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Research Article

Ergodic Capacity for the SIMO Nakagami-*m* Channel

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This paper presents closed-form expressions for the ergodic channel capacity of SIMO (single-input and multiple output) wireless systems operating in a Nakagami-*m* fading channel. As the performance of SIMO channel is closely related to the diversity combining techniques, we present closed-form expressions for the capacity of maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC), and switch and stay (SSC) diversity systems operating in Nakagami-*m* fading channels. Also, the ergodic capacity of a SIMO system in a Nakagami-*m* fading channel without any diversity technique is derived. The latter scenario is further investigated for a large amount of receive antennas. Finally, numerical results are presented for illustration.

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1. Introduction

In recent years, the use of multiantenna systems provides large spectral efficiency for wireless communications in the presence of multipath fading environments. Multiple antennas can be used at the transmitter (MISO), the receiver (SIMO), or at both of them (MIMO). A SIMO system can be viewed as an antenna diversity scheme (diversity in space). Also, diversity combining is known to be a powerful technique to improve system performance in the presence of fading [1].

Several papers have been published regarding the capacity of SIMO systems operating in Nakagami environments. In [2], the channel capacity of a SIMO system in a Nakagami-*m* fading channel is presented with the assumption that all links between transmit and receive antennas are independent and identically distributed (i.i.d.). In [3], capacity with MRC and optimal power and rate adaptation is presented while in [4] Shannon capacity with MRC is derived. In both of them, the assumption that all links are correlated and not identically distributed Nakagami holds. In [5], capacity of Nakagami-*m* multipath fading channels with MRC was studied for different power and rate adaptation policies. Also, simple capacity formulas for correlated SIMO Nakagami-*m* channels were derived in [6]. In [7], an analytical expression for the capacity of SIMO systems over nonidentically independent Nakagami-*m* channels was derived. Significant work has been done in [8], where ergodic capacities of MRC, EGC, SC, and SSC of dual branch diversity systems are presented in closed-form expressions. The capacity expressions were obtained by assuming correlated and identically distributed Nakagami-*m* links.

In this paper, we examine the ergodic capacity of a SIMO system operating in independent Nakagami-*m* channels. Specifically we derive closed-form expressions for the ergodic capacity of dual EGC, SC, and SSC systems. For the EGC and SSC cases, we extend the work in [8] by allowing the parameter m of the Nakagami-*m* distribution to take noninteger values. Moreover for the SSC case, a compact and quite flexible formula of the ergodic capacity for integer values of m is presented. For the SC case, we present a new expression for the ergodic capacity with the assumption that the Nakagami branches are not identical. Finally, we present for the first time in international literature the ergodic capacity of a SIMO system without using any diversity combining technique over independent nonidentical Nakagami-*m* branches. In addition, it is shown that when the number of receive antennas is large, the ergodic capacity of such a system can be very well approximated by the ergodic capacity of a Rayleigh channel.

The remaining of this paper is organized as follows. Section 2 introduces a SIMO system. Section 3 examines the ergodic capacity for each diversity scheme and for the case where none of diversity technique is applied. Section 4 presents some results, and Section 5 draws the conclusion.

2. System Description

Consider a SIMO system with L receive antennas operating in independent Nakagami-m channels. The total power of the complex transmitted signal at a symbol period is constrained to be $\overline{S}$. The received signal vector r at a random symbol period, assuming that the channel is constant over a symbol period, is given in a baseband representation as

$$r = \mathbf{a} \cdot (\mathbf{h} \cdot s + \mathbf{n}), \tag{1}$$

where $\mathbf{h} = [h_1, h_2, \dots, h_L]^T$ is the $L \times 1$ complex channel-gain vector (T means matrix transposition), $\mathbf{a} = [a_1, a_2, \dots, a_L]$ is the $1 \times L$ complex antenna-gain vector, and $\mathbf{n}$ is the $L \times 1$ zero-mean complex additive white Gaussian (AWGN) vector with i.i.d. entries and variance N. The received signal can be written as

$$\begin{aligned} r &= s \cdot \sum_{i=1}^{L} h_i \cdot a_i + \sum_{i=1}^{L} n_i a_i \\ &= s \cdot \sum_{i=1}^{L} |h_i| \cdot |a_i| \cdot e^{j(\theta_i + \phi_i)} + \sum_{i=1}^{L} n_i a_i \\ &= s \cdot g + \sum_{i=1}^{L} n_i a_i, \end{aligned} \tag{2}$$

where $h_i = |h_i| \cdot e^{j\theta_i}$, $a_i = |a_i| \cdot e^{j\phi_i}$ and $g = \sum_{i=1}^{L} |h_i| \cdot |a_i| \cdot e^{j(\theta_i+\phi_i)}$. Thus, the received signal-to-noise ratio (SNR) γ over a symbol period is

$$\gamma = \frac{\overline{S}}{N} \frac{|g|^2}{\sum_{i=1}^{L} |a_i|^2}. \tag{3}$$

Assuming that the fading process is ergodic, the ergodic channel capacity is

$$C = W \cdot E\left\langle \log_2(1+\gamma) \right\rangle = W \cdot \int_0^{\infty} \log_2(1+\gamma) p_\gamma(\gamma) d\gamma, \tag{4}$$

where W is the bandwidth of the channel, $E\langle \cdot \rangle$ denotes the ensemble average over γ, and $p_\gamma(\gamma)$ is the probability density function (PDF) of the SNR γ.

3. Ergodic Channel Capacity

For all the cases listed below, we will assume that all links between transmitter and receivers $|h_i|$ are Nakagami-m distributed. From [9], their PDF is

$$p_{|h_i|}(x) = \frac{2{m_i}^{m_i}}{\Gamma(m_i)\Omega_i^{m_i}} x^{2m_i-1} \exp\left(-\frac{m_i}{\Omega_i} x^2\right), \tag{5}$$

where $\Gamma(x)$ is the Gamma function [10, eqution (8.310.1)], $\Omega_i = \mathrm{E}\langle |h_i|^2 \rangle$, and $m_i = \Omega_i^2 / \mathrm{E}\langle (|h_i|^2 - \Omega_i)^2 \rangle \geq 1/2$.

3.1. Ergodic Channel Capacity of Nakagami-m Fading Channel with MRC. Taking into account the above system description and assuming that the receiver has full channel state information (CSI), we choose the phases ϕ_i of a_i, appeared in (2), as $\phi_i = -\theta_i$. Also we choose $|a_i| = |h_i|$. This means that all the signals at the receiver can be added cophasely and weighted according to the channel gain. Thus, (3) becomes

$$\gamma_{\mathrm{MRC}} = \frac{\overline{S}}{N} \sum_{i=1}^{L} |h_i|^2. \tag{6}$$

This SNR arises from the MRC diversity technique [1, equation (9.1)]. Substituting the PDF of (6) in (4) gives the ergodic capacity of the Nakagami-m fading channel using MRC. Closed-form expressions have been presented in [3, equation (20)], [4, equation (16)] for the general case where links are correlated and not identically Nakagami-m distributed. If the links follow i.i.d. Nakagami-m variables, the referred equations reduce to [2, equation (36)]. When the parameter m of i.i.d Nakagami-m branches takes integer values, the ergodic capacity is given by [5, equation (26)]. A good approximation for the ergodic capacity, where links are independent and not identically distributed, was given in [7, equation (16)]. Also a useful expression for the PDF of γ_{MRC} for correlated and not identically distributed links can be found in [11, equation (18)].

3.2. Ergodic Channel Capacity of Nakagami-m Fading Channel with Coherent EGC. Taking into account the system description discussed in Section 2 and assuming that the receiver has full CSI, we choose phases ϕ_i and modulus $|a_i|$, appeared in (2), as $\phi_i = -\theta_i$, $|a_i(k)| = 1$ for all i. Thus (3) becomes

$$\gamma_{\mathrm{EGC}} = \frac{\overline{S}}{L \cdot N} \left(\sum_{i=1}^{L} |h_i| \right)^2. \tag{7}$$

This SNR arises using the coherent EGC diversity technique [1, equation (9.188)]. Substituting the PDF of (7) in (4) gives the ergodic capacity of the Nakagami-m fading channel using coherent EGC diversity technique. Finding analytically the PDF of (7) and consequently the channel capacity seems to be a very difficult problem. In [12], the sum of i.i.d Nakagami-m variables was studied.

The PDF of the sum of two i.i.d. Nakagami-m variables is given by [12, equation (4)]. From [13, page 130], we obtain the PDF transformation $p_Y(x) = p_X(\sqrt{x}/a)/2a\sqrt{x}$ for two random variables Y, X related as $Y = aX^2$. Using that PDF transformation in (7) (here $a = \overline{S}/(L \cdot N)$) with the help of [12, equation (4)], we calculate the PDF of the SNR γ_{EGC}. Thus, the PDF of the SNR of a dual branch EGC system over i.i.d. Nakagami-m fading channels can be written as

$$\begin{aligned} p_{\gamma_{\mathrm{EGC}}}(x) = {} & \frac{\sqrt{\pi}\,\Gamma(2m)}{\Gamma^2(m)\Gamma(2m+1/2)4^{m-1}} \\ & \times \left(\frac{1}{\overline{\gamma}}\right)^{2m} x^{2m-1} e^{-(2/\overline{\gamma})x} {}_1F_1\left(2m, 2m+\frac{1}{2}, \frac{x}{\overline{\gamma}}\right), \end{aligned} \tag{8}$$

Table 1

Notation	Description
$\overline{S}$	Total power of a transmitted symbol
N	Variance of Gaussian noise
γ	Received SNR
C	Ergodic capacity
W	Bandwidth
$\overline{\gamma}$	Average received SNR
γ_{th}	predetermined threshold of SNR
p_γ	PDF of received SNR
${}_pF_q(\alpha_1,\ldots,a_p;\beta_1,\ldots,\beta_q;x)$	Generalized hypergeometric series, [10, equation (9.14.1)]
$\Psi(x)$	Digamma function, [10, equation (8.36.1)]
$G_{p,q}^{m,n}(x\vert_{b_1,\ldots,b_q}^{a_1,\ldots,a_p})$	Meijer's-G function, [10, page 1032]
$\gamma(\alpha,x)$	Lower incomplete gamma function, [10, equation (8.350.1)]
$\Gamma(\alpha,x)$	Upper incomplete gamma function, [10, equation (8.350.2)]
$\Gamma(x)$	Gamma function, [10, equation (8.310.1)]

where $\overline{\gamma} = \Omega\overline{S}/mN$ is the average received SNR, and ${}_1F_1(\alpha_1;\beta_1;x)$ denotes the confluent hypergeometric series, as in [10, equation (9.21.1)]. Closed-form expressions for the capacity of a dual branch equal gain combiner with correlated identically distributed Nakagami-m branches have been presented in [8, equation (8)]. In the following paragraph, we extend that ergodic capacity expression for i.i.d. Nakagami-m branches where the Nakagami parameter m is not necessarily an integer.

The ergodic capacity of a dual-branch EGC system over i.i.d. Nakagami-m fading channels ($\Omega_1 = \Omega_2 = \Omega$ and $m_1 = m_2 = m$) is given by (see Appendix A)

$$\frac{C}{W} = \frac{\sqrt{\pi}}{\ln(2)\Gamma^2(m)4^{2m-1}}\sum_{k=0}^{\infty}\frac{\Gamma^2(k+2m)2^{-k}}{\Gamma(k+2m+0.5)k!} \times\Bigg[\left(\frac{2}{\overline{\gamma}}\right)^{2m+k}\frac{\Gamma(1-k-2m)}{k+2m}\,{}_1F_1\left(2m+k,2m+k+1,\frac{2}{\overline{\gamma}}\right) - \ln\left(\frac{2}{\overline{\gamma}}\right)+\Psi(k+2m) + \frac{2/\overline{\gamma}}{2m+k-1}\,{}_2F_2\left(1,1;2,2-2m-k,\frac{2}{\overline{\gamma}}\right)\Bigg], \tag{9}$$

where ${}_pF_q(\alpha_1,\ldots,a_p;\beta_1,\ldots,\beta_q;x)$ is a generalized hypergeometric series, [10, equation (9.14.1)], and $\Psi(x)$ denotes the digamma function, [10, equation (8.36.1)]. For integer values of the parameter m, (9) is reduced to (see Appendix A)

$$\frac{C}{W} = \frac{\sqrt{\pi}e^{2/\overline{\gamma}}}{\Gamma^2(m)\ln(2)4^{2m-1}}\sum_{k=0}^{\infty}\frac{\Gamma^2(k+2m)2^{-k}}{\Gamma(k+2m+1/2)k!} \times\sum_{i=0}^{2m+k-1}\Gamma\left(-i,\frac{2}{\overline{\gamma}}\right)\left(\frac{2}{\overline{\gamma}}\right)^{i}, \tag{10}$$

where $\Gamma(\alpha,x)$ is the upper incomplete gamma function, as in [10, equation (8.350.2)]. Taking into account [12, equations (8), (9) and (10)] and following the same procedure as in Appendix A (derivation of (9), (10)), we can derive the ergodic capacity of an equal gain combiner for three, four, and M i.i.d Nakagami-m branches. However it is impractical for the purpose of this paper to present all those tedious mathematical formulations. Nevertheless the impact of diversity on the capacity can be clearly depicted by using two branch schemes.

3.3. Ergodic Channel Capacity of Nakagami-m Fading Channel with SC. We assume a combiner that chooses the branch with the highest SNR (or equivalently with the strongest signal assuming equal noise power among the branches). Thus, we choose the antenna gains appearing in (2) as $a_i = 1$ if $|h_i| > |h_j|$ for all $j \neq i$ and 0 otherwise. Thus, (3) becomes

$$\gamma_{\text{SC}} = \frac{\overline{S}}{N}|h_i|^2, \quad |h_i| > \left|h_j\right| \ \forall j \neq i. \tag{11}$$

This is the widely known SC diversity technique as in [1, Chapter (9.8)]. The PDF of the SNR of two correlated identically distributed Nakagami-m channels is given in [1, equation (9.235)], and the resulting ergodic capacity for m integer is presented in [8, equation (19)].

From [14, equation (14)] and using the PDF transformation $p_Y(x) = p_X(\sqrt{x}/a)/2a\sqrt{x}$ for two random variables Y, X related as $Y = aX^2$ (here $a = \overline{S}/N$), we can calculate the PDF of the SNR of two independent and not identically distributed Nakagami-m branches as follows:

$$p_{\gamma_{\text{SC}}}(x) = \frac{x^{m_1-1}}{\Gamma(m_1)\Gamma(m_2)}\left(\frac{1}{\overline{\gamma}_1}\right)^{m_1}e^{-(1/\overline{\gamma}_1)x}\gamma\left(m_2,\frac{1}{\overline{\gamma}_2}x\right) + \frac{x^{m_2-1}}{\Gamma(m_1)\Gamma(m_2)}\left(\frac{1}{\overline{\gamma}_2}\right)^{m_2}e^{-(1/\overline{\gamma}_2)x}\gamma\left(m_1,\frac{1}{\overline{\gamma}_1}x\right), \tag{12}$$

where $\overline{\gamma}_1 = \Omega_1\overline{S}/m_1N$, $\overline{\gamma}_2 = \Omega_2\overline{S}/m_2N$, and $\gamma(\alpha,x)$ is the lower incomplete gamma function, [10, equation (8.350.1)].

In order to find the ergodic capacity of a dual SC system, we have to solve the integral resulting by substituting (12) in (4). Unfortunately, this integral cannot be solved analytically when the Nakagami parameters m_1, m_2 take noninteger values. But, assuming that m_1, m_2 take integer values, the ergodic capacity of a dual-branch SC system over independent nonidentically distributed Nakagami-m fading channels is given by (see Appendix A)

$$\begin{aligned}\frac{C}{W} &= \frac{e^{1/\overline{\gamma}_1}}{\ln(2)}\sum_{k=0}^{m_1-1}\left(\frac{1}{\overline{\gamma}_1}\right)^k\Gamma\left(-k,\frac{1}{\overline{\gamma}_1}\right)\\ &\quad+\frac{e^{1/\overline{\gamma}_2}}{\ln(2)}\sum_{k=0}^{m_2-1}\left(\frac{1}{\overline{\gamma}_2}\right)^k\Gamma\left(-k,\frac{1}{\overline{\gamma}_2}\right)\\ &\quad-\frac{e^{1/\overline{\gamma}_1+1/\overline{\gamma}_2}}{\ln(2)\Gamma(m_1)}\left(\frac{\overline{\gamma}_2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{m_1}\sum_{k=0}^{m_2-1}\frac{\Gamma(k+m_1)}{k!}\left(\frac{\overline{\gamma}_1}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^k\\ &\quad\times\sum_{i=0}^{m_1-1+k}\left(\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right)^i\Gamma\left(-i,\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right)\\ &\quad-\frac{e^{1/\overline{\gamma}_1+1/\overline{\gamma}_2}}{\ln(2)\Gamma(m_2)}\left(\frac{\overline{\gamma}_1}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^{m_2}\sum_{k=0}^{m_1-1}\frac{\Gamma(k+m_2)}{k!}\left(\frac{\overline{\gamma}_2}{\overline{\gamma}_1+\overline{\gamma}_2}\right)^k\\ &\quad\times\sum_{i=0}^{m_2-1+k}\left(\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right)^i\Gamma\left(-i,\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right).\end{aligned} \tag{13}$$

If the branches are identically distributed ($\Omega_1 = \Omega_2 = \Omega$ and $m_1 = m_2 = m$), (13) reduces to

$$\begin{aligned}\frac{C}{W} &= \frac{2e^{1/\overline{\gamma}}}{\ln(2)}\sum_{k=0}^{m-1}\Gamma\left(-k,\frac{1}{\overline{\gamma}}\right)\left(\frac{1}{\overline{\gamma}}\right)^k\\ &\quad-\frac{2^{1-m}e^{2/\overline{\gamma}}}{\ln(2)\Gamma(m)}\sum_{k=0}^{m-1}\frac{\Gamma(m+k)}{2^k k!}\sum_{i=0}^{m+k-1}\Gamma\left(-i,\frac{2}{\overline{\gamma}}\right)\left(\frac{2}{\overline{\gamma}}\right)^i.\end{aligned} \tag{14}$$

3.4. Ergodic Channel Capacity of Nakagami-m Fading Channel with SSC. We consider a diversity system, for which, when the SNR of the currently connected branch falls below a predetermined threshold, the receiver switches to and stays with another branch, regardless of whether the SNR of that branch is above or below the predetermined threshold. This is the widely known SSC diversity technique as in [1, page 419]. In particular, we choose the antenna gains appearing in (2) as $a_i = 1$ if the SNR at the i branch is above a predetermined threshold γ_{th} and 0 otherwise. If the SNR at the i branch is below the predetermined threshold γ_{th}, then we choose randomly an $a_j = 1$ where $j \in (1, L)$, $j \neq i$.

The PDF of the resulting SNR of a dual-branch SSC system over two i.i.d. Nakagami-m channels is given in [1, equation (9.276)]

$$p_{\gamma_{\mathrm{SSC}}} = \begin{cases} A\dfrac{1}{\Gamma(m)}\left(\dfrac{1}{\overline{\gamma}}\right)^m x^{m-1}e^{-x/\overline{\gamma}}, & x < \gamma_{\mathrm{th}},\\[2ex] (A+1)\dfrac{1}{\Gamma(m)}\left(\dfrac{1}{\overline{\gamma}}\right)^m x^{m-1}e^{-x/\overline{\gamma}}, & x \geq \gamma_{\mathrm{th}}, \end{cases} \tag{15}$$

where $A = 1 - \Gamma(m, (m/\Omega)\gamma_{\mathrm{th}})/\Gamma(m)$.

The ergodic capacity of a dual-branch SSC system over i.i.d. Nakagami-m fading channels is given by (see Appendix A)

$$\begin{aligned}\frac{C}{W} &= \frac{A}{\ln(2)}\left[\left(\frac{1}{\overline{\gamma}}\right)^m\frac{\Gamma(1-m)}{m}\,{}_1F_1\left(m,m+1,\frac{1}{\overline{\gamma}}\right)\right.\\ &\qquad-\ln\left(\frac{1}{\overline{\gamma}}\right)+\Psi(m)\\ &\qquad\left.+\frac{1}{m-1}\left(\frac{1}{\overline{\gamma}}\right){}_2F_2\left(1,1;2,2-m,\frac{1}{\overline{\gamma}}\right)\right]+\frac{e^{1/\overline{\gamma}}}{\ln(2)}\\ &\quad\times\sum_{k=0}^{\infty}\frac{1}{\Gamma(m-k)k!}\left(-\frac{1}{\overline{\gamma}}\right)^k\\ &\quad\times\left[\ln\left(\gamma_{\mathrm{th}}\frac{\overline{S}}{N}+1\right)\Gamma\left(m-k,\gamma_{\mathrm{th}}\frac{m}{\Omega}+\frac{1}{\overline{\gamma}}\right)\right.\\ &\qquad\left.+G_{2,3}^{3,0}\left(\gamma_{\mathrm{th}}\frac{m}{\Omega}+\frac{1}{\overline{\gamma}}\,\middle|\,{}^{1,1}_{0,0,m-k}\right)\right],\end{aligned} \tag{16}$$

where $G_{p,q}^{m,n}(x|^{a_1,\ldots,a_p}_{b_1,\ldots,b_q})$ is Meijer's-G function as defined in [10, page 1032]. For integer values of the parameter m, (16) is reduced to (see Appendix A)

$$\begin{aligned}\frac{C}{W} &= \frac{e^{1/\overline{\gamma}}}{\ln(2)}A\cdot\sum_{k=0}^{m-1}\Gamma\left(-k,\frac{1}{\overline{\gamma}}\right)\left(\frac{1}{\overline{\gamma}}\right)^k+\frac{1-A}{\ln(2)}\ln\left(\gamma_{\mathrm{th}}\frac{\overline{S}}{N}+1\right)\\ &\quad+\frac{1}{\ln(2)}\cdot\sum_{k=0}^{m-1}\frac{\Gamma(k,\gamma_{\mathrm{th}}(m/\Omega)+1/\overline{\gamma})}{k!}\cdot\frac{\Gamma(m-k,-1/\overline{\gamma})}{\Gamma(m-k)}.\end{aligned} \tag{17}$$

3.5. Ergodic Channel Capacity of Nakagami-m Fading Channel with No Diversity Combining Technique. We suppose that the receiver has no CSI and no complexity (cannot make any signal processing). Thus, the system operates without any diversity technique used. Thus, $a_i = 1$ for all i and the random variable g (appearing in (3)) is a sum of Nakagami-m random phase vectors. Consequently, (3) becomes

$$\gamma_{\mathrm{nodiversity}} = \frac{\overline{S}}{L\cdot N}\left|\sum_{i=1}^{L}h_i\right|^2. \tag{18}$$

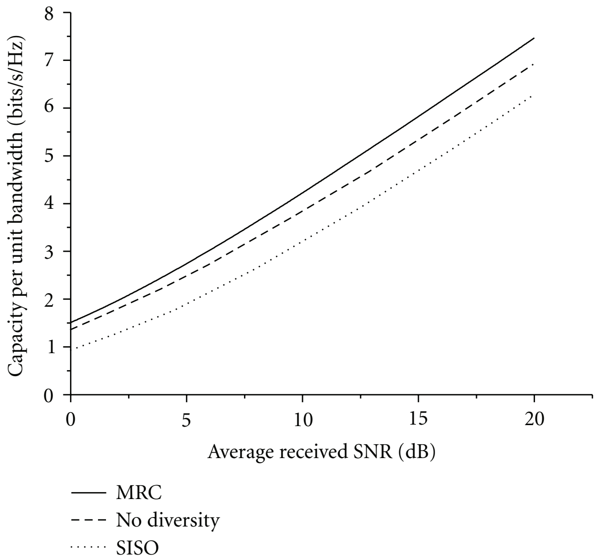

Figure 1: Capacity per unit bandwidth of a dual branch Nakagami-m fading channel with MRC diversity as a function of the average received SNR.

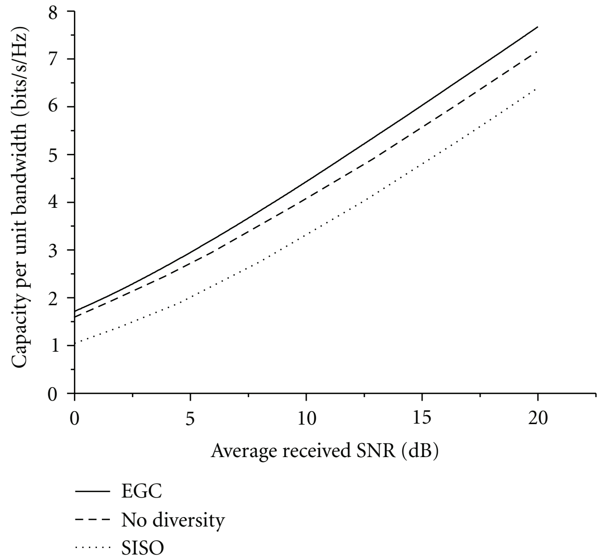

Figure 2: Capacity per unit bandwidth of a dual branch Nakagami-m fading channel with EGC diversity as a function of the average received SNR.

In order to find the PDF of (18), the PDF of the modulus of the sum of Nakagami-m random phase vectors is necessary. In [15], that PDF was derived for integer values of the Nakagami parameter m. Using that result, we write the modulus of the sum of Nakagami-m random phase vectors as a sum of weighted Nakagami-m PDFs (see Appendix B).

Thus, using in (18) the PDF transformation $p_Y(x) = p_X(\sqrt{x}/a)/2a\sqrt{x}$ for two random variables Y, X related as $Y = aX^2$ (here $a = \overline{S}/(L\cdot N)$), we can derive with the help of (B.6) the PDF of the SNR as a sum of weighted gamma PDFs, that is,

$$p_{\gamma_{\text{nodiversity}}}(x) = \sum_{j=0}^{\text{sum}} j!\cdot c_j \cdot f\left(x; j+1, \frac{U_L\overline{S}}{L\cdot N}\right), \tag{19}$$

where $f(x;k,\theta) = x^{k-1}(e^{-x/\theta}/\theta^k\Gamma(k))$ is the PDF of a gamma-distributed random variable as in [13, page 87]. The ergodic channel capacity of a SIMO system without any diversity technique, over independent nonidentically distributed Nakagami-m branches, is given by (see Appendix A)

$$\frac{C}{W} = \frac{e^{L\cdot N/U_L\overline{S}}}{\ln 2}\sum_{j=0}^{\text{sum}} j!\cdot c_j \sum_{i=0}^{j}\Gamma\left(-i, \frac{L\cdot N}{U_L\overline{S}}\right)\left(\frac{L\cdot N}{U_L\overline{S}}\right)^i. \tag{20}$$

Herein we will examine the case that the number of receive antennas is large. In that case the random variable g (appearing in (3)) tends to be a complex Gaussian random variable, according to the Central Limit Theorem [13, page 278], that is,

$$g = \sum_{i=1}^{L} h_i = \sum_{i=1}^{L} x_i + j\sum_{i=1}^{L} y_i \approx X + jY, \tag{21}$$

where x_i and y_i are the quadrature components of a Nakagami-m vector which follow the PDF according to [16, equation (6)]. That PDF has zero mean, and its variance equals to $\Omega_i/2$. According to the Central Limit Theorem, X and Y are zero mean Gaussian random variables with variance $\sum_{i=1}^{L}(\Omega_i/2)$. Thus, $|g|$ can be approximated by a Rayleigh distribution, as defined in [13, page 90], with its parameter $\sigma = \sqrt{\sum_{i=1}^{L}(\Omega_i/2)}$. Taking into account the random variables transformation $Y = aX^2$ in (18), the resulting SNR follows an exponential distribution [13, page 85], that is,

$$p_{\gamma_{\text{nodiversity}}}(x) = \frac{1}{\lambda}e^{-x/\lambda}, \tag{22}$$

where $\lambda = (\overline{S}/N\cdot L)\sum_{i=1}^{L}\Omega_i$. Substituting (22) in (4) and using Theorem 3, we obtain

$$\frac{C}{W} = \frac{e^{1/\lambda}}{\ln(2)}\Gamma\left(0, \frac{1}{\lambda}\right). \tag{23}$$

Thus, when L is large (asymptotic analysis), the ergodic capacity of the SIMO system can be approximated by the simple formula of (23), which is in fact the capacity of a Rayleigh channel [2, equations (21), (22)].

4. Results

Figure 1 shows the capacity per unit bandwidth for a dual branch MRC system over i.i.d. Nakagami-m fading channels (see [2, equation (36)]) for the case $m_1 = m_2 = m = 2$, $\Omega_1 = \Omega_2 = \Omega = 1$. Also, channel capacity without any diversity technique (see (20)) and SISO channel capacity (see [2, equation (21)]) are presented for comparison. It is clear that MRC improves the capacity of a SIMO Nakagami-m fading channel. It is also remarkable that channel capacity with none diversity technique is much greater than SISO capacity.

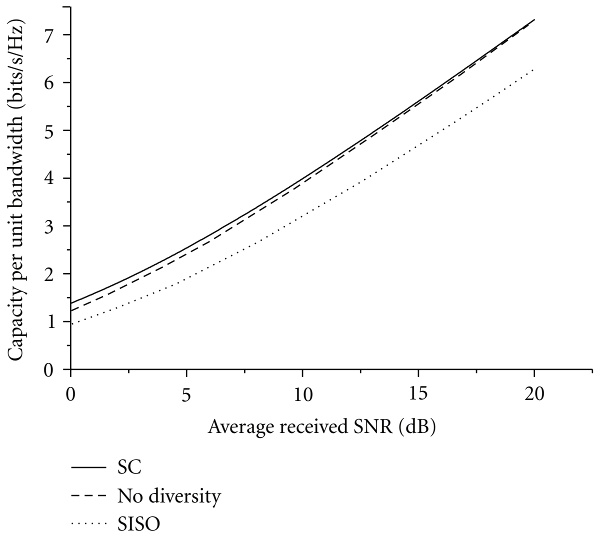

Figure 3: Capacity per unit bandwidth of a dual branch Nakagami-m fading channel with SC diversity as a function of the average received SNR.

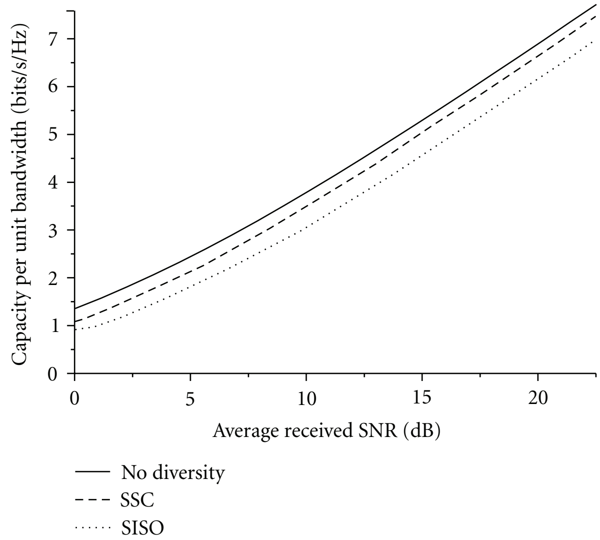

Figure 4: Capacity per unit bandwidth of a dual branch Nakagami-m fading channel with SSC diversity as a function of the average received SNR.

Figure 2 shows the capacity per unit bandwidth for a dual branch EGC system over i.i.d. Nakagami-m fading channels (see (10)) for the same parameter set as in Figure 1. Again, channel capacity without any diversity technique and SISO channel capacity are presented for comparison. It is obvious that EGC improves the capacity of a SIMO Nakagami-m fading channel.

Figure 3 shows the capacity per unit bandwidth for a dual branch SC system over independent Nakagami-m fading channels (see (13)) for the same parameter set as in Figures 1 and 2. Channel capacity without any diversity technique and SISO channel capacity are presented for comparison. We can see that SC performs slightly better than the case of none diversity technique.

Figure 4 shows the capacity per unit bandwidth for a dual branch SSC system over independent Nakagami-m fading channels (see (17)), for the same parameter set as in Figures 1, 2, and 3. The optimum adaptive switching threshold was determined as in [8, equation (31)] and was found $\gamma_{\text{th}} = 0.8$. Channel capacity without any diversity technique and SISO channel capacity are presented for comparison. It is clear that SSC improves the capacity of a SIMO Nakagami-m fading channel, but the case of none diversity technique performs slightly better than SSC. This is because SSC does not exploit both branches simultaneously. Essentially, SSC cannot have diversity gain. It simply uses one branch which has a smaller possibility to fall below a predetermined threshold (as it hops to the other).

Figure 5 shows the capacity per unit bandwidth for a five and a dual branch system where none diversity technique is used. Each branch is assumed to be independent and is characterized as Nakagami-m fading channel with parameters $m_i = 2$ and $\Omega_i = 1$. Figure 5 shows that (23) gives a very good approximation of (20) for the channel capacity of a SIMO Nakagami-m fading channel even with $L = 5$ receive antennas.

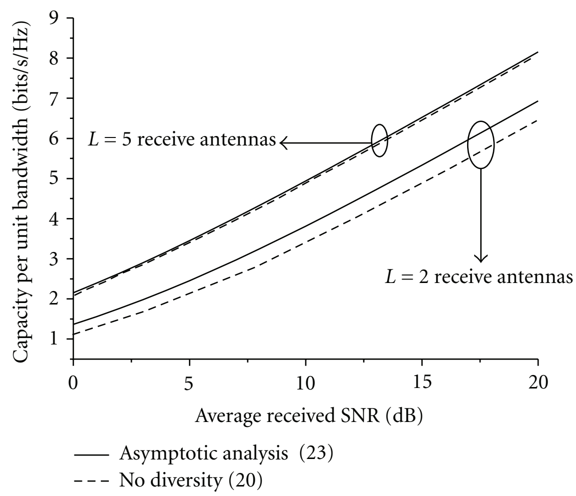

Figure 5: Capacity per unit bandwidth of a Nakagami-m fading channel with $L = 5$ and $L = 2$ receive antennas without any diversity technique as a function of the average received SNR.

5. Conclusions

We have investigated the ergodic capacity of a SIMO system operating in a Nakagami-m fading channel. We have derived a closed-form expression for the ergodic capacity of a dual branch EGC system over i.i.d Nakagami-m branches, where the Nakagami parameter m can be any real number with $m \geq 1/2$. Also, a closed-form formula is presented for m integer. A simple closed-form expression for the ergodic capacity of a dual SC diversity system operating in two independent but not identically distributed Nakagami-m branches has been derived. For that case, we have assumed that the Nakagami parameters m_i take integer values. A closed-form expression for the ergodic capacity of a dual SSC diversity system operating in two i.i.d. Nakagami-m branches has been derived for the case where the Nakagami parameter m can be any real number with $m \geq 1/2$. Also, a simple closed-form formula is presented for m integer. It was observed that all diversity combining techniques that are considered in this paper provide improvement to the capacity of Nakagami-m fading channels. Finally, the ergodic capacity of a SIMO system without any diversity technique has been presented in a closed-form expression. For that case, we have assumed L independent but not identically distributed Nakagami-m branches, where the parameters m_i take integer values. It was shown that when the number of receive antennas is large ($L \geq 5$), the ergodic capacity of a SIMO system without any diversity technique can be very well approximated by the capacity of a Rayleigh fading channel.

Appendices

A.

Theorem 1. *One has*

$$\int_c^\infty \ln(1+x)x^{n-1}e^{-bx}dx = e^b\Gamma(n)\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(n-k)k!} \times \left(\ln(c+1)\Gamma(n-k,b(c+1))+G_{2,3}^{3,0}\left(b(c+1)\big|_{10,0,n-k}^{1,1}\right)\right) \tag{A.1}$$

for $n \in \mathbb{R}$, $b > 0$ and $c > -1$.

Theorem 2. *One has*

$$\int_c^\infty \ln(1+x)x^{n-1}e^{-bx}dx = \ln(c+1)\Gamma(n,bc)b^{-n} + b^{-n}\Gamma(n)\sum_{q=0}^{n-1}\frac{\Gamma(q,b(c+1))}{q!}\frac{\Gamma(n-q,-b)}{\Gamma(n-q)} \tag{A.2}$$

for $n \in \mathbb{N}$, $b > 0$ and $c > -1$.

Theorem 3. *One has*

$$\int_0^\infty \ln(1+x)x^{n-1}e^{-bx}dx = \Gamma(n)e^b b^{-n}\sum_{k=0}^{n-1}b^k\Gamma(-k,b) \quad \text{for } b>0,\ n\in\mathbb{N}. \tag{A.3}$$

Proof of Theorem 1. We define $I_c(n,b) = \int_c^\infty \ln(1+x)x^{n-1}e^{-bx}dx$. With a change of variables we have

$$I_c(n,b) = e^b\int_{c+1}^\infty \ln(x)(x-1)^{n-1}e^{-bx}dx. \tag{A.4}$$

We write the term $(x-1)^{n-1}$ to its Taylor series expansion as $(x-1)^{n-1} = \sum_{k=0}^\infty(\Gamma(n)(-1)^k/\Gamma(n-k)k!)x^{n-1-k}$, and (A.4) becomes

$$I_c(n,b) = e^b\Gamma(n)\sum_{k=0}^\infty\frac{(-1)^k}{\Gamma(n-k)k!}\int_{c+1}^\infty \ln(x)x^{n-1-k}e^{-bx}dx. \tag{A.5}$$

The integral of (A.5) is solved by Mathematica Toolbox as

$$\int_c^\infty \ln(x)x^{n-1}e^{-bx}dx = b^{-n}\left(\ln(c)\Gamma(n,bc) + G_{2,3}^{3,0}\left(bc\big|_{0,0,n}^{1,1}\right)\right). \tag{A.6}$$

Substituting (A.6) in (A.5), we obtain (A.1). □

Proof of Theorem 2. Using partial integration and [10, equation (8.356.4)], the integral $I_c(n,b)$ can be written as

$$\begin{aligned} I_c(n,b) &= -\int_c^\infty \ln(1+x)\left(\frac{\Gamma(n,bx)}{b^n}\right)' dx \\ &= \ln(c+1)\Gamma(n,bc)b^{-n} + b^{-n}\int_c^\infty\frac{\Gamma(n,bx)}{x+1}dx. \end{aligned} \tag{A.7}$$

If n is an integer ($n \in \mathbb{N}$), using [10, equation (8.352.7)], (A.7) becomes

$$I_c(n,b) = \ln(c+1)\Gamma(n,bc)b^{-n} + b^{-n}\Gamma(n)\sum_{k=0}^{n-1}\frac{b^k}{k!}\int_c^\infty\frac{x^k e^{-bx}}{x+1}dx. \tag{A.8}$$

The integral of (A.8) can be written with the help of [10, equation (1.111)] as

$$\begin{aligned} \int_c^\infty\frac{x^k e^{-bx}}{x+1}dx &= e^b\int_{c+1}^\infty\frac{(x-1)^k e^{-bx}}{x}dx \\ &= e^b k!\sum_{q=0}^k\frac{(-1)^{k-q}}{q!(k-q)!}\int_{c+1}^\infty x^{q-1}e^{-bx}dx \end{aligned} \tag{A.9}$$

and by the definition of the upper incomplete gamma function [10, equation (8.350.2)], (A.9) becomes

$$\int_c^\infty\frac{x^k e^{-bx}}{x+1}dx = (-1)^k e^b k!\sum_{q=0}^k\frac{(-b)^{-q}}{q!(k-q)!}\Gamma(q,b(c+1)). \tag{A.10}$$

Replacing (A.10) in (A.8), we have that

$$I_c(n,b) = \ln(c+1)\Gamma(n,bc)b^{-n} + e^b b^{-n}\Gamma(n)\sum_{k=0}^{n-1}\sum_{q=0}^{k}\frac{(-b)^{k-q}}{q!(k-q)!}\Gamma(q,b(c+1)). \tag{A.11}$$

Reversing the double sum of (A.11) and using [10, equation (8.352.4)], we obtain (A.2). □

Proof of Theorem 3. We set $c = 0$ in (A.8) and we have

$$I_0(n,b) = \Gamma(n)b^{-n}\sum_{k=0}^{n-1}\frac{b^k}{k!}\int_0^\infty \frac{x^k e^{-bx}}{x+1}dx. \quad \text{(A.12)}$$

Solving the integral in the second part of (A.12) with the help of [10, equation (8.353.3)], we obtain (A.3). □

Derivation of (9) *and* (10). Substituting (8) in (4) and replacing the confluent hypergeometric function of (8) with its infinite series representation [10, equation (9.210.1)], we obtain after some manipulations

$$\begin{aligned}\frac{C}{W} &= \frac{\sqrt{\pi}}{\ln(2)\Gamma^2(m)4^{m-1}}\left(\frac{1}{\bar{\gamma}}\right)^{2m} \\ &\quad\times\sum_{k=0}^{\infty}\frac{\Gamma(k+2m)}{\Gamma(k+2m+0.5)k!}\left(\frac{1}{\bar{\gamma}}\right)^{k} \\ &\quad\times\int_0^\infty \ln(1+x)x^{2m-1+k}e^{-(2/\bar{\gamma})x}dx.\end{aligned} \quad \text{(A.13)}$$

The integral appearing in (A.13) was solved in [17, equation (2.6.23.4)]. Using this result, after some manipulations, (9) is derived. If m in (A.13) is an integer, we evaluate the integral of (A.13) by using Theorem 3 and after some manipulations, (10) arises.

Derivation of (13). Assuming that m_1 and m_2 take integer values, we are able to replace the lower incomplete gamma function $\gamma(\alpha,x)$, appearing in (12), with its finite sum representation [10, equation (8.352.1)]. Substituting (12) in (4) with the help of [10, equation (8.352.1)], we obtain after some manipulations

$$\begin{aligned}\frac{C}{W} &= \frac{1}{\ln(2)\Gamma(m_1)}\left(\frac{1}{\bar{\gamma}_1}\right)^{m_1} I_0\left(m_1,\frac{1}{\bar{\gamma}_1}\right) \\ &\quad+\frac{1}{\ln(2)\Gamma(m_2)}\left(\frac{1}{\bar{\gamma}_2}\right)^{m_2} I_0\left(m_2,\frac{1}{\bar{\gamma}_2}\right)-\frac{1}{\ln(2)\Gamma(m_1)}\left(\frac{1}{\bar{\gamma}_1}\right)^{m_1} \\ &\quad\times\sum_{k=0}^{m_2-1}\frac{1}{k!}\left(\frac{1}{\bar{\gamma}_2}\right)^{k} I_0\left(m_1+k,\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}\right)-\frac{1}{\ln(2)\Gamma(m_2)}\left(\frac{1}{\bar{\gamma}_2}\right)^{m_2} \\ &\quad\times\sum_{k=0}^{m_1-1}\frac{1}{k!}\left(\frac{1}{\bar{\gamma}_1}\right)^{k} I_0\left(m_2+k,\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2}\right).\end{aligned} \quad \text{(A.14)}$$

Since m_1 and m_2 in (A.14) are integers, we use Theorem 3 of the current Appendix to solve the integral $I_0(n,b)$ and finally, we obtain (13).

Derivation of (16) *and* (17). Substituting (15) in (4), with the help of (A.4), we obtain

$$\begin{aligned}\frac{C}{W} &= \frac{A}{\ln(2)\Gamma(m)}\left(\frac{1}{\bar{\gamma}}\right)^{m} I_0\left(m,\frac{1}{\bar{\gamma}}\right) \\ &\quad+\frac{1}{\ln(2)\Gamma(m)}\left(\frac{1}{\bar{\gamma}}\right)^{m} I_{\gamma_{\text{th}}}\left(m,\frac{1}{\bar{\gamma}}\right).\end{aligned} \quad \text{(A.15)}$$

Using Theorem 1 to evaluate the integral $I_c(n,b)$ and considering that the integral $I_0(n,b)$ was solved in [17, equation (2.6.23.4)], from (A.15) we obtain (16).

For integer values of the Nakagami parameter m in (A.15), we are able to use Theorems 2 and 3 to solve the integrals $I_c(n,b)$ and $I_0(n,b)$, respectively. After some manipulations, from (A.15) we obtain (17).

Derivation of (20). Replacing (19) in (4), we obtain after some manipulations

$$\frac{C}{W} = \frac{1}{\ln(2)}\sum_{j=0}^{\text{sum}}\frac{c_j(-1)^j}{j!}\left(\frac{N}{U_L\bar{S}}\right)^{j+1}\int_0^\infty \ln(1+x)x^j e^{-(N/U_L\bar{S})x}dx. \quad \text{(A.16)}$$

Using Theorem 3, we can solve the integral of (A.16) and finally we obtain (20).

B.

If $H = \sum_{i=1}^{L} h_i$ is a sum of Nakagami-m random phase vectors, the PDF of its modulus has been derived in [15, equation (8)] as

$$p_{|H|}(r) = \mathtt{G}\left\langle\frac{2r^{2j+1}e^{-r^2/U_L}}{U_L^{j+1}}\right\rangle, \quad \text{(B.1)}$$

where $U_L = \sum_{k=1}^{L}(\Omega_k/m_k)$ and $\mathtt{G}\langle\mathtt{Z}\rangle$ is an operator defined as [15, equation (4)]

$$\mathtt{G}\langle\mathtt{Z}\rangle \triangleq \sum_{i_1=0}^{m_1-1}\cdots\sum_{i_L=0}^{m_L-1}\sum_{j=0}^{S_L}\frac{\prod_{k=1}^{L}\left((1-m_k)_{i_k}/(i_k!)^2\right)(-S_L)_j S_L! Y_L\cdot\mathtt{Z}}{(j!)^2(U_L/4)^{S_L}} \quad \text{(B.2)}$$

with $S_L = \sum_{k=1}^{L} i_k$, $Y_L = \prod_{k=1}^{L}(\Omega_k/4m_k)^{i_k}$. $(z)_n$ is the Pochhammer symbol, [18, equation (6.1.22)]. Reversing the multiple sum in (B.2), we can rewrite $\mathtt{G}\langle\mathtt{Z}\rangle$ as

$$\begin{aligned}\mathtt{G}\langle\mathtt{Z}\rangle &= \sum_{j=0}^{\text{sum}}\mathtt{Z}\cdot\sum_{i_1=0}^{m_1-1}\cdots\sum_{i_L=0}^{m_L-1}\frac{\prod_{k=1}^{L}\left((1-m_k)_{i_k}/(i_k!)^2\right)(-S_L)_j S_L! Y_L}{(j!)^2(U_L/4)^{S_L}} \\ &= \sum_{j=0}^{\text{sum}}\mathtt{Z}\cdot\mathtt{c}_j\end{aligned} \quad \text{(B.3)}$$

with the constraint that $\sum_{k=1}^{L} i_k \geq j$ and sum $= \sum_{k=1}^{L} m_k - L$. Thus, c_j is defined as

$$c_j \triangleq \sum_{\substack{i_1=0 \\ \sum_{k=1}^{L} i_k\geq j}}^{m_1-1}\cdots\sum_{i_L=0}^{m_L-1}\frac{(-S_L)_j S_L! Y_L}{(j!)^2(U_L/4)^{S_L}}\prod_{k=1}^{L}\left(\frac{(1-m_k)_{i_k}}{(i_k!)^2}\right). \quad \text{(B.4)}$$

Using the property $(-z)_n = (-1)^n(z-n+1)_n$ for the Pochhammer symbol, (B.4) can be written as

$$c_j = \frac{(-1)^j}{(j!)^2} \sum_{\substack{i_1=0 \\ \sum_{k=1}^{L} i_k \geq j}}^{m_1-1} \cdots \sum_{i_L=0}^{m_L-1} (-U_L)^{-S_L} \frac{(S_L!)^2}{(S_L-j)!} \times \prod_{k=1}^{L} \left(\frac{\Gamma(m_k)}{(i_k!)^2 \Gamma(m_k - i_k)} \left(\frac{\Omega_k}{m_k} \right)^{i_k} \right). \quad \text{(B.5)}$$

Substituting (B.3) in (B.1) with the help of (B.5) we obtain

$$p_{|H|}(r) = \sum_{j=0}^{\text{sum}} \frac{2r^{2j+1} e^{-r^2/U_L}}{U_L^{j+1}} c_j, \quad \text{(B.6)}$$

which is, in fact, a sum of weighted Nakagami-m PDFs, as the author of [15] has noted.

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