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Editorial

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In 1695, the concept of the fractional derivative appeared for the first time in a famous correspondence between G.A. de L'Hospital and G.W. Leibniz. Many mathematicians have further developed this area and we can mention the studies of L. Euler, J.L. Lagrange, P.S. Laplace, J.B.J. Fourier, N.H. Abel, J. Liouville, B. Riemann, O. Heaviside, H. Weyl, M. Riesz and W. Feller. In the past forty years, fractional calculus had played a very important role in various fields such as mechanics, electricity, chemistry, biology, economics, control theory, and signal and image processing [1–15].

In the last decade, fractional calculus has been recognised as one of the best tools to describe long-memory processes. Such models are interesting for engineers and physicists but also for pure mathematicians. The most important among such models are those described by partial differential equations (PDEs) containing fractional derivatives. Their evolutions behave in a much more complex way than in the classical integer-order case and the study of the corresponding dynamics is a hugely demanding task. Although some results of qualitative analysis for fractional partial differential equations (FPDEs) can be similarly obtained, many classical PDEs' methods are hardly applicable directly to FPDEs. New theories and methods are thus required to be specifically developed for FPDEs, whose investigation becomes more challenging. Comparing with PDEs' classical theory, the research on FPDEs is only at an initial stage of development.

This special issue on Dynamics of Fractional Partial Differential Equations consists of 20 original articles covering various aspects of FPDEs and their applications. These papers could be grouped into four categories, namely, survey of fractional calculus [22], numerical methods to solve FPDEs [17,23,26–28,32–34], qualitative analysis of FPDEs [16,18,21,24,30,35] and application of FPDEs in various fields [19,20,25,29,31].

In paper [16] the theory of Hausdorff measure of noncompactness and fixed point theorems are applied to study the abstract nonlocal Cauchy problem of a class

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of fractional evolution equations with Caputo derivatives when the semigroup is compact or noncompact. Some interesting existence theorems of mild solutions are derived under some weak conditions via the theory of measure of noncompactness.

In paper [17] the Laplace transform, finite Hankel transform and Mittag-Leffer functions are used to investigate the solution of the axisymmetric time-fractional diffusion-wave equation in a cylinder under Robin boundary conditions with the prescribed linear combination of the values of the sought function and the values of its normal derivative at the boundary.

Paper [18] deals with the fractional Schrödinger equation with the quantum Riesz-Feller derivative for a particle that moves in a potential field. With the help of the fractional Fourier transform, this equation is solved for the case of a free particle in terms of the Fox H-function. The authors show that, in the general case, the quantum Riesz-Feller derivative is a non-Hermitian operator that possesses no real eigenvalues that makes the fractional Schrödinger equation with the fractional Riesz-Feller derivative unattractive as a quantum mechanics model.

Paper [19] develops a new application of fractional calculus and extends the application of the method to the Fermi-Pasta-Ulam problem. The study starts by addressing the classical formulation, based on the standard integer order differential calculus and evaluates the time and frequency responses.

Paper [20] proposes a fractional Taylor series method for the fractional Boussinesq equation by assuming power-law changes of flux in a control volume. Furthermore, it is assumed that the average thickness of the watery layer of an unconfined aquifer is constant, and a linear fractional Boussinesq equation is derived.

Paper [21] investigates multidimensional integration by parts formulae for generalised fractional derivatives and integrals. The new results allow the authors to obtain optimality conditions for multidimensional fractional variational problems with Lagrangians depending on generalised partial integrals and derivatives.

In paper [22] a survey of useful and established formulae from recent developments in fractional calculus is presented. Many expressions of fractional calculus have been published, but such results are scattered over the literature and use different notations. This paper intends to gather systematically some of the most useful formulae for reference purposes.

Paper [23] discusses the exact solution of fractional Fokker-Planck equations for anomalous diffusion in an external potential by using both ordinary and matrix continued fractions. The procedure is illustrated by solving various problems concerning the anomalous translational diffusion in both periodic and double-well potentials.

Paper [24] studies fractional Langevin equations involving two Mittag-Leffler functions and impulsive terms. Formulae of solutions involving Mittag-Leffler functions and impulsive terms of such equations are successively derived by studying the corresponding linear Langevin equations with two different fractional derivatives. Meanwhile, existence results of solutions are established.

Paper [25] discusses a realistic model for estimation of the medical effect of brain cancer (glioma) treatment by a radio-frequency electric field. It is shown that the efficiency of the medical treatment by the tumor-treating field depends essentially on the mass fractal dimension of the cancer in the outer-invasive region.

Paper [26] develops high-order finite difference/element methods for the nonlinear anomalous diffusion equations of subdiffusion and superdiffusion. The stability and error estimates are proved for both cases of superdiffusion and subdiffusion. Numerical examples are provided to confirm the theoretical analysis.

Paper [27] derives series expansion solutions for the multi-term time and space fractional partial differential equations in two- and three-dimensions. The authors derive series expansion solutions based on a spectral representation of the Laplacian operator on a bounded region. Some applications are given for the two- and three-dimensional telegraph equation, power law wave equation and Szabo wave equation.

Paper [28] describes the application of exponential integrators to time-fractional partial differential equations after the discretisation of spatial derivatives. The aim of this approach is to overcome the stability issues due to the stiffness in the resulting semi-discrete system.

Paper [29] introduces the generalised classical mechanics developed as a classical counterpart of fractional quantum mechanics. The Lagrangian of generalised classical mechanics is introduced, and equation of motion is obtained. Lagrange, Hamilton and Hamilton-Jacobi frameworks are implemented.

Paper [30] constructs a fundamental solution of a multi-time diffusion equation with the Dzhrbashyan-Nersesyan time fractional differentiation operator. The authors give a representation for a solution of the Cauchy problem and prove the uniqueness theorem in the class of functions of fast growth. The corresponding results for equations with Riemann-Liouville and Caputo derivatives are obtained too.

Paper [31] explores the acoustic-elastodynamic interaction in isotropic fractal media. The authors consider two situations where in the first, the fractal medium is enclosed within a thin spherical shell, while in the second, the fractal medium extends infinitely outside the shell.

Paper [32] proposes a new numerical algorithm for solving the two dimensional fractional reaction subdiffusion equation. The stability and convergence of this method are investigated by the Fourier analysis. Theoretical analysis and numerical experiment demonstrate that the proposed method is effective for solving the two dimensional fractional reaction subdiffusion equation.

Paper [33] develops a matrix approach for partial differential equations with Riesz space fractional derivatives. The Shift-and-Invert method is applied to approximate the solution of the partial differential equation as the action of the matrix exponential on a suitable vector which mimics the given initial conditions.

Paper [34] applies a finite difference method with non-uniform time steps for fractional diffusion and diffusion-wave equations. The non-uniformity of the time-steps allows one to adapt their size to the behaviour of the solution, which leads to large reductions in the computational time required to obtain the numerical solution without loss of accuracy.

In the final paper [35] the existence and multiplicity of weak solutions for a fractional boundary value problem involving left and right fractional derivatives are proved. Based on variational methods and critical point theory, the authors provide an effective tool for studying the existing of infinitely many solutions for fractional bounded value problems.

Thus, this special issue provides a wide spectrum of current research in the development of FPDEs, and we hope that the related researchers in these fields will find it useful. We would like to thank Professors D. Baleanu, K. Balachandran, Y.Q. Chen, I. Vasundhara Devi, M. Fečkan, F. Liu, J.A.T. Machado, S. Momani, D.F.M. Torres and J.R. Wang for their support and help. We wish to express our appreciation to the authors of all the articles in this special issue for the excellent contributions as well as many reviewers for their high-quality work on reviewing the manuscripts.

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