Review

Creation of electron-positron plasma with superstrong laser field

N.B. Narozhny and A.M. Fedotov

National Research Nuclear University MEPHI, 115409 Moscow, Russia

Abstract. We present a short review of recent progress in studying QED effects within the interaction of ultra-relativistic laser pulses with vacuum and e^-e^+ plasma. Current development in laser technologies promises very rapid growth of laser intensities in the near future. Two exawatt class facilities (ELI and XCELS, Russia) in Europe are already in the planning stage. Realization of these projects will make available a laser intensity of $\sim 10^{26} \, \mathrm{W/cm}^2$ or even higher. Therefore, discussion of nonlinear optical effects in vacuum are becoming compelling for experimentalists and are currently gaining much attention. We show that, in spite of the fact that the expected field strength is still essentially less than $E_S = m^2 c^3 / e\hbar = 1.32 \cdot 10^{16} \, \mathrm{V/cm}$, the nonlinear vacuum effects will be accessible for observation at the ELI and XCELS facilities. The most promissory effect for observation is pair creation by a laser pulse in vacuum. It is shown, that at intensities $\sim 5 \cdot 10^{25} \, \mathrm{W/cm}^2$, creation even of a single pair is accompanied by the development of an avalanche QED cascade. There exists a distinctive feature of the laser-induced cascades, as compared with the air showers arising due primarily to cosmic rays entering the atmosphere. In our case the laser field plays not only the role of a target (similar to a nucleus in the case of air showers) but is also responsible for the acceleration of slow particles. It is shown that the effect of pair creation imposes a natural limit for the attainable laser intensity and, apparently, the field strength $E \sim E_S$ is not accessible for a pair-creating electromagnetic field at all.

1 Introduction

The invention of the CPA (Chirped Pulse Amplification) technique [1] in the second half of the 80 s and its subsequent development has resulted in the creation of petawatt laser facilities generating short pulses of coherent optical radiation with peak intensities up to $10^{22} \, \mathrm{W/cm}^2$ [2]. Currently, two European projects, ELI (Extreme Light Infrastructure [3]) and XCELS (Exawatt Center for Extreme Light Studies [4]), both aiming at the generation of femtosecond laser pulses with intensities of more than $10^{24} \, \mathrm{W/cm}^2$, and in the long term – up to $10^{26} \, \mathrm{W/cm}^2$, are in the planning stage. This will open novel possibilities for the investigation of nonlinear effects in the interaction of electromagnetic radiation with matter (for details, see the reviews [5,6]).

In 2011, a special network called IZEST (International Center for Zetta-Exawatt Science and Technology) was established [7] in order to stimulate the exchange of ideas and coordinate the worldwide activities of different groups involved in the projects related to the field of fundamental and high-energy physics with high intensity lasers.

One of the most challenging research trends is the experimental investigation of non-linear QED effects in the presence of very strong electromagnetic fields. Dozens of theoretical works on Intense Field QED (IFQED) have been published starting from the early 1960s. The probabilities of basic IFQED processes, such as emission and absorption of a photon by an electron, pair photoproduction and annihilation of a pair into a single photon, photon splitting in the field of a plane monochromatic electromagnetic wave and in a constant field, etc., have been calculated. It was predicted theoretically that QED vacuum polarized by a strong field acquires characteristics of a non-linear optical media, thus giving rise to a number of new optical effects, e.g., birefringence and dichroism of vacuum [8–11], photon splitting [11–13], Cherenkov radiation in vacuum [14–16], mutual and self-focusing in vacuum [17–19], light-by-light scattering [20,21], harmonic generation in vacuum [20,22,23], etc. A comprehensive overview of the work performed prior to 1985, as well as a detailed list of references can be found in [24–27].

So far, we have only the one experimental test of IFQED, namely the famous E144 experiment performed at SLAC in 1996-97. The scattering of ultra-relativistic electrons with the energy of 46.6 Gev by laser pulses of the intensity 10^{18} W/cm^2 (nonlinear Compton effect) [28] and electron-positron pair production by backscattered hard photons have been observed [29]. The results of the experiments proved to be in quantitative agreement with the theory, see [30] for the details. As it was mentioned above, the intensities of modern laser facilities have increased already by four orders of magnitude, and are planned to be increased further by 2–4 orders more in the near future. This opens unique opportunities for the experimental investigation of the already observed IFQED effects at a new intensity level, as well as for completely new, hitherto unexplored experimentally, nonlinear vacuum effects.

In this paper, we focus on a brief review of some recently published fundamental theoretical results related to the IFQED studies with the forthcoming laser facilities.

2 Pair creation by laser field in vacuum

Perhaps, the most interesting among the non-linear IFQED effects is the creation of electron-positron pairs by a strong classical electromagnetic field in vacuum. Field strength for this process is scaled by the so-called critical QED electric field strength $E_{cr} = m^2 c^3 / e\hbar = 1.32 \times 10^{16}$V/cm first introduced by F. Sauter [31,32], see [33]. An electron gains the energy equal to mc^2 across its Compton length $l_C = \hbar/mc = 3.86 \cdot 10^{-11}$cm under the action of such a field. For a long time it was mistakenly believed that the probability of the pair creation effect was solely determined by the Schwinger exponent $\exp(-\pi E_{cr}/E)$, and thus was exponentialy small at $E < E_{cr}$. However, the Schwinger exponent nominally determines the probability of pair creation only in the Compton 4-volume $V_C V_T = l_C^4/c$. If the peak electric field strength E_0 is small compared with E_S, $E_0 \ll E_{cr}$, the probability of pair creation in a Compton 4-volume remains exponentially suppressed. However, if the field strength E_0 is created in 4-volume VT which is much greater than the Compton volume, then the probability of pair creation acquires a large pre-exponential factor with the magnitude of the order of the ratio $VT/V_C T_C$. For an optical laser pulse with a wavelength $\lambda = 1\,\mu$m and duration of 10 fs, focused to the diffraction limit, this ratio is of the order of 10^{25}. This factor is so large that it can compensate for the small Schwinger exponent at some $E_0 \ll E_S$. This sort of enhancement singles out

the pair creation process among other vacuum polarization effects, thus nominating it as the most probable to be an easily observable candidate among nonlinear vacuum IFQED processes.

Our method for calculating the probability of pair creation by the laser field [34–36] is based on the fact that the typical formation length l_f and time T_f for this process with fields close to critical are of the order of the Compton length and time respectively [37]. Since for laser radiation in the optical range the relation $l_f \sim l_C \ll \lambda$ holds, such a field can be locally considered as constant and homogeneous with respect to the process of pair creation. This means that the mean number of produced pairs can be calculated with the following formula [34–36]

$$N_{e^+e^-} = \frac{e^2 E_S^2}{4\pi^2} \int dV \int_{-\infty}^{\infty} dt \ \epsilon\eta \coth\left(\frac{\pi\eta}{\epsilon}\right) \exp\left(-\frac{\pi}{\epsilon}\right), \tag{1}$$

where integration is performed over the whole volume occupied by the field and its duration, the invariant quantities $\epsilon = \mathcal{E}/E_S$, $\eta = \mathcal{H}/E_S$ are defined by

$$\mathcal{E}, \mathcal{H} = \sqrt{(\mathcal{F}^2 + \mathcal{G}^2)^{1/2} \pm \mathcal{F}}, \ \mathcal{F} = \frac{\boldsymbol{E}^2 - \boldsymbol{H}^2}{2}, \ \mathcal{G} = \boldsymbol{E} \cdot \boldsymbol{H},$$

and have the meaning of the electric and magnetic field strengths respectively in a reference frame in which they are parallel. To compute the invariants ϵ and η, we use the analytical model of a laser pulse based on an exact solution of Maxwell's equations [38].

It turns out [34–36], that the feasibility for the observation of the pair creation effect in the case of collision of only two counter-propagating focused laser pulses requires an intensity $\sim 10^{27}$ W/cm^2 which is two orders of magnitude smaller than the critical one. However, it still exceeds the capabilities of the forthcoming facilities, ELI and XCELS. Therefore it was important to find the ways to reduce this threshold intensity, I_{th}, for the creation of at least a single pair. To solve this problem a multi-beam technology was suggested in [39]. It was shown that for n laser pulses with a fixed total power focused coherently at one point, I_{th} would decrease with increasing n.

Use of a multi-beam setup allows for the redistribution of the electromagnetic field in the focal region. In the course of the interference of the colliding pulses the resulting field develops a spiked spatio-temporal structure. Thus, the total 4-volume in the focal region occupied by the field is reduced, but the peak value of the field strength increases. The number of created pairs depends on the peak strength of the electric field exponentially, while on the effective focal 4-volume occupied by the field as a power and these dependencies account for the overall decrease in the threshold intensity. For example, two identical linearly polarized laser pulses can always be superposed in such a way that in the resulting antinodes of the standing wave the electric fields of the two pulses are additive but the magnetic fields compensate each other. Obviously, the most preferable multi-beam configuration is such that the central axis of the colliding pulses lie in the same plane, and the pulses are linearly polarized in the direction orthogonal to the plane of their propagation. Assuming the geometry of the experiment is organized so that all the pulses propagate in one plane and are grouped into head-on colliding pairs, the magnetic fields in the common focal region will cancel each other and the electric fields will stack. In this configuration, the peak intensity of the electric field will be proportional to $\sqrt{n_{p_1}}$, where n_{p_1} is the total number of pulses but, of course, n_{p_1} is limited by the aperture of the colliding pulses. In order to decrease I_{th} further, albeit with less efficiency, it is possible to add n_{p_2} pulses, also grouped into counterpropagating couples with propagation axes constituting an angle θ with the plane in which the initial n_{p_1} pulses are located. In this case, the

Fig. 1. The concept of multi-beam technology: planer setup with a) $n = 2$, b) $n = 4$, c) $n = 8$ colliding pulses and 3D setup d) with $n = 24$ colliding pulses (as proposed in [39]).

Table 1. The number $N_{e^+e^-}$ of electron-positron pairs created by different numbers of pulses (each with numerical aperture $\Delta = 0.3$, and the total energy $10 \, \text{kJ}$) and the threshold value for the total energy W_{th}.

n	$N_{e^+e^-}$ at $W = 10 \, \text{kJ}$	W_{th}, kJ
2	9.0×10^{-19}	40
4	3.0×10^{-9}	20
8	1.0	10
24	4.2×10^{6}	5.1

resultant intensity in the focus will be proportional to $(n_{p_1} + n_{p_2} \cos \theta)/\sqrt{n_p}$, where $n_p = n_{p_1} + n_{p_2}$, [39].

Consider a setup in which the number of colliding focused pulses reaches the value $n = 24$, see Fig. 1. The first eight are focused in the plane (yz) to form four pairs of mutually counterpropagating pulses along/opposite axes y and z, and two bisecting lines of the coordinate quadrants of the plane yz. Additional groups of eight counterpropagating pulse pairs can be added along the lines constituting $45°$ with the plane (yz). The resulting collision geometry corresponding to 24 pulses grouped into three belts is shown in Fig. 1. Note that this setup allows one to approach very closely to the theoretical focusing limit discussed in Refs. [40–42].

Table 2 shows the results of numerical calculations for the number of pairs produced by various numbers n of colliding pulses from Ref. [39]. It is assumed that all the pulses have the same duration of $10 \, \text{fs}$, wavelength $\lambda = 1 \mu \text{m}$ and numerical aperture $\Delta = 0.3$ (the latter corresponds to the focal spot size of the order λ). The total energy of the pulses was taken to be $10 \, \text{kJ}$ independent of n. These results show that the use of multi-beam technology allows for a significant decrease in the threshold energy for pair creation as compared with the cases of a single pulse or two colliding focused pulses. In the arrangement with 24 pulses the threshold energy is almost twice lower. The effect is hoped to be observed experimentally at the main pillar of the ELI facility if the setup with 8 colliding laser pulses of the total energy $10 \, \text{kJ}$ is chosen.

3 QED cascades

Ultra-high-intensity laser pulses, which are planned to be obtained with the forth-coming ELI and XCELS projects, open the possibility to reproduce the SLAC-like experiments at a new level [43–45]. Owing to much higher laser intensity, collisions of high-energy electrons with laser pulses will be accompanied by the development of long chains (cascades) of secondary processes, instead of single events as was the case for the SLAC experiment. These chains (Fig. 2) will be formed by sequential events of hard photon emissions by electrons (Fig. 3a) and creations of electron-positron pairs by these photons (Fig. 3b), which may last until the charged particles totally
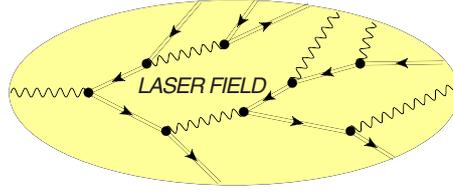
Fig. 2. A conceptual view of QED cascade in a laser field (as an example, seeded by a hard photon arriving from the left).
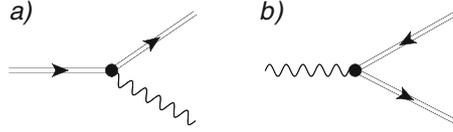

Fig. 3. First-order QED processes in an external field: a) photon emission; b) pair photoproduction. Double solid lines directed rightwards/leftwards are electrons/positrons, dressed by the field; wavy lines are the hard photons.

lose their energy. In order to design experiments under such conditions it is highly desirable to simulate the laser-matter interaction processes in this novel regime.

Note that the cascade theory is well developed, especially in the context of extensive air showers (EAS) which are generated in a medium (i.e. planet atmosphere) by cosmic rays [46]. Cascades in an external magnetic field are very similar and have been well studied already [49,50]. For the case of a laser field, the cascade development is governed [51] by the so-called cascade Eqs. (2a), (2b), where $f_\pm(\boldsymbol{r}, \boldsymbol{p}_e, t)$ and $f_\gamma(\boldsymbol{r}, \boldsymbol{p}_\gamma, t)$ are the distributions of positrons, electrons and photons respectively in the phase space, $\boldsymbol{v}_e = \boldsymbol{p}_e/\varepsilon_e$ is the velocity and $\varepsilon_e = \sqrt{p_e^2 + m^2}$ is the energy of an electron or positron. $dW_{rad}(\boldsymbol{p}_e \to \boldsymbol{p}_\gamma) = w_{rad}(\boldsymbol{p}_e \to \boldsymbol{p}_\gamma) d^3 p_\gamma$ and $dW_{cr}(\boldsymbol{p}_\gamma \to \boldsymbol{p}_e) = w_{cr}(\boldsymbol{p}_\gamma \to \boldsymbol{p}_e) d^3 p_e$ are the differential probability rates for photon emission (Fig. 3a) and pair photoproduction (Fig. 3b) in the external electromagnetic field, $W_{rad}(\boldsymbol{p}_e)$ and $W_{cr}(\boldsymbol{p}_\gamma)$ are the respective total probability rates for these processes.

$$
\frac{\partial f_\pm(\boldsymbol{r}, \boldsymbol{p}_e, t)}{\partial t} \pm e\left[\boldsymbol{E}(\boldsymbol{r}, t) + \boldsymbol{v}_e \times \boldsymbol{H}(\boldsymbol{r}, t)\right] \cdot \frac{\partial f_\pm(\boldsymbol{r}, \boldsymbol{p}_e, t)}{\partial \boldsymbol{p}_e}
$$
$$
= \int w_{rad}(\boldsymbol{p}_e + \boldsymbol{p}_\gamma \to \boldsymbol{p}_\gamma) f_\pm(\boldsymbol{r}, \boldsymbol{p}_e + \boldsymbol{p}_\gamma, t) \, d^3 p_\gamma - W_{rad}(\boldsymbol{p}_e) f_\pm(\boldsymbol{r}, \boldsymbol{p}_e, t)
$$
$$
+ \int w_{cr}(\boldsymbol{p}_\gamma \to \boldsymbol{p}_e) f_\gamma(\boldsymbol{r}, \boldsymbol{p}_\gamma, t) \, d^3 p_\gamma, \tag{2a}
$$

$$
\frac{\partial f_\gamma(\boldsymbol{r}, \boldsymbol{p}_\gamma, t)}{\partial t} = \int w_{rad}(\boldsymbol{p}_e \to \boldsymbol{p}_\gamma)[f_+(\boldsymbol{r}, \boldsymbol{p}_e, t) + f_-(\boldsymbol{r}, \boldsymbol{p}_e, t)]
$$
$$
\times d^3 p_e - W_{cr}(\boldsymbol{p}_\gamma) f_\gamma(\boldsymbol{r}, \boldsymbol{p}_\gamma, t). \tag{2b}
$$

Note that the classical part of radiation reaction need not be included separately because it is already taken into account by the first two terms of Eq. (2a), see e.g. [51–53] for the details. For the case of laser field of optical frequency ($\lambda \gg l_C$) and ultra-relativistic particles, one can use these probability rates in the approximation of a locally constant crossed ($\mathcal{F}, \mathcal{G} \approx 0$) field, which are well known [24–27]. Then, they

are exclusively determined by the dynamical quantum parameters of the participating particles

$$\chi_{e,\gamma} = \frac{\gamma_{e,\gamma}}{E_S} \sqrt{(\boldsymbol{E} + \boldsymbol{v}_{e,\gamma} \times \boldsymbol{H})^2 - (\boldsymbol{v}_{e,\gamma} \cdot \boldsymbol{E})^2}, \tag{3}$$

where $\gamma = \varepsilon/m$ is the dimensionless energy (Lorentz factor for the charged particles), $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{H}(\boldsymbol{r},t)$ are the local values of the electric and magnetic fields. Note that χ is proportional to the product of γ and the component of the Lorentz force, orthogonal to the momentum of a charged particle. As for the total rates, they are determined by the value of χ of the incoming particle, e.g.

$$W_{e,\gamma} \sim \frac{\alpha m^2}{\varepsilon_{e,\gamma}} \chi_{e,\gamma}^{2/3}, \quad \chi_{e,\gamma} \gg 1. \tag{4}$$

In the classical limit $\chi_\gamma \lesssim 1$ pair photoproduction is exponentially suppressed.

Cascade pair production in a media or magnetic field requires high energy for the initial seed particle and lasts until the secondary particles totally lose their kinetic energy. The multiplicity of such a cascade is always limited. For instance, in a magnetic field it can be estimated simply as $N_{e^+e^-} \sim \chi_{in}$, where χ_{in} is the quantum dynamical parameter of the seed particle, compare [54]. Cascades in a laser field are very different. In the regions where $\mathcal{F} > 0$, the laser field is capable of acceleration of the charged particles [55–57], thus restoring their energy, or the value of the quantum dynamical parameter. As it was first noted in [55,56], the laser field can play a dual role. In addition to being a target for high energy particles, it is a continuously working particle accelerator. Therefore, the multiplicity of QED cascades can be limited by the duration of the laser pulse rather than by the value of χ_{in} under some conditions, and the number of created secondary particles can become very large [57]. In this regard, QED cascades resemble electron avalanches occurring due to impact ionization in a dielectric-filled trench used for electrical isolation of semiconductor devices [58].

Simple estimates allowing for some insight into the physical mechanism of the self-sustaining regime of cascade pair production were suggested in Ref. [57]. It is useful to ignore for a moment the effect of the magnetic field and consider a model of a planar uniformly rotating electric field $\boldsymbol{E}(t) = \{E_0 \cos \Omega t, E_0 \sin \Omega t\}$ (such a field is very similar to the field in the anti-nodes of a circularly polarized monochromatic standing wave). According to the equation of motion $\dot{\boldsymbol{p}}_e = e\boldsymbol{E}(t)$, the momentum of an electron initially at rest primarily grows linearly,

$$\varepsilon_e \sim p_e \sim eE_0 \delta t, \tag{5}$$

but retards with respect to the rotating direction of the field. So that, the angle between them also grows linearly with t, $\theta_e \sim \Omega \delta t$. Hence, the increasing transverse, with respect to the electron momentum, component of the electric field $E_\perp \sim E_0 \theta_e \sim E_0 \Omega \delta t$ arises. As a result the dynamical quantum parameter starts to grow

$$\chi_e \sim \gamma_e E_\perp / E_S \sim m\Omega (E_0/E_S)^2 \delta t^2, \tag{6}$$

and reaches the value ~ 1 for $\delta t \sim t_{acc}$, where $t_{acc} = (E_S/E_0)(m\Omega)^{-1/2}$ is called the acceleration time. On the other hand, substituting (5) and (6) into Eq. (4) one can find that the mean free time $t_{e,\gamma}$, defined by $W_{e,\gamma} t_{e,\gamma} \sim 1$, is given by $t_{e,\gamma} \sim (E_S/\alpha^3 E_0)^{1/4}(m\Omega)^{-1/2}$. Hence, if $E_0 \gtrsim E_* = \alpha E_S \approx (1/137)E_S$, $t_{acc} \lesssim t_{e,\gamma}$. This means that the basic QED events occur with $\chi_{e,\gamma} \gtrsim 1$, or even $\chi_{e,\gamma} \gg 1$. Therefore: (i) usage of Eq. (4) is indeed legal, at least qualitatively; (ii) pair photoproduction is not suppressed. The latter is obviously a necessary condition for the cascade development. Multiplicity of a cascade initiated by a single seed particle at the time t can now be

estimated as $N_{e_+e^-}(t) \sim \exp(t/t_{e,\gamma})$. If one assumes that all the particles, being ultra-relativistic, vacate the focal region with almost the speed of light, then in the worst case $t \sim \Omega^{-1}$ and $t/t_{e,\gamma} \sim \sqrt{m\alpha^2/\Omega}(E_0/E_*)^{1/4}$. For an optical laser $\sqrt{m\alpha^2/\Omega} \sim 1$, therefore it turns out that the value E_* is indeed the threshold strength for cascade development. It corresponds to an intensity $\sim 10^{25}\,\mathrm{W/cm}^2$. Note that according to (6), the effective expansion parameter of IFQED perturbation theory is always $\ll 1$ for all conceivable experimental setups at the moment since even for $\chi_{e,\gamma} \gg 1$, according to Ref. [59], it is $\sim \alpha[\chi_{e,\gamma}(t_{e,\gamma})]^{2/3} \sim E_0/E_S \ll 1$. Hence, the QED processes of higher orders do not compete with the basic processes of the first order.

Unlike the standard theory of cascades [46, 49, 50], the acceleration mechanism of self-sustaining cascades essentially requires $2D$ or $3D$ treatment, which makes numerical calculations much more complicated. By now, Eqs. (2a), (2b) were solved numerically by the Monte-Carlo method [51, 60–64] in combination with the particle-in-cell (PIC) scheme [65] for the cases when it was necessary to take into account plasma effects. These simulations seem to confirm the above described qualitative picture of cascade development. As was expected, the multiplicity primarily grows exponentially until the moment when the plasma density reaches a value $a_0 n_{cr}$, and hence the plasma becomes non-transparent and the rapid depletion of the laser field's trailing edges for colliding pulses occurs and the growth of the number of pairs ceases. It is worth noting that self-sustaining cascades cannot occur for some configurations of the external field. It happens for example in the nodes of the magnetic field of a standing wave [66] because electrons follow the direction of the field and are not deflected. A plane wave field serves as a second example because the dynamical quantum parameter χ is an integral of motion in such a field and thus a mechanism of acceleration does not exist. However, the cascades always arise for a focused laser pulse or for any combination of several such pulses. Moreover, the threshold intensity observed in simulations was essentially lower than $10^{25}\,\mathrm{W/cm}^2$ because of the large size of the focal region and presumably fluctuations in the parameters χ and mean free times $t_{e,\gamma}$ [51, 55, 56, 60, 62, 64]. One should bear in mind that for simplicity most of the authors have used the probability rates for non-polarized particles in their simulations, but spin effects may be of some importance and it was shown recently that not taking them into account introduces a 10% error in the final results [67].

It is interesting whether the impact of cascades is in general crucial for the dynamics of laser-target interaction at intensities that will be attained at the ELI and XCELS facilities. In Refs. [68–70] such interaction was simulated for moderate intensities $\sim 10^{23-24}\,\mathrm{W/cm}^2$ for the case of normal or inclined incidence of weakly focused laser pulses onto foil targets. Contrary to what was initially anticipated, the yield of the pairs, although observed, remained small, so that such interactions mainly result in the generation of hard photons in quantitative agreement with [71], where the pair photoproduction effect was ignored. The reason is simple. Namely, the electrons and positrons carried in the propagation direction of the laser field. Such a situation is known to be unfavourable for cascade development. However, the situation must certainly be different if the target is pressed by several pulses as in Fig. 1, or if the laser field is tightly focused. Note that for a realistic setup of the laser-target interaction (e.g., for solid target) pair photoproduction on ions [72] would seemingly exceed the one in the laser field, so that both channels must be taken into account in further simulations.

4 Discussion

IFQED processes at the next generation of high intensity laser facilities is one of the topics which is widely discussed in literature [43–45, 73, 74]. In our opinion, observation

of pair production from vacuum and self-sustaining QED cascades could be among the most important goals. The issue of principal importance is whether the intensity attainable with an optical laser is limited, as it was deduced in [57]. Let us remind that this limitation originates from the assertion that such high intensities seem only to be obtained with short tightly focused laser pulses. Such fields would create pairs in vacuum, which in turn would seed the self-sustaining QED cascades catalyzing the pair creation process and thus depleting the original laser pulse. Consequently, there would be no way to overcome some threshold intensity value, which was roughly estimated to be about $3 \cdot 10^{26}\,\mathrm{W/cm}^2$. Just such an intensity level was the original aim for ELI and XCELS.

To understand the physics of laser-matter interactions at the next intensity level deeper, several problems still need to be solved. Let us mention here just a few of them. First, there are some indications [75–77] that when radiation reaction dominates, the charged particles could be trapped in the focal region and stay there much longer than anticipated. Second, for now it is not evident what may happen at the final stage of the cascade. On the one hand, some instabilities associated with the presently employed numerical algorithms make simulations at extreme intensity levels unreliable. On the other hand, at high enough plasma densities, which are typical for the final stage of the cascade, relaxation processes can come into play that thermalize the plasma and preventing further depletion of the laser field. Finally, in order to make more definite predictions, specifically on the attainability of extreme intensities, pair creation from vacuum should be systematically and precisely incorporated into the general framework of the kinetic approach, based on Eqs. (2a), (2b). This task is still far from completion despite the many attempts to do so; see [78] for the most recent reconsideration of the problem, and references therein. These and other associated problems are now under active investigation.

References

1. D. Strickland, G. Mourou, Opt. Commun. **56**, 219 (1985)
2. V. Yanovsky, et al., Opt. Expr. **16**, 2109 (2008)
3. http://www.extreme-light-infrastructure.eu
4. http://www.xcels.iapras.ru
5. G.A. Mourou, T. Tajima, S.V. Bulanov, Rev. Mod. Phys. **78**, 309 (2006)
6. A. Di Piazza, C. Müller, K.Z. Hatsagortsyan, C.H. Keitel, Rev. Mod. Phys. **84**, 1177 (2012)
7. http://www.izest.polytechnique.edu
8. R. Baier, P. Breitenlohner, Acta Phys. Austr. **25**, 212 (1967)
9. N.B. Narozhnyi, Sov. Phys. JETP **28**, 371 (1969)
10. I.A. Batalin, A.E. Shabad, Sov. Phys. JETP **33**, 483 (1971)
11. S.L. Adler, Ann. Phys. (NY) **67**, 599 (1971)
12. Z. Bialynicka-Birula, I. Bialynicka-Birula, Phys. Rev D **2**, 2341 (1970)
13. V.O. Papanyan, V.I. Ritus, Sov. Phys. JETP **34**, 1195 (1972)
14. T. Erber, Rev. Mod. Phys. **38**, 626 (1966)
15. V.I. Ritus, Sov. Phys. JETP **30**, 1181 (1970)
16. I.M. Dremin, JETP Lett. **76** 151 (2002)
17. N.N. Rozanov, JETP **86**, 284 (1998)

18. M. Soljačić, M. Segev, Phys. Rev. A **62**, 043817 (2000)
19. D. Kharzeev, K. Tuchin, Phys. Rev. A **75**, 043807 (2007)
20. A. Di Piazza, K.Z. Hatsagortsyan, C.H. Keitel, Phys. Rev. D **72**, 085005 (2005)
21. B.E. Lundström, et al., Phys. Rev. Lett. **96**, 083602 (2006)
22. A.E. Kaplan, Y.J. Ding, Phys. Rev. A **62**, 043805 (2000)
23. A.M. Fedotov, N.B. Narozhny, Phys. Lett. A **362**, 1 (2007)
24. V.I. Ritus, J. Sov. Laser Res. **6**, 497 (1985)
25. A.I. Nikishov, J. Sov. Laser Res. **5**, 619 (1985)
26. A.I. Nikishov, V.I Ritus, Trudy FIAN **168**, (1986) [in Russian]
27. V.N. Baier, V.M. Katkov, V.M. Strakhovenko, *Electromagnetic processes at high energies in oriented single crystals* (World Scientific, Singapore, 1998)
28. C. Bula, et al., Phys. Rev. Lett. **76**, 3116 (1996)
29. D.L. Burke, et al., Phys. Rev. Lett. **79**, 1626 (1997)
30. C. Bamber, et al., Phys. Rev. D **60**, 092004 (1999)
31. F. Sauter, Z. Phys. **69**, 742 (1931)
32. F. Sauter, Z. Phys. **73**, 547 (1932)
33. J. Schwinger, Phys. Rev. **82**, 664 (1951)
34. N.B. Narozhny, S.S. Bulanov, V.D. Mur, V.S. Popov, Phys. Lett. A **330**, 1 (2004)
35. S.S. Bulanov, N.B. Narozhny, V.D. Mur, V.S. Popov, JETP **102**, 9 (2006)
36. A.M. Fedotov, Laser Phys. **19**, 214 (2009)
37. A.I. Nikishov, Sov. Phys. JETP **30**, 660 (1970)
38. N.B. Narozhny, M.S. Fofanov, JETP **90**, 753 (2000)
39. S.S. Bulanov, V.D. Mur, N.B. Narozhny, J. Nees, V.S. Popov, Phys. Rev. Lett. **104**, 220404 (2010)
40. I.M. Bassett, Optica Acta: Int. J. Opt. **33**, 279 (1986)
41. I. Gonoskov, A. Aiello, S. Heugel, G. Leuchs, Phys. Rev. A **86**, 053836 (2012)
42. A. Gonoskov, et al., Phys. Rev. Lett. **111**, 060404 (2013)
43. I.V. Sokolov, N.M. Naumova, J.A. Nees, G.A. Mourou, Phys. Rev. Lett. **105**, 195005 (2010)
44. S.S. Bulanov, M. Chen, C.B. Schroeder, et al., AIP Conf. Proc. **1507**, 825 (2012)
45. S.S. Bulanov, C.B. Schroeder, E. Esarey, W.P. Leemans, Phys. Rev. A **87**, 062110 (2013)
46. P. Auger, et al., Rev. Mod. Phys. **11**, 288 (1939)
47. T.K. Gaisser, *Cosmic Rays and Particle Physics* (Cambridge University Press, 1990)
48. A.T. Bharucha-Reid, *Elements of the Theory of Markov Processes and Their Applications* (McGraw-Hill, New York, 1960)
49. A.I. Akhiezer, N.P. Merenkov, A.P. Rekalo, J. Phys. G **20**, 1499 (1994)
50. V. Anguelov, H. Vankov, J. Phys. G: Nucl. Part. Phys. **25**, 1755 (1999)
51. N.V. Elkina, et al., Phys. Rev. STAB **14**, 054401 (2011)
52. A. Di Piazza, K.Z. Hatsagortsyan, C.H. Keitel, Phys. Rev. Lett. **105**, 220403 (2010)
53. N. Neitz, A. Di Piazza, Phys. Rev. Lett. **111**, 054802 (2013)
54. W. Heitler, *The quantum theory of radiation*, 3rd edition (Oxford: Clarendon Press, 1954)
55. A.R. Bell, J.G. Kirk, Phys. Rev. Lett. **101**, 200403 (2008)
56. J.G. Kirk, A.R. Bell, I. Arka, Plasma Phys. Control. Fusion **51**, 085008 (2009)
57. A.M. Fedotov, N.B. Narozhny, G. Mourou, G. Korn, Phys. Rev. Lett. **105**, 080402 (2010)
58. B.G. Streetman, S. Banerjee, *Solid State Electronic Devices* (New Jersey: Prentice Hall, 2000)
59. N.B. Narozhny, Phys. Rev. D **21**, 1176 (1980)
60. M.V. Legkov, A.M. Fedotov, N. Elkina, H. Ruhl, Proc. SPIE **7994**, LAT 2010: Int. Conf. on Lasers, Appl. Technol., 799423 (2011)
61. E.N. Nerush, I.Yu. Kostyukov [`arXiv:1101.4385v1`]
62. R. Duclous, J.G. Kirk, A.R. Bell, Plasma Phys. Control. Fusion **53**, 015009 (2011)
63. E.N. Nerush, V.F. Bashmakov, I.Yu. Kostyukov, Phys. Plasmas **18**, 083107 (2011)
64. V.F. Bashmakov, et al. [`arXiv:1310.4077v1`]
65. E.N. Nerush, et al., Phys. Rev. Lett. **106**, 035001 (2011)

66. S.S. Bulanov, et al., Phys. Rev. Lett. **105**, 220407 (2010)
67. B. King, N. Elkina, H. Ruhl, Phys. Rev. A **87**, 042117 (2013)
68. C.P. Ridgers, et al., Phys. Plasmas **20**, 056701 (2013)
69. J.G. Kirk, A.R. Bell, C.P. Ridgers, Plasma Phys. Control. Fusion **55**, 095016 (2013)
70. E.N. Nerush, I.Yu. Kostyukov, L. Ji, A. Pukhov [`arXiv:1309.1648v1`]
71. T. Nakamura, et al., Phys. Rev. Lett. **108**, 195001 (2012)
72. E.P. Liang, S.C. Wilks, M. Tabak, Phys. Rev. Lett. **81**, 4887 (1998)
73. S.V. Bulanov, et al., Nucl. Instr. Methods Phys. Res. A **660**, 31 (2011)
74. S.V. Bulanov, et al., Plasma Phys. Control. Fusion **53**, 124025 (2011)
75. N.V. Elkina (private communication)
76. A. Gonoskov, et al. [`arXiv:1306.5734v2`]
77. A. Zhidkov, et al. [`arXiv:1308.1608v1`]
78. F. Hebenstreit, R. Alkofer, H. Gies, Phys. Rev. D **82**, 105026 (2010)