Eur. Phys. J. Special Topics **228**, 2351–2355 (2019)

https://doi.org/10.1140/epjst/e2019-900201-4

THE EUROPEAN PHYSICAL JOURNAL
SPECIAL TOPICS

Editorial

Diffusion dynamics and information spreading in multilayer networks: An overview

Matjaž Perc[1,2,3,a]

[1] Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, 2000 Maribor, Slovenia
[2] Center for Applied Mathematics and Theoretical Physics, University of Maribor, Mladinska 3, 2000 Maribor, Slovenia
[3] Complexity Science Hub Vienna, Josefstädterstraße 39, 1080 Vienna, Austria

Received 19 September 2019
Published online 28 October 2019

Abstract. Recent years have seen multilayer networks taking the helm at network science research. Diffusion dynamics and information spreading in multilayer networks are thereby two of the most explored topics, with numerous applications in social, technological, and biological systems. Almost a decade ago, groundbreaking research has shown that even small and seemingly unimportant changes in one network layer can have catastrophic consequences in many other network layers. Such failure cascades across different network layers can, for example, shut down power grids, arrest traffic, idle virality, or impair cooperation. Different spreading processes are often the common denominator of such phenomena, and in the light of their importance, we here present a brief overview of this subject.

1 Network science

During the past two decades network science has emerged as a central paradigm behind some of the most fascinating discoveries of the 21st century [1–4]. The impetus for this fascinating development came with the discovery that seemingly very different networks have universal properties that pervade across social, biological, and technological systems. In 1998, Watts and Strogatz [5] termed this the collective dynamics of "small-world" networks, observing that electric power grids, food chains, brain networks, protein networks, transcriptional networks, and social networks are all highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs. Hence the name "small-world" networks. Their simple and intuitive mathematical model was a hit, and as such it was soon applied prolifically across the social and natural sciences.

One year later Barabási and Albert proposed the growth and preferential attachment model to describe the universal scaling in degree distributions of many realistic networks [6], and their model too enjoyed, and still enjoys, fantastic popularity. Perhaps the third major discovery came with the realization that many networks have groups of nodes that are more strongly interconnected with one another than

[a] e-mail: matjaz.perc@um.si

they are to other nodes of the network, which today is well-known as community structure [7].

These major breakthroughs, combined with numerous insightful and timely applications, propelled network science towards the top of the hottest fields of research of the still young 21st century, as evidenced by several highly influential and cited reviews [8–16].

2 Temporal and multilayer networks

Network science also provides methods and tools for studying and taking into account network evolution over time. This can be due to changes in external factors that affect a particular network, targeted attack, or simply due to random failure or network growth. Such changes can be studied in the realm of temporal networks [13,17,18], where the theoretical framework accounts for the addition or removal of nodes, or similarly for the changes in the links between nodes, over time.

Networks also exist between different layers of each studied system, and this is particularly apparent in social systems, where one person can be simultaneously member of many different networks that are to various degrees interdependent [19–25]. This can be accommodated in the theoretical framework of multilayer networks, or more generally networks of networks, which acknowledge that not only is the range of our interactions limited and thus inadequately described by well-mixed models, but also that the networks that should be an integral part of such models are often interconnected, thus making the processes that are unfolding on them interdependent [14,15,26–28]. From the world economy and transportation systems to the spread of epidemics, it is clear that processes taking place in one network can significantly affect what is happening in many other networks.

Various phenomena have been studied on multilayer networks, including the evolution of cooperation [29–41], for example. This research has revealed interdependent network reciprocity, probabilistic interconnectedness, and information transmission across network layers as potent facilitators of cooperation in social dilemmas. Research has also addressed cascading failures [19,22,42,43], competitive percolation [20,44,45], transport [46], neuronal synchronization [47], epidemic spreading [48], robustness against attack and assortativity [49,50], stability [51], growth [52], entropy and overlap [53], as well as abrupt transition in the structural formation [54]. Multilayer networks have indeed become a leading paradigm of network science in recent years [14,15,26].

3 Diffusion dynamics and information spreading

It is thus clear that the consideration of multilayer or interdependent networks is crucial also for a comprehensive treatment of diffusion dynamics and information spreading. In their seminal paper, Gómez et al. [25] have studied diffusion processes on multilayer networks in which nodes were conserved through the different layers. Although many real multilayer networks consist of layers that have their own, and usually different, temporal and structural properties, their research paved the way for an analysis of diffusion processes that may unfold on such structures. Notably, diffusive processes are a good approximation for different types of dynamical processes, like synchronization [55], which can be described by the behavior of the eigenvalues of the Laplacian matrix. For example, the time needed to synchronize phase oscillators in a network is related to the second smallest eigenvalue of the Laplacian λ_2 [56], and the stability of the synchronized state is determined by the eigenratio λ_N/λ_2 [57].

Indeed, the spectral analysis of complex networks remains a vibrant research area [58,59].

Following a perturbative analysis of the spectra [60], the approach by Gómez et al. [25] has revealed new physical insights into the diffusion processes through the analytical determination of the asymptotic behavior of the eigenvalues of the Laplacian of the multiplex, i.e., the supra-Laplacian, when the coupling between layers is either small or large. Research has shown that the emergent physical behavior of the diffusion process in multilayer networks is far from trivial. In some cases, the coupling of networks shows a super-diffusive behavior, meaning that diffusive processes in the multiplex are faster than in any of the networks that form it separately.

4 Special issues and outlook

With this in mind, the European Physical Journal Special Topics has hosted a Special Issue titled Diffusion Dynamics and Information Spreading in Multilayer Networks, which has seen the publication of 11 research papers [61–71]. Previously, the New Journal of Physics also hosted a Focus on Multilayer Networks, where a total of 23 papers have been published [72–94]. Subjects that were covered range from social contagion, interlayer competition, public cooperation, belief percolation, explosive synchronization, and epidemic spreading to pattern formation, cascading dynamics, consensus ranking, congestion, and meme spreading.

Given the diversity of the topics that were covered, it is challenging to pull a common thread through the contributions that have been published. Interested readers are therefore cordially invited to browse through the collections and discover for themselves the full breadth of research that has been covered and to select their favorite papers.

Evidently, the field is still very much alight and vibrant, and the future will certainly see further attention devoted to multilayer networks. The aims remain to more accurately describe and understand collective social phenomena that are due to the interactions among individuals, groups, and governments, as well as to more efficiently tackle technological challenges that arise due to the interconnectedness of different components of information, computing, and transportation infrastructure. The hope is to ultimately develop better social systems, more efficient policies, and more resilient technologies for a sustainable future.

We would like to thank the publishing team at the European Physical Journal Special Topics for all their support and assistance. We would also like to thank all the authors and referees for their valuable contributions and help. This work was supported by the Slovenian Research Agency (Grants J4-9302, J1-9112 and P1-0403).

References

1. A. Barrat, M. Barthélemy, A. Vespignani, *Dynamical Processes on Complex Networks* (Cambridge University Press, Cambridge, 2008)
2. M.E.J. Newman, *Networks: An Introduction* (Oxford University Press, Oxford, 2010)
3. E. Estrada, *The Structure of Complex Networks: Theory and Applications* (Oxford University Press, Oxford, 2012)
4. A.-L. Barabási, *Network Science* (Cambridge University Press, Cambridge, 2015)
5. D.J. Watts, S.H. Strogatz, Nature **393**, 440 (1998)
6. A.-L. Barabási, R. Albert, Science **286**, 509 (1999)
7. M. Girvan, M.E. Newman, Proc. Natl. Acad. Sci. USA **99**, 7821 (2002)

8. R. Albert, A.-L. Barabási, Rev. Mod. Phys. **74**, 47 (2002)
9. M.E.J. Newman, SIAM Rev. **45**, 167 (2003)
10. S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, D. Hwang, Phys. Rep. **424**, 175 (2006)
11. S. Fortunato, Phys. Rep. **486**, 75 (2010)
12. M. Barthelemy, Phys. Rep. **499**, 1 (2011)
13. P. Holme, J. Saramäki, Phys. Rep. **519**, 97 (2012)
14. S. Boccaletti, G. Bianconi, R. Criado, C. del Genio, J. Gómez-Gardeñes, M. Romance, I. Sendiña-Nadal, Z. Wang, M. Zanin, Phys. Rep. **544**, 1 (2014)
15. M. Kivelä, A. Arenas, M. Barthelemy, J.P. Gleeson, Y. Moreno, M.A. Porter, J. Complex Networks **2**, 203 (2014)
16. L. Lü, D. Chen, X.-L. Ren, Q.-M. Zhang, Y.-C. Zhang, T. Zhou, Phys. Rep. **650**, 1 (2016)
17. T. Gross, B. Blasius, J. R. Soc. Interface **5**, 259 (2008)
18. M. Perc, A. Szolnoki, BioSystems **99**, 109 (2010)
19. S.V. Buldyrev, R. Parshani, G. Paul, H.E. Stanley, S. Havlin, Nature **464**, 1025 (2010)
20. R. Parshani, S.V. Buldyrev, S. Havlin, Phys. Rev. Lett. **105**, 048701 (2010)
21. P.J. Mucha, T. Richardson, K. Macon, M.A. Porter, J.-P. Onnela, Science **328**, 876 (2010)
22. R. Parshani, S.V. Buldyrev, S. Havlin, Proc. Natl. Acad. Sci. USA **108**, 1007 (2011)
23. J. Gao, S.V. Buldyrev, H.E. Stanley, S. Havlin, Nat. Phys. **8**, 40 (2012)
24. S. Havlin, D.Y. Kenett, E. Ben-Jacob, A. Bunde, H. Hermann, J. Kurths, S. Kirkpatrick, S. Solomon, J. Portugali, Eur. Phys. J. Special Topics **214**, 273 (2012)
25. S. Gómez, A. Díaz-Guilera, J. Gómez-Gardeñes, C. Pérez-Vicente, Y. Moreno, A. Arenas, Phys. Rev. Lett. **110**, 028701 (2013)
26. M. De Domenico, A. Solé-Ribalta, E. Cozzo, M. Kivelä, Y. Moreno, M.A. Porter, S. Gómez, A. Arenas, Phys. Rev. X **3**, 041022 (2013)
27. Z. Wang, L. Wang, A. Szolnoki, M. Perc, Eur. Phys. J. B **88**, 124 (2015)
28. D.Y. Kenett, M. Perc, S. Boccaletti, Chaos Soliton Fract. **80**, 1 (2015)
29. Z. Wang, A. Szolnoki, M. Perc, EPL **97**, 48001 (2012)
30. J. Gómez-Gardeñes, I. Reinares, A. Arenas, L.M. Floría, Sci. Rep. **2**, 620 (2012)
31. J. Gómez-Gardeñes, C. Gracia-Lázaro, L.M. Floría, Y. Moreno, Phys. Rev. E **86**, 056113 (2012)
32. B. Wang, X. Chen, L. Wang, J. Stat. Mech. **2012**, P11017 (2012)
33. Z. Wang, A. Szolnoki, M. Perc, Sci. Rep. **3**, 1183 (2013)
34. Z. Wang, A. Szolnoki, M. Perc, Sci. Rep. **3**, 2470 (2013)
35. L.-L. Jiang, M. Perc, Sci. Rep. **3**, 2483 (2013)
36. A. Szolnoki, M. Perc, New J. Phys. **15**, 053010 (2013)
37. Z. Wang, L. Wang, M. Perc, Phys. Rev. E **89**, 052813 (2014)
38. F. Battiston, M. Perc, V. Latora, New J. Phys. **19**, 073017 (2017)
39. C. Shen, C. Chu, L. Shi, M. Jusup, M. Perc, Z. Wang, EPL **124**, 48003 (2018)
40. C. Xia, X. Li, Z. Wang, M. Perc, New J. Phys. **20**, 075005 (2018)
41. L. Shi, C. Shen, Y. Geng, C. Chu, H. Meng, M. Perc, S. Boccaletti, Z. Wang, Nonlinear Dyn. **96**, 49 (2019)
42. W. Li, A. Bashan, S. V. Buldyrev, Phys. Rev. Lett. **108**, 228702 (2012)
43. C.D. Brummitt, R.M. D'Souza, E.A. Leicht, Proc. Natl. Acad. Sci. USA **109**, E680 (2012)
44. J. Nagler, A. Levina, M. Timme, Nat. Phys. **7**, 265 (2011)
45. D. Cellai, E. López, J. Zhou, J.P. Gleeson, G. Bianconi, Phys. Rev. E **88**, 052811 (2013)
46. R.G. Morris, M. Barthelemy, Phys. Rev. Lett. **109**, 128703 (2012)
47. X. Sun, J. Lei, M. Perc, J. Kurths, G. Chen, Chaos **21**, 016110 (2011)
48. C. Granell, S. Gómez, A. Arenas, Phys. Rev. Lett. **111**, 128701 (2013)
49. X. Huang, J. Gao, S.V. Buldyrev, S. Havlin, H.E. Stanley, Phys. Rev. E **83**, 065101(R) (2011)
50. D. Zhou, H.E. Stanley, G. D'Agostino, A. Scala, Phys. Rev. E **86**, 066103 (2012)
51. E. Cozzo, A. Arenas, Y. Moreno, Phys. Rev. E **86**, 036115 (2012)
52. V. Nicosia, G. Bianconi, V. Latora, M. Barthelemy, Phys. Rev. Lett. **111**, 058701 (2013)

53. G. Bianconi, Phys. Rev. E **87**, 062806 (2013)
54. F. Radicchi, A. Arenas, Nat. Phys. **9**, 717 (2013)
55. A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, C. Zhou, Phys. Rep. **469**, 93 (2008)
56. J.A. Almendral, A. Díaz-Guilera, New J. Phys. **9**, 187 (2007)
57. M. Barahona, L.M. Pecora, Phys. Rev. Lett. **89**, 054101 (2002)
58. S. Jalan, G. Zhu, B. Li, Phys. Rev. E **84**, 046107 (2011)
59. E. Estrada, N. Hatano, M. Benzi, Phys. Rep. **514**, 89 (2012)
60. P. Van Mieghem, *Graph spectra for complex networks* (Cambridge University Press, Cambridge, 2010)
61. F. Geier, W. Barfuss, M. Wiedermann, J. Kurths, J.F. Donges, Eur. Phys. J. Special Topics **228**, 2357 (2019)
62. Y. Feng, A.J.M. Khalaf, F.E. Alsaadi, T. Hayat, V.-T. Pham, Eur. Phys. J. Special Topics **228**, 2371 (2019)
63. N.S. Frolov, V.A. Maksimenko, M.V. Khramova, A.N. Pisarchik, A.E. Hramov, Eur. Phys. J. Special Topics **228**, 2381 (2019)
64. Z. Wang, F.E. Alsaadi, V.-T. Pham, Eur. Phys. J. Special Topics **228**, 2391 (2019)
65. Y. Shaverdi, S. Panahi, T. Kapitaniak, S. Jafari, Eur. Phys. J. Special Topics **228**, 2405 (2019)
66. X. Li, T. Xu, J. Li, Eur. Phys. J. Special Topics **228**, 2419 (2019)
67. S. Kundu, S. Majhi, D. Ghosh, Eur. Phys. J. Special Topics **228**, 2429 (2019)
68. B.K. Bera, S. Rakshit, D. Ghosh, Eur. Phys. J. Special Topics **228**, 2441 (2019)
69. M. Ge, L. Lu, Y. Xu, X. Zhan, L. Yang, Y. Jia, Eur. Phys. J. Special Topics **228**, 2455 (2019)
70. F. Parastesh, C.-Y. Chen, H. Azarnoush, S. Jafari, B. Hatef, Eur. Phys. J. Special Topics **228**, 2465 (2019)
71. F. An, X. Gao, J. Guan, M. Jiang, Q. Liu, Eur. Phys. J. Special Topics **228**, 2475 (2019)
72. M. Pósfai, N. Braun, B.A. Beisner, B. McCowan, R.M. DSouza, New J. Phys. **21**, 055001 (2019)
73. B. Min, M. San Miguel, New J. Phys. **21**, 035004 (2019)
74. J. Choi, K.-I. Goh, New J. Phys. **21**, 035005 (2019)
75. J. Van Lidth de Jeude, T. Aste, G. Caldarelli, New J. Phys. **21**, 025002 (2019)
76. J.D. O'Brien, I.K. Dassios, J.P. Gleeson, New J. Phys. **21**, 025001 (2019)
77. R.-R. Liu, C.-X. Jia, Y.-C. Lai, New J. Phys. **21**, 045002 (2019)
78. G. Cencetti, F. Battiston, New J. Phys. **21**, 035006 (2019)
79. H. Wang, C. Qu, C. Jiao, W. Rusze, New J. Phys. **21**, 035001 (2019)
80. Y. Zhou, J. Zhou, G. Chen, H.E. Stanley, New J. Phys. **21**, 035002 (2019)
81. A. Solé-Ribalta, A. Arenas, S. Gómez, New J. Phys. **21**, 035003 (2019)
82. J. Liu, Y. Fan, J. Zhang, Z. Di, New J. Phys. **21**, 015007 (2019)
83. A.D. Kachhvah, S. Jalan, New J. Phys. **21**, 015006 (2019)
84. H.-J. Li, L. Wang, New J. Phys. **21**, 015005 (2019)
85. E. Estrada, New J. Phys. **21**, 015004 (2019)
86. D. Han, X. Li, New J. Phys. **21**, 015002 (2019)
87. N. An, H. Chen, C. Ma, H. Zhang, New J. Phys. **20**, 125006 (2018)
88. W. Dang, Z. Gao, D. Lv, M. Liu, Q. Cai, X. Hong, New J. Phys. **20**, 125005 (2018)
89. L. Valdez, H.A. Rêgo, H. Stanley, S. Havlin, L. Braunstein, New J. Phys. **20**, 125003 (2018)
90. X. Zhu, H. Tian, X. Chen, W. Wang, S. Cai, New J. Phys. **20**, 125002 (2018)
91. G.F. de Arruda, E. Cozzo, F.A. Rodrigues, Y. Moreno, New J. Phys. **20**, 095004 (2018)
92. H.-C.H. Chang, F. Fu, New J. Phys. **20**, 095001 (2018)
93. E. Pitsik, V. Makarov, D. Kirsanov, N. Frolov, M. Goremyko, X. Li, Z. Wang, A. Hramov, S. Boccaletti, New J. Phys. **20**, 075004 (2018)
94. C. Xia, X. Li, Z. Wang, M. Perc, New J. Phys. **20**, 075005 (2018)