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Review

Accelerating Universe viewed from the Scalar-Tensor theory

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Abstract. We briefly review how successfully the Scalar-Tensor-Theory applies to the Accerelating Universe, particularly through the unique way in resolving the Fine-Tuning Problem. An emphasis is placed also on the role of the scalar field as a (pseudo) Nambu-Goldstone boson (dilaton) for the mass generation of the Higgs field.

1 Introduction

Toward the end of the last millenium, our cosmology entered the new era featuring the Accelerating Universe, likely described in terms of the re-newed concept of the Cosmological Constant (CC), with its observational value estimated to be [1–3]

$$\Lambda_{\rm obs} \sim 10^{-120}, \tag{1}$$

where we have used the Planckian unit system defined by

$$c = \hbar = M_{\rm P}(= (8\pi G)^{-1/2}) = 1, \tag{2}$$

with the Newton's gravitational constant G. The units of length, time and energy are given by

$$8.10 \times 10^{-33}{\rm cm}, \quad 2.70 \times 10^{-43}{\rm sec}, \quad 2.44 \times 10^{18}{\rm GeV}, \tag{3}$$

respectively, in the conventional units.

As another observational result, we find [1–3]

$$\Omega_\Lambda \equiv \frac{\Lambda_{\rm obs}}{\rho + \Lambda_{\rm obs}} = 0.72 \pm 0.06, \tag{4}$$

where ρ is the matter density.

Comparing (1) with $\Lambda_{\rm th} \sim 1$ expected from almost any of the theoretical models of Unification, we find a surprise

$$\Lambda_{\rm obs} \ll \Lambda_{\rm th}, \tag{5}$$

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one of the worst records of disagreement between theory and observation, thus facing the well-known Fine-Tuning Problem.

According to the second of (3), however, we find today's age t_0 of the universe;

$$t_0 = 1.37 \times 10^{10}\text{y} \approx 10^{60.2}, \tag{6}$$

expressed in Planckian units, hence [4,5]

$$\Lambda_{\text{obs}} \sim 10^{-120} \sim t_0^{-2}. \tag{7}$$

Suppose this agreement is hardly a mere coincidence. We are then *free from* the Fine-Tuning Problem; Today's CC is this small only because we happen to be old cosmologically. We explore here the Scalar-Tensor Theory (STT) in which (7) derives nearly automatically. In the current article, we attempt a short and simplified summary of the references [4–6], to which readers are advised to refer for more details.

2 Scalar-Tensor Theory

The history of STT might be traced back to the Large-Numbers Hypothesis (LNH) due to Dirac in 1937 [7]. He raised the question why gravitation between two electrons, for example, is so weak compared with the electromagnetic coupling, ultimately why $Gm_{\text{e}}^2 \ll \alpha = e^2/(4\pi)$. As one of the simplest ways to understand this huge discrepancy, he assumed G falling off like $G \sim t^{-1}$ with t the cosmic time; with sufficiently small t, G might be large in the early era of the Universe. Unfortunately, LNH appears to have been hampered by the inability to reach an observational success. He nevertheless left a tremendous impact by pioneering an approach in which a constant in the conventional sense might vary as the Universe evolves.

He also suffered from a theoretical issue; Einstein's General Relativity never allowed a time-dependent G. But some years later, Jordan came to the rescue with his STT [8]. By respecting Einstein's spacetime geometry, he proposed how the Einstein-Hilbert term $\mathcal{L}_{\text{EH}}$ is to be replaced by his *nonminimal coupling* together with the kinetic-energy term of the scalar field ϕ;

$$\mathcal{L}_{\text{EH}} = \sqrt{-g}\frac{1}{16\pi G}R \Longrightarrow \bar{\mathcal{L}}_{\text{EH}} = \sqrt{-g}\left(\frac{1}{2}\xi\phi^2 R - \epsilon\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right), \tag{8}$$

with R the scalar curvature and $\epsilon = \pm 1$. We define the *effective gravitational constant* as:

$$8\pi G_{\text{eff}} = \left(\xi\phi^2(x)\right)^{-1}, \tag{9}$$

naturally allowing a variable G_{eff}, where ξ is a constant.

However, this non-minimal coupling term can be eliminated by a local transformation

$$g_{\mu\nu} \to g_{*\mu\nu} = \Omega^2(x)g_{\mu\nu}, \tag{10}$$

with an arbitrary spacetime function $\Omega(x)$. Often called a *conformal transformation*, this is a natural extension of a global *scale transformation* $x^\mu \to \lambda x^\mu$, with a constant λ to be useful only in flat spacetime. By substituting the inverse of (10), the same Lagrangian $\bar{\mathcal{L}}_{\text{EH}}$ on the RHS of (8) can be re-expressed as a function of $g_{*\mu\nu}$. Also with a particular choice

$$\Omega = \xi^{1/2}\phi, \tag{11}$$

we reach

$$\bar{\mathcal{L}}_{\text{EH}} = \sqrt{-g_*}\left(\frac{1}{2}R_* - \frac{1}{2}g_*^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right), \tag{12}$$

where the new canonical scalar field σ is defined by

$$\phi = \xi^{-1} e^{\zeta\sigma}, \quad \text{with} \quad \zeta^{-2} = 6 + \epsilon\xi^{-1}. \tag{13}$$

Notice the constant multiplier before R_*, the same scalar curvature as a function of $g_{*\mu\nu}$, implies a constant, $G_* = 1$.

The above procedure will be summarized; we move from the Jordan conformal frame (JCF), with the Lagrangian (8), also with a variable G, to the Einstein conformal frame (ECF), with (12) with a constant gravitational constant G_*. As an important lesson, we learn; whether a quantity is constant or variable, depends on what CF we live in. The whole argument does not require the invariance of the theory under the conformal transformation. This reminds us how important the concept of the inertial frame should be in Newtonian mechanics, for example representing different physics on a rotating frame or in a free-fall frame. Also to be pointed out, the concept of the conformal frame is unique in STT, to be distinguished from GR.

As another unique issue, Brans and Dicke [9] raised the question of whether to include ϕ in the matter Lagrangain, $\mathcal{L}_{\rm matter}$, to be added to $\bar{\mathcal{L}}_{\rm EH}$. They advocated the constraint in which ϕ is decoupled from $\mathcal{L}_{\rm matter}$, because then ϕ couples to the whole system in JCF only through the nonminimal coupling term in $\bar{\mathcal{L}}_{\rm EH}$, thus allowing no direct ϕ-matter coupling. As a consequence, the matter coupling inherits the same of GR, including Weak Equivalence Principle, WEP, the coupling strength determined uniquely by the total mass of the matter system independently of its content, often called composition-independence.

This constraint was probably invented because Dicke was at that time trying to probe WEP experimentally following Eötvös but with higher precision. Although in the microscopic approach we explore later, the BD constraint will be shown not likely to be obeyed.

In order to obtain the realistic behavior as suggested by the recent discovery of an Accelerating Universe, we add another Lagrangian to (8) in the JCF:

$$\mathcal{L}_\Lambda = -\sqrt{-g}\Lambda = -\sqrt{-g_*}V(\sigma) = \sqrt{-g_*}\Lambda e^{-4\zeta\sigma}, \tag{14}$$

where Λ is chosen to be a constant, while in the last two expressions for ECF, the field σ occurs with a slowly falling potential $V(\sigma)$. This can be an advantage over what is called the Quintessence approach which shares the same type of potential but without any theoretical basis [10,11].

3 Simplified cosmology

After these preparations, we now apply STT to the JCF Universe, assumed to be uniform and isotropic, to be described by the Robertson-Walker geometry, and to be radiation-dominated under the BD constraint.

Skipping all the details, we find an asymtotic and attractor solution of the cosmological equations, but with

$$H = \dot{a}/a = 0, \tag{15}$$

implying a static universe, obviously against the expanding Universe as we see it, and that we are not in a JCF. As a next step, we then try an ECF for which the cosmological equations are

$$3H_*^2 = \rho_\sigma + \rho_*, \tag{16}$$

$$\ddot{\sigma} + 3H_*\dot{\sigma} + V'(\sigma) = 0, \tag{17}$$

$$\dot{\rho}_* + 4H_*\rho_* = 0, \tag{18}$$

where the subscript $*$ is used for the ECF, while the dot is for differentiation with respect to t_*, the cosmic time in the ECF. Also

$$\rho_* = e^{-4\zeta\sigma}\rho_1, \quad \rho_1 = \text{const}, \tag{19}$$

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + V(\sigma) = \Lambda_{\text{eff}}, \tag{20}$$

where ρ_σ defined by (20) might be interpreted as an effective CC, also to be called Dark Energy.

We then obtain the asymptotic and attractor solutions;

$$a_* = \Omega a \sim \Omega \sim t_*^{1/2}, \tag{21}$$

$$\sigma = \bar{\sigma} + \frac{1}{2}\zeta^{-1}\ln t_*, \quad \Lambda e^{-4\zeta\bar{\sigma}} = \frac{1}{16}\zeta^{-2}, \tag{22}$$

$$\rho_\sigma = \frac{3}{16}t_*^{-2}, \quad \rho_* = \frac{3}{4}\left(1 - \frac{1}{4}\zeta^{-4}\right)t_*^{-2}, \tag{23}$$

According to (21), the Universe now expands, allowing us to qualify the ECF to be a physical CF, at least in the current aspect. Moreover, the first of (23) does show the behavior,

$$\Lambda_{\text{eff}} \sim t_*^{-2}, \tag{24}$$

which might be called a *decaying cosmological constant*, our goal, as was suggested following (7) before. One might raise, however, an immediate objection; the above statement suffers from a semantic issue; a "constant" is supposed not to vary! To reply, we must go beyond an overall behavior. A numerical solution is shown in Fig. 1, taken from Fig. 5.8 of [4] and Fig. 9 of [5].

In the bottom diagram, we plot $\log\rho_*$ and $\log\rho_{\sigma\chi}$ as the dashed and the solid curves, respectively against $\log t_*$ so that the present era is about 60.2. Notice that $\rho_{\sigma\chi}$ is similar to but slightly different from the foregoing ρ_σ for the reason to be explained shortly. Due to the non-linear nature of the RHS of (20), we find *sporadic* occurrence of the leveling-off behavior, or *hesitation behavior*, of ρ_σ playing the same role as the CC, at least during relatively short periods of t_*, hence causing *mini-inflations*, represented by sharp rises of $\ln a_*$ in the upper diagram drawn by a dotted curve, a desired behavior corresponding to the Accelerating Universe. Needless to say, we have chosen one of such behaviors to occur around today.

In spite of this convenient feature, our $\rho_\sigma = \Lambda_{\text{eff}}$ shows a symptom to bend itself downward immediately before it approaches ρ_* that comes down from the above, hence leaving Ω_Λ defined by (4) short of reaching the value 0.5. In order to have the observed value $\Omega_\Lambda > 0.5$, we are forced to accept what we call the *Trapping Mechanism* to be described in Sect. 5.4 [4] and Sect. 6.1 [5], implemented phenomenologically by means of yet another scalar field χ. In this way, our two *continuous* curves, ρ_* and $\rho_{\sigma\chi}$, fall off with an approximately common behavior $\sim t_*^{-2}$, interlacing each other.

Suppose the leveling-off behaviors occur at certain discrete values t_{*i} with the heights Λ_i for $i = 1, 2, \cdots$, obeying the rule:

$$\Lambda_i \sim t_{*i}^{-2}, \quad i = 1, 2, \cdots. \tag{25}$$

This is precisely what we mean by a decaying CC.

One might still criticize an artificial nature of the above mechanism. We nevertheless emphasize that this is an additional rule supplementing a more fundamental and natural law formulated by the asymptotic and overall solutions obtained before.

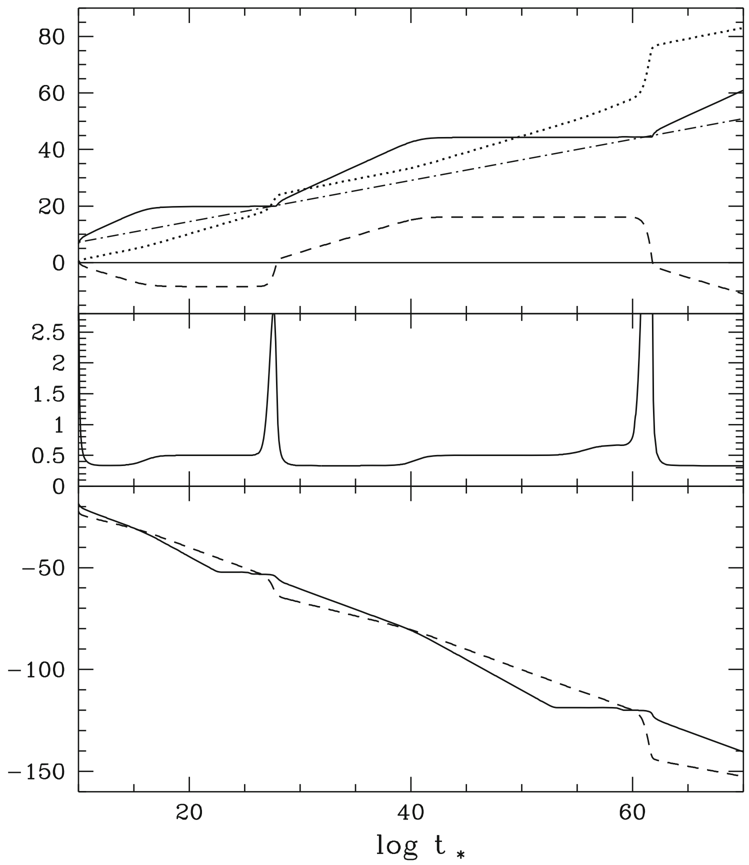

Fig. 1. Upper diagram: $\ln a_*$ (dotted); σ (solid); and 2χ (dashed) another scalar field in Trapping Mechanism. Middle diagram: p_* with $a_* \sim t_*^{p_*}$. Lower diagram: $\log \rho_{\sigma\chi}$ (solid); $\log \rho_*$ (dashed).

4 Masses of fundamental particles

We are finally moving to discuss the origin of masses of the fundamental particles. This is not only the issue of how the microscopic physics meets gravitation in the very early Universe, but also of what meter-stick is used by today's astronomers who measure the size of the Universe. In connection with the second issue above, we recall that an astronomers's routine begins with spectral analysis of the incoming light from distant objects, to be compared with the earth-bound spectrometers. In this way they measure the red-shifts of the given atomic transitions, and thus estimate how fast the objects recede from us. Obviously, they measure the cosmological distances in reference to the microscopic *units*, or basically Rydberg constants. At the same time, we have no way to detect possible changes of the units themselves, unless other units are employed. We call this the Own-Unit-Insensitivity-Principle (OUIP). The question is how our approach can be made consistent with this practice in modern cosmology.

As before we start with JCF in which the matter Lagrangian of the matter fields, illustrated by a free neutral massive scalar field Φ, looks like

$$\mathcal{L}_{\rm matter} = -\frac{1}{2}\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + m_\natural^2\Phi^2\right). \qquad (26)$$

With the BD constraint; the absence of the gravitational scalar field ϕ implies a truly *constant* mass $m_\natural$.

We then move to the ECF, the physical frame. On the basis of the standard procedure [4,5], we reach basically the same form as (26), but with the $*$ everywhere, we obtain the mass $m_* = \Omega^{-1} m_\natural \sim t_*^{-1/2}$, obviously in contradiction with OUIP emphasized above.

A remedy is then offered by *the scale-invariant model*, with the last term, the mass term, of (26) replaced by the coupling term

$$\mathcal{L}_{\phi\Phi} = -\frac{1}{2}\sqrt{-g} f^2 \phi^2 \Phi^2, \tag{27}$$

which fails to respect the BD constraint by violating WEP, but featuring an invariance under global scale transformation, sometimes called *dilatation*, at least in 4-dimensions, because the coupling constant f is then dimensionless. Again moving to the ECF, we go through some of the complications as elaborated in [4,5], finding the ECF mass, $m_\sharp$, computed as

$$m_\sharp = f\xi^{-1/2} \sim f, \tag{28}$$

which is *constant* as expected. Also for a reasonable choice $\xi \sim 1$, we expect $f \sim 10^{-16}$ if Φ_* is identified with the Higgs field of the mass $\sim(126 \sim 10^2)$ GeV, for example.[1]

The mass generation here is a result of the scale-invariance in (27) broken *spontaneously* with the help of σ playing the role of a Nambu-Goeldtone boson [12,13], called a *dilaton*, as understood precisely in terms of the Noether current, as elaborated in Sect. 6.2 in [4] and Sect. 5.2 of [5]. This also *derives* the masslessness of the scalar component of STT for the first time.

However, Nambu himself discussed a pseudo-NG boson [12], which acquires a nonzero mass because of no immunity against a self-mass, like the massive pion arising from spontaneous breaking of the axial-vector current. As we find, our realistic pseudo-dilaton is likely massive according to the quantum loop effects, typically of the order of $m_\sigma^2 \sim (m_{\mathrm{q},1} M_{\mathrm{ssb}})^2 \sim (10^{-9}\mathrm{eV})^2$, or the force-range $\sim$ 100m, representing what we once called non-Newtonian gravity [14,15].

In summation, we achieve an understanding of two major issues: mass generation of particles in the utmost microscopic world and how we observe the cosmological evolution. The two analyses have been woven together into a single formulation, enhancing the idea of Unification from a renewed and more fundamental point of view.

References

1. A.G. Riess, et al., Astron. J. **116**, 1009 (1998)
2. S. Perlmutter, et al., Astrophys. J. **517**, 565 (1999)
3. D.N. Spergel, et al., Astrophys. J. Suppl. **148**, 175 (2003)
4. Y. Fujii, K. Maeda, *The scalar-tensor theory of gravitation* (Cambridge University Press, 2003)
5. Y. Fujii, *Entropy*, **14** (2012) 1997: `www.mdpi.com/journal/entropy`
6. Y. Fujii, K. Homma, Prog. Theor. Phys. **126**, 531 (2011)
7. P.A.M. Dirac, Proc. Roy. Soc. **A165**, 199 (1938)
8. P. Jordan, *Schwerkraft und Weltall* (Friedrich Vieweg und Sohn, Braunschweig, 1955)

[1] Using a more realistic double-well potential leaves the basic points un-affected, as shown in Sect. 6.2 of [4] and Sect. 5.2 of [5].

9. C. Brans, R.H. Dicke. Phys. Rev. **124**, 925 (1961)
10. R.R. Caldwell, R. Dave, P.J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998)
11. L. Wang, R.R. Caldwell, J.P. Ostriker, P.J. Steinhardt, Astroph. J. **530**, 17 (2000)
12. Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960)
13. J. Goldstone, Nuovo Cim. **19**, 154 (1961)
14. Y. Fujii, Nature Phys. Sci. **234**, 5 (1971)
15. E. Fischbach, C. Talmadge. *The search for non-Newtonian gravity* (AIP Press, Springer-Verlag, New York, 1998)