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Erratum

A Note to Paper "On the Stability of Cubic Mappings and Quartic Mappings in Random Normed Spaces"

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Recently, Baktash et al. (2008) proved the stability of the cubic functional equation $f(2x+y)+f(2x-y) = 2f(x+y)+2f(x-y)+12f(x)$ and the quartic functional equation $f(2x+y)+f(2x-y) = 4f(x+y)+4f(x-y)+24f(x)-6f(y)$ in random normed spaces. In this note, we correct the proofs.

1. Introduction and Preliminaries

If $\inf\{t > 0 : F(t) > a\} \le \inf\{t > 0 : G(t) > a\}$, in general we cannot conclude that $F(t) \ge G(t)$. For example, let $F(t) = 3/4$, $G(t) = t/(t+1)$ and $a = 1/2$. We know that $\inf\{t > 0 : 3/4 > 1/2\} = 0 \le \inf\{t > 0 : t/(t+1) > 1/2\} = 1$ but $F(4) = 3/4 < G(4) = 4/5$. This example shows that in [1], inequalities (2.13) and (3.13) do not follow from inequalities (2.12) and (3.12).

The functional equation

$$f(2x+y) + f(2x-y) = 2f(x+y) + 2f(x-y) + 12f(x) \tag{1.1}$$

is said to be the cubic functional equation since the function $f(x) = cx^3$ is its solution. Every solution of the cubic functional equation is said to be a cubic mapping. The stability problem for the cubic functional equation was solved by Jun and Kim [2] and Lee [3] for mappings $f : X \to Y$, where X is a real normed space and Y is a Banach space. Later, a number of mathematicians have worked on the stability of some types of the cubic equation [4]. The functional equation

$$f(2x+y) + f(2x-y) = 4f(x+y) + 4f(x-y) + 24f(x) - 6f(y) \tag{1.2}$$

is said to be the quartic functional equation since the function $f(x) = cx^4$ is its solution. Every solution of the quartic functional equation is said to be a quartic mapping. The stability problem for the quartic functional equation first was solved by Rassias [5] and Lee and Chung [6] for mappings $f : X \to Y$, where X is a real normed space and Y is a Banach space.

In the sequel, we shall adopt the usual terminology, notations and conventions of the theory of random normed spaces as in [7–15]. Throughout this paper, the space of all probability distribution functions is denoted by

$$\Delta^+ = \{F : \mathbb{R} \cup \{-\infty, +\infty\} \to [0,1] : F \text{ is left-continuous and nondecreasing on } \mathbb{R} \text{ and } F(0) = 0,\ F(+\infty) = 1\} \tag{1.3}$$

and the subset $D^+ \subseteq \Delta^+$ is the set $D^+ = \{F \in \Delta^+ : l^- F(+\infty) = 1\}$, where $l^- f(x)$ denotes the left limit of the function f at the point x. The space Δ^+ is partially ordered by the usual point-wise ordering of functions, that is, $F \le G$ if and only if $F(t) \le G(t)$ for all t in $\mathbb{R}$. The maximal element for Δ^+ in this order is the distribution function given by

$$\varepsilon_0(t) = \begin{cases} 0, & \text{if } t \le 0, \\ 1, & \text{if } t \le 0. \end{cases} \tag{1.4}$$

Definition 1.1 (see [13]). A function $T : [0,1] \times [0,1] \to [0,1]$ is a continuous triangular norm (briefly, a t-norm) if T satisfies the following conditions:

(a) T is commutative and associative;

(b) T is continuous;

(c) $T(a,1) = a$ for all $a \in [0,1]$;

(d) $T(a,b) \le T(c,d)$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d \in [0,1]$.

Three typical examples of continuous t-norms are $T(a,b) = ab$, $T(a,b) = \max(a+b-1, 0)$ and $T(a,b) = \min(a,b)$.

Recall that, if T is a t-norm and $\{a_n\}$ is a given sequence of numbers in $[0,1]$, $T_{i=1}^{n} a_i$ is defined recursively by $T_{i=1}^{1} a_i = a_1$ and $T_{i=1}^{n} a_i = T(T_{i=1}^{n-1} a_i, a_n)$ for $n \ge 2$.

Definition 1.2. A *random normed space* (briefly, RN-space) is a triple (X, μ, T), where X is a vector space, T is a continuous t-norm and μ is a mapping from X into D^+ such that the following conditions hold:

(PN1) $\mu_x(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $x = 0$;

(PN2) $\mu_{\alpha x}(t) = \mu_x(t/|\alpha|)$ for all x in X, $\alpha \neq 0$ and $t \ge 0$;

(PN2) $\mu_{x+y}(t+s) \ge T(\mu_x(t), \mu_y(s))$ for all $x, y \in X$ and $t, s \ge 0$.

Definition 1.3. Let (X, μ, T) be an RN-space.

(1) A sequence $\{x_n\}$ in X is said to be *convergent* to x in X if, for every $t > 0$ and $\varepsilon > 0$, there exists a positive integer N such that $\mu_{x_n - x}(t) > 1 - \varepsilon$ whenever $n \ge N$.

(2) A sequence $\{x_n\}$ in X is called *Cauchy sequence* if, for every $t > 0$ and $\varepsilon > 0$, there exists a positive integer N such that $\mu_{x_n - x_m}(t) > 1 - \varepsilon$ whenever $n \ge m \ge N$.

(3) An RN-space (X, μ, T) is said to be *complete* if and only if every Cauchy sequence in X is convergent to a point in X.

Theorem 1.4 (see [13]). *If (X, μ, T) is an RN-space and $\{x_n\}$ is a sequence such that $x_n \to x$, then* $\lim_{n\to\infty}\mu_{x_n}(t) = \mu_x(t)$.

In this paper, we establish the stability of the cubic and quartic functional equations in the setting of random normed spaces.

2. On the Stability of Cubic Mappings in RN-Spaces

Theorem 2.1. *Let X be a linear space, $(Z, \mu', \min)$ be an RN-space, $\varphi : X \times X \to Z$ be a function such that for some $0 < \alpha < 8$,*

$$\mu'_{\varphi(2x,0)}(t) \geq \mu'_{\alpha\varphi(x,0)}(t), \quad \forall x \in X,\ t > 0, \tag{2.1}$$

$f(0) = 0$ and $\lim_{n\to\infty}\mu'_{\varphi(2^n x, 2^n y)}(8^n t) = 1$ for all $x, y \in X$ and $t > 0$. Let $(Y, \mu, \min)$ be a complete RN-space. If $f : X \to Y$ is a mapping such that

$$\mu_{f(2x+y)+f(2x-y)-2f(x+y)-2f(x-y)-12f(x)}(t) \geq \mu'_{\varphi(x,y)}(t), \quad \forall x \in X,\ t > 0, \tag{2.2}$$

then there exists a unique cubic mapping $C : X \to Y$ such that

$$\mu_{f(x)-C(x)}(t) \geq \mu'_{\varphi(x,0)}(2(8-\alpha)t). \tag{2.3}$$

Proof. Putting $y = 0$ in (2.2), we get

$$\mu_{(f(2x)/8)-f(x)}(t) \geq \mu'_{\varphi(x,0)}(16t), \quad \forall x \in X. \tag{2.4}$$

Replacing x by $2^n x$ in (2.4) and using (2.1), we obtain

$$\begin{aligned}\mu_{(f(2^{n+1}x)/8^{n+1})-(f(2^n x)/8^n)}(t) &\geq \mu'_{\varphi(2^n x,0)}(16 \times 8^n) \\ &\geq \mu'_{\varphi(x,0)}\left(\frac{16 \times 8^n}{\alpha^n}\right).\end{aligned} \tag{2.5}$$

It follows from $(f(2^n x)/8^n) - f(x) = \sum_{k=0}^{n-1}((f(2^{k+1}x)/8^{k+1}) - (f(2^k x)/8^k))$ and (2.5) that

$$\mu_{(f(2^n x)/8^n)-f(x)}\left(t\sum_{k=0}^{n-1}\frac{\alpha^k}{16 \times 8^k}\right) \geq T_{k=0}^{n-1}\left(\mu'_{\varphi(x,0)}(t)\right) = \mu'_{\varphi(x,0)}(t), \tag{2.6}$$

that is,

$$\mu_{(f(2^n x)/8^n)-f(x)}(t) \geq \mu'_{\varphi(x,0)}\left(\frac{t}{\sum_{k=0}^{n-1}(\alpha^k/(16\times 8^k))}\right). \tag{2.7}$$

By replacing x with $2^m x$ in (2.7), we observe that

$$\mu_{(f(2^{n+m}x)/8^{n+m})-(f(2^m x)/8^m)}(t) \geq \mu'_{\varphi(x,0)}\left(\frac{t}{\sum_{k=m}^{n+m}(\alpha^k/(16\times 8^k))}\right). \tag{2.8}$$

As $\mu'_{\varphi(x,0)}(t/\sum_{k=m}^{n+m}(\alpha^k/16\times 8^k))$ tends to 1 as m,n tend to ∞, then $\{f(2^n x)/8^n\}$ is a Cauchy sequence in $(Y,\mu,\min)$. Since $(Y,\mu,\min)$ is a complete RN-space, this sequence converges to some point $C(x)\in Y$. Fix $x\in X$ and put $m=0$ in (2.8). Then we obtain

$$\mu_{(f(2^n x)/8^n)-f(x)}(t) \geq \mu'_{\varphi(x,0)}\left(\frac{t}{\sum_{k=0}^{n-1}(\alpha^k/(16\times 8^k))}\right) \tag{2.9}$$

and so, for every $\delta>0$, we have

$$\begin{aligned}\mu_{C(x)-f(x)}(t+\delta) &\geq T\left(\mu_{C(x)-(f(2^n x)/8^n)}(\delta),\mu_{(f(2^n x)/8^n)-f(x)}(t)\right)\\ &\geq T\left(\mu_{C(x)-(f(2^n x)/8^n)}(\delta),\mu'_{\varphi(x,0)}\left(\frac{t}{\sum_{k=0}^{n-1}(\alpha^k/(16\times 8^k))}\right)\right).\end{aligned} \tag{2.10}$$

Taking the limit as $n\to\infty$ and using (2.10), we get

$$\mu_{C(x)-f(x)}(t+\delta) \geq \mu'_{\varphi(x,0)}(2t(8-\alpha)). \tag{2.11}$$

Since δ was arbitrary, by taking $\delta\to 0$ in (2.11), we get

$$\mu_{C(x)-f(x)}(t) \geq \mu'_{\varphi(x,0)}(2t(8-\alpha)). \tag{2.12}$$

Replacing x and y by $2^n x$ and $2^n y$ in (2.2), respectively, we get

$$\begin{aligned}&\mu_{(f(2^n(2x+y))/8^n)+(f(2^n(2x-y))/8^n)-(2f(2^n(x+y))/8^n)-(2f(2^n(x-y))/8^n)-(12f(2^n(x))/8^n)}(t)\\ &\quad\geq \mu'_{\varphi(2^n x,2^n y)}(8^n t)\end{aligned} \tag{2.13}$$

for all $x,y\in X$ and for all $t>0$. Since $\lim_{n\to\infty}\mu'_{\varphi(2^n x,2^n y)}(8^n t)=1$, we conclude that C fulfills (1.1). To prove the uniqueness of the cubic mapping C, assume that there exists a cubic mapping $D : X \to Y$ which satisfies (2.3). Fix $x \in X$. Clearly, $C(2^n x) = 8^n C(x)$ and $D(2^n x) = 8^n D(x)$ for all $n \in \mathbb{N}$. It follows from (2.3) that

$$\begin{aligned}\mu_{C(x)-D(x)}(t) &= \lim_{n\to\infty} \mu_{(C(2^n x)/8^n)-(D(2^n x)/8^n)}(t),\\ \mu_{(C(2^n x)/8^n)-(D(2^n x)/8^n)}(t) &\geq \min\left\{\mu_{(C(2^n x)/8^n)-(f(2^n x)/8^n)}\left(\frac{t}{2}\right), \mu_{(D(2^n x)/8^n)-(f(2^n x)/8^n)}\left(\frac{t}{2}\right)\right\}\\ &\geq \mu'_{\varphi(2^n x,0)}(8^n(8-\alpha)t)\\ &\geq \mu'_{\varphi(x,0)}\left(\frac{8^n(8-\alpha)t}{\alpha^n}\right).\end{aligned} \tag{2.14}$$

Since $\lim_{n\to\infty}(8^n(8-\alpha)t/\alpha^n) = \infty$, we get $\lim_{n\to\infty}\mu'_{\varphi(x,0)}(8^n(8-\alpha)t/\alpha^n) = 1$. Therefore, it follows that $\mu_{C(x)-D(x)}(t) = 1$ for all $t > 0$ and so $C(x) = D(x)$. This completes the proof. □

3. On the Stability of Quartic Mappings in RN-Spaces

Theorem 3.1. *Let X be a linear space, $(Z, \mu', \min)$ be an RN-space, $\varphi : X \times X \to Z$ be a function such that for some $0 < \alpha < 16$,*

$$\mu'_{\varphi(2x,0)}(t) \geq \mu'_{\alpha\varphi(x,0)}(t), \quad \forall x \in X,\ t > 0, \tag{3.1}$$

$f(0) = 0$ and $\lim_{n\to\infty}\mu'_{\varphi(2^n x, 2^n y)}(16^n t) = 1$ for all $x, y \in X$ and $t > 0$. Let $(Y, \mu, \min)$ be a complete RN-space. If $f : X \to Y$ is a mapping such that

$$\mu_{f(2x+y)+f(2x-y)-4f(x+y)-4f(x-y)-24f(x)+6f(y)}(t) \geq \mu'_{\varphi(x,y)}(t), \quad \forall x \in X,\ t > 0, \tag{3.2}$$

then there exists a unique quartic mapping $Q : X \to Y$ such that

$$\mu_{f(x)-Q(x)}(t) \geq \mu'_{\varphi(x,0)}(2(16-\alpha)t). \tag{3.3}$$

Proof. The proof is the same as Theorem 2.1. □

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