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Editorial

Fractional Dynamical Systems: Recent Trends in Theory and Applications

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Abstract. In this EPJ Special Topics issue selected contributions on some recent developments in the theory and applications of fractional dynamical systems are presented. Results on topics involving fundamental theory, qualitative theory, iterative methods and numerous applications of fractional-order equations are reported.

The fractional dynamic is a field of study in mathematics and physics investigating the behavior of objects and systems by using differentiations of fractional orders. Due to its widespread applications in science and technology, research within the fractional dynamical systems has led to new developments that have attracted the attention of considerable audience of professionals such as mathematicians, physicists, applied researches and practitioners. Unlike integer-order models, fractional-order models have the potential to capture nonlocal relations in time and space with power law memory kernels. This makes them providing more realistic and adequate descriptions for many real-world phenomena.

In spite of the tremendous amount of published results focused on fractional differential equations and dynamical systems, there remain many challenging open problems. Indeed, the theory and applications of these systems are still very active areas of research. The main aim of this Special Issue for the EPJ Special Topics is to fill a void in the literature by making relevant information available for an important area of research. The Special Issue on "Fractional Dynamical Systems: Recent Trends in Theory and Applications" provides an international forum for researchers to contribute with original research focusing on the latest achievements in the theory and applications of fractional dynamical systems. In it, 32 regular research articles are published that can be grouped into four categories, namely:

1. Fundamental Theory [1–16]
2. Qualitative Theory [17–20]
3. Iterative and Numerical Methods [21–23]
4. Applications and Modelling [24–32]

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The papers in the category "Fundamental Theory" present recent developments on fundamental properties of solutions of fractional differential and difference equations. New definitions of fractional derivatives are also proposed. Properties of Mittag-Leffer function and fractional integral formulas have been investigated.

Ten papers are dedicated to some fundamental properties of the solutions for fractional-order differential systems. The paper [3] authored by B. Ahmad, N. Alghamdi, A. Alsaedi and S.K. Ntouyas discusses existence and uniqueness of solutions for a new class of multi- point boundary value problems of multi-term fractional differential equations. The properties of solution sets for Sobolev type fractional differential inclusions via resolvent family of operators are investigated by Y.-K. Chang and R. Ponce in [4]. Existence results for a class of boundary value problems for fractional differential equations with the Riesz-Caputo derivative, which holds two-sided nonlocal effects are presented by F. Chen, D. Baleanu and G.-C. Wu in the paper [5]. The article [7] authored by J. Henderson, C. Nelms Jr., D. Wang and A. Yang is concerned with characterizing extremal points for a Riemann-Liouville fractional boundary value problem. A comparative analysis among the possible types of initial conditions including (or not) derivatives in the Riemann-Liouville sense for incommensurate fractional differential systems with distributed delays is proposed by H. Kiskinov and A. Zahariev in [9]. The provided analysis is essentially based on the possibility to attribute physical meaning to the initial conditions expressed in terms of Riemann-Liouville fractional derivatives. This allows the values of the initial functions for the mentioned initial conditions to be obtained by appropriate measurements or observations. In the paper [10] K. Mekhalfi and D.F.M. Torres introduce more general concepts of Riemann-Liouville fractional integral and derivative on time scales, of a function with respect to another function. Sufficient conditions for existence and uniqueness of solution to an initial value problem described by generalized fractional order differential equations on time scales are also proved. Sufficient conditions for the non-existence of global solutions for a certain class of sequential fractional differential inequalities involving Riemann-Liouville left-sided fractional derivatives with different orders are provided in [12]. The article [13] authored by M. Seghier, A. Ouahab and J. Henderson offers some new results on the existence of solutions and the compactness of solution sets of a random system of fractional differential equations via the Hadamard-type derivative. The existence of positive solutions for a system of nonlinear fractional differential equations with sign-changing nonlinearities in investigated in [14] by D. Xie, C. Bai, H. Zhouand Y. Liu. In the paper of H. Zhou, J. Alzabut and L. Yang [16] existence criteria for the solutions of p-Laplacian fractional Langevin differential equations with anti-periodic boundary conditions are provided.

In two papers new fractional-order derivatives are defined. In the paper [8] of F. Jarad, T. Abdeljawad and J. Alzabut, the Caputo and Riemann-Liouville generalized proportional fractional derivatives involving exponential functions in their kernels are generated. The authors X.-J. Yang, F. Gao, J.A.T. Machado and D. Baleanu of the article [15] propose a new fractional derivative involving the normalized sinc function without singular kernel.

Two papers addressed important questions on the fundamental theory of discrete fractional equations. The paper by T. Abdeljawad, F. Jarad and J. Alzabut [1] formulates nabla fractional sums and differences and the discrete Laplace transform on a time scale. Basing on a local type of a proportional difference (without memory), the authors generate new types of fractional sums and differences with memory in two parameters which are generalizations to the formulated fractional sums and differences. The article [2] studies a Lyapunov inequality for fractional difference operators with discrete Mittag-Leffler kernel of order $2 \leq \alpha < 5/2$.

In the paper [6] the properties of Mittag-Leffer function are reviewed within the framework of an umbral formalism by G. Dattoli, K. Gorska, A. Horzela, S. Licciardi and R.M. Pidatella.

In the paper [11] the authors S. Salahshour, A. Ahmadian and D. Baleanu introduce a variation of the constant formula for fractional interval differential equations under interval fractional conformable derivative.

Four papers in the category "Qualitative Theory" study the stability, stabilization, asymptotic behavior and control of solutions of certain fractional differential equations. In the paper [17] by M. Bohner, I. Stamova and G. Tr. Stamov, using vector multivariable piecewise continuous Lyapunov functions, stability criteria with respect to a manifold are established for nonlinear impulsive functional differential systems involving Caputo fractional derivatives. In the paper [18] authored by A.A. Martynyuk, I. Stamova and A. Martynyuk-Chernienko the stability analysis of the set of trajectories for differential equations with fractional dynamics is provided. The problem of stability and synchronization of fractional-order complex-valued neural networks with time delay is investigated in [19] by K. Udhayakumar, R. Rakkiyappan and G. Velmurugan using Lyapunov-Krasovskii functional approach and some linear matrix inequality conditions. The article [20] of A. Zafer and S.R. Grace deals with the asymptotic behavior of nonoscillatory solutions of a class of fractional differential equations.

The papers in the category "Iterative and Numerical Methods" consist of three papers that address new methods to achieve the solutions of fractional equations explicitly. The novelty of the paper [21] authored by G.A. Anastassiou and I.K. Argyros is in the design of suitable iterative methods for generating a sequence approximating solutions of equations on Banach spaces. The authors M. Fečkan and K. Marynets of [22] give a new approach of investigation and approximation of solutions of fractional differential systems subjected to periodic boundary conditions. In the paper [23], numerical solutions of fractional telegraph differential equations by theta-method are presented by M. Modanli and A. Akgül.

Nine papers in the category "Applications and Modelling" presented different applications and modelling techniques related to fractional dynamical systems.

The paper [24] by D.P. Ahokpossi, A. Atangana and D.P. Vermeulen has considered the groundwater flow in dual media with fractional differentiation with power and generalized Mittag-Leffler laws kernels, where the media could be elastic, heterogeneous and viscos-elastic. For each model, using a numerical scheme known as Upwind, numerical simulations are presented.

In three papers, fractional-order chaotic systems are studied. In the article [25] of A. Bayani, M.A. Jafari, K. Rajagopal, H. Jiang and S. Jafari, a new fractional chaotic system with specific topological properties has been introduced and a Field Programmable Gate Arrays (FPGA) implementation has been developed. Design and implementation of fractional-order hyper-chaotic multi-scroll systems based on hysteresis series are offered by L. Chen, W. Pan, J.A. Tenreiro Machado, A.M. Lopes, R. Wu and Y. He in [27]. In the paper [31] a novel fractional chaotic system which can have self-excited or hidden attractor depending on the parameters is derived and investigated by K. Rajagopal, F. Nazarimehr, A. Akgul, S. Jafari and A. Karthigeyan.

The paper [26] authored by M. Borah and B.K. Roy proposes four fractional-order systems with diverse dynamical behaviour and their switching-parameter hybrid-synchronization are analyzed.

Time-fractional derivatives with non-singular kernel have been applied by I. Khan, M. Saqib and F. Ali in [28] to study the generalized convective flow of Casson fluid passing through a vertical microchannel with constant walls temperature. A newly introduced fractional derivative namely Caputo-Fabrizio fractional derivative is adopted for the generalization of classical partial differential equations that govern the flow.

The paper of P. Moghaddam and J.A. Tenreiro Machado [29] presents a new discretization technique for the forced Van der Pol oscillator with variable order

derivatives. The study introduces variable-order fractional time derivatives into the state space model and investigates their influence upon the system dynamics.

In the paper [30] an impulsive time-varying model for the dynamics of price adjustment in a single commodity market using the Caputo fractional-order derivative is developed by J.J. Nieto, G. Tr. Stamov and I. Stamova. The authors investigate the existence and stability of an almost periodic solution of the model.

In [32] the authors P. Veeraian, U. Gandhi and U. Mangalanathan developed fractional order models of inductive transducers and their characteristics are analyzed. The impact of fractional order parameter on the sensitivity and nonlinearity of the fractional order transducer is also studied.

The editors of the special issue have the pleasure to express their sincere gratitude to all authors for their important contributions, and also to the reviewers for their outstanding efforts in reviewing of the articles. We believe that the selected papers will enrich the readers' knowledge and will stimulate the continuing efforts to develop the theory and applications of fractional dynamical systems.

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