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Editorial

Special issue on nonlinear phenomena in physics: new techniques and applications

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This special topics issue of the European Physical Journal contains a selection of articles that highlight recent developments in applied dynamical systems and complexity, both in the theory and in experimental studies. These articles cover topics of interest that arise in various research areas in Nonlinear Physics: fluid flow problems, conservative systems, statistical fluctuations in non-equilibrium models, networks of coupled oscillators, and numerical analysis of nonlinear oscillations. These topics are presented in individual articles by physicists, computational scientists, and applied mathematicians, with expertise in computational methods and experimental techniques, who have contributed extensively to this intrinsically multidisciplinary field. We are fortunate to have had the opportunity to select the contributions to this Special Issue from the invited presentations at the International Conference "Dynamics Days Latin America and the Caribbean", which was held in Puebla, Mexico, from October 24 to November 01, 2016. Our selection consists of sixteen regular papers and a mini-review, organized in four categories, namely,

1 Fluid Flow Instabilities

2 Nonlinear Systems with Conservation Laws

3 Numerical Analysis of Nonlinear Oscillations

4 Dynamics of Complex Systems

The first article in the category *Fluid Flow Instabilities*, by van Veen et al. [1], considers the incompressible Navier–Stokes equations using the Smagorinsky model. The authors present the first detailed analysis of the transition from laminar to weakly turbulent behavior in Large Eddy Simulation (LES) flows in a three-dimensional, periodic domain. The system is analyzed by computing unstable families of equilibria and periodic orbits, which show how the large-amplitude turbulence approaches the unstable invariant objects.

The second article, by Sánchez Umbría and Net [2], considers the dynamics of a fluid in a cavity with heating on the sides, as modeled by realistic boundary conditions. Using numerical continuation methods, the authors study the influence of the Prandtl number on the stable stationary and periodic flows of the fluid. Their approach enables the detection of long stable periodic oscillations that are isolated in small regions of the parameter space, which are difficult to capture by chance via time integration.

The next two articles [3,4] in the category *Fluid Flow Instabilities* are contributed by Inomoto, Müller, Kobayashi and Hauser, and focus on the dynamics of chemical reaction fronts. The first of these articles concerns reaction fronts in the iodate-arsenous acid reaction. It is observed that depending on the stoichiometry of the reactants, the speed of the front can be slow and constant, or it can accelerate significantly. To model the experimental results the authors propose a simple one-dimensional reaction-diffusion model, with an extra term that relates to the surface tension. In their companion article, the authors focus on an additional effect due to the adsorption of iodine from the gas phase. For this purpose they extend their one-dimensional chemo-hydrodynamic model to include the coupling to iodine on a gas layer.

The final article in the *Fluid Flow Instabilities* category is by Guzman et al. [5]. They study chemical reaction fronts inside a two-dimensional fluid layer, as described by the Kuramoto–Sivashinsky equation coupled to a surface tension driven flow. Different shapes of reaction fronts are found, depending on the magnitude and direction of the gradients in the surface tension, as represented by the Marangoni number. Large regions of bistability are observed, where the limit of strong Marangoni flow favors the formation of stable non-axisymmetric structures of relatively high speeds.

Current studies of nonlinear waves include the varied phenomenology of the interplay between spatial inhomogeneities and nonlinearity. These effects are especially relevant to optics and photonics, and to fluid mechanics. In the second category, *Nonlinear Systems with Conservation Laws*, the first article by Lutsky and Malomed [6] examines two-dimensional optical vector solitons in a medium with a quadratic nonlinearity that is modulated in space. The authors add a spatial modulation of the nonlinear coupling with a triangular symmetry. They show that this produces multi-soliton states having the same symmetry. They also identify stable and unstable multi-solitons, showing the possibility of controlling nonlinear localization. The second article by Calaça et al. [7] considers the modulation instability in a one-dimensional cubic nonlinear Schrödinger equation with spatial modulation. Their model contains a derivative nonlinearity that arises in fluid and plasma waves. In the constant coefficient case, the authors find that the instability generates a cascade of solitonic structures that are similar to the Peregrine solitons. The Peregrine solutions are homoclinic to the unstable plane wave, and are localized in space and time. The cascading structure persists under modulations of the parameters, even when the individual peaks undergo additional large oscillations of their center of mass.

Within the context of *Nonlinear Systems with Conservation Laws*, a related area of active research is that of nonlinear lattice dynamics, where discrete versions of problems in continuous systems are studied. Some difficult issues related to long-term dynamics, and their connection to statistical behavior, are explored in the next two articles. There is significant current interest in understanding how variants of the interaction between sites affect the dynamics. The article by Christodoulidi et al. [8] considers the Klein–Gordon and Flach–Gorbach lattices with an on-site quartic potential, and with linear and nonlinear long-range interactions. The main novelty is the combination these two effects. The authors also consider the effect of increasing the size of the lattice. On-site nonlinearity enhances localization, and the paper shows that such localization persists under the addition of long-range interactions. The authors consider general initial conditions, and they find evidence that long-range interactions also lead to a maximum Lyapunov exponent that decreases with the size of the lattice. This suggests an integrable-like behavior in the limit of infinite size.

The final article in this category, by Martinez-Farias and Panayotaros [9], studies the quartic Fermi–Pasta–Ulam model with a site-dependent number of interacting neighbors. This work is motivated by simplified models of protein vibrations. The inhomogeneity leads to localized linear modes, and the main question is whether this spatial localization persists in the nonlinear problem. Their work considers new examples where the variable connectivity is random, and the authors provide evidence that this randomness enhances the stability of nonlinear spatially localized motions.

In the category *Numerical Analysis of Nonlinear Oscillators*, the first article is by Słowiński et al. [10]. It presents a detailed analysis of the mechanism that leads to the appearance of relaxation oscillations in the Jirsa–Kelso excitator model, which is a simple model that describes the generation of rhythmic and discrete human movements. The authors demonstrate the existence of the canard phenomenon in this model. Using a blow-up method and projection onto the Poincaré sphere, the authors exhibit different mechanisms for global transitions in the Jirsa–Kelso excitator and in FitzHugh–Nagumo models. Their study demonstrates that the slow-fast nature of the transition between discrete and rhythmic movements, as observed in the Jirsa–Kelso excitator model, can be analysed and understood in an experimentally relevant phase space.

The second article in this category, by Farjami et al. [11], is devoted to the numerical analysis of the spike-adding transition in the Morris–Lecar neural membrane model. This is a 3D slow-fast ordinary differential equation, with two fast variables and one slow variable. The transition from 2-spike to 3-spike periodic bursting under increasing stiffness is studied in detail. This is accomplished by determining the location of a corresponding periodic orbit with respect to the 2D stable invariant manifold of the saddle slow manifold of the system. The authors use advanced computational methods for the determination of the manifolds, based on boundary value continuation of appropriate orbit segments.

The third article in the category *Numerical Analysis of Nonlinear Oscillators*, is by Calleja et al. [12]. It investigates the existence of choreographies along families of periodic solutions of the discrete nonlinear Schrödinger equations (DNLSE), configured in a periodic lattice. The periodic solutions bifurcate from a symmetric polygonal relative equilibrium. Using specialized numerical continuation techniques for conservative systems with symmetries, the authors can locate periodic orbits of interest, including an unlimited number of choreographies. A small, representative selection of such choreographies is presented, of which many are linearly stable.

The final article in this category, by Senyange and Skokos [13], presents a detailed numerical analysis of one and two-dimensional disordered Klein–Gordon lattices. The authors compare the performance of several symplectic integration schemes used for the equations of motion of Hamiltonian lattice models, and for the variational equations needed to find the maximum Lyapunov exponent. The implementation of symplectic integration schemes for fast and reliable results is useful in many fields of research. In particular, the numerical schemes used in their article are relevant to the numerical investigation of important questions that arise in the study of nonlinear lattices.

In the final category, *Dynamics of Complex Systems*, the article by Velarde and Robledo [14] reviews fundamental routes to chaos in low-dimensional nonlinear systems, as displayed by the classical logistic and circle maps. This topic developed independently from the perspective and concepts of nonlinear dynamics, and the authors consider analogies and links between these different fields. It includes sharp collective phenomena (glass formation, critical points and localization transitions) in condensed matter physics, as well as in networks, game theory, and rank distributions in complex systems with a large number of degrees of freedom. The authors argue that the connections between low-dimensional nonlinear dynamics and high dimensional many-body or agent systems may involve a drastic reduction of variables. This is evocative of the generality of the few-variable formalism of the thermodynamics of equilibrium states in ordinary statistical-mechanical systems.

The second article is by Soriano-Panos et al. [15]. It considers a generalization of the Ross-Macdonald model for the transmission of vector-borne diseases. Human-to-human contagion is taken into account, as in the case for ZIKV epidemics. This generalized model is formulated in terms of a mean field theory, which deals with the case where human contacts are described by a complex network. To validate the mean field model, the authors have compared its predictions to the results of numerical Monte Carlo simulations, where epidemic thresholds were found. The authors argue that their generalized model may pave the way to the formulation of more accurate ZIKV models.

The third article in this category is by Lameu et al. [16]. It studies the effects of the synaptic delay on the coupling strength and synchronization in a neuronal network with synaptic plasticity. Neuroplasticity, also known as brain plasticity, refers to the brain's ability to change neuronal connections as a result of environmental stimuli, new experiences, or damage. The authors show that alterations in the synchronization and connectivity in a plastic network depend on the synaptic delay. When time-delay is activated in the network, an interesting form of weak synchronization is observed. This contrasts with the synchronous spiking that is related to brain disorders. The authors consider a Hodgkin–Huxley model of 100 coupled heterogeneous neurons in an all-to-all network, in which the weights evolve according to specific plasticity dynamics. They observe that plasticity allows an evolving network that converges to a directed network pattern. The latter connects excitatory neurons with inhibitory neurons, with synaptic links that are strengthened and others that are weakened in the process.

The last article in this category, by Abe [17], develops a Hamiltonian formulation of the fractional Fokker–Planck equation, and recasts this equation into a Liouville form. This nontrivial problem for the fractional kinetic equation, which arises from the temporal nonlocality and nonuniqueness of the Hamiltonian, is resolved in this contribution. As a result, phenomena such as non-Gaussian normal diffusion in super-cooled liquids may be dealt from a different perspective.

In summary, this special issue highlights the wide spectrum of current research in Nonlinear Dynamics and Complex Systems, including both theory and experiments, and offers state of the art computational techniques and experimental methods. We hope that the articles will be valuable to physicists, engineers, and applied mathematicians who are interested in nonlinear phenomena and complex systems. We thank all authors in this special issue for their outstanding contributions. We also wish to express our gratitude to the referees of the articles, for their often thorough and thoughtful reviews.

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