Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2007, Article ID 56471, 12 pages
doi:10.1155/2007/56471

Research Article

Capacity Performance of Adaptive Receive Antenna Subarray Formation for MIMO Systems

Panagiotis Theofilakos and Athanasios G. Kanatas

Wireless Communications Laboratory, Department of Technology Education and Digital Systems, University of Piraeus, 80 Karaoli & Dimitriou Street, 18534 Piraeus, Greece

Received 15 November 2006; Accepted 1 August 2007

Recommended by R. W. Heath Jr.

Antenna subarray formation is a novel RF preprocessing technique that reduces the hardware complexity of MIMO systems while alleviating the performance degradations of conventional antenna selection schemes. With this method, each RF chain is not allocated to a single antenna element, but instead to the complex-weighted and combined response of a subarray of elements. In this paper, we derive tight upper bounds on the ergodic capacity of the proposed technique for Rayleigh i.i.d. channels. Furthermore, we study the capacity performance of an analytical algorithm based on a Frobenius norm criterion when applied to both Rayleigh i.i.d. and measured MIMO channels.

1. INTRODUCTION

The interest in multiple-input multiple-output (MIMO) antenna systems has exploded over the last years because of their potential of achieving remarkably high spectral efficiency. However, their practical application has been limited by the increased manufacture cost and energy consumption of the RF chains (performing the frequency transition between microwave and baseband) and analog-to-digital converters, the number of which is proportional to the number of antenna elements.

This high degree of hardware complexity has motivated the introduction of antenna selection schemes, which judiciously choose a subset from all the available antenna elements for processing and thus decrease the number of necessary RF chains. Both analytical [1–11] and stochastic [12] algorithms for antenna selection have been proposed. However, when a limited number of frequency converters are available, antenna selection schemes suffer from severe performance degradations in most fading channels.

In order to alleviate the performance degradations of conventional antenna selection, antenna subarray formation (ASF) has been recently introduced [13]. With this method, each RF chain is not allocated to a single antenna element, but instead to a combined and complex-weighted response of a subarray of antenna elements. Even though additional RF switches (for selecting the antenna elements that participate in each subarray), variable RF phase shifters, or/and variable gain-linear amplifiers (performing the complex-weighting) are required with respect to antenna selection schemes, the proposed method achieves decreased receiver hardware complexity, since less frequency converters and analog-to-digital converters are required with respect to the full system.

Antenna subarray formation actually performs a linear transformation in the RF domain in order to reduce the number of necessary RF chains while taking advantage of the responses of all antenna elements. Since it is a linear preprocessing technique that can be generally applied jointly to both receiver and transmitter, antenna subarray formation can be viewed as a special case of linear precoder-decoder joint designs [14–19]. Indeed, the fundamental mathematical models for both techniques are exactly the same; however, in conventional linear precoding-decoding schemes, preprocessing is performed in the baseband by digital signal processors that are not subject to the practical constraints and hardware nonidealities imposed by the RF components (namely the number of available RF chains, variable phase shifters, or/and variable gain-linear amplifiers) and thus no restrictions on the structure of the preprocessing matrices are required. Instead of decoupling the MIMO channel into independent subchannels (eigenmodes), ASF aims at constructing subchannels (namely, subarrays) that are as mutually independent as possible and deliver the largest receive power gain, under the aforementioned constraints. Note that an RF preprocessing technique for reducing hardware costs has also been introduced in [20], but without grouping antenna elements into subarrays.

Initially, antenna subarray formation was introduced with the restriction that each antenna element participates in one subarray only. For this special case of ASF, the problem of selecting the elements and the weights for the subarray formation has been addressed in [13], where an evolutionary optimization technique is used. In [21], we have introduced an analytical algorithm based on a Frobenius norm criterion. Recognizing that cost-effective analog amplifiers in RF with satisfactory noise figure are practically unavailable, we have also suggested a phase-shift-only design of the technique [22]. Taking into consideration that the performance of ASF may be adversely affected by hardware nonidealities, such as insertion loss, calibration, and phase-shifting errors (which are not an issue in conventional precoder-decoder schemes), we have presented simulation results in [23] that indicate the robustness of ASF to such nonidealities.

In this paper, we elaborate on the capacity performance of ASF and the Frobenius-norm-based algorithm. In particular, we derive a theoretical upper bound on the ergodic capacity of the technique for Rayleigh i.i.d. channels. Moreover, we demonstrate the performance of the technique and the algorithm through extensive computer simulations and application to measured channels.

The rest of the paper is organized as follows: Section 2 explains the proposed technique and its mathematical formulation in more detail, provides capacity calculations for the resulted system and introduces some special ASF schemes. In Section 3, tight theoretical upper bounds on the ergodic capacity of the technique are derived. Section 4 presents an analytical algorithm for ASF and its extensions for several ASF schemes. The capacity performance of the technique and the proposed algorithm is demonstrated in Section 5 through extensive computer simulations. Finally, the paper is concluded with a summary of results.

2. THE ANTENNA SUBARRAY FORMATION TECHNIQUE

In this section, we first present the antenna subarray formation technique and its mathematical formulation. Afterwards, we provide capacity calculations for the resulted system. Finally, some special schemes of ASF are introduced, which are dependent on the number of phase shifters or/and variable gain-linear amplifiers available at the receiver.

2.1. MIMO system model

Consider a flat fading, spatial multiplexing MIMO system with M_T elements at the transmitter and $M_R > M_T$ elements at the receiver. Unless otherwise stated, the $M_R \times M_T$ channel transfer matrix $\mathbf{H}$ is assumed to be perfectly known to the receiver, but unknown to the transmitter.

In spatial multiplexing systems, independent data streams are transmitted simultaneously by each antenna. The received vector for M_R receive elements is given by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is the zero-mean circularly symmetric complex Gaussian noise vector with covariance matrix $\mathbf{R_n} = N_0 \mathbf{I}_{M_R}$ and $\mathbf{s}$ is the transmitted vector. Assuming that the total transmitter power is P, the covariance matrix for the transmitted vector is constrained as

$$\operatorname{tr}\{E[\mathbf{s}\mathbf{s}^{\mathrm{H}}]\} = P, \tag{2}$$

and the intended average signal-to-noise ratio per antenna at the receiver is

$$\rho = \frac{P}{N_0}. \tag{3}$$

2.2. General mathematical formulation of antenna subarray formation

Antenna Subarray Formation can be applied with any number of RF chains available at the receiver. However, without loss of generality, we assume that the receiver is equipped with exactly M_T RF chains. This assumption is frequently made in antenna selection literature and is justified by the well-known fact that, when the number of receiving RF chains becomes larger than the number of transmit antennas, the number of parallel spatial data pipes that can be opened is constrained by the number of transmit antennas. Thus, the receiver RF chains in excess cannot be exploited to increase the throughput, but can only offer increased diversity order [24]. This assumption is meaningful when the full system channel matrix is of full column rank.

The process of subarray formation, complex weighting and combining at the receiver is linear and thus can be adequately described by the transformation matrix $\mathbf{A}$. In particular, the received vector after antenna subarray formation $\widetilde{\mathbf{y}}$ is found by left multiplying the received vector for M_R antenna elements with $\mathbf{A}^{\mathrm{H}}$, that is,

$$\widetilde{\mathbf{y}} = \mathbf{A}^{\mathrm{H}}\mathbf{y}. \tag{4}$$

Thus, the response of the jth subarray $\widetilde{y}_j$ (i.e., the jth entry of $\widetilde{\mathbf{y}}$) is

$$\widetilde{y}_j = \boldsymbol{\alpha}_j^{\mathrm{H}}\mathbf{y} = \sum_{i=1}^{M_R} a_{ij}^* y_i, \tag{5}$$

where $\boldsymbol{\alpha}_j$ denotes the jth column of $\mathbf{A}$. Clearly, the response of the jth subarray $\widetilde{y}_j$ is a linear combination of the responses of the M_R receiving antenna elements and the conjugated entries of $\boldsymbol{\alpha}_j$ are the corresponding complex weights. Thus, (4) is an adequate mathematical formulation of the subarray formation process, provided that we furthermore enforce the following restriction on the entries of $\mathbf{A}$:

$$a_{ij} = 0, \quad \text{if } i \notin \mathcal{S}_j, \tag{6}$$

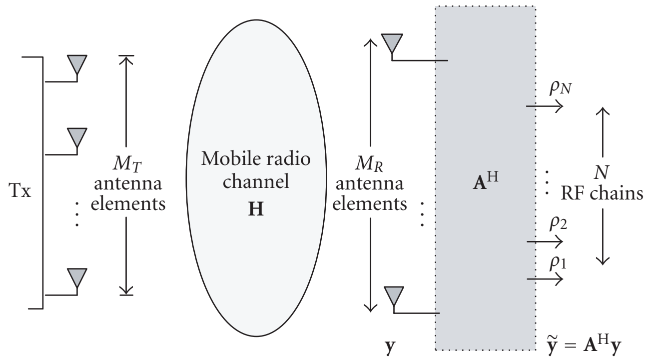

Figure 1: System model of receive antenna subarray formation.

with $\mathcal{S}_j$ denoting the set of receive antenna element indices that participate in the jth subarray.

Throughout this paper we assume that the transformation matrix $\mathbf{A}$ is adapted to the instantaneous channel state. Thus, we should have written $\mathbf{A}(\mathbf{H})$, denoting the dependence on the full system channel matrix $\mathbf{H}$. However, to facilitate notation, we just write $\mathbf{A}$ which henceforth implies $\mathbf{A}(\mathbf{H})$.

By substituting (1) into (4), the received vector after subarray formation becomes

$$\tilde{\mathbf{y}} = \mathbf{A}^{\mathrm{H}}\mathbf{H}\mathbf{s} + \mathbf{A}^{\mathrm{H}}\mathbf{n}. \tag{7}$$

Apparently, the combined effect of the propagation channel and the receive antenna subarrays on the transmitted signal is described by the effective channel matrix

$$\tilde{\mathbf{H}} = \mathbf{A}^{\mathrm{H}}\mathbf{H}. \tag{8}$$

The effective noise component in (7) is

$$\tilde{\mathbf{n}} = \mathbf{A}^{\mathrm{H}}\mathbf{n}, \tag{9}$$

which is zero-mean circularly symmetric complex Gaussian vector (ZMCSCGV) [25] with covariance matrix:

$$R_{\tilde{\mathbf{n}}\tilde{\mathbf{n}}} = \mathrm{E}[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^{\mathrm{H}}] = N_0\mathbf{A}^{\mathrm{H}}\mathbf{A}. \tag{10}$$

The block model of the resulted system is displayed in Figure 1.

2.3. Capacity of receive antenna subarray formation

Depending on the time-variation of the channel, there are different quantities that characterize the capacity of the resulted system. In this paragraph we apply well-known information-theoretic results for MIMO systems to RASF systems and elaborate the capacity of the proposed technique when different assumptions for channel-time variation are made.

2.3.1. Deterministic capacity

Deterministic capacity is a meaningful quantity when the static channel model is adopted, which implies that the channel matrix, despite being random, once chosen it is held fixed for the whole transmission. In this case, the Shannon capacity of RASF is given in terms of mutual information between the transmitter vector $\mathbf{s}$ and the received vector after subarray formation $\tilde{\mathbf{y}}$ as

$$C_{\mathrm{RASF}} = \max_{\substack{p(\mathbf{s}) \\ \mathrm{tr}(\mathbf{R}_\mathbf{s})=P}} I(\mathbf{s};\tilde{\mathbf{y}}) = \max_{p(\mathbf{s})}\left[H(\tilde{\mathbf{y}} \mid \mathbf{H}) - H(\tilde{\mathbf{y}} \mid \mathbf{s},\mathbf{H})\right], \tag{11}$$

where $H(\mathbf{x})$ is the entropy of $\mathbf{x}$, $p(\mathbf{s})$ denotes the distribution of $\mathbf{s}$ and $\mathrm{tr}(\mathbf{R}_\mathbf{s}) = P$ is the power constraint on the transmitter. Recognizing that the transmitted symbols are independent from noise, assuming that $\mathbf{s}$ is ZMCSCGV [25, 26] and taking into account that $\tilde{\mathbf{n}} \sim \mathcal{N}_C(\mathbf{0}, N_0\mathbf{A}^{\mathrm{H}}\mathbf{A})$, we find that

$$\begin{aligned} C_{\mathrm{RASF}} &= \max_{\substack{p(\mathbf{s}) \\ \mathrm{tr}(\mathbf{R}_\mathbf{s})=P}} I(\mathbf{s};\tilde{\mathbf{y}}) \\ &= \log_2 \det(\pi e \mathbf{R}_{\tilde{\mathbf{y}}}) - \log_2 \det(\pi e N_0 \mathbf{A}^{\mathrm{H}}\mathbf{A}), \end{aligned} \tag{12}$$

where $\mathbf{R}_{\tilde{\mathbf{y}}} = \mathrm{E}[\tilde{\mathbf{y}}\tilde{\mathbf{y}}^{\mathrm{H}}] = \mathbf{A}^{\mathrm{H}}\mathbf{H}\mathbf{R}_\mathbf{s}\mathbf{H}^{\mathrm{H}}\mathbf{A} + N_0\mathbf{A}^{\mathrm{H}}\mathbf{A}$ is the covariance matrix of $\tilde{\mathbf{y}}$. After some mathematical manipulations, (12) becomes

$$C_{\mathrm{RASF}} = \max_{\substack{\mathbf{R}_\mathbf{s} \\ \mathrm{tr}(\mathbf{R}_\mathbf{s})=P}} \log_2 \det\left[\mathbf{I}_{M_T} + \frac{1}{N_0}\mathbf{R}_\mathbf{s}\mathbf{H}^{\mathrm{H}}\mathbf{A}(\mathbf{A}^{\mathrm{H}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{H}}\mathbf{H}\right]. \tag{13}$$

Since the transmitter does not know the channel and taking into account the power constraint, it is reasonable to assume that

$$\mathbf{R}_\mathbf{s} = \frac{P}{M_T}\mathbf{I}_{M_T}. \tag{14}$$

Thus, the Shannon capacity of receive antenna subarray formation with equal power allocation at the transmitter is

$$C_{\mathrm{RASF}} = \log_2 \det\left[\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{H}^{\mathrm{H}}\mathbf{A}(\mathbf{A}^{\mathrm{H}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{H}}\mathbf{H}\right]. \tag{15}$$

The capacity of the resulted system is upper bounded by the capacity of the full system, that is

$$C_{\mathrm{RASF}} \le C_{\mathrm{FS}} = \log_2 \det\left(\mathbf{I}_{M_R} + \frac{\rho}{M_T}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right). \tag{16}$$

Proof of this result is given in Appendix A.

2.3.2. Ergodic capacity

In time-varying channels with no delay constraints, ergodic capacity is a meaningful quantity, defined as the probabilistic average of the static channel capacity over the distribution of the channel matrix $\mathbf{H}$. The ergodic capacity for RASF is given by

$$\overline{C}_{\mathrm{RASF}} = \mathrm{E}_\mathbf{H}\left[\log_2 \det\left(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{H}^{\mathrm{H}}\mathbf{A}(\mathbf{A}^{\mathrm{H}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{H}}\mathbf{H}\right)\right]. \tag{17}$$

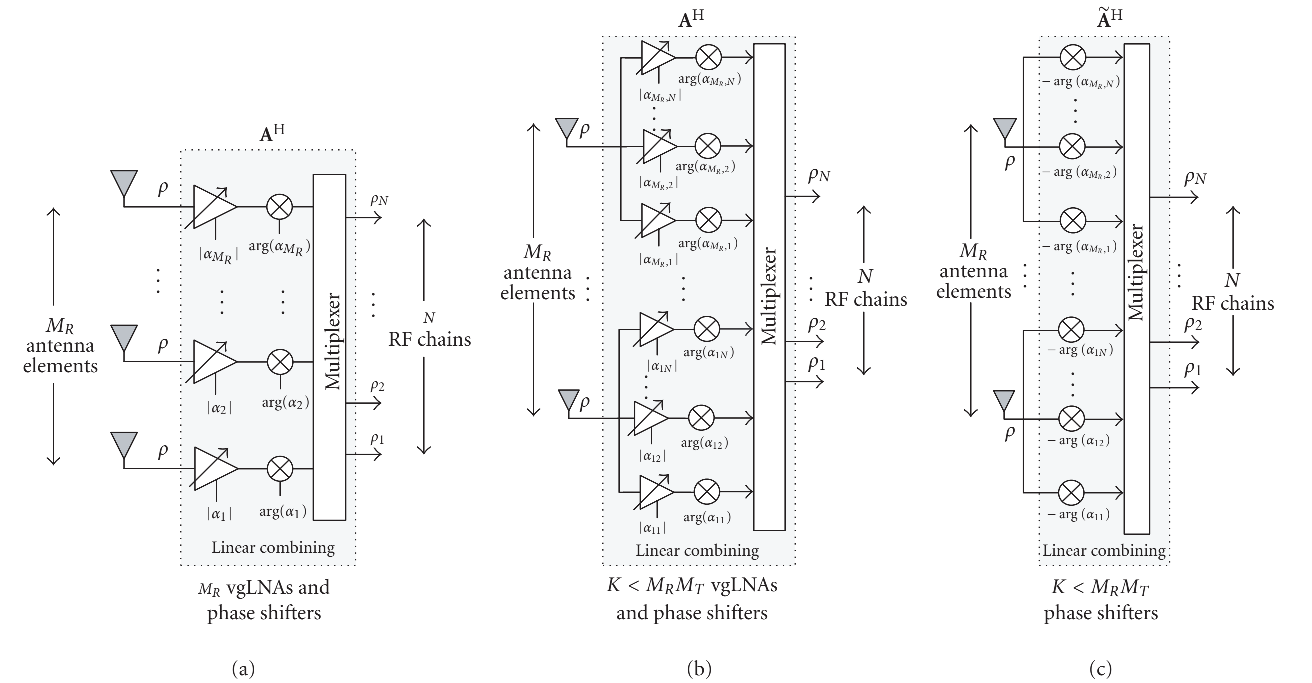

Figure 2: Receiver structures for several receive antenna subarray formation (ASF) schemes: (a) strictly-structured ASF (SS-ASF), (b) relaxed-structured ASF (RS-ASF) and (c) reduced hardware complexity ASF (RHC-ASF).

2.3.3. *Outage capacity*

Outage capacity is a meaningful quantity in slowly varying channels. Assuming a fixed transmission rate R, there is an associated probability P_{out} (bounded away from zero) that the received data will not be received correctly, or equivalently that mutual information will be less than transmission rate R. Outage capacity for RASF is therefore defined as

$$C_{\text{RASF}} = R : \Pr\left\{\log_2 \det\left(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{H}^{\mathrm{H}}\mathbf{A}\left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}}\mathbf{H}\right) < R\right\} = P_{\text{out}}. \tag{18}$$

2.4. *Receive antenna subarray formation schemes*

In general, no more constraints on the transformation matrix $\mathbf{A}$ are required. However, depending on the number of available phase shifters or/and variable gain-linear amplifiers (which determine the number of its nonzero entries), further restrictions on matrix $\mathbf{A}$ may be necessary. Motivated by these practical considerations, we have introduced several variations of antenna subarray formation [22], namely, the following.

(1) *Strictly-Structured* ASF (SS-ASF), in which each antenna element is allowed to participate in one subarray only. Thus, each row of the transformation matrix $\mathbf{A}$ may contain only one nonzero element, whereas no restriction is enforced on the columns of $\mathbf{A}$. With this scheme, exactly M_R phase shifters and variable gain-linear amplifiers are required at the receiver.

(2) *Relaxed-Structured* ASF (RS-ASF), where no restrictions on matrix $\mathbf{A}$ are imposed, except for the number of its nonzero entries, which is a fixed system design parameter that determines the number of phase shifters and variable gain-linear amplifiers available to the receiver.

(3) *Reduced Hardware Complexity* ASF (RHC-ASF), which is a phase-shift-only design of the technique. While cost-effective variable gain-linear amplifiers with satisfactory noise figure are not practically available, the economic design and manufacture of variable phase-shifters for the microwave frequency is feasible due to the rapid advances in MMIC technology. Therefore, this scheme reduces even further the hardware complexity of the receiver with negligible capacity loss, as it will be demonstrated in Section 5.

An efficient algorithm for determining the transformation matrix $\mathbf{A}$ for all the aforementioned schemes will be presented in detail in Section 4. Figure 2 presents the receiver architecture for each of the ASF schemes.

3. AN UPPER BOUND ON THE ERGODIC CAPACITY OF ANTENNA SUBARRAY FORMATION FOR I.I.D. RAYLEIGH CHANNELS

In this section, we derive an upper bound on the ergodic capacity of the technique for i.i.d. Rayleigh fading channels, the tightness of which will be verified by extensive computer simulations in Section 5.

A well-known upper bound on the (deterministic) capacity of the full system is given by

$$C_{\mathrm{FS}} \leq \sum_{i=1}^{M_T} \log_2\left(1 + \frac{\rho}{M_T}\gamma_i\right), \tag{19}$$

where γ_i are independent chi-squared variates with $2M_R$ degrees of freedom. The equality holds in the "very artificial case" when the transmitted signal vector components "are conveyed over M_T "channels" that are uncoupled and each channel has a separate set of M_R receive antennas" [27]. In other words, when the full MIMO system is consisted of M_T separable and independent parallel SIMO systems, each performing maximum ratio combining (MRC) at the receiver.

In our case, we consider as well that the resulted system is consisted of M_T separable and independent parallel SIMO systems. We suppose that the jth SIMO system is formed by the jth transmit antenna element and the jth receive subarray; thus, for each subarray, only one signal component is received and processed without any interference from the others. Of course, this scheme is practically infeasible; however, it must lead to an upper bound of the resulted system capacity.

A subarray corresponds to an independent SIMO system and is actually formed by choosing a subset of antenna elements, the responses of which are linearly combined and fed to an RF chain. Thus, generalized selection combining (i.e., combining the responses of a subset of antenna elements) is performed in each SIMO system. The maximum SNR (which also achieves maximum capacity) in this case is obtained with the hybrid selection maximum ratio combining scheme (HS/MRC). Furthermore, in this section, we assume that each subarray is formed using a predefined and fixed number of antenna elements (let it be k_j antenna elements for the jth subarray). Therefore, a capacity bound for antenna subarray formation can be obtained by

$$C_{\mathrm{bound}} = \sum_{j=1}^{M_T} \log_2\left(1 + \xi_j\right). \tag{20}$$

Assuming that there are no delay constraints, the channel is *ergodic* and therefore it is meaningful to derive an upper bound on *ergodic capacity* as

$$\overline{C}_{\mathrm{bound}} = \sum_{j=1}^{M_T} \mathrm{E}\left[\log_2\left(1 + \xi_j\right)\right]. \tag{21}$$

The expectation in (21) can be found [28] by

$$\overline{c}_j \triangleq \mathrm{E}\left[\log_2\left(1 + \xi_j\right)\right] = \int_0^\infty \log_2(1+\xi)\cdot p_{\xi_j}(\xi)d\xi. \tag{22}$$

Since ξ_j is actually the postprocessing SNR of HS/MRC when k_j out of M_R elements are chosen, its probability density function is [29]

$$\begin{aligned} p_{\xi_j}(\xi) = \binom{M_R}{k_j}\Bigg[&\left(\frac{M_T}{\rho}\right)^{k_j}\frac{\xi^{k_j-1}e^{-(M_T/\rho)\xi}}{(k_j-1)!} \\ &+\frac{M_T}{\rho}\sum_{l=1}^{M_R-k_j}(-1)^{k_j+l-1}\binom{M_R-k_j}{l} \\ &\times\left(\frac{k_j}{l}\right)^{k_j-1}e^{-(M_T/\rho)\xi} \\ &\times\left(e^{-(M_T l/\rho k_j)\xi} - \sum_{m=0}^{k_j-2}\frac{1}{m!}\left(-\frac{l\cdot M_T}{\rho\cdot k_j}\xi\right)^m\right)\Bigg]. \end{aligned} \tag{23}$$

Substituting (23) into (22) and defining the integral

$$\mathcal{I}_n(x) \triangleq \int_0^\infty t^{n-1}\ln(1+t)e^{-xt}dt \qquad x>0;\ n=1,2,\ldots, \tag{24}$$

we get

$$\begin{aligned} \overline{c}_j = \frac{1}{\ln 2}\binom{M_R}{k_j}\Bigg[&\left(\frac{M_T}{\rho}\right)^{k_j}\frac{\mathcal{I}_{k_j}(M_T/\rho)}{(k_j-1)!} \\ &+\frac{M_T}{\rho}\sum_{l=1}^{M_R-k_j}(-1)^{k_j+l-1}\binom{M_R-k_j}{l}\left(\frac{k_j}{l}\right)^{k_j-1} \\ &\times\Bigg[\mathcal{I}_1\left(\frac{M_T}{\rho}\left\{1+\frac{l}{k_j}\right\}\right) - \sum_{m=0}^{k_j-2}\frac{1}{m!} \\ &\times\left(-\frac{l\cdot M_T}{\rho\cdot k_j}\right)^m\mathcal{I}_{m+1}(M_T/\rho)\Bigg]\Bigg], \end{aligned} \tag{25}$$

which, in fact, is the average channel capacity achieved when employing HS/MRC in a SIMO system with M_R receiving antenna elements and k_j branches.

The integral $\mathcal{I}_n(x)$ can be evaluated by [30]

$$\mathcal{I}_n(x) = (n-1)!\cdot e^x\cdot\sum_{q=1}^{n}\frac{\Gamma(-n+q,x)}{x^q}, \tag{26}$$

which for $n=1$ reduces to

$$\mathcal{I}_1(x) = e^x\frac{E_1(x)}{x}. \tag{27}$$

Note that $E_1(x)$ is the exponential integral of first-order function defined by

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t}dt \tag{28}$$

and $\Gamma(\alpha,x)$ is the complementary incomplete gamma function (or Prym's function) defined as

$$\Gamma(\alpha,x) = \int_x^\infty t^{\alpha-1}e^{-t}dt. \tag{29}$$

For q positive integer, $\Gamma(-q,x)$ can be calculated by

$$\Gamma(-q,x) = \frac{(-1)^n}{n!}\left[E_1(x) - e^{-x}\sum_{m=0}^{q-1}(-1)^m\frac{m!}{x^{m+1}}\right]. \tag{30}$$

Thus, the ergodic capacity bound for receive antenna subarray formation can be analytically obtained by

$$\begin{aligned} C_{\text{bound}} = {} & \frac{1}{\ln 2}\sum_{j=1}^{M_T}\binom{M_R}{k_j} \\ & \times\Bigg[\left(\frac{M_T}{\rho}\right)^{k_j}\frac{\mathcal{I}_{k_j}(M_T/\rho)}{(k_j-1)!} + \frac{M_T}{\rho}\sum_{l=1}^{M_R-k_j}(-1)^{k_j+l-1} \\ & \times\binom{M_R-k_j}{l}\left(\frac{k_j}{l}\right)^{k_j-1} \\ & \times\Bigg[\mathcal{I}_1\left(\frac{M_T}{\rho}\left\{1+\frac{l}{k_j}\right\}\right) - \sum_{m=0}^{k_j-2}\frac{1}{m!} \\ & \times\left(-\frac{l\cdot M_T}{\rho\cdot k_j}\right)^m\mathcal{I}_{m+1}(M_T/\rho)\Bigg]\Bigg]. \end{aligned} \tag{31}$$

A simpler expression than (25) can be derived by recognizing that $\log_2(\cdot)$ is a concave function and applying Jensen's inequality to (21),

$$\bar{c}_j = \mathrm{E}\left[\log_2\left(1+\xi_j\right)\right] \le \log_2\left(1+\mathrm{E}\left[\xi_j\right]\right). \tag{32}$$

It is known for HS/MRC [29] that

$$\mathrm{E}[\xi_j] = \frac{\rho}{M_T}k_j\left(1+\sum_{l=k_j+1}^{M_R}\frac{1}{l}\right). \tag{33}$$

Thus, (21) becomes

$$\overline{C}_{\text{bound}} \le \sum_{j=1}^{M_T}\log_2\left[1+\frac{\rho}{M_T}k_j\left(1+\sum_{l=k_j+1}^{M_R}\frac{1}{l}\right)\right], \tag{34}$$

which has a much simpler form than (31) while being almost as tight as computer simulations have demonstrated.

Before concluding this section, we note that analyzing the resulted system into parallel SIMO systems each performing HS/MRC results into capacity bounds of RS-ASF, since HS/MRC requires both phase shifters and variable gain amplifiers. Capacity bounds for RHC-ASF could be derived in a similar manner by considering M_T parallel SIMO systems each performing HS/EGC. Since HS/MRC delivers the best performance amongst all hybrid selection schemes, the upper bound on the ergodic capacity of RS-ASF is also an upper bound on the ergodic capacity of any ASF scheme, including RHC-ASF.

4. ALGORITHM FOR ANTENNA SUBARRAY FORMATION

In this section, we present a novel, analytical algorithm for receive antenna subarray formation, based on a Frobenius norm criterion. We first develop the algorithm for SS-ASF and then provide extensions for RS-ASF and RHC-ASF. The capacity performance of the algorithms will be demonstrated in Section 5.

4.1. Starting point for the algorithm

The starting point for determining the transformation matrix $\mathbf{A}$ will be an optimal solution to the *unconstrained* problem of maximizing the deterministic capacity in (15). As shown in Appendix A, (15) can be maximized when $\mathbf{A}_o = \mathbf{U}$, where the columns of $\mathbf{U}$ are the M_T dominant left singular vectors of the full channel matrix $\mathbf{H}$. Therefore, the entries of the transformation matrix $\mathbf{A}$ will be

$$a_{ij} = \begin{cases} u_{ij} & \text{if } i \in \mathcal{S}_j \\ 0 & \text{otherwise,} \end{cases} \tag{35}$$

with u_{ij} being the (i,j) entry of matrix $\mathbf{U}$. Alternatively,

$$\mathbf{A} = \mathbf{S}\odot\mathbf{U}, \tag{36}$$

where $\odot$ denotes the Hadamard (elementwise) matrix product and the entries of $\mathbf{S}$ are

$$s_{ij} = \begin{cases} 1 & i \in \mathcal{S}_j \\ 0 & \text{otherwise.} \end{cases} \tag{37}$$

4.2. Frobenius norm based algorithm for SS-ASF

We first develop an algorithm for SS-ASF and afterwards extend it for other receive ASF schemes. Due to the additional constraints of SS-ASF, the capacity of the resulted system is given by

$$\begin{aligned} C_{\text{RASF}} &= \log_2\det\left(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{H}^{\mathrm{H}}\mathbf{A}\mathbf{A}^{\mathrm{H}}\mathbf{H}\right) \\ &= \log_2\det\left(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\widetilde{\mathbf{H}}^{\mathrm{H}}\widetilde{\mathbf{H}}\right). \end{aligned} \tag{38}$$

In order to retain the capacity calculations to the intended system SNR measured at the output of every receiver antenna element, $\mathbf{A}$ is now subject to the following normalization:

$$\mathbf{A}^{\mathrm{H}}\mathbf{A} = \mathbf{I}_{M_T}. \tag{39}$$

Intuitively, the desired transformation matrix $\mathbf{A}$ should be such that the distance between the two subspaces defined by $\widetilde{\mathbf{H}}_{\text{opt}} = \mathbf{U}^{\mathrm{H}}\mathbf{H}$ (i.e., the effective channel matrix obtained from the optimal solution to the unconstrained problem) and $\widetilde{\mathbf{H}} = \mathbf{A}^{\mathrm{H}}\mathbf{H}$ is minimized. As a result, we employ the following minimum distance distortion metric:

$$\varepsilon(\mathbf{A}) = \left\|\widetilde{\mathbf{H}}_{\text{opt}} - \widetilde{\mathbf{H}}\right\|_{\mathrm{F}}^2 = \left\|(\mathbf{U}-\mathbf{A})^{\mathrm{H}}\mathbf{H}\right\|_{\mathrm{F}}^2. \tag{40}$$

Defining $\mathbf{E} \triangleq \mathbf{U}-\mathbf{A}$ and $\mathbf{F} \triangleq \mathbf{E}^{\mathrm{H}}\mathbf{H}$, (40) can be written as

$$\varepsilon(\mathbf{A}) = \|\mathbf{F}\|_{\mathrm{F}}^2 = \sum_{j=1}^{N}\left(\sum_{i=1}^{M_T}|f_{ji}|^2\right) = \sum_{j=1}^{M_T}\|\mathbf{f}_j\|^2, \tag{41}$$

Table 1: Frobenius-norm-based algorithm for RASF.

Algorithm steps (K, M_R, M_T, and $\mathbf{H}$ are given) (In case of SS-ASF, $K := M_R$)		Complexity
Obtain the SVD of full system channel matrix $\mathbf{H}$.	$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}$	$O(12M_T M_R^2 + 9M_R^3)$
Compute the decision metrics g_{ij} that will determine if the ith antenna element will participate in the jth subarray.	For i: = 1 to M_R For j: = 1 to M_T $g_{ij} := U(i,j) \cdot \lVert \mathbf{H}(i,:) \rVert^2$ end end	$O(M_T^2 M_R)$
Initialize with every $a_{ij} = 0$ and all S_j empty. S_j: set of indices of antenna elements that participate in the jth subarray.	$S_j := \varnothing$ ($\forall\ j = 1, \ldots, M_T$) $\mathbf{A} := \mathbf{0}_{M_R \times M_T}$; n: = 0	
Repeat the following until matrix $\mathbf{A}$ is filled with K nonzero elements: (i) let (i_0, j_0) be the indices of the largest g_{ij} element over $1 \le i \le M_R$ and $1 \le j \le M_T$, provided that $a_{ij} = 0$; *for SS-ASF only*, $i \notin \bigcup_j S_j$; (ii) set $a_{i_0 j_0} = u_{i_0 j_0}$, that is, the i_0th antenna element participates in the j_0th subarray;	While $n < K$ $(i_0, j_0) = \arg\max_{\substack{(i,j) \\ a_{ij}=0}} (g_{ij})$ $S_{j_0} := S_{j_0} \cup \{i_0\}$ $A(i_0, j_0) := U(i_0, j_0)$ n: = $n + 1$ end	$O(K M_R M_T)$
for SS-ASF only, normalize $\mathbf{A}$ so that $\mathbf{A}^{\mathrm{H}}\mathbf{A} = \mathbf{I}_{M_T}$.	*For SS-ASF only*: For j = 1:M_T $\mathbf{A}(:,j) := \mathbf{A}(:,j)/\lVert \mathbf{A}(:,j) \rVert$ end	

where $\mathbf{f}_j$ denotes the jth row of $\mathbf{F}$, being equal to $\mathbf{f}_j = \mathbf{e}_j^{\mathrm{H}}\mathbf{H}$, and $\mathbf{e}_j$ is the jth column of matrix $\mathbf{E}$.

Recognizing that the ith row of matrix $\mathbf{F}$ can be written as a linear combination of the rows $\mathbf{h}_i$ of the full system channel matrix $\mathbf{H}$ and taking into account that

$$e_{ij} \triangleq u_{ij} - a_{ij} = \begin{cases} u_{ij} & i \notin \mathcal{S}_j \\ 0 & i \in \mathcal{S}_j, \end{cases} \tag{42}$$

the distortion metric becomes

$$\varepsilon(\mathbf{A}) = \sum_{j=1}^{M_T} \left\lVert \sum_{i \in \mathcal{S}_j} e_{ij}^{*}\mathbf{h}_i \right\rVert^2 = \sum_{j=1}^{M_T} \left\lVert \sum_{i \notin \mathcal{S}_j} u_{ij}^{*}\mathbf{h}_i \right\rVert^2 \le \sum_{j=1}^{M_T} \sum_{i \notin \mathcal{S}_j} |u_{ij}|^2 \lVert \mathbf{h}_i \rVert^2, \tag{43}$$

where the upper bound on the right-hand side follows from the triangular inequality. As a result, the objective is to minimize the upper bound on the distortion metric in (43).

Since the selection of the elements of the transformation matrix $\mathbf{A}$ is based on matrix $\mathbf{U}$, it is trivial to conclude that minimizing the upper bound in (43) is equivalent to maximizing

$$\tilde{p} = \sum_{j=1}^{M_T} \sum_{i \in \mathcal{S}_j} |u_{ij}|^2 \lVert \mathbf{h}_i \rVert^2, \tag{44}$$

which upper-bounds the power of the effective channel matrix $\lVert \widetilde{\mathbf{H}} \rVert_{\mathrm{F}}^2$. Indeed, after mathematical manipulations similar to those in (41)–(43), it follows that

$$\lVert \widetilde{\mathbf{H}} \rVert_F^2 = \sum_{j=1}^{M_T} \left\lVert \sum_{i \in \mathcal{S}_j} u_{ij}^{*}\mathbf{h}_i \right\rVert^2 \le \sum_{j=1}^{M_T} \sum_{i \in \mathcal{S}_j} |u_{ij}|^2 \lVert \mathbf{h}_i \rVert^2 = \tilde{p}, \tag{45}$$

where $\tilde{\mathbf{h}}_j$ denotes the jth row of $\widetilde{\mathbf{H}}$ and $\boldsymbol{\alpha}_j$ is the jth column of matrix $\mathbf{A}$. Consequently, minimizing an upper bound on the minimum distance distortion metric is equivalent to maximizing an upper bound on the power of the effective channel matrix. The latter may not be the optimal way to maximize capacity in spatial multiplexing systems, but it should result into an increased capacity performance, since it is known that [24]

$$C_{\text{SS-ASF}} \ge \log_2 \det\left(1 + \frac{\rho}{M_T} \lVert \widetilde{\mathbf{H}} \rVert_{\mathrm{F}}^2\right). \tag{46}$$

The proposed algorithm appoints the receiver antenna elements to the appropriate subarray, so that the metric (44) is maximized. Finally, $\mathbf{A}$ is normalized as in (39). Table 1 presents the algorithm steps in more detail.

4.3. Extension of the algorithm for RS-ASF

The capacity of RS-ASF given by (15) is lower bounded by the capacity formula (38) for SS-ASF, that is,

$$C_{\text{RS-ASF}} \geq \log_2 \det\left(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{H}^{\mathrm{H}}\mathbf{A}\mathbf{A}^{\mathrm{H}}\mathbf{H}\right). \tag{47}$$

Proof of this result and indications for the tightness of the bound are provided in Appendix B.

Thus, in the case of RS-ASF we also use the Frobenius norm based algorithm initially developed for SS-ASF. The algorithm terminates when the transformation matrix $\mathbf{A}$ contains exactly K nonzero elements, where $K < M_R M_T$ is a system design parameter that determines the number of variable gain-linear amplifiers and phase shifters available to the receiver.

The computational complexity of the proposed algorithm (see Table 1) is dominated by the initial cost of the singular value decomposition, that is, $O(M_R^3)$ when $M_R \gg M_T$, whereas the complexity of Gorokhov et al. algorithm [4] and of the alternative implementation proposed in [5] for antenna selection is $O(M_T^2 M_R^2)$ and $O(M_T^2 M_R)$, respectively.

4.4. Extention of the algorithm for RHC-ASF

The transformation matrix $\breve{\mathbf{A}}$ for RHC-ASF (a phase-shift-only design of antenna subarray formation) can be obtained from the transformation matrix $\mathbf{A}$ for RS-ASF by applying the following formula to its entries:

$$\breve{a}_{ij} = \begin{cases} \exp\left(-j \angle a_{ij}\right) & \text{if } i \in \mathcal{S}_j \\ 0 & \text{otherwise.} \end{cases} \tag{48}$$

Intuitively, RHC-ASF follows the notion of equal gain combining. A similar procedure for obtaining a phase-shift-only RF preprocessing technique has been followed in [20].

5. SIMULATION RESULTS

In this section, we present extensive computer simulation results that demonstrate the capacity performance of receive ASF technique, the tightness of the ergodic capacity bounds derived in Section 3, and the performance of the proposed algorithm.

5.1. Upper bound on ergodic capacity for ASF

We first deal with the ergodic capacity bounds of ASF for Rayleigh i.i.d. channels derived in Section 3, namely, (31) and (34). Henceforth, we refer to (34) as "simpler theoretical capacity bound," in order to distinguish it from (31). We consider a flat-fading Rayleigh i.i.d. MIMO channel with $M_R = 8$ receiving and $M_T = 2$ transmitting antenna elements and assume that the receiver is equipped with $N = M_T = 2$ RF chains.

Figure 3 presents the ergodic capacity bounds of RS-ASF over a wide range of SNRs when $K = 8$ variable gain-linear amplifiers and phase shifters are available at the receiver and

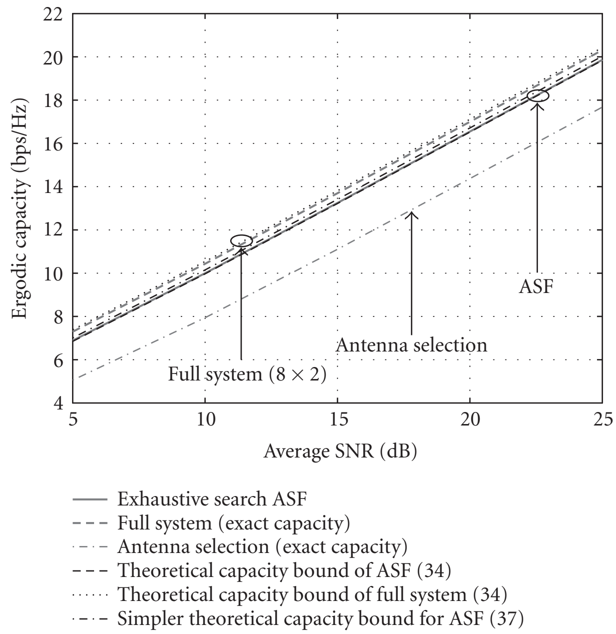

Figure 3: Ergodic capacity bounds for ASF and capacity of exhaustive search ASF when $M_R = 8$, $M_T = 2$, and $K = 8$ variable gain-linear amplifiers and phase shifters are available at the receiver (4 antenna elements in each subarray). Results are compared to an ergodic capacity bound and exact ergodic capacity of the full system.

exactly $k \triangleq K/N = 4$ receiving antenna elements participate in each subarray. For purposes of reference, the ergodic capacity of the exhaustive search solution of RS-ASF is also shown. The exhaustive search solution is obtained by considering all the $\binom{M_R}{k}^N$ possible combinations of subarray formation, that is, all possible combinations for the structure of matrix $\mathbf{S}$ as defined in (37), assuming that $\mathbf{A}$ is obtained as in (36). Apparently, both capacity bounds are very tight to the exhaustive search solution.

When each subarray contains M_R antenna elements, the capacity bound of the MIMO system is found by analyzing it into M_T parallel SIMO systems. Each of these parallel systems reduces to a MRC diversity system and therefore the ergodic capacity bound of the full system will be obtained by (31). This observation is verified in Figure 3.

5.2. Frobenius-norm-based algorithm

In this paragraph we demonstrate the capacity performance of the Frobenius-norm-based algorithm for various schemes of receive ASF in terms of outage capacity (when the slowly-varying block fading channel model is adopted) and ergodic capacity (when the channel is assumed ergodic). The proposed algorithm is applied to both Rayleigh i.i.d. and measured MIMO channels.

5.2.1. Rayleigh i.i.d. channels

We consider Rayleigh i.i.d. MIMO channels with $M_T = 2$ elements at the transmitter and assume that the receiver is

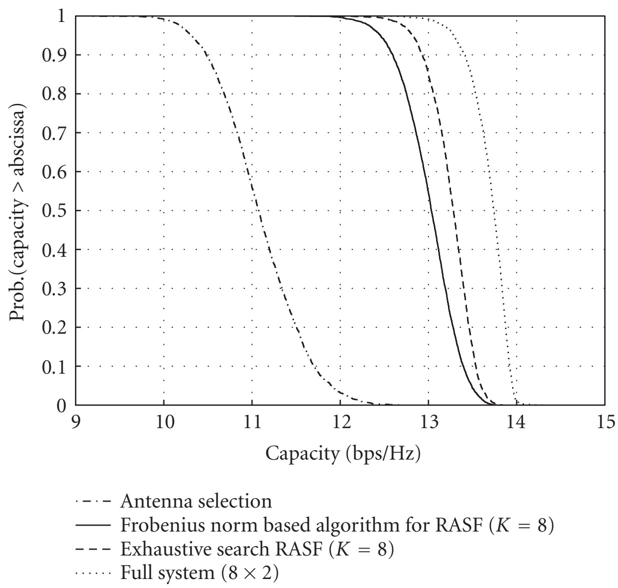

Figure 4: Empirical complementary cdf of the capacity of the resulted system when the Frobenius-norm-based algorithm for strictly structured receive antenna subarray formation (SS-ASF) is applied to a 8×2 Rayleigh i.i.d. channel with SNR = 15 dB. The performance of the algorithm is compared with the exhaustive search solution for SS-ASF, the full system (8×2), and Gorokhov et al. decremental algorithm for antenna selection.

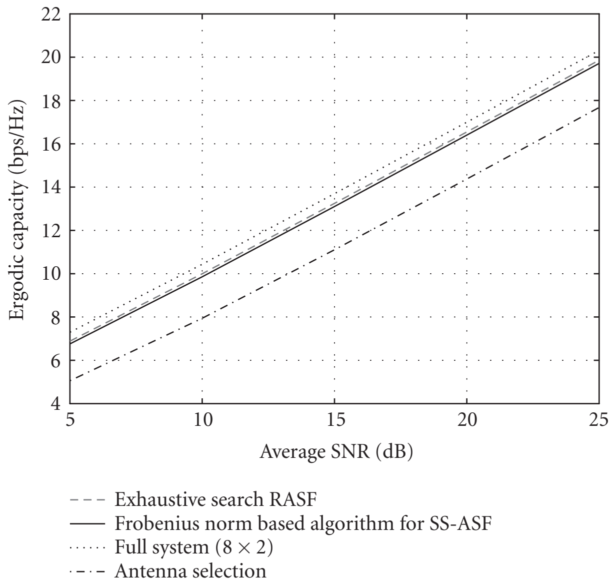

Figure 5: Performance evaluation of strictly structured ASF (SS-ASF) applied to an 8×2 MIMO Rayleigh i.i.d. channel, in terms of ergodic capacity. The performance of the algorithm is compared to the exhaustive search solution for receive ASF, the full system (8×2), and Gorokhov et al. decremental algorithm for antenna selection.

equipped with $M_T = 8$ elements, $N = M_T = 2$ RF chains, and $K = 8$ phase shifters or/and variable gain-linear amplifiers.

Figure 4 presents the complementary cdf of the capacity of the resulted system for SS-ASF when the SNR is at 15 dB. Clearly, SS-ASF outperforms Gorokhov et al. algorithm for antenna selection [4], which is quasi optimal in terms of capacity performance. Moreover, the performance of the proposed algorithm is very close to the exhaustive search solution. Thus, the SS-ASF technique delivers a significant capacity increase with respect to conventional antenna selection schemes. The same results are verified in Figure 5, where the ergodic capacity of the resulted system over a wide range of SNRs is plotted.

5.2.2. Measured channel

In order to examine the performance in realistic conditions, we have applied the proposed algorithm to measured MIMO channel transfer matrices. Measurements were conducted using a vector channel sounder operating at the center frequency of 5.2 GHz with 120 MHz measurement bandwidth in short-range outdoor environments with LOS propagation conditions. A more detailed description of the measurement setup can be found in [31]. The transmitter has $M_T = 4$ equally spaced antenna elements and the receiver is equipped with $M_R = 16$ receiving elements and $N = M_T = 4$ RF chains. The interelement distance for both the transmitting and receiving antenna arrays is $d = 0,4\lambda$.

Figure 6 displays the complementary cdf of the capacity of the resulted system when the Frobenius-norm-based algorithm is applied to several schemes of receive ASF and for various values of K (i.e., the number of phase shifters or/and variable gain-linear amplifiers). Clearly, all ASF schemes outperform conventional antenna selection.

Solid black lines correspond to RS-ASF (or SS-ASF for $K = M_R = 16$) and dashed black lines to RHC-ASF. Comparing the solid with the dashed lines for the same value of K, it is evident that RHC-ASF delivers capacity performance very close to RS-ASF. Therefore, the expensive variable gain-linear amplifiers can be abolished from the design of ASF with negligible capacity loss.

For $K = 48$, the capacity performance of RS-ASF and RHC-ASF is very close to the full system, despite the fact that in ASF the receiver is equipped with only $N = M_T = 4$ RF chains (whereas the full system has $M_R = 16$ RF chains). Even when $K = 32$, the capacity loss with respect to the full system is still quite low (10% outage capacity loss of RHC-ASF is less than 1.5 bps/Hz at 15 dB). Similar results are observed for a wide range of signal-to-noise ratios (Figure 7). Consequently, the proposed algorithm can deliver near-optimal capacity performance with respect to the full system while reducing drastically the number of necessary RF chains.

6. CONCLUSIONS

In this paper, we have developed a tight theoretical upper bound on the ergodic capacity of antenna subarray formation and have presented an analytical algorithm for

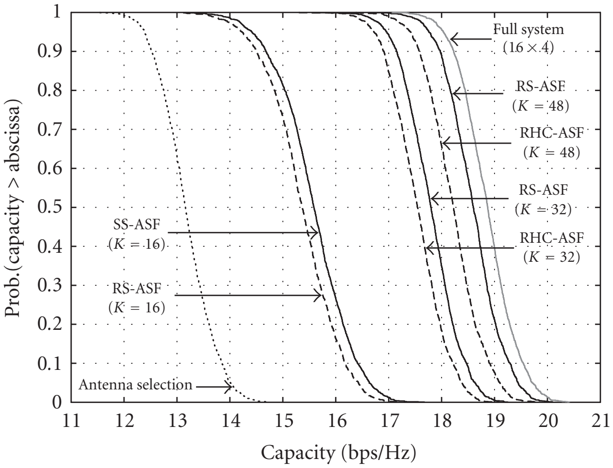

Figure 6: Empirical complementary cdf of the capacity of the resulted system when the Frobenius-norm-based algorithm for several schemes of receive antenna subarray formation (ASF) is applied to a 16 × 4 measured channel with SNR = 15 dB. In particular, the RASF schemes studied are strictly structured ASF (SS-ASF), relaxed-structured ASF (RS-ASF), and reduced hardware complexity ASF (RHC-ASF). K denotes the number of phase shifters or/and variable gain-linear amplifiers available to the receiver. The performance of the algorithm is compared to the full system (16 × 4) and Gorokhov et al. decremental algorithm for antenna selection.

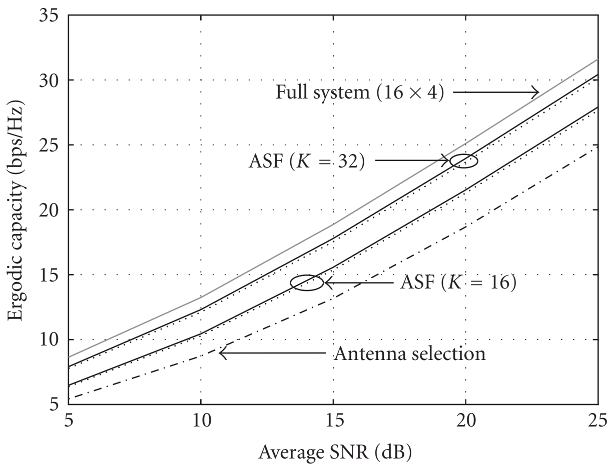

Figure 7: Performance evaluation of Frobenius-norm-based algorithm for several schemes of receive antenna subarray formation (RASF) applied to a 16×4 MIMO measured channel, in terms of ergodic capacity. In particular, the RASF schemes studied are strictly structured ASF (SS-ASF), relaxed-structured ASF (RS-ASF) (solid lines), and reduced hardware complexity ASF (RHC-ASF) (dotted lines). K denotes the number of phase shifters or/and variable gain-linear amplifiers available to the receiver. The performance of the algorithm is compared to the full system (16 × 4) and Gorokhov et al. decremental algorithm for antenna selection.

adaptively grouping receive array elements to subarrays. Application in Rayleigh i.i.d. and measured channels demonstrates significant capacity performance, which can become near optimal with respect to the full system, depending on the number of available phase shifters or/and variable gain-linear amplifiers. Furthermore, it has been shown that a phase-shift-only design of the technique is feasible with negligible performance penalty. Thus, it has been established that antenna subarray formation is a promising RF preprocessing technique that reduces hardware costs while achieving incredible performance enhancement with respect to conventional antenna selection schemes.

APPENDICES

A.

Let $\mathbf{A} = \mathbf{U}_{\mathrm{A}}\boldsymbol{\Sigma}_{\mathrm{A}}\mathbf{V}_{\mathrm{A}}^{\mathrm{H}}$ be a singular value decomposition [32] of matrix $\mathbf{A}$. We get

$$\begin{aligned}\mathbf{A}\left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}} &= \mathbf{U}_{\mathrm{A}}\boldsymbol{\Sigma}_{\mathrm{A}}\mathbf{V}_{\mathrm{A}}^{\mathrm{H}}\left(\mathbf{V}_{\mathrm{A}}\boldsymbol{\Sigma}_{\mathrm{A}}^{2}\mathbf{V}_{\mathrm{A}}^{\mathrm{H}}\right)^{-1}\mathbf{V}_{\mathrm{A}}\boldsymbol{\Sigma}_{\mathrm{A}}\mathbf{U}_{\mathrm{A}}^{\mathrm{H}}\\ &= \mathbf{U}_{\mathrm{A}}\boldsymbol{\Sigma}_{\mathrm{A}}\mathbf{V}_{\mathrm{A}}^{\mathrm{H}}\mathbf{V}_{\mathrm{A}}\boldsymbol{\Sigma}_{\mathrm{A}}^{-2}\mathbf{V}_{\mathrm{A}}^{\mathrm{H}}\mathbf{V}_{\mathrm{A}}\boldsymbol{\Sigma}_{\mathrm{A}}\mathbf{U}_{\mathrm{A}}^{\mathrm{H}}\\ &= \mathbf{U}_{\mathrm{A}}\mathbf{U}_{\mathrm{A}}^{\mathrm{H}}.\end{aligned} \tag{A.1}$$

Thus, the capacity formula in (15) becomes

$$C_{\mathrm{RASF}} = \log_2 \det\left(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{H}^{\mathrm{H}}\mathbf{U}_{\mathrm{A}}\mathbf{U}_{\mathrm{A}}^{\mathrm{H}}\mathbf{H}\right). \tag{A.2}$$

Applying the known formula for determinants [32]

$$\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA}) \tag{A.3}$$

to (A.2), we get

$$C_{\mathrm{RASF}} = \log_2 \det\left(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{U}_{\mathrm{A}}^{\mathrm{H}}\mathbf{H}\mathbf{H}^{\mathrm{H}}\mathbf{U}_{\mathrm{A}}\right) \tag{A.4}$$

which can be written as

$$C_{\mathrm{RASF}}=\sum_{m=1}^{M_T}\log_2\left(1+\frac{\rho}{M_T}\lambda_m\left(\mathbf{U}_{\mathrm{A}}^{\mathrm{H}}\mathbf{H}\mathbf{H}^{\mathrm{H}}\mathbf{U}_{\mathrm{A}}\right)\right), \tag{A.5}$$

where $\lambda_m(\mathbf{X})$ denotes the mth eigenvalue of square matrix $\mathbf{X}$ in descending order. Poincare separation theorem [32] states that

$$\lambda_m(\mathbf{U}_{\mathrm{A}}^{\mathrm{H}}\mathbf{H}\mathbf{H}^{\mathrm{H}}\mathbf{U}_{\mathrm{A}}) \le \lambda_m(\mathbf{H}\mathbf{H}^{\mathrm{H}}) \tag{A.6}$$

with equality occurring when the columns of $\mathbf{U}_A$ are the M_T dominant left singular vectors of $\mathbf{H}$. Thus,

$$\begin{aligned}C_{\mathrm{RASF}} &\le \sum_{k=1}^{M_T}\log_2\left(1+\frac{\rho}{M_T}\lambda_k\left(\mathbf{H}\mathbf{H}^{\mathrm{H}}\right)\right)\\ &= \log_2\det\left(\mathbf{I}_{M_R}+\frac{\rho}{M_T}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right) = C_{\mathrm{FS}},\end{aligned} \tag{A.7}$$

where equality occurs when

$$\mathbf{U}_A = [\mathbf{u}_1 \;\; \mathbf{u}_2 \;\; \cdots \;\; \mathbf{u}_{M_T}] \qquad \text{(A.8)}$$

and $\mathbf{u}_k$ is the kth dominant singular vector of $\mathbf{H}$. Therefore, an *optimal* solution to the *unconstrained* (i.e., without the subarray formation constraints in (6) capacity maximization problem is

$$\mathbf{A}_o = [\mathbf{u}_1 \;\; \mathbf{u}_2 \;\; \cdots \;\; \mathbf{u}_{M_T}]\mathbf{Q}, \qquad \text{(A.9)}$$

where $\mathbf{Q} = \mathbf{\Sigma}_A\mathbf{V}_A^H$ is a matrix with orthogonal rows and columns.

B.

Let $\mathbf{A} = \mathbf{U}_A\mathbf{\Sigma}_A\mathbf{V}_A^H$ be a singular value decomposition of the transformation matrix $\mathbf{A}$. Exploiting Hadarmard's inequality for determinants [32] and after some trivial mathematical manipulations, it follows that

$$\begin{aligned}\det\left(\mathbf{\Sigma}_\mathbf{A}^2\right) &= \det\left(\mathbf{V}_\mathbf{A}\mathbf{\Sigma}_\mathbf{A}^2\mathbf{V}_\mathbf{A}^H\right) = \det\left(\mathbf{A}^H\mathbf{A}\right) \le \prod_{k=1}^{M_T}\left[\mathbf{A}^H\mathbf{A}\right]_{kk} \\ &= \prod_{k=1}^{M_T}\mathbf{a}_k^H\mathbf{a}_k = \prod_{k=1}^{M_T}\|\mathbf{a}_k\|^2 \le 1,\end{aligned} \qquad \text{(B.1)}$$

where $\mathbf{a}_k$ denotes the kth column of the transformation matrix $\mathbf{A}$. The last inequality in (B.1) follows from $\|\mathbf{a}_k\| \le \|\mathbf{u}_k\| = 1$, with $\mathbf{u}_k$ being the kth left singular vector of the full system channel matrix, and it is justified by the fact that the entries of matrix $\mathbf{A}$ are obtained as in (35).

In the high SNR regime, after substituting for $\mathbf{A} = \mathbf{U}_A\sum{}_A\mathbf{V}_A^H$ and taking into account (B.1), it is valid to write

$$\begin{aligned}\det\left(\mathbf{I}_{M_T} + \frac{\rho}{M_T}\mathbf{H}^H\mathbf{A}\mathbf{A}^H\mathbf{H}\right) &\approx \det\left(\frac{\rho}{M_T}\mathbf{H}^H\mathbf{U}_A\mathbf{\Sigma}_\mathbf{A}^2\mathbf{U}_A^H\mathbf{H}\right) \\ &= \det\left(\mathbf{\Sigma}_\mathbf{A}^2\right)\det\left(\frac{\rho}{M_T}\mathbf{H}^H\mathbf{U}_A\mathbf{U}_A^H\mathbf{H}\right) \\ &\le \det\left(\frac{\rho}{M_T}\mathbf{H}^H\mathbf{U}_A\mathbf{U}_A^H\mathbf{H}\right).\end{aligned} \qquad \text{(B.2)}$$

Recognizing that the right-hand side of (B.2) is an approximation of (A.2), that is, the capacity of the RASF system, in the high SNR regime, the validity of the bound in (47) is proven.

Note that the same approximation for the capacity of MIMO systems at high SNR has been widely used (see, e.g., [24]). Simulation results in Figure 8 demonstrate that the bound is quite tight.

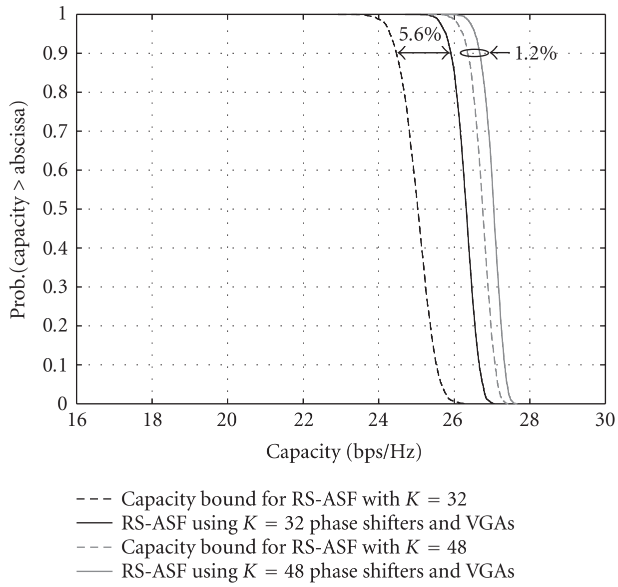

Figure 8: Comparison between capacity bound (47) for relaxed structured ASF and true capacity (15) of the resulted system in terms of empirical complementary cdf, when applied to a 16×4 MIMO Rayleigh i.i.d. channel with SNR = 15 dB. Proof of this bound can be found in Appendix B.

Acknowledgment

This work has been partially funded by Antenna Centre of Excellence (ACE2) research programme, under the EU 6th Framework Programme.

References

[1] D. A. Gore, R. U. Nabar, and A. J. Paulraj, "Selecting an optimal set of transmit antennas for a low rank matrix channel," in *Proceedings of IEEE Interntional Conference on Acoustics, Speech, and Signal Processing (ICASSP '00)*, vol. 5, pp. 2785–2788, Istanbul, Turkey, June 2000.

[2] R. S. Blum and J. H. Winters, "On optimum MIMO with antenna selection," *IEEE Communications Letters*, vol. 6, no. 8, pp. 322–324, 2002.

[3] A. F. Molisch, M. Z. Win, Y.-S. Choi, and J. H. Winters, "Capacity of MIMO systems with antenna selection," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1759–1772, 2005.

[4] A. Gorokhov, D. A. Gore, and A. J. Paulraj, "Receive antenna selection for MIMO spatial multiplexing: theory and algorithms," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2796–2807, 2003.

[5] M. Gharavi-Alkhansari and A. B. Gershman, "Fast antenna subset selection in MIMO systems," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 339–347, 2004.

[6] D. A. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2580–2588, 2002.

[7] R. W. Heath Jr., S. Sandhu, and A. J. Paulraj, "Antenna selection for spatial multiplexing systems with linear receivers," *IEEE Communications Letters*, vol. 5, no. 4, pp. 142–144, 2001.

[8] D. A. Gore, R. W. Heath Jr., and A. J. Paulraj, "Transmit selection in spatial multiplexing systems," *IEEE Communications Letters*, vol. 6, no. 11, pp. 491–493, 2002.

[9] M. A. Jensen and M. L. Morris, "Efficient capacity-based antenna selection for MIMO Systems," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 1, pp. 110–116, 2005.

[10] A. F. Molisch, M. Z. Win, and J. H. Winter, "Reduced-complexity transmit/receive-diversity systems," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2729–2738, 2003.

[11] L. Dai, S. Sfar, and K. B. Letaief, "Receive antenna selection for MIMO systems in correlated channels," in *Proceedings of the IEEE International Conference on Communications (ICC '04)*, vol. 5, pp. 2944–2948, Paris, France, June 2004.

[12] P. D. Karamalis, N. D. Skentos, and A. G. Kanatas, "Selecting array configurations for MIMO systems: an evolutionary computation approach," *IEEE Transactions on Wireless Communications*, vol. 3, no. 6, pp. 1994–1998, 2004.

[13] P. D. Karamalis, N. D. Skentos, and A. G. Kanatas, "Adaptive antenna subarray formation for MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 5, no. 11, pp. 2977–2982, 2006.

[14] G. G. Raleigh and J. M. Cioffi, "Spatio-temporal coding for wireless communication," *IEEE Transactions on Communications*, vol. 46, no. 3, pp. 357–366, 1998.

[15] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant filterbank precoders and equalizers—I: unification and optimal designs," *IEEE Transactions on Signal Processing*, vol. 47, no. 7, pp. 1988–2006, 1999.

[16] H. Sampath, P. Stoica, and A. J. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Transactions on Communications*, vol. 49, no. 12, pp. 2198–2206, 2001.

[17] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Transactions on Signal Processing*, vol. 50, no. 5, pp. 1051–1064, 2002.

[18] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Transactions on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, 2003.

[19] C. Mun, J.-K. Han, and D.-H. Kim, "Quantized principal component selection precoding for limited feedback spatial multiplexing," in *Proceedings of the IEEE International Conference on Communications (ICC '06)*, pp. 4149–4154, Istanbul, Turkey, June 2006.

[20] X. Zhang, A. F. Molisch, and S.-Y. Kung, "Variable-phase-shift-based RF-baseband codesign for MIMO antenna selection," *IEEE Transactions on Signal Processing*, vol. 53, no. 11, pp. 4091–4103, 2005.

[21] P. Theofilakos and A. G. Kanatas, "Frobenius norm based receive antenna subarray formation for MIMO systems," in *Proceedings of the1st European Conference on Antennas and Propagation (EuCAP '06)*, vol. 626, Nice, France, November 2006.

[22] P. Theofilakos and A. G. Kanatas, "Reduced hardware complexity receive antenna subarray formation for MIMO systems based on frobenius norm criterion," in *Proceedings of the 3rd International Symposium on Wireless Communication Systems (ISWCS '06)*, Valencia, Spain, September 2006.

[23] P. Theofilakos and A. G. Kanatas, "Robustness of receive antenna subarray formation to hardware and signal non-idealities," in *Proceedings of the 65th IEEE Vehicular Technology Conference (VTC '07)*, pp. 324–328, Dublin, Ireland, April 2007.

[24] O. Oyman, R. U. Nabar, H. Bölcskei, and A. J. Paulraj, "Characterizing the statistical properties of mutual information in MIMO channels," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2784–2795, 2003.

[25] F. D. Neeser and J. L. Massey, "Proper complex random processes with applications to information theory," *IEEE Transactions on Information Theory*, vol. 39, no. 4, pp. 1293–1302, 1993.

[26] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, John Wiley & Sons, New York, NY, USA, 1991.

[27] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, 1998.

[28] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, New York, NY, USA, 4th edition, 2002.

[29] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, John Wiley & Sons, New York, NY, USA, 1st edition, 2000.

[30] M.-S. Alouini and A. J. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 4, pp. 1165–1181, 1999.

[31] N. D. Skentos, A. G. Kanatas, P. I. Dallas, and P. Constantinou, "MIMO channel characterization for short range fixed wireless propagation environments," *Wireless Personal Communications*, vol. 36, no. 4, pp. 339–361, 2006.

[32] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, Cambridge, UK, 1985.