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Review

Extreme quantum field theory and particle physics with IZEST

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Abstract. The prospect of next-generation ultra-high-intensity laser sources has prompted recent renewed study of nonlinear QED processes, such as the Schwinger effect, in which the instability of the QED vacuum is probed by external fields. Experimental observation of these nonlinear QED effects would provide unprecedented controlled access to non-perturbative processes in quantum field theory under extreme conditions, which is of direct interest in particle physics and astrophysical applications. I summarize important theoretical issues, both conceptual and computational, related to these nonlinear QED effects.

1 Introduction

Quantum vacuum fluctuations have the consequence that the QED vacuum behaves like a polarizable medium that modifies classical behavior, leading to novel quantum effects [1–9]. Some of these effects, such as the Casimir effect or vacuum birefringence, are perturbative and are well described by perturbative quantum field theory. There are also truly nonperturbative processes, such as the Schwinger effect: the nonperturbative production of electron/positron pairs from vacuum subjected to an electric field. A rough order-of-magnitude estimate of the scale at which such nonperturbative processes should become significant is given by comparing the "binding energy", $2\,\mathrm{mc}^2$, with the work done to separate a virtual electron/positron pair by a Compton wavelength, giving the "Schwinger critical field": $\mathcal{E}_c = \frac{m^2c^3}{e\,\hbar} \approx 10^{16}$ V/cm, corresponding to a critical intensity, $I_c = \frac{c}{8\pi}\,\mathcal{E}_c^2 \approx 4\times 10^{29}$ W/cm^2. These are truly extreme intensities, defining a new regime of extreme field physics. An important motivation for the IZEST (International Zeta-ExaWatt Science and Technology) program is to probe new physics at these extreme intensities. This has prompted theoretical progress (discussed below) showing that in fact even this estimate [which is based on the assumption of a constant applied electric field] can be significantly lowered by the systematic shaping and combining of laser pulses, and also by interacting laser beams with electron beams. IZEST involves aspects of laser physics, plasma physics, particle physics, atomic physics and accelerator physics, making it a highly multi-disciplinary enterprise. The corresponding intense magnetic fields exceeding 10^{14} Gauss are indirectly observed to exist in neutron stars and magnetars, raising fundamental questions for quantum field theory and astro-particle physics concerning how matter and radiation behave in such extreme environments.

An incomplete list of new nonlinear processes to be probed and studied includes: (i) photon-photon scattering and quantum reflection; (ii) the Schwinger effect and trident processes; (iii) searches for new physics with lasers; (iv) vacuum birefringence; (v) photon splitting in an intense magnetic field; (vi) nonlinear Compton scattering; (vii) radiation reaction (quantum and classical); (viii) nonequilibrium physics associated with pair cascades; (ix) effect of intense electromagnetic fields on quantum chromodynamical processes; and many more.

There are many theoretical puzzles here, and it will be important to have systematic and controlled experimental input, opening a new window into this largely unexplored regime of extreme field physics and nonperturbative quantum field theory. The consequences go beyond QED, with implications for vacuum energy and dark energy, cosmological particle production, axion and dark matter searches, nonequilibrium quantum field theory and the quark-gluon plasma, and black hole physics and Hawking radiation. Some of these topics are discussed in other chapters in this Special Issue.

2 The QED effective action

In quantum field theory, the quantum corrections to classical Maxwell electrodynamics are encoded in the "effective action" $\Gamma[A]$ [10,11]. For example, the polarization tensor $\Pi_{\mu\nu} = \frac{\delta^2\Gamma}{\delta A_\mu \delta A_\nu}$ contains the electric permittivity ϵ_{ij} and the magnetic permeability μ_{ij} of the quantum vacuum, and is obtained by varying the effective action $\Gamma[A]$ with respect to the external probe $A_\mu(x)$. $\Gamma[A]$ is defined in terms of the vacuum-vacuum persistence amplitude

$$\langle 0_{\rm out}\,|\,0_{\rm in}\rangle = \exp\left[\frac{i}{\hbar}\left\{{\rm Re}(\Gamma) + i\,{\rm Im}(\Gamma)\right\}\right] \tag{1}$$

${\rm Re}(\Gamma[A])$ describes dispersive effects, such as vacuum birefringence, while ${\rm Im}(\Gamma[A])$ describes absorptive effects, such as vacuum pair production. The imaginary part encodes the probability of vacuum par production as

$$\begin{aligned} P_{\rm production} &= 1 - |\langle 0_{\rm out}\,|\,0_{\rm in}\rangle|^2 \\ &= 1 - \exp\left[-\frac{2}{\hbar}\,{\rm Im}\,\Gamma\right] \approx \frac{2}{\hbar}\,{\rm Im}\,\Gamma. \end{aligned} \tag{2}$$

From a computational perspective, the effective action is defined as [10,11]

$$\Gamma[A] = \hbar\,\ln\,\det\left[i\not{D} - m\right] = \hbar\,{\rm tr}\,\ln\left[i\not{D} - m\right]. \tag{3}$$

Here, $\not{D} \equiv \gamma^\mu D_\mu$, where the covariant derivative operator, $D_\mu = \partial_\mu - i\frac{e}{\hbar c}A_\mu$, defines the coupling between electrons and the electromagnetic field A_μ. When the gauge field A_μ is such that the field strength, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, is constant, this effective action was computed exactly [and non-perturbatively] by Heisenberg and Euler [12,13]. For example, for a constant electric field $\mathcal{E}$:

$$\frac{\Gamma^{\rm HE}[\mathcal{E}]}{\hbar\,{\rm Vol}_4} = -\frac{e^2\mathcal{E}^2}{8\pi^2}\int_0^\infty \frac{ds}{s^2}\,e^{-\frac{m^2}{e\mathcal{E}}s}\left(\cot(s) - \frac{1}{s} + \frac{s}{3}\right). \tag{4}$$

The leading imaginary part comes from the first pole of the cot(s) function:

$$\frac{{\rm Im}\,\Gamma^{\rm HE}}{{\rm Vol}_4} \sim \hbar\frac{e^2\,\mathcal{E}^2}{8\pi^3}\exp\left[-\frac{\pi\,m^2\,c^3}{e\,\mathcal{E}\,\hbar}\right] \tag{5}$$

consistent with the order-of-magnitude estimate of the crticial field mentioned previously.

2.1 Keldysh approach for inhomogeneous background fields

It is important, both experimentally and theoretically, to understand how this constant field result (5) is modified for more realistic inhomogeneous fields, such as those describing ultra-short pulse focussed lasers. This is difficult. The first step in this direction is motivated by a seminal result of Keldysh [14] for the ionization of atoms in a monochromatic time dependent electric field $\mathcal{E}(t) = \mathcal{E}\cos(\omega t)$. This introduces a new scale, ω, to the problem, and Keldysh computed the ionization probability as a function of the dimensionless *adiabaticity parameter* $\gamma_K = \frac{\omega\sqrt{2mE_b}}{e\mathcal{E}}$, that characterized the fast $[\gamma_K \gg 1]$ and slow $[\gamma_K \ll 1]$ regimes. [The Keldysh parameter is related to the standard laser field strength parameter a_0 as $a_0 = 1/\gamma_K$.] Keldysh's WKB result interpolates smoothly between the non-perturbative tunnel-ionization regime where $\gamma_K \ll 1$, and the perturbative multi-photon regime where $\gamma_K \gg 1$. This formalism was generalized to the Schwinger effect in QED [15,16], with an analogous *adiabaticity parameter*, $\gamma_K \equiv \frac{mc\omega}{e\mathcal{E}}$,

$$P_{\text{pair prod.}} \sim \begin{cases} \exp\left[-\dfrac{\pi m^2 c^3}{e\mathcal{E}\hbar}\right], & \gamma_K \ll 1 \\ \left(\dfrac{e\mathcal{E}}{m\omega}\right)^{2mc^2/\hbar\omega}, & \gamma_K \gg 1 \end{cases} \tag{6}$$

The $\gamma_K \ll 1$ regime corresponds to nonperturbative tunneling, while $\gamma_K \gg 1$ is the perturbative multiphoton regime.

2.2 Trident process

In the perturbative multi-photon regime, this QED pair production effect has been observed in a beautiful experiment (E-144) at SLAC [17], in which a laser pulse collided with the highly relativistic SLAC electron beam, leading to nonlinear Compton scattering involving 4–5 photons, producing a high energy gamma photon that decays into an electron-positron pair. A significant recent theoretical advance has been a complete QED analysis of the trident process [18], mapping out the physics of combining an intense laser beam with an intense laser beam. The important conclusion is that by combining such beams, within accessible ranges of electron energies (1–100 GeV) and laser intensities (10^{16}–10^{21} W/cm^2, it should be possible to scan across the boundary between perturbative multi-photon physics and non-perturbative tunneling physics. There are also many interesting phenomena to be probed concerning Compton scattering at ultra-high intensities [19]. These analyses define a set of experiments that should be done, with precision, to confirm that we really do understand this unexplored corner of QED. There may be surprises, both experimental and theoretical, so precision agreement would be an important milestone on the path towards the extreme physics regime.

2.3 Pulse-shaping effects

The Keldysh approach captures an enormous amount of important physical information. Various methods, such as the quantum kinetic approach [20–23], numerical

Dirac equation and dispersion relations [9,24], Green's functions methods [25], the Dirac-Heisenberg-Wigner formalism [26,27], quantum mechanical instantons [28], and the worldline instanton approach [29–31], have been developed to compute the pair production probability when the background electric field has more substructure. Many of these are ultimately based on Feynman's physical analogy between the pair-production process and quantum scattering in space-time, in which positrons can be viewed as electrons propagating backwards in time [32]. This opens the door to techniques from scattering theory and semiclassical analysis. These are now well understood when the field inhomogeneity is one-dimensional, but both conceptual and computational challenges arise once we try to go beyong one dimension. This is a familiar roadblock in many areas of semiclassical physics, and new ideas and methods are needed in order to address the important optimization problem:

> *How should experimentalists tailor the laser pulse profile, both temporally and spatially, in order to obtain the maximal, and most distinct, signal for a given cost of laser intensity?*

While a general answer to this question is still beyond our current reach, some interesting ideas have arisen recently. For example, the effective critical field intensity for the Schwinger effect can be lowered by several orders of magnitude by taking into account the importance of the spatial spot-size, large compared to the Compton wavelength scale, and the possibility of combining many coherent laser beams [33]. Another idea, the "dynamically assisted Schwinger effect" [34], proposes to combine two laser pulses, one slow (e.g., optical) and one fast (e.g., X-ray), so that there is a combination of both multi-photon and tunneling physics. This provides an exponential enhancement, as the fast field effectively lowers the tunneling barrier by partial mutli-photon excitement, so that the combined field produces an appreciable signal even though each individual pulse cannot. Here too, estimates lower the critical field intensity by several orders of magnitude [34–36], bringing it into the 10^{26} W/cm^2 regime, dramatically lower than the 10^{29} W/cm^2 "critical field", and potentially accessible within the IZEST program.

2.4 Worldline instanton formalism

One approach with the potential to analyze both spatial and temporal inhomogeneities is the worldline instanton method [29], based on Feynman's worldline formulation of the QED effective action [37,38], as a quantum mechanical path integral over closed loops $x_\mu(\tau)$ in four dimensional spacetime, with the closed loops being parametrized by the proper time τ. For simplicity, consider scalar QED. The effective action for a scalar charged particle in a Euclidean classical gauge background $A_\mu(x)$ is

$$\Gamma[A] = \int_0^\infty \frac{dT}{T}\, e^{-m^2 T} \int d^4x^{(0)} \int_{x(T)=x(0)=x^{(0)}} \mathcal{D}x \times \exp\left[-\int_0^T d\tau \left(\frac{\dot{x}_\mu^2}{2} + e\, A_\mu \dot{x}_\mu\right)\right].$$

The main technical difficulty is to compute the quantum mechanical path integral, either by direct Monte Carlo [39], or by a semiclassical approximation to the path integral [29], solving the classical Euclidean equations of motion:

$$\ddot{x}_\mu = F_{\mu\nu}(x)\, \dot{x}_\nu, \quad (\mu,\nu = 1\ldots 4). \tag{7}$$

In this semiclassical approximation, the path integral is dominated by a classical closed loop solution, called a "worldline instanton", together with quantum fluctuations about this loop. The dominant exponential factor in the imaginary part of the effective action is $\exp(-S[x_{\rm classical}])$, with the action evaluated on the worldline instanton trajectory. The prefactor contributions can also be physically significant, and can be computed with a generalization of the Gutzwiller trace formula [40]. The main technical challenge in this worldline instanton approach is finding the closed classical trajectories in a given (Euclidean) background field [29,41].

2.5 Quantum interference: Towards optimization and quantum control

For laser pulses with sub-cycle structure, quantum interference effects become important for the Schwinger process. This interference phenomenon is extremely sensitive to the temporal profile of the laser pulse, such as the carrier phase or chirp, so we can take advantage of this in order to enhance the signal. Such phenomena are familiar from strong-field atomic and molecular physics, discussed long ago in the theory of atomic ionization [42,43], and observed experimentally in photoionization spectra [44,45]. Technically, such laser pulses have multiple semiclassical saddle points, and their relative phases can produce dramatic interference effects. For example, for an electric field with a carrier phase ϕ,

$$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t + \phi) \exp\left(-\frac{t^2}{2\tau^2}\right) \tag{8}$$

when $\phi = \pi/2$, one finds oscillatory behavior of the longitudinal electron-positron momentum spectrum [46,47]. Indeed, there are values of the electron-positron momentum at which the probability of production is enhanced, and others where it vanishes. This is a physical manifestation of the Stokes phenomenon [48,49]. A single-pulse field has just one dominant saddle point, but when $\phi = \pi/2$ there are two saddle points of comparable amplitude, so

$$P \approx e^{-2S_c^{(1)}} + e^{-2S_c^{(2)}} \pm 2\cos(2\alpha)\, e^{-S_c^{(1)} - S_c^{(2)}}. \tag{9}$$

Here α is an integral connecting different saddle points, characterizing the interference. The $\pm$ in (9) refers to scalar/spinor QED, reflecting the expected opposite sign dependence of interference effects on quantum statistics.

This is a QED vacuum realization of the temporal double-slit experiment, analogous to one performed for atomic ionization [45]. This fact can be further exploited to construct a Ramsey-style set-up with a train of N alternating sign electric field pulses producing an N-slit interference pattern for the momentum spectrum [50]. Thus, the central mode is enhanced by a factor of N^2, as the particle production probability is redistributed between the momentum modes in a coherent manner. In addition to this enhancement effect, the acute sensitivity of the momentum spectra to the laser pulse shape may provide distinctive experimental signatures of the Schwinger effect, and eventually provide sub-cycle pulse resolution at extremely short time scales, using the quantum vacuum fluctuations. There are also many interesting analogies in condensed matter physics and quantum optics [51].

These results also suggest to adapt and apply the methods of 'optimal quantum control' to the Schwinger effect and other nonlinear QED processes. Such quantum control techqniues are now widely used in many areas of physics and chemistry [52,53], and their relativistic generalization could give insight into nonlinear QED. First steps have been taken [54,55], but much remains to be done. It is, however, clear that quantum interference is a powerful guiding principle. For example, most fine details of the pulse shape are not relevant: the important thing is the location of the semiclassical saddle points, and this fact can be used for optimzation.

2.6 Backreaction and cascade effects

One of the most tantalizing theoretical and conceptual problems concerns the possibility of backreaction from the produced electron/positron pairs, and the potential for resulting cascades. This could lead to prolific pair production [56,57], and it has been suggested that this may even place a fundamental limit on the physically attainable electric field strength [58,59]. This is a very difficult theoretical subject, and a fully quantum treatment is still needed. Indeed, the back-reaction process in QED raises important conceptual issues of non-equilibrium physics [22], with implications for black-hole physics and cosmology. Furthermore, in both classical and quantum electrodynamics, there are many interesting open puzzles concerning radiation reaction [9,60]. Experimental input from intense QED studies would be extremely valuable.

3 Conclusion

The IZEST project offers deep and fundamental challenges, both experimental and theoretical, as we approach our first glimpse into this new regime of extreme field physics. We anticipate that the IZEST ultra-high intensity regime will provide a complementary new approach to particle physics, as the original laser revolution did for atomic physics, discovering many surprising new phenomena and as-yet-unimagined new applications.

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