected based on the received channel information and, since the quantization region information at the transmitter and receiver is assumed to be error-free, the input power constraint, $EX^2 = P$, changes to

$$\sum_{i=1}^{2^N} T_i p_i = P. \quad (18)$$



(a) Nakagami, $\mu = 2$

(b) Nakagami, $\mu = 3$

(c) Log−normal

Figure 4: Increase in achievable rate bounds relative the "no knowledge" lower bound using 1 and 2 bits equal probability quantization, short-term power constraint.

Consequently, considering the lower bound of channel capacity, the optimal input power allocation can be found by maximization of (16) with the constraint (18), that is, maximization of

$$\theta = \sum_{i=1}^{2^N} p_i \log\left(1 + \frac{T_i}{\delta^2 E_i}\right)\Bigg) + \rho\sum_{i=1}^{2^N} T_i p_i, \quad (19)$$

over N unknown values of T_i and the Lagrange multiplier ρ determined by the input power constraint. Taking the derivatives of (19) with respect to T_i's and setting them equal

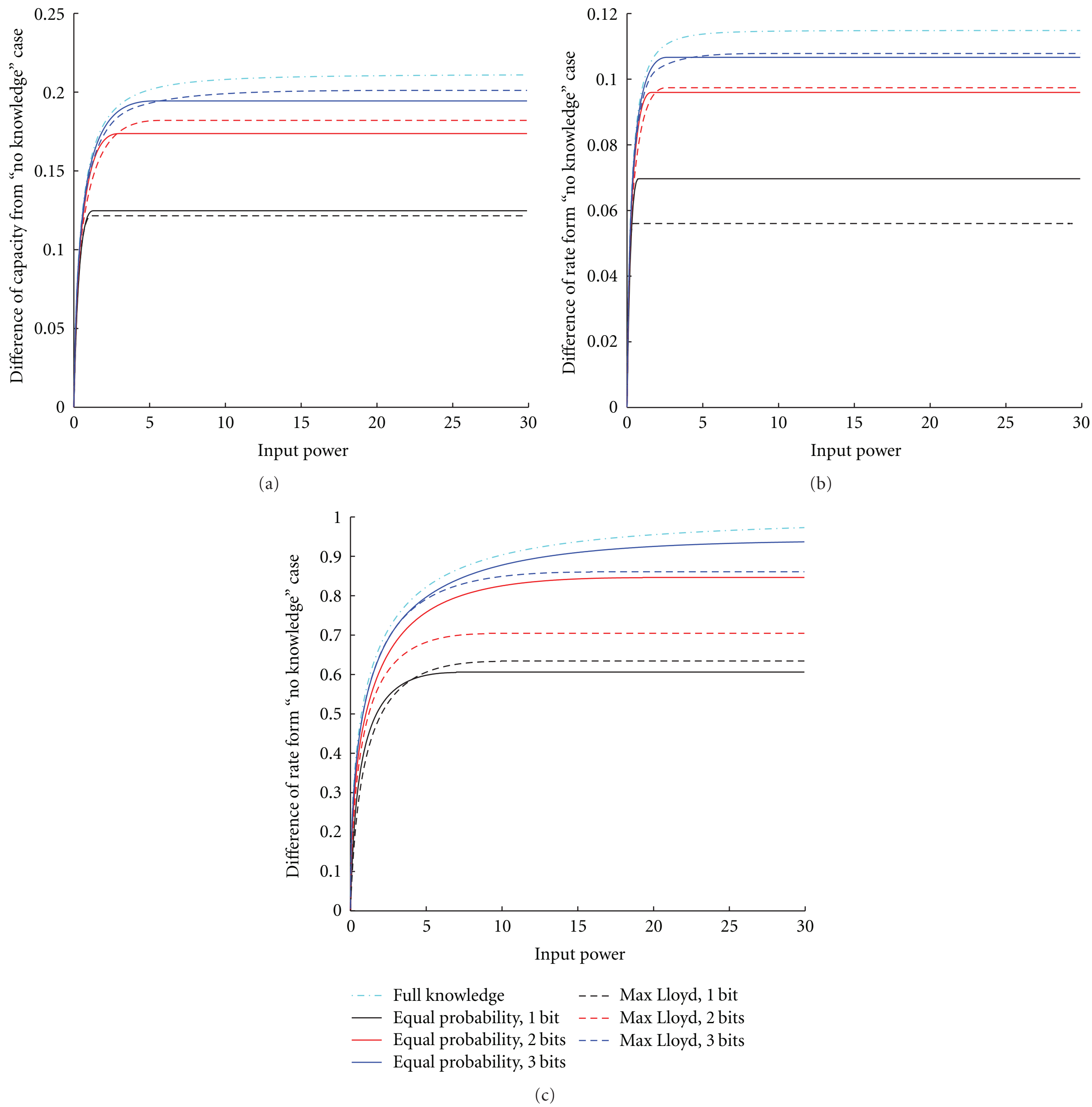


FIGURE 5: Increase in achievable rates relative the "no knowledge" lower bound using Max-Lloyd and equal probability quantizations. (a) Nakagami $\mu = 2$, (b) Nakagami $\mu = 3$, (c) Log-normal.

to zero yields the following set of equations:

$$\frac{\partial\theta}{\partial T_i} = p_i \frac{1}{T_i + \delta^2 E_i} + \rho p_i = 0$$
$$\Longrightarrow T_i + \delta^2 E_i = \frac{-1}{\rho} = \lambda, \tag{20}$$

which, due to the fact that T_i's are nonnegative, is the famous water-filling problem [48]. In order to find the constant λ, one can write

$$P = \sum_{i;T_i>0} T_i p_i = \sum_{i;T_i>0} p_i \left(\lambda - \delta^2 E_i\right) \tag{21}$$

and so,

$$\lambda = \frac{1}{\sum_{i;T_i>0} p_i}\left(P + \delta^2 \sum_{i;T_i>0} p_i E_i\right). \tag{22}$$

As shown in Figure 5, considering long-term power constraint and the three mentioned gain distributions, the lower bounds of the capacity relative the lower bound of "no knowledge" case have been found for the cases where $N = 1, 2$, and 3 bits Max-Lloyd or equal probability quantizers are implemented for channel quantization.

As an alternative lower bound, the *worst case* approach may be used, which has been proven to be optimal under block fading assumption [30]. However, we show that our proposed lower bound outperforms this approach under outage-free condition. Considering the worst (highest) value (in [29, 30], $Y = UX + N$ has been considered as the channel model and so, the worst case corresponds to a_{i-1} in each region $S_i : [a_{i-1}, a_i])$ of the channel gain in each quantization interval $S_i : [a_{i-1}, a_i]$ results in transmission rate of

$$R_2 = \frac{1}{2}\sum_{i=1}^{2^N} p_i \log\left(1 + \frac{T'_i}{\delta^2 a_i^2}\right), \qquad \sum_{i=1}^{2^N} p_i T'_i = P, \tag{23}$$

where $T'_i = EX_i'^2$ is the power allocated to the ith input distribution X'_i and again the optimal power allocation is obtained by

$$T'_i + \delta^2 a_i^2 = \frac{-1}{\rho'} = \varepsilon, \tag{24}$$

where $\rho' = -1/\varepsilon$ is the Lagrange multiplier constant [30]. Now, based on the fact that

$$\forall i : E_i = \frac{1}{p_i}\int_{a_{i-1}}^{a_i} g^2 f_G(g)\mathrm{d}g \le \frac{1}{p_i}\int_{a_{i-1}}^{a_i} a_i^2 f_G(g)\mathrm{d}g = a_i^2, \tag{25}$$

for a given total power P, quantization regions S_i and a set of strictly positive powers $\{T'_k \mid k \in K,\ \sum_{k\in K} p_k T'_k = P\}$ optimally allocated based on (24), we have

$$\begin{aligned} R_2 &= \frac{1}{2}\sum_{i=1}^{2^N} p_i \log\left(1 + \frac{T'_i}{\delta^2 a_i^2}\right) \\ &\overset{(a)}{=} \frac{1}{2}\sum_{k\in K} p_k \log\left(1 + \frac{T'_k}{\delta^2 a_k^2}\right) \\ &\overset{(b)}{\le} \frac{1}{2}\sum_{k\in K} p_k \log\left(1 + \frac{T'_k}{\delta^2 E_k}\right) \\ &\overset{(c)}{\le} \frac{1}{2}\sum_{i=1}^{2^N} p_i \log\left(1 + \frac{T_i}{\delta^2 E_i}\right) = R_1, \end{aligned} \tag{26}$$

where R_1 is the lower bound obtained in (16) and T_i's are determined based on (20). Here, (a) is obtained by removing the zero-power terms, (b) is based on (25), and (c) follows from the fact that, although being an acceptable power distribution, $\{T'_k \mid k \in K\}$ is not necessarily the optimal power allocation strategy determined by (20). Hence, it is concluded that, under long-term power constraint and for every given total power, our proposed lower bound results in higher transmission rate in comparison to the *worst case* approach. Based on (25), it is obvious that the conclusion is also valid when the same powers are allocated to corresponding input distributions, for example, for the short-term power constraint. Finally, note that, since the main focus of this work is on the capacity lower bounds, we do not discuss the upper bounds any further.

5. Optimal Design of Channel Quantization: Reliable Quantization Region Detection

5.1. Long-Term Power Constraint. Under the long-term power constraint, in order to design an optimal quantizer reaching the maximum lower bound of capacity, one can consider (19) in which

$$p_i = \int_{a_{i-1}}^{a_i} f_G(g)\mathrm{d}g, \qquad E_i = \frac{1}{p_i}\int_{a_{i-1}}^{a_i} g^2 f_G(g)\mathrm{d}g \tag{27}$$

are functions of quantization boundaries a_i, $i = 0, \ldots, 2^N$ ($a_0 = 0$, $a_{2^N} = \infty$).

While the optimal power allocation constraint given by (20) is still valid, taking derivative with respect to a_i, it can be written as

$$\begin{aligned} \frac{\partial\theta}{\partial a_i} &= \frac{\partial p_i}{\partial a_i}\log\left(1 + \frac{T_i}{\delta^2 E_i}\right) + \frac{\partial p_{i+1}}{\partial a_i}\log\left(1 + \frac{T_{i+1}}{\delta^2 E_{i+1}}\right) \\ &\quad + p_i \frac{-T_i\delta^2(\partial E_i/\partial a_i)/\delta^4 E_i^2}{1 + T_i/\delta^2 E_i} \\ &\quad + p_{i+1}\frac{-T_{i+1}\delta^2(\partial E_{i+1}/\partial a_i)/\delta^4 E_{i+1}^2}{1 + T_{i+1}/\delta^2 E_{i+1}} \\ &\quad + \rho T_i \frac{\partial p_i}{\partial a_i} + \rho T_{i+1}\frac{\partial p_{i+1}}{\partial a_i} \end{aligned} \tag{28}$$

which, considering the following properties, can be simplified to(30)

$$\begin{aligned} \frac{\partial p_i}{\partial a_i} &= f_G(a_i) \\ \frac{\partial p_{i+1}}{\partial a_i} &= -f_G(a_i) \\ \frac{\partial E_i}{\partial a_i} &= \frac{a_i^2 f_G(a_i)p_i - f_G(a_i)\sigma_i}{p_i^2} \\ \frac{\partial E_{i+1}}{\partial a_i} &= \frac{-a_i^2 f_G(a_i)p_{i+1} + f_G(a_i)\sigma_{i+1}}{p_{i+1}^2} \\ \sigma_i &= \int_{a_{i-1}}^{a_i} g^2 f_G(g)\mathrm{d}g \end{aligned} \tag{29}$$

$$\begin{aligned} &\log\left(1 + \frac{T_i}{\delta^2 E_i}\right) - \log\left(1 + \frac{T_{i+1}}{\delta^2 E_{i+1}}\right) - \frac{T_i}{E_i}\frac{(a_i^2 - E_i)}{T_i + \delta^2 E_i} \\ &\quad + \frac{T_{i+1}}{E_{i+1}}\frac{(a_i^2 - E_{i+1})}{T_{i+1} + \delta^2 E_{i+1}} + \rho T_i - \rho T_{i+1} = 0. \end{aligned} \tag{30}$$

Furthermore, since for every positive T_i, we have

$$T_i + \delta^2 E_i = \frac{-1}{\rho}, \tag{31}$$

Equation (30) can be further simplified to

$$\log \frac{E_{i+1}}{E_i} = \rho a_i^2 \left(\frac{T_{i+1}}{E_{i+1}} - \frac{T_i}{E_i} \right), \tag{32}$$

which can be rewritten as

$$\log \frac{E_{i+1}}{E_i} = -a_i^2 \left(\frac{1}{E_{i+1}} - \frac{1}{E_i} \right). \tag{33}$$

Equation (33) shows that, under a long-term power constraint, the quantization boundaries can be determined independent of individual and total input powers if the optimal T_i's are all strictly positive. It is worth noting that, using the *worst case* approach the optimal quantization boundaries will depend on the total input power [29].

The quantization boundaries and the input powers can be found using Max-Lloyd-like algorithms [30] or numerically. Algorithm 1 illustrates an example of an efficient numerical method maximizing the capacity lower bound under long-term power constraint. Although time consuming, the algorithm has been shown to be efficient in complex optimization problems dealing with, large number of optimization parameters [49]. Also, note that, with some straightforward modifications, the algorithm can be applied for other optimization problems as well.

Considering 1- and 2-bit quantization and the three mentioned channel gain distributions, the optimal quantization boundaries maximizing the capacity lower bounds have been determined for various input powers which can be reviewed in Tables 1, 2, and 3. As it can be seen, (except the case of log-normal gain distribution and 2-bit quantization, in which up to input power 30 still one of the input transmitters remains off) when the power is increased above the threshold where all T_i's are strictly positive, the optimal quantization boundaries become independent of input power and are determined based just on the channel gain distribution. Moreover, in order to get better insight of the effect of optimal quantization, considering 1- and 2-bit quantization optimized for input power 15, the increase in channel capacity lower bounds relative the lower bound of no knowledge case, that is, (11), have been found for the three gain distributions as presented in Figure 6. Based on the Figures 4, 5, and 6, it can be concluded that, while channel side information results in significant rate increment in severe fading conditions (log-normal pdf), as the fading severity decreases (Nakagami $\mu = 2$ and $\mu = 3$ pdfs) the effect of side information and adaptive transmission is also reduced. Also, the figure shows that, considering different fading conditions, it is possible to reach near-full-knowledge performance using 2-bit channel quality information feedback. Finally, considering 2-bit feedback and various gain distributions, Figure 7 demonstrates the superiority of the proposed lower bound in comparison to the *worst case* approach even when transmission outage is permitted [30]. In this case, the optimal channel gain is found to be some value in the middle of the region if it falls in the last quantization interval, leading to slightly higher rates in comparison to (23). However, there is always positive outage probability in data transmission. This does not happen in our approach which increases the transmission reliability.

Table 1: Optimal quantization boundaries, reliable quantization region detection, long-term power constraint, and CG distribution: Nakagami, $\mu = 2$, $w = 1$.

Input power	1 bit	Input power	2 bits
0.5	1.044	0.5	0.738, 0.935, 1.162
1	1.169	1	0.781, 1.022, 1.331
2	1.322	2	0.823, 1.118, 1.563
≥2.7	1.394	3	0.855, 1.193, 1.728
		5	0.888, 1.276, 1.972
		≥9.6	0.922, 1.373, 2.327

Table 2: Optimal quantization boundaries, reliable quantization region detection, long-term power constraint, and CG distribution: Nakagami, $\mu = 3$, $w = 1$.

Input power	1 bit	Input power	2 bits
0.5	1.060	0.5	0.782, 0.961, 1.165
1	1.178	1	0.822, 1.041, 1.327
≥1.3	1.220	2	0.863, 1.135, 1.535
		3	0.885, 1.190, 1.688
		≥3.7	0.898, 1.216, 1.766

Table 3: Optimal quantization boundaries, reliable quantization region detection, long-term power constraint, and CG distribution: Log-normal, $\gamma = 1$.

Input power	1 bit	Input power	2 bits
0.5	0.656	0.5	0.236, 0.487, 0.868
1	0.784	1	0.271, 0.575, 1.079
2	0.937	2	0.306, 0.670, 1.331
3	1.040	3	0.328, 0.738, 1.523
5	1.183	5	0.357, 0.819, 1.778
10	1.403	10	0.396, 0.958, 2.223
≥12.8	1.488	15	0.421, 1.034, 2.508

5.2. Short-Term Power Constraint. Under the short-term power constraint, the optimization criterion is simplified to

$$\theta = \sum_{i=1}^{2^N} p_i \log\left(1 + \frac{P}{\delta^2 E_i}\right) \tag{34}$$

which, taking the derivative with respect to a_i, leads to

$$\log\left(\frac{\xi_i}{\xi_{i+1}}\right) = a_i^2 \left(\frac{\xi_i - 1}{\xi_i E_i} - \frac{\xi_{i+1} - 1}{\xi_{i+1} E_{i+1}} \right) + \frac{1}{\xi_i} - \frac{1}{\xi_{i+1}},$$
$$\xi_i = 1 + \frac{P}{\delta^2 E_i}. \tag{35}$$

Here, it is obvious that, in general, the optimal quantization boundaries are not independent of total input power. However, using the approximation $1 + P/\delta^2 E_i \approx P/\delta^2 E_i$, (35) can be simplified to (33) which shows that, for high SNR's, the optimal quantization can be obtained by (33)

(I) For a given power P, consider M, for example, $M = 20$, randomly generated gain vectors $\overline{a}_m = [a_{0,m}\ a_{1,m} \cdots a_{2^N,m}]$, $a_{0,m} = 0 < a_{1,m} \leq \cdots \leq a_{2^N,m} = \infty$.
(II) For each vector, find the optimal power allocation and the capacity lower bound c_m based on (16), (18) and (20)–(22).
(III) Determine the vector which results in highest capacity lower bound, that is, $\overline{a}_l$ where $c_m \leq c_l$, for all $m = 1, \dots, M$.
(IV) $\overline{a}_1 \leftarrow \overline{a}_l$.
(V) Generate $t \ll M$, for example, $t = 5$, new vectors $\overline{a}_m^{\text{new}}$, $m = 1, \dots, t$, around $\overline{a}_1$. These vectors should also satisfy the constraints introduced in (I).
(VI) $\overline{a}_{m+1} \leftarrow \overline{a}_m^{\text{new}}$, $m = 1, \dots, t$.
(VII) Regenerate the remaining vectors $\overline{a}_m$, $m = t+2, \dots, M$ randomly such that $a_{0,m} = 0 < a_{1,m} \leq \cdots \leq a_{2^N,m} = \infty$, for all $m = t+2, \dots, M$.
(VIII) Go to (II) and continue until convergence.

Algorithm 1: Capacity lower bound optimization.

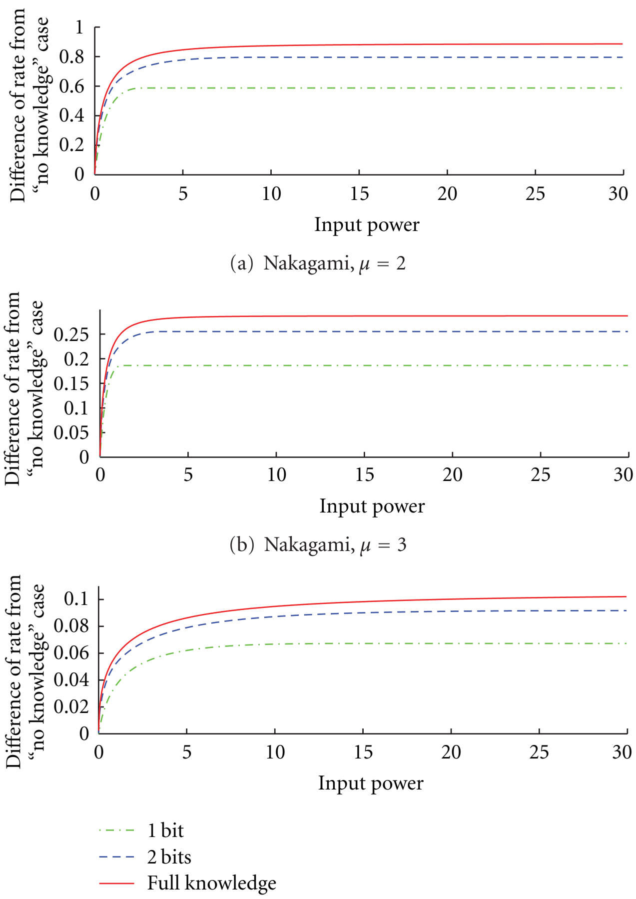


Figure 6: Increase in channel capacity relative the "no knowledge" lower bound using optimal quantization.

independent of whether the power constraint is short term or long term. For more comparison, considering the three channel gain distributions, the quantization boundaries have been determined for different powers based on (35) which can be reviewed in Table 4. As it can be seen, as the input power increases, the quantization boundaries converge to the ones obtained under long-term power constraint.

Table 4: Optimal quantization boundaries, reliable quantization region detection, and short-term power constraint.

pdf, input power	1 bit	2 bits
Nakagami $\mu = 2$, $P = 1$	1.244	0.843, 1.192, 1.859
Nakagami $\mu = 2$, $P = 5$	1.350	0.897, 1.307, 2.127
Nakagami $\mu = 2$, $P = 10$	1.376	0.912, 1.340, 2.212
Nakagami $\mu = 2$, $P = 15$	1.384	0.916, 1.355, 2.252
Nakagami $\mu = 3$, $P = 1$	1.153	0.852, 1.126, 1.591
Nakagami $\mu = 3$, $P = 5$	1.205	0.887, 1.188, 1.708
Nakagami $\mu = 3$, $P = 10$	1.214	0.894, 1.207, 1.744
Nakagami $\mu = 3$, $P = 15$	1.217	0.896, 1.209, 1.745
Log-normal, $P = 1$	0.861	0.332, 0.756, 1.739
Log-normal, $P = 5$	1.190	0.403, 0.983, 2.398
Log-normal, $P = 10$	1.313	0.417, 1.023, 2.458
Log-normal, $P = 15$	1.370	0.419, 1.032, 2.459

6. Channel Capacity Bounds: Unreliable Quantization Region Detection

In contrast to the previous case, where there are 2^N different desired input distributions, each of which selected specifically based on their corresponding channel gain quantization, in the presence of quantization region detection error, the transmitter and the receiver may be wrongly informed about the channel gain and so, for a given input distribution X_j, any of the partitioned channel gain distributions, $f_{G_i}(g) = (1/p_i) f_G(g)$, $g \in S_i$, $i = 1, \dots, 2^N$, may be selected. In this case, the capacity can be written as

$$C = \max \sum_{j=1}^{2^N} p\left(X_j\right) I\left(X_j; Y_j \mid \hat{q}_j\right), \qquad (36)$$

where $p(X_j)$ is the probability of jth input selection, $Y_j = X_j + Z\hat{G}_j$ is the channel output in response to the jth input distribution passing through the channel having gain distribution $\hat{G}_j$, and $\hat{q}_j$ denotes the unreliable quantization region detection information available to the transmitter and the receiver. As illustrated in Figure 8, considering the 2^N nonoverlapping channel gain distributions mentioned in the

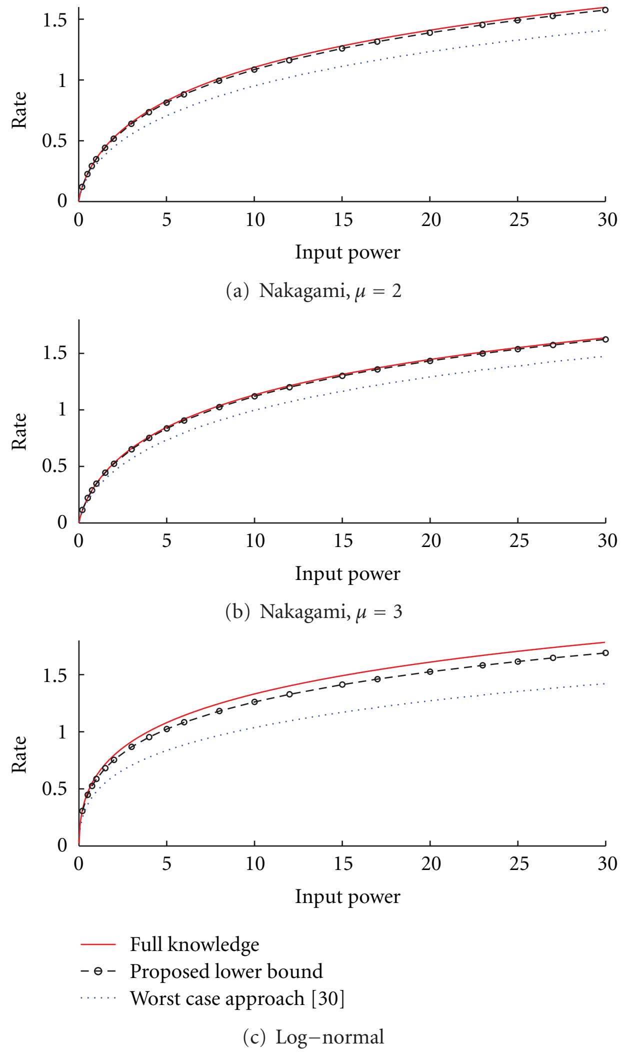


Figure 7: Comparison between the two proposed channel capacity lower bounds.

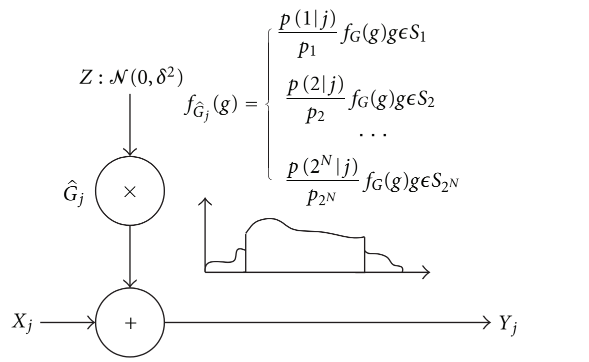


Figure 8: Channel model for a given X_j in the presence of unreliable quantization region detection.

previous section, given an input distribution index j, the channel gain distribution is changed to

$$f_{\hat{G}_j}(g) = \begin{cases} \dfrac{p(1 \mid j)}{p_1} f_G(g) & g \in S_1 \\ \dfrac{p(2 \mid j)}{p_2} f_G(g) & g \in S_2 \\ \vdots & \\ \dfrac{p(2^N \mid j)}{p_{2^N}} f_G(g) & g \in S_{2^N}, \end{cases} \quad (37)$$

where again $p_i = \int_{S_i} f_G(g)\mathrm{d}g$ is probability of the ith quantization interval and $p(i \mid j)$ is the probability of being in the ith quantization region when the jth input distribution is selected. In this way, for a given X_j, it can be written as

$$\begin{aligned} E\hat{G}_j^2 &= \int_{\bigcup_k S_k} g^2 f_{\hat{G}_j}(g)\mathrm{d}g \\ &= \sum_{k=1}^{2^N} \frac{p(k \mid j)}{p_k} \int_{S_k} g^2 f_G(g)\mathrm{d}g \\ &= \sum_{k=1}^{2^N} p(k \mid j) E_k, \\ E_k &= \frac{1}{p_k} \int_{S_k} g^2 f_G(g)\mathrm{d}g \end{aligned} \quad (38)$$

and so, according to (11) and (12), it follows that

$$\begin{aligned} &\frac{1}{2}\log\left(1 + \frac{T_j}{\delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k}\right) \\ &\quad \le I\left(X_j; Y_j \mid \hat{q}_j\right) \\ &\quad \le \frac{1}{2}\log\left(2\pi e\left(T_j + \delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k\right)\right) - h\left(Z\hat{G}_j\right). \end{aligned} \quad (39)$$

Therefore, the total achievable rate is bounded to

$$\begin{aligned} &\frac{1}{2}\sum_{j=1}^{2^N} p\left(X_j\right) \log\left(1 + \frac{T_j}{\delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k}\right) \\ &\quad \le C \\ &\quad \le \sum_{j=1}^{2^N} p\left(X_j\right) \left(\frac{1}{2}\log\left(2\pi e\left(T_j + \delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k\right)\right) \right. \\ &\qquad \left. - h\left(Z\hat{G}_j\right)\right). \end{aligned} \quad (40)$$

Again, in order to find the optimal input power allocation maximizing the capacity lower bound, on which it is focused, we introduce the Lagrangian

$$\theta = \sum_{j=1}^{2^N} p(X_j) \log\left(1 + \frac{T_j}{\delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k}\right)\Bigg) + \rho \sum_{j=1}^{2^N} T_j p(X_j) \quad (41)$$

which, taking the derivative with respect to T_j, leads to the same water-filling results, that is,

$$T_j + \delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k = \frac{-1}{\rho} = \lambda$$
$$\lambda = \frac{1}{\sum_{j;\, T_j>0} p(X_j)} \left(P + \delta^2 \sum_{j;\, T_j>0} p(X_j) \sum_{k=1}^{2^N} p(k \mid j) E_k \right). \quad (42)$$

Equation (41) can also be considered as the optimization criterion finding the optimal channel quantization boundaries maximizing the lower bound; based on the facts that

$$\frac{\partial p(X_j)}{\partial a_i} = \frac{\partial}{\partial a_i} \sum_{k=1}^{2^N} p(j \mid k) p_k = \sum_{k=1}^{2^N} p(j \mid k) \frac{\partial p_k}{\partial a_i}$$
$$= \{p(j \mid i) - p(j \mid i+1)\} f_g(a_i)$$
$$\frac{\partial}{\partial a_i} \sum_{k=1}^{2^N} p(k \mid j) E_k = \frac{\partial}{\partial a_i} \sum_{k=1}^{2^N} \frac{p(j \mid k) p_k E_k}{p(X_j)}$$
$$= \frac{\{p(j \mid i) - p(j \mid i+1)\} f_g(a_i)}{p(X_j)} \times \left\{ a_i^2 - \sum_{k=1}^{2^N} p(k \mid j) E_k \right\}, \quad (43)$$

setting the $\partial\theta/\partial a_i$ equal to zero, for every i, it follows that

$$\sum_{j=1}^{2^N} \psi_{ji} \log\left(1 + \frac{T_j}{\delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k}\right) + \sum_{j=1}^{2^N} \frac{-T_j \psi_{ji} \left[a_i^2 - \sum_{k=1}^{2^N} p(k \mid j) E_k \right]}{\left(T_j + \delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k\right)\left(\sum_{k=1}^{2^N} p(k \mid j) E_k\right)} + \rho \sum_{j=1}^{2^N} T_j \psi_{ji} = 0$$
$$\psi_{ji} = p(j \mid i) - p(j \mid i+1) \quad (44)$$

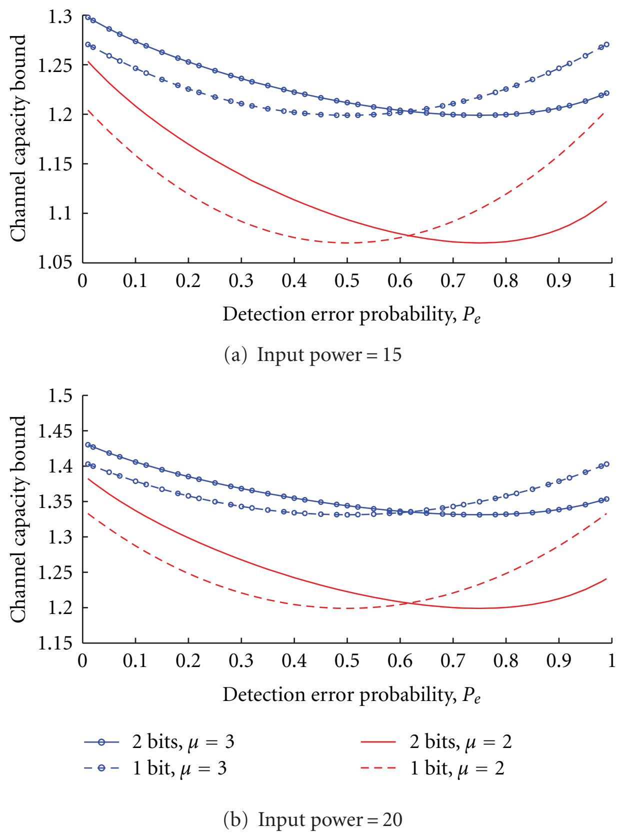


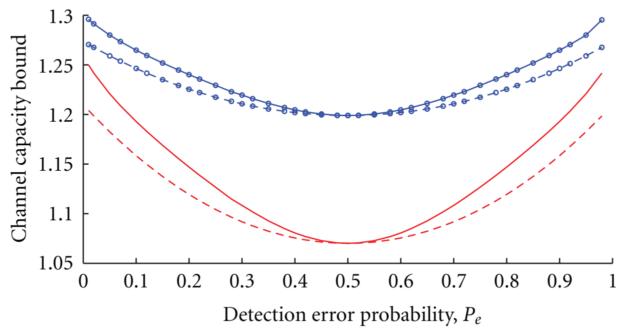
FIGURE 9: Channel capacity lower bound versus detection error probability. Quantization region detection model: (46).

which, considering $T_j + \delta^2 \sum_{k=1}^{2^N} p(k \mid j) E_k = -1/\rho = \lambda$ for high input powers, is simplified to

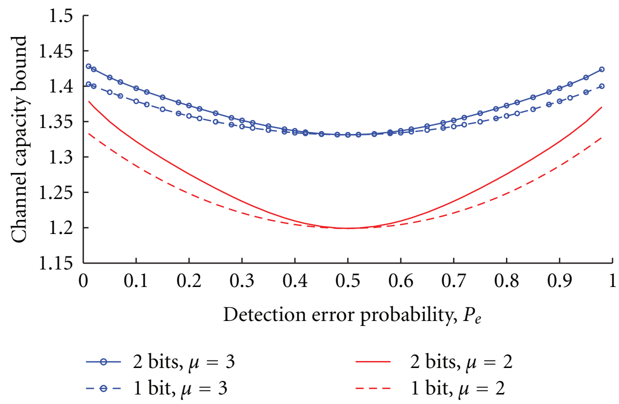
$$\sum_{j=1}^{2^N} [p(j \mid i) - p(j \mid i+1)] \log\left(\sum_{k=1}^{2^N} p(k \mid j) E_k\right) = -a_i^2 \sum_{j=1}^{2^N} \frac{p(j \mid i) - p(j \mid i+1)}{\sum_{k=1}^{2^N} p(k \mid j) E_k}. \quad (45)$$

These equations, along with (42), provide the nonlinear equations determining the optimal quantization boundaries and input power allocation maximizing the proposed lower bound of achievable rates under long-term power constraint and for any gain detector. Finally, removing the last term of (41), the same approach can be implemented for optimizing the quantization boundaries under short-term power constraint, and we do not discuss it any further.

It is obvious that, under unreliable quantization region detection conditions, the achievable rates are functions of the channel detector which can be optimized as well. However, for simplicity and just as illustrative examples, we have



(a) Input power = 15



(b) Input power = 20

FIGURE 10: Channel capacity lower bound versus detection error probability. Quantization region detection model: $p(i \mid j) = P_e^{m_{ij}}(1-P_e)^{(N-m_{ij})}$.

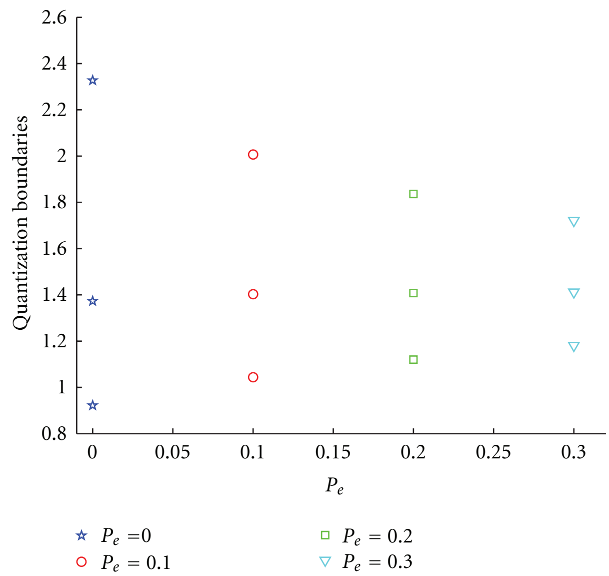


FIGURE 11: Power-independent optimal quantization boundaries for different values of error probability in the detection model, CG distribution: Nakagami, $\mu = 2$.

TABLE 5: Optimal quantization boundaries, unreliable quantization region detection, long-term power constraint, and CG distribution: Nakagami, $\mu = 2, w = 1$, $Pe = 0.1$.

Input power	1 bit	Input power	2 bits
0.5	1.063	0.5	0.895, 1.013, 1.147
1	1.182	1	0.937, 1.094, 1.296
1.5	1.268	2	0.970, 1.181, 1.501
≥1.9	1.405	3	1.025, 1.303, 1.550
		≥4.8	1.044, 1.403, 2.007

considered two different unreliable detection models. The first one is a symmetric model mathematically given by

$$P(i \mid j) = \begin{cases} 1-P_e & j = i \\ \dfrac{P_e}{2^N-1} & j \neq i. \end{cases} \tag{46}$$

Although simple, the model can still give us insight about the channel capacity bounds for the case of unreliable quantization region detection. Considering 1- and 2-bit quantization and different values of detection error probability P_e, the quantization intervals maximizing the lower bound are found for Nakagami ($\mu = 2$) channel gain distribution which can be seen in Tables 5, 6, and 7. Again, the tables emphasize that, for high input powers, the optimal quantization boundaries become independent of the input power allocation, in harmony with (45). Also, considering fixed input powers $P = 15$ and $P = 20$, the effect of detection error probabilities, P_e, on the lower bound of channel capacity has been investigated for two Nakagami distributions, as illustrated in Figure 9. As can be seen, the minimum of achievable rates for 1- and 2 bit quantization occurs at the points $P_e = 0.5$ and $P_e = 0.75$ (the maximum entropy points of the detection error model), respectively, which, considering Figures 2 and 9, correspond to the case of no channel information. For more clarity, the same figure is plotted for the case where the detection error probability is modeled as $p(i \mid j) = P_e^{m_{ij}}(1-P_e)^{(N-m_{ij})}$ where m_{ij} is the number of different bits between the binary representations of the ith and the jth quantization indices (Figure 10). Here, for 1- and 2 bit quantization and for both distributions, the minimum achievable rates, which are associated with no knowledge case, are obtained at $P_e = 0.5$. This is again the point that leads to maximum entropy of the detection model. Finally, Figure 11 summarizes the optimal 2-bit power-independent quantization boundaries of Tables 1 and 5–7. The figure shows that, increasing the quantization region detection error probability, the quantization boundaries converge together. This result is valid for the other distributions also.

7. Conclusion

The aim of this paper is to investigate the effect of channel side information on the achievable rates of continuous power-limited non-Gaussian channels. Lower and upper bounds of channel capacity are determined for the case where (1) there is quantized channel state information

Table 6: Optimal quantization boundaries, unreliable quantization region detection, long-term power constraint, and CG distribution: Nakagami, $\mu = 2$, $w = 1$, $Pe = 0.2$.

Input power	1 bit	Input power	2 bits
0.5	0.955	0.5	0.730, 0.818, 0.890
0.75	1.091	1	0.910, 1.026, 1.166
1	1.239	2	1.115, 1.352, 1.391
≥1.3	1.410	3	1.116, 1.404, 1.776
		≥3.2	1.118, 1.408, 1.836

Table 7: Optimal quantization boundaries, unreliable quantization region detection, long-term power constraint, and CG distribution: Nakagami, $\mu = 2$, $w = 1$, $Pe = 0.3$.

Input power	1 bit	Input power	2 bits
0.5	1.091	0.5	0.709, 0.811, 0.910
0.7	1.356	1	0.808, 0.954, 1.116
≥0.8	1.412	2	1.179, 1.405, 1.576
		≥2.2	1.181, 1.412, 1.721

available at the transmitter and receiver, respectively, and (2) having in-region variations, the channel gain remains in the same quantization region for a long time. The results are found for two scenarios. In the first scenario, it is supposed that the channel gain quantization region information is correctly detected and fed back to the transmitter. In this case, first the channel capacity bounds were found for two different Max-Lloyd and equal probability quantization approaches and three different log-normal and Nakagami-based distributions and then, the optimal quantizations maximizing the bounds of achievable rates were obtained. In the second scenario, the quantization region detection is not reliable, and so the transmitter and the receiver may be wrongly informed about the channel status. In this case, the general formula for the capacity bounds and the optimal quantizations were presented which were evaluated via some simulations.

In summary, the results emphasize the following points.

(i) The proposed lower bound leads to higher achievable rates in comparison to the *worst case* approach which has been previously developed for block fading channels and can be considered as a lower bound for this channel condition also. Moreover, considering different fading conditions, it is possible to reach near-full-knowledge performance using 2-bit channel quality information feedback.

(ii) For high SNR's, it is possible to determine a power-independent optimal quantization maximizing the capacity lower bound which is identical for both short- and long-term power constraints.

(iii) While channel side information results in significant rate increment in severe fading conditions, as the fading severity decreases the effect of channel side information and adaptive transmission is also reduced.

(iv) Increasing the detection error probability, the quantization boundaries converge together reducing the effect of channel side information.

Finally, considering the effect of feedback signaling overhead on the optimization problems is left for the future.

References

[1] C. E. Shannon, "Channels with side information at the transmitter," *IBM Journal of Research and Development*, vol. 2, pp. 289–293, 1958.

[2] M. Médard, "The effect upon channel capacity in wireless communications of perfect and imperfect knowledge of the channel," *IEEE Transactions on Information Theory*, vol. 46, no. 3, pp. 933–946, 2000.

[3] J. C. Roh and B. D. Rao, "Transmit beamforming in multiple-antenna systems with finite rate feedback: a VQ-based approach," *IEEE Transactions on Information Theory*, vol. 52, no. 3, pp. 1101–1112, 2006.

[4] T. Eriksson and T. Ottosson, "Compression of feedback for adaptive transmission and scheduling," *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2314–2321, 2007.

[5] T. Eriksson and T. Ottosson, "Compression of feedback in adaptive OFDM-based systems using scheduling," *IEEE Communications Letters*, vol. 11, no. 11, pp. 859–861, 2007.

[6] B. Makki and T. Eriksson, "Efficient channel quality feedback signaling using transform coding and bit allocation," in *Proceedings of the 71st IEEE Vehicular Technology Conference (VTC '10)*, Taipei, Taiwan, May 2010.

[7] http://www.3gpp.org/.

[8] 3GPP, "Technical specification group radio access network, physical layer," Tech. Spec. 3GPP ts 25.214 V3.10.0, 3rd Generation Partnership Project, 2002-2003.

[9] R. Dobrushin, "Transmission of information in channels with feedback," *Theory of Probability and Its Applications*, vol. 3, no. 4, pp. 395–412, 1958.

[10] J. Wolfowitz, *Coding Theorems of Information Theory*, Prentice-Hall, Berlin, Germany, 1961.

[11] A. J. Goldsmith and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Transactions on Information Theory*, vol. 43, no. 6, pp. 1986–1992, 1997.

[12] G. Caire and S. Shamai, "On the capacity of some channels with channel state information," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 2007–2019, 1999.

[13] A. Das and P. Narayan, "Capacities of time-varying multiple-access channels with side information," *IEEE Transactions on Information Theory*, vol. 48, no. 1, pp. 4–25, 2002.

[14] S. C. Tatikonda, *Control under communication constraints*, Ph.D. dissertation, MIT, Cambridge, Mass, USA, 2000.

[15] S. Verdu and T. S. Han, "General formula for channel capacity," *IEEE Transactions on Information Theory*, vol. 40, no. 4, pp. 1147–1157, 1994.

[16] A. Lapidoth and S. Shamai, "Fading channels: how perfect need "perfect side information" be?" *IEEE Transactions on Information Theory*, vol. 48, no. 5, pp. 1118–1134, 2002.

[17] M. Skoglund and G. Jöngren, "On the capacity of a multiple-antenna communication link with channel side information," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 3, pp. 395–405, 2003.

[18] V. K. N. Lau, Y. Liu, and T. A. Chen, "Capacity of memoryless channels and block-fading channels with designable cardinality-constrained channel state feedback," *IEEE*

Transactions on Information Theory, vol. 50, no. 9, pp. 2038–2049, 2004.

[19] S. Yüksel and S. Tatikonda, "Capacity of Markov channels with partial state feedback," in *Proceedings of IEEE International Symposium on Information Theory (ISIT '07)*, pp. 1861–1865, June 2007.

[20] R. Gallager, *Information Theory and Reliable Communication*, John Wiley & Sons, New York, NY, USA, 1968.

[21] J. Chen and T. Berger, "The capacity of finite-state Markov channels with feedback," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 780–798, 2005.

[22] S. Yang, A. Kavčić, and S. Tatikonda, "Feedback capacity of finite-state machine channels," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 799–810, 2005.

[23] S. Jafar, "Capacity with causal and noncausal side information: a unified view," *IEEE Transactions on Information Theory*, vol. 52, no. 12, pp. 5468–5474, 2006.

[24] Y. H. Kim, A. Lapidoth, and T. Weissman, "The Gaussian channel with noisy feedback," in *Proceedings of the IEEE International Symposium on Information Theory (ISIT '07)*, pp. 1416–1420, June 2007.

[25] N. C. Martins and T. Weissman, "Coding for additive white noise channels with feedback corrupted by quantization or bounded noise," *IEEE Transactions on Information Theory*, vol. 54, no. 9, pp. 4274–4282, 2008.

[26] R. Dabora and A. Goldsmith, "Capacity theorems for the finite-state broadcast channel with feedback," in *Proceedings of the IEEE International Symposium on Information Theory (ISIT '08)*, pp. 1716–1720, July 2008.

[27] H. H. Permuter, T. Weissman, and J. Chen, "Capacity region of the finite-state multiple-access channel with and without feedback," *IEEE Transactions on Information Theory*, vol. 55, no. 6, pp. 2455–2477, 2009.

[28] S. Tatikonda and S. Mitter, "The capacity of channels with feedback," *IEEE Transactions on Information Theory*, vol. 55, no. 1, pp. 323–349, 2009.

[29] X. Liu, H. Yang, W. Guo, and D. Yang, "Capacity of fading channels with quantized channel side information," *IEICE Transactions on Communications*, vol. E89-B, no. 2, pp. 590–593, 2006.

[30] T. T. Kim and M. Skoglund, "On the expected rate of slowly fading channels with quantized side information," *IEEE Transactions on Communications*, vol. 55, no. 4, pp. 820–829, 2007.

[31] S. Ekbatani, F. Etemadi, and H. Jafarkhani, "Transmission over slowly fading channels using unreliable quantized feedback," in *Proceedings of the Data Compression Conference (DCC '07)*, pp. 353–362, Snowbird, Utah, USA, March 2007.

[32] P. L. Brockett, M. Hinich, G. R. Wilson et al., "Robust multiuser detection in non-Gaussian channels," *Journal of the Acoustical Society of America*, vol. 82, pp. 1286–1399, 1987.

[33] D. Hui and R. Ramésh, "Iterative channel and parameter estimation for noise with non-Gaussian distributions," in *Proceedings of IEEE Global Telecommunications Conference (GLOBECOM '03)*, vol. 5, pp. 2426–2431, December 2003.

[34] X. Wang and H. V. Poor, "Robust multiuser detection in non-gaussian channels," *IEEE Transactions on Signal Processing*, vol. 47, no. 2, pp. 289–305, 1999.

[35] M. S. Pinsker, V. V. Prelov, and S. Verdu, "Sensitivity of channel capacity," *IEEE Transactions on Information Theory*, vol. 41, no. 6, pp. 1877–1888, 1995.

[36] V. V. Prelov and E. C. van der Meulen, "Higher order asymptotics of mutual information for nonlinear channels with non-Gaussian noise," in *Proceedings IEEE International Symposium on Information Theory (ISIT '03)*, p. 83, Yokohama, Japan, June-July 2003.

[37] C. R. Baker, "Coding capacity for a class of additive channels," *IEEE Transactions on Information Theory*, vol. 37, no. 2, pp. 233–243, 1991.

[38] J. Binia, "Divergence and minimum mean-square error in continuous-time additive white Gaussian noise channels," *IEEE Transactions on Information Theory*, vol. 52, no. 3, pp. 1160–1163, 2006.

[39] A. Lapidoth, "Nearest neighbor decoding for additive non-Gaussian noise channels," *IEEE Transactions on Information Theory*, vol. 42, no. 5, pp. 1520–1529, 1996.

[40] A. Das, "Capacity-achieving distributions for non-Gaussian additive noise channels," in *Proceedings of IEEE International Symposium on Information Theory (ISIT '00)*, p. 432, June 2000.

[41] A. Tchamkerten, "On the discreteness of capacity-achieving distributions," *IEEE Transactions on Information Theory*, vol. 50, no. 11, pp. 2773–2778, 2004.

[42] J. S. Smith, *On the information capacity of peak and average power constrained Gaussian channels*, Ph.D dissertation, Department of Electrical Engineering, University of California, 1969.

[43] C. Luschi and B. Mulgrew, "Nonparametric trellis equalization in the presence of non-Gaussian interference," *IEEE Transactions on Communications*, vol. 51, no. 2, pp. 229–239, 2003.

[44] D. Hui and R. Ramesh, "Maximum likelihood sequence estimation in the presence of constant envelope interference," in *Proceedings of the 58th IEEE Vehicular Technology Conference (VTC '03)*, vol. 2, pp. 1060–1064, Orlando, Fla, USA, October 2003.

[45] R. Schober and L. Lampe, "Sequence detection and adaptive channel estimation for ISI channels under Class-A impulsive noise," *IEEE Transactions on Communications*, vol. 52, no. 9, pp. 1523–1531, 2004.

[46] Y. Wang and L. Wei, "Turbo TCM coded OFDM systems for non-Gaussian channels," in *Proceedings of the IEEE International Symposium on Information Theory (ISIT '06)*, pp. 1389–1393, July 2006.

[47] S. Lloyd, "Least squares quantization in PCM," *IEEE Transactions on Information Theory*, vol. 28, no. 2, pp. 129–137, 1982.

[48] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, John Wiley & Sons, New York, NY, USA, 2006.

[49] B. Makki, M. N. Hosseini, S. A. Seyyedsalehi, and N. Sadati, "Unaligned training for voice conversion based on a local nonlinear principal component analysis approach," *Neural Computing and Applications*, vol. 19, no. 3, pp. 437–444, 2010.