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Review

A guide to the QCD light-cone sum rules for b-quark decays

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Abstract We overview the current status and future perspectives of the QCD-based method of light-cone sum rules. The two main versions of these sum rules, using light-meson and B-meson distribution amplitudes are introduced and the most important applications of the method are discussed. We also outline open problems and future perspectives of this method.

1 Introduction

The QCD-based method of light-cone sum rules (LCSRs) has already a long history. It was originally developed in Refs. [1, 2] for the process of a hyperon radiative decay and, independently, in Ref. [3], where LCSRs were for the first time applied to B-meson exclusive decays. An application to the D-meson semileptonic decays followed then in Ref. [4]. The usefulness of this method for the calculation of B-meson transition form factors was recognized already in the early applications [5–8].

The LCSRs, in their different versions, offer a realistic possibility to calculate various form factors of semileptonic or radiative B-meson decays in the large recoil region of the final light hadron. The method can be extended to other hadronic amplitudes involved in B physics, such as strong couplings and nonleptonic decay amplitudes. It is also applicable to the decays containing two light hadrons in the final state and involving broad resonances. Neither the large recoil region, nor resonance final states are currently accessible to the lattice QCD calculations, and therefore LCSRs in several cases are the only available QCD-based method.

To describe the method in a nutshell: each LCSR for a hadronic transition form factor starts from a correlator which is a vacuum-to-hadron matrix element of two quark currents, chosen to match a form factor we are interested in. The correlator contains a quark-level transition current, and a current interpolating the other hadron involved in this transition. For this correlator, an operator product expansion (OPE) near the light-cone is applied, provided the external momenta of quark currents are far off-shell. The OPE yields an analytical and factorizable expression for the correlator, written in a form of a convolution of the calculable hard-scattering kernel with universal light-cone distribution amplitudes (DAs) of the hadron entering the vacuum-to-hadron matrix element. These DAs absorb nonperturbative effects, and their parameters represent a universal, process-independent input. The factorized form of the OPE has its roots in the early studies of the QCD asymptotics of hadron form factors [9–12]. There is a crucial difference, though: in the LCSR approach factorization is applied at the correlator level. To access the form factor, the tools of conventional QCD (SVZ) sum rule method [13] are then applied to the same correlator, including the hadronic dispersion relation and the quark-hadron duality approximation. Combining these three main elements: the light-cone OPE, dispersion relation and duality, leads to the resulting LCSR, which is an approximate analytical formula for the form factor depending on a set of universal inputs.

Without going into further details in this brief introduction, we only mention that the method of LCSRs allows us to calculate the so called soft contribution to a form factor. It describes a transition between the initial and final

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hadrons which is realized as an overlap of two configurations with low-momenta spectator quarks, hence, without a hard-gluon exchange. The soft contributions in a LCSR, being not accessible in purely perturbative QCD-based methods, correspond to the leading-order (LO), $O(\alpha_s^0)$ kernel of the OPE. The hard-scattering part of the same form factor is then provided by the next-to-leading (NLO) perturbative gluon corrections to the same kernel. As a result, the LCSR allows one to access the complete form factor within one method and input, albeit in an indirect way, via dispersion relation.

We emphasize that the main goal of this review is not to dwell on the derivational details of the LCSR method. These details can be found in several reviews (see e.g., Refs. [14–17]). We also do not cover here the subtle aspects of twist expansion for nonlocal operators near light-cone and the emergence of the corresponding light-cone distribution amplitudes. These issues essential for LCSRs can be found in dedicated papers and reviews and the relevant ones will be cited below. Here we rather aim at presenting a short "user guide" to the LCSR applications to various exclusive decays of B-meson, mostly to their form factors, but also to other related hadronic quantities. We have made a selection of all—from our point of view—important and useful papers in this field, with a brief assessment of their main results. We also tried to outline unsolved problems and future perspectives of the LCSR applications.

The rest of the review is organized as follows. In Sects. 2 and 3 we overview the two main versions of the LCSR method for B meson transition form factors, employing, respectively, the light-meson and B-meson DAs. In Sect. 4, we collect all important results concerning the B decays into photons and leptons. Section 5 contains a brief guide to other perspective applications of LCSRs for heavy hadron decays. The concluding discussion is presented in Sect. 6.

2 Sum rules with light-meson distribution amplitudes

2.1 Standard application: the $B \to \pi$ form factors and $B^*B\pi$ coupling

The $B \to \pi$ form factors, generated in SM by the weak $b \to u$ transition, were among the first applications of LCSRs. The vector and scalar form factors[1] $f^+_{B\pi}$ and $f^0_{B\pi}$ determine the weak semileptonic decay $\bar{B}^0 \to \pi^+\ell\nu_\ell$. To describe the rare $B \to \pi\ell^+\ell^-$ decay, generated by the flavour-changing neutral current (FCNC) $b \to d$ transition, one needs, in addition to $f^{+,0}_{B\pi}$, also a form factor $f^T_{B\pi}(q^2)$ of the tensor $b \to d$ quark current.

In the rest of this subsection, we briefly recall the derivation of LCSR for the most important form factor $f^+_{B\pi}$. The underlying correlator is defined as a vacuum-to-pion matrix element of the time-ordered product of two currents:

$$\begin{aligned} F_\mu(p,\, q) &= i \int d^4x\; e^{iq\cdot x} \langle \pi^+(p)|T\big\{\bar{u}(x)\gamma_\mu b(x),\, m_b\bar{b}(0)i\gamma_5 d(0)\big\}|0\rangle \\ &= F((p+q)^2,\, q^2)\, p_\mu + \widetilde{F}((p+q)^2,\, q^2)\, q_\mu, \end{aligned} \tag{1}$$

where q and $(p+q)$ are, respectively, the momenta of the $b \to u$ weak current and of the B-meson interpolating current. Only the first invariant amplitude F in the Lorentz-decomposition is relevant for $f^+_{B\pi}$. The amplitude $\widetilde{F}$ is used to obtain a second LCSR, so that a linear combination of the two sum rules yields the scalar form factor. The tensor form factor $f^T_{B\pi}$ is accessed by replacing the vector weak current in the correlator by the tensor $b \to d$ transition current. Ability to access different form factors by varying currents or invariant amplitudes in the correlator reveals flexibility and universality of the LCSR method.

The correlator in Eq. (1) is calculated at the external momenta squared far below the open b-flavour thresholds, that is, at

$$(p+q)^2 \ll m_b^2, \quad q^2 \ll m_b^2. \tag{2}$$

These conditions induce a strongly oscillating exponent in the correlator, retaining only the region near the light-cone $x^2 \simeq 0$ in the four-coordinate integral. Hence, the product of currents can be expanded near $x^2 = 0$. In particular, a highly virtual b-quark is replaced with a light-cone expansion [19] of its propagator. Contraction of b-quark fields generates a perturbatively calculable kernel which is factorized from the remaining long-distance

[1] We use standard definitions of these form factors, see, e.g., Ref. [18].

part. The resulting expression for the invariant amplitude F (or similarly for $\tilde{F}$) has the following schematic form:

$$F((p+q)^2,\, q^2) = i\int d^4x\, e^{iqx}\Big\{[S_0(x^2,\, m_b,\, \mu) + \alpha_s S_1(x^2,\, m_b,\, \mu) + \cdots] \\ \otimes \langle \pi(p) \mid \bar{u}(x)\Gamma d(0)\,|\,0\rangle_{|\mu} + \int_0^1 dv\; [\tilde{S}_0(x^2,\, m_b,\, \mu,\, v) + \cdots] \\ \otimes \langle \pi(p) \mid \bar{u}(x)G(vx)\tilde{\Gamma}d(0)\} \mid 0\rangle_{|\mu} + \cdots \Big\}, \tag{3}$$

where S_0 and S_1 are the LO and NLO parts of the perturbative kernel. They correspond, respectively, to the free b-quark propagator and $O(\alpha_s)$ perturbative gluon corrections. This kernel is convoluted with the vacuum-to-pion matrix element of the operator formed by a product of the $\bar{u}$ and d quark fields at a near-light-cone separation, with a generic Dirac structure Γ. The term with $\tilde{S}_0$ corresponds to a low-virtuality gluon emitted at a light-cone separation from the b-quark propagator and absorbed, together with a quark-antiquark pair, in the final pion state. The ellipsis in Eq. (3) denotes higher-order terms, e.g., the NNLO, $O(\alpha_s^2)$ corrections to S_0, NLO corrections to $\tilde{S}_0$ or the terms with two quarks and two antiquarks entering the vacuum-to-pion matrix element. The diagrams and detailed expressions for this correlator at NLO are given in Ref. [18], (see also the introductory review [15] for a description of derivation at the LO level).

The scale μ indicated in Eq. (3) corresponds to a separation between the short-distance and long-distance parts in this convolution. The optimal choice is $\mu \sim \sqrt{\Lambda m_b}$, where an intermediate scale $\Lambda \sim 1$ GeV is parametrically larger than $\Lambda_{\rm QCD}$, but at the same time does not scale with the heavy mass m_b. This choice guarantees that the average interval $|x^2| \sim 1/\mu^2$, between the emission points of the light-quark and gluon fields in the vacuum-to-pion matrix elements, can still be considered small. The subsequent transition of light constituents into an on-shell pion state includes all nonperturbative effects at energy-momenta below μ.

The formula (3) reveals a general structure of the light-cone OPE, emerging after expanding the correlator in terms of (i) quark-gluon coupling and (ii) multiplicity of light-quark and gluon fields entering the vacuum-to-pion matrix elements. The possibility to retain the lowest terms in the expansions (i) and (ii) is based on the suppression, respectively, due to extra powers of α_s (for which the scale μ is a natural choice) and due to inverse powers of μ.

The complete OPE of the correlator outlined in Eq. (3) is in fact more involved. Within each vacuum-to-pion matrix element, an additional expansion emerges after expressing these matrix elements in terms of the pion light-cone DAs with growing twists, starting from the lowest twist-2 DA:

$$\langle \pi(p)|\bar{u}(x)[x,\, 0]\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2\sim 0} = -ip_\mu f_\pi \int_0^1 du\, e^{iup\cdot x}\varphi_\pi(u) + O(x^2) + \cdots \tag{4}$$

Here u and $1-u$ are the fractions of the pion momentum p distributed among the two partons (quark and antiquark) in the pion, and $[x,\, 0]$ is the gauge factor. Two similar vacuum-to-pion matrix elements emerging in the light-cone expansion of the correlator contain (instead of $\gamma_\mu\gamma_5$) the Dirac-matrices $\Gamma = i\gamma_5$ and $\Gamma = \sigma_{\mu\nu}$, yielding, respectively, two additional DAs of the next-to-lowest twist-3. In addition, the $O(x^2)$ terms in Eq. (4) generate two more two-particle (quark-antiquark) DAs of twist-4. The second hadronic matrix element in Eq. (3) is expressed via three-particle (quark-antiquark-gluon) pion DAs, starting from twist 3. Currently, altogether five two-particle DAs with $t = 2,\, 3,\, 4$ and five three-partlce DAs with $t = 3,\, 4$ are taken into account in the OPE. Their expressions, normalization parameters and other important features, such as the relations between two and three-particle DAs due to QCD equations of motion, can be found in the most updated form in Ref. [20]. A short summary of these DAs and convenient defining formulas for the two- and three-particle vacuum-to-pion matrix elements are also given in Appendix A of Ref. [18].

After substitution of DAs and the coordinate integration, the OPE (3) for the invariant amplitude F transforms into a sum of separate DA contributions. In a compact generic form we have:

$$F_{OPE}((p+q)^2,\, q^2) = \sum_{m=1}^{\infty}\int \mathcal{D}u_{\{m\}} \sum_{t\geq 2}\Big[T_0^{(m,\,t)}((p+q)^2,\, q^2,\, u_1,\, \ldots,\, u_m,\, \bar{m}_b,\, \mu) \\ +\, \alpha_s T_1^{(m,\,t)}((p+q)^2,\, q^2,\, u_1,\, \ldots,\, u_m,\, \bar{m}_b,\, \mu) + O(\alpha_s^2)\Big]\phi_\pi^{(m,\,t)}(u_1,\, u_2,\, \ldots,\, u_m,\, \mu), \tag{5}$$

Table 1 The calculated terms in the OPE expansion (5). The multiplicities $m = 2$, 3 and $m = 4$ correspond, respectively to the quark-antiquark, quark-antiquark-gluon DAs and to the diquark-antidiquark DAs in the factorized approximation (quark condensate $\otimes$ two-particle DA)

Multiplicity of DA	Order in α_s	Twist	References
$m = 2$	LO	$t = 2, 3, 4$	[5, 6]
	NLO	$t = 2$	[21, 22]
		$t = 3$	[18, 23]
	partial NNLO	$t = 2$	[24]
$m = 3$	LO	$t = 3, 4$	[5, 6]
$m = 4$	LO	$t = 5, 6$	[25]

where $\phi_\pi^{(m,\,t)}$ is a pion DA with the light-parton multiplicity m and twist t. This DA depends on the pion momentum fractions $u_1, u_2, \ldots, u_m$, and we use the notation $\mathcal{D}u_{\{m\}} = \int(\prod_{i=1}^m du_i)\,\delta(1-\sum_{i=1}^m u_i)$ for the integration element. In the above expression, the twist-2 DA defined in Eq. (4) corresponds to $\phi^{(2,\,2)}(u,\,1-u) = f_\pi\varphi_\pi(u)$. A nontrivial feature of the light-cone OPE is the factorization proven at NLO for the leading power twist-2 terms, so that the μ dependence of the perturbative kernel $T_0^{(2,\,2)} + \alpha_s T_1^{(2,\,2)}$ is compensated by the perturbative evolution of $\phi_\pi^{(2,\,2)}$, well known as the ERBL evolution [10, 12]. Also the choice of the $\overline{\mathrm{MS}}$ scheme for the b-quark mass $\overline{m}_b$ is justified for a correlator with a highly-virtual b-quark. The currently known terms in Eq. (5) are listed in Table 1 with references to the papers in which they have been computed and where further important details can be found.

Finally, one more expansion implicitly present in the OPE (5) concerns the dependence of each DA on the fractions u_i of the pion momenta shared between light degrees of freedom. Here the formalism of conformal partial-wave expansion in QCD is used. Comprehensive reviews on that subject can be found, e.g. in Refs. [26, 27]. The conformal expansion represents a given pion DA in a form of series in orthogonal polynomials. For the twist-2 DA it is the well familiar series in Gegenbauer polynomials with multiplicatively renormalizable coefficients:

$$\varphi_\pi(u,\,\mu) = 6u(1-u)\Big(1 + a_2^\pi(\mu)C_2^{3/2}(2u-1) + a_4^\pi(\mu)C_4^{3/2}(2u-1) + \cdots\Big). \tag{6}$$

The polynomial expansion for this and other DAs, is usually taken up to the second conformal partial wave, that is, retaining only a_2^π and a_4^π in the above formula. A usual motivation is that the anomalous dimensions of polynomial coefficients grow with n, suppressing terms with larger n, so that at sufficiently large scale μ, the DA (6) is not far from its asymptotic form, which is $6u(1-u)$. The Gegenbauer moments in φ_π and similar polynomial coefficients for the higher-twist pion DAs, all taken at a reference scale $\mu = 1.0$ GeV, represent universal, process independent inputs for the OPE. Various methods to determine these parameters include lattice QCD computation and two-point QCD sum rules. Another useful strategy applied to determine the shape of the lowest twist-2 pion DA is to fit to their measured values the pion transition form factor $\gamma\gamma^* \to \pi^0$ and the pion electromagnetic form factor, both calculated from LCSRs (see, respectively, e.g., Refs. [28–30] and Refs. [31, 32]).

Having at hand the OPE result (5) for the invariant amplitude as an analytical function of the variable $(p+q)^2$ at fixed q^2, one then applies the standard tools of the conventional QCD sum rule method:

- equating this result to the hadronic dispersion relation in the variable $(p+q)^2$:

$$F^{OPE}((p+q)^2,\,q^2) = \frac{2f_B m_B^2 f_{B\pi}^+(q^2)}{m_B^2-(p+q)^2} + \cdots, \tag{7}$$

where the ellipsis indicates all heavier states with the B-meson quantum numbers, starting from the lowest threshold at $m_{B^*}+m_\pi$. Note that the hadronic dispersion relation is valid at any $(p+q)^2$, whereas the relation (7) is only used in the region (2) of the OPE validity;

- using quark-hadron semi-local duality approximation:

$$\int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\,\frac{\mathrm{Im}\,F(s,\,q^2)}{s-(p+q)^2} = \int\limits_{s_0^B}^{\infty} ds\,\frac{[\,\mathrm{Im}\,F(s,\,q^2)]_{OPE}}{s-(p+q)^2}, \tag{8}$$

where the effective channel-specific threshold s_0^B is introduced;

• subtracting the heavier state contributions from Eq. (7) with the help of Eq. (8) and applying the Borel transform $(p+q)^2 \to M^2$.

The final form of LCSR is then obtained:

$$m_B^2 f_B f^+_{B\pi}(q^2) e^{-m_B^2/M^2} = \int\limits_{m_b^2}^{s_0^B} ds\, e^{-s/M^2} [\,\mathrm{Im}\, F(s,\, q^2)]_{\mathrm{OPE}}. \tag{9}$$

This sum rule has three different sets of input parameters, ordered according to their universality. The first one includes the b-quark mass and α_s, both determined with great accuracy. The second set contains universal parameters of the pion DAs, including their normalizations and polynomial coefficients. Finally, the third set includes parameters specific for the B-meson channel: (a) the ranges of the scale μ and Borel parameter M; (b) the decay constant of B-meson f_B, that can be taken from lattice QCD average or, with a larger uncertainty, from the two-point QCD sum rule calculation (see e.g., Ref. [33]); and (c) the threshold s_0^B which is usually extracted from a derivative sum rule obtained from LCSR. The latter procedure was systematically used in more recent analyses (see e.g., Ref. [34]).

The latest numerical results for the $B \to \pi$ form factors at $q^2 = 0$ are given in the next subsection, in Table 3, together with the form factors of all other B transitions to light pseudoscalar mesons. The standard z-expansion (here preferred is the BCL version suggested in Ref. [35]) allows one to extrapolate the $B \to \pi$ form factors from the region of LCSR validity, typically at $q^2 \leq 12 - 15$ GeV^2 to larger q^2, that is, to the low recoil region of the pion, where this extrapolation can be compared with the lattice QCD predictions.[2]

The uncertainties quoted in Table 3 are parametric and they are usually estimated in quadratures. A more advanced Bayesian analysis of LCSRs was initiated in Ref. [34]. There are however not many possibilities left to substantially decrease these uncertainties in the future. Indeed, the still missing parts of OPE such as the NLO corrections to the nonasymptotic twist-3 terms and to the twist-4 terms are expected to be very small. Also the factorizable twist-5, 6 contributions estimated in Ref. [25] turned out to be negligible. On the other hand, there is still a relatively large uncertainty in the parameters of the pion twist-2 DA. Further efforts are desirable to improve the knowledge of this key element of LCSRs, combining all available methods.

The most elusive uncertainty of systematic origin in LCSRs remains the one related to the application of quark-hadron duality. The attempts in the literature to quantify this uncertainty by varying the threshold parameter s_0^B with the Borel scale or in any other way are only remedies, because these analyses still use duality as a basic assumption. In the future, one has to find alternative methods to estimate the integrated hadronic spectral density of higher states in the sum rules.

As an indirect way to assess the actual accuracy of LCSR predictions, we suggest to increase the accuracy of the q^2-shape in the measured differential (binned) width of the $B \to \pi \ell \nu_\ell$ decays. This observable, being directly proportional to the squared shape of the form factor $f^+_{B\pi}(q^2)$, provides a direct test of a QCD method, independent of the value of CKM parameter V_{ub}.

Finally, let us mention that straightforward byproducts of LCSRs with pion DAs considered in this subsection are the analogous sum rules for the $D \to \pi$ form factors. Switching to the charmed sector is straightforward, and demands only a replacement of b quark by the c quark in the correlator and a corresponding adjustment of all channel-specific inputs. The last LCSR calculation of these form factors in Ref. [37] deserves an update, e.g., including the twist 5,6 terms in the OPE. A comparison with experimental data on $D \to \pi \ell \nu_\ell$ semileptonic decay will allow one to tune the universal input parameters of LCSRs and to further increase the accuracy of the $B \to \pi$ form factor determination.

The $B \to \pi$ form factor $f^+_{B\pi}(q^2)$ at large q^2 (near the zero recoil of the pion), where the B^*-pole dominates, is determined by the strong $B^* B\pi$ coupling. It is defined as the invariant constant parametrizing the hadronic matrix element

$$\langle B^*(q)\pi(p)|B(p+q)\rangle = -g_{B^*B\pi}\, p^\mu \epsilon_\mu^{(B^*)}, \tag{10}$$

where $\epsilon_\mu^{(B^*)}$ is the polarization vector of B^* meson. The coupling $g_{B^*B\pi}$ is not measurable, since the $B^* \to B\pi$ decay is kinematically forbidden, contrary to its analog in charm sector, the $D^* \to D\pi$ decay.

As originally suggested in Ref. [6], the coupling $g_{B^*B\pi}$ can be obtained from a LCSR, considering the same correlator (1) and using its OPE. For the hadronic representation of the invariant function $F(q^2,\, (p+q)^2)$ a double

[2] For brevity, we do not quote for comparison the lattice QCD results here, they can be found in the updated and averaged form in Ref. [36], see also the review on lattice QCD applications to b quark physics in this volume.

Table 2 LCSR results [39] for the strong coupling of the bottom mesons for the two choices of the decay constants and of the pion twist-2 DA

φ_π	Decay constants	$g_{B^*B\pi}$	$\hat{g}$	δ [GeV]
Model 1	2-point sum rule	$24.1^{+4.5}_{-3.8}$	$0.18^{+0.02}_{-0.03}$	$3.28^{+0.62}_{-0.17}$
	Lattice QCD	$30.0^{+2.6}_{-2.4}$	$0.30^{+0.02}_{-0.02}$	$1.17^{+0.04}_{-0.04}$
Model 2	2-point sum rule	$23.0^{+4.5}_{-3.8}$	$0.17^{+0.03}_{-0.03}$	$3.31^{+0.30}_{-0.01}$
	Lattice QCD	$28.6^{+3.0}_{-2.8}$	$0.29^{+0.03}_{-0.03}$	$1.18^{+0.00}_{-0.02}$

dispersion relation in both variables p^2 and $(p+q)^2$ should then be used:

$$F(q^2,\,(p+q)^2) = \frac{m_B^2 m_{B^*} f_B f_{B^*}\, g_{B^*B\pi}}{(m_B^2-(p+q)^2)(m_{B^*}^2-q^2)} + \frac{1}{\pi^2}\iint ds_2 ds_1 \frac{\mathrm{Im}_{s_1}\mathrm{Im}_{s_2}F(s_1,\,s_2)}{(s_2-(p+q)^2)(s_1-q^2)}, \tag{11}$$

where the lowest double-pole term contains the $B^*B\pi$ coupling multiplied by the decay constants of pseudoscalar (f_B) and vector (f_{B^*}) bottom mesons. The duality approximation has to be defined for a two-dimensional region in the $\{s_1,\,s_2\}$ plane, adding an uncertainty, related to the freedom to choose the shape of that region, whereas the parametric accuracy of the OPE is the same as in the LCSRs for $B\to\pi$ form factors. Instead of a single Borel transform, as in Eq. (9), the double Borel transform is then performed, removing all subtraction terms that are not shown in Eq. (11) for brevity, including single-variable dispersion integrals. Due to the approximate mass degeneracy of B^* and B mesons, usually the two Borel parameters, $q^2\to M_1^2$ and $(p+q)^2\to M_2^2$, are taken equal, $M_1^2=M_2^2=2M^2$.

The strong coupling $g_{B^*B\pi}$ is then extracted from the sum rule:

$$f_B\, f_{B^*}\;g_{B^*B\pi} = \frac{1}{m_B^2 m_{B^*}} e^{\frac{m_B^2+m_{B^*}^2}{2M^2}} \iint\limits^{\Sigma(s_0)} ds_2 ds_1 e^{-\frac{s_2+s_1}{2M^2}}\frac{1}{\pi^2}\mathrm{Im}_{s_1}\mathrm{Im}_{s_2}F_{OPE}(s_1,\,s_2)\,, \tag{12}$$

where $\Sigma_0(s_0)$ indicates the duality region parametrized with the effective threshold s_0.

In Ref. [6] the LO result for this sum rule at twist-4 accuracy was obtained. The NLO correction to the twist-2 contribution was computed in Ref. [38]. The most recent and substantially improved analysis of the LCSR (12) for the $B^*B\pi$ coupling and (replacing $b\to c$ quark in the correlator) for the $D^*D\pi$ coupling is in Ref. [39], where also the NLO twist-3 contributions were calculated.

In Table 2, the results obtained in Ref. [39] for the $B^*B\pi$ strong coupling are displayed for two choices of the decay constants f_B and f_B^*: from two-point sum rules, and from lattice QCD. Also, two models for the leading twist-2 pion DA are considered. Additional details of this analysis and references relevant for the choice of the input parameters can be found in Ref. [39].

It is important to stress that the coupling $g_{B^*B\pi}$ is calculated from LCSR at a finite b-quark mass. Hence, replacing $b\to c$ in the sum rule provides also the charmed meson coupling $g_{D^*D\pi}$. The infinitely heavy-quark limit of this coupling, known as the static coupling $\hat{g}$, and serving as a key parameter in the Heavy-Meson Chiral Perturbation Theory (HMχPT), can also be obtained from the same LCSR. In Ref. [39] this limit was estimated, together with the inverse heavy mass correction combining the couplings for both heavy mesons and fitting them to the parametrization:

$$g_{H^*H\pi} = \frac{2m_H\,\hat{g}}{f_\pi}\left(1+\frac{\delta}{m_H}\right), \quad (H=D,\,B). \tag{13}$$

The results are given in Table 2.

2.2 The $B_{(s)}$-meson transitions to various light mesons

Analogously to the $B\to\pi$ form factors, the other $B_{(s)}\to P$ form factors ($P=K,\,\eta,\,\eta'$) can be obtained from LCSRs. These form factors are of a particular interest, for an alternative V_{ub} determination, for various rare $B_{(s)}\to P\ell^+\ell^-$ decays, and also for testing the factorization approximation in nonleptonic $B_{(s)}$ decays.

The initial correlator for a $B_{(s)}\to K$ transition is obtained from Eq. (1), replacing the pion state with the kaon state and making necessary changes in the interpolation and transition quark currents. It is also important that LCSRs allow for a complete account of $SU(3)_{fl}$-breaking effects, originating from a nonvanishing s-quark mass and revealing themselves in differences between the kaon and pion DAs, starting from the ratio f_K/f_π as well as in the ratios of other hadronic parameters entering the sum rules, e.g., m_{B_s}/m_B and f_{B_s}/f_B.

The lowest twist-2 DA of a kaon has an expansion in Gegenbauer polynomials similar to Eq. (6), but including also the odd moments $a^K_{1,\,3,\,\ldots}$, in contrast to the pion DA, where due to the G-parity conservation, the odd moments vanish. The set of higher twist kaon DAs are worked out in Ref. [20].

The $B_{(s)} \to K$ form factors were calculated from LCSRs first in Ref. [40] and more recently in Ref. [41], with the same accuracy as for $B \to \pi$ form factors discussed in the previous subsection. The calculation of the $B_{(s)} \to \eta,\, \eta'$ transition form factors is somewhat more complicated due to the $\eta - \eta'$ mixing and a related $U(1)_A$ QCD anomaly contribution to that mixing. The first calculation of the $f^+_{B\eta}$ transition form factors at NLO level for the leading twist-2 was done in Ref. [42]. This calculation was further improved in Ref. [43], where also the $B \to \eta'$ transition was considered. The $U(1)_A$ anomaly induces, in addition to flavour-singlet quark-antiquark DA the two-gluon DA which contributes to the $B \to \eta,\, \eta'$ transitions, at NLO level, and has to be taken into account. This introduces additional uncertainty in the calculation since the coefficients in the Gegenbauer expansion of the twist-2 gluon DA are not known. The most recent application of LCSRs to all the $B_{(s)} \to \eta,\, \eta'$ form factors (as well as to the $D_{(s)} \to \eta,\, \eta'$ form factors), at NLO and including two-gluon DAs is in Ref. [44] (see also Ref. [45]). The results for all form factors of the B-meson transitions to pseudoscalar mesons at $q^2 = 0$ are summarized in Table 3.

Extension of the LCSR method to the $B \to V$ form factors, where $V = \rho,\, \omega,\, \phi,\, K^*$, demands a vacuum-to-$V$ correlator which otherwise has the same structure as the vacuum-to-π correlator in Eq. (1). Correspondingly, the OPE is obtained in terms of vector meson distribution amplitudes. Importantly, these DAs are defined neglecting the widths of vector mesons. As an example, one of the two twist-2 DAs of ρ-meson is defined as:

$$\langle \rho(p)|\bar{u}(x)\sigma_{\mu\nu}[x,\,0]d(0)|0\rangle = -if^\perp_\rho(\epsilon^{*(\rho)}_\mu p_\nu - p_\mu \epsilon^{*(\rho)}_\nu)\int_0^1 du\, e^{iup\cdot x}\phi^{(\rho)}_\perp(u), \tag{14}$$

where $\epsilon^{(\rho)}$ is the polarization vector of ρ. The DA $\phi^{(\rho)}_\perp$ corresponds to the transversely polarized ρ-meson and $f^\perp_\rho$ is the transverse decay constant. Note that the shape of this DA is also determined by the Gegenbauer polynomial expansion. A comprehensive analysis of the twist-3 and twist-4 DAs was done, respectively, in Refs. [48] and [49].

The very first LCSRs for $B \to V$ form factors were derived in Ref. [7], where the radiative $B \to V\gamma$ decays were considered. The first application of this method to the semileptonic $B \to \rho\ell\nu$ decay was done, at LO level, in Refs. [8, 50], where also the advantages of the LCSR approach over the three-point QCD sum rules for the calculation of the heavy-to-light transition form factors were discussed in detail. A major update including the NLO twist-2 contributions and extending the method to almost all $B \to V$ channels was made in Ref. [51]. After that, in Ref. [52] the analysis was improved by adding to OPE the NLO twist-3 terms and obtaining LCSRs also for the $B \to \omega$ transition. The most recent update of LCSRs with vector meson DAs for all $B_{(s)} \to V$ form factors, and with OPE based on the results of Ref. [52], can be found in Ref. [53]. In the latter paper, the form factors are also extrapolated to the whole semileptonic decay region after fitting them to the z-expansion. Their results for the $B \to V$ transition form factors at $q^2 = 0$ are summarized in Table 4.

Table 3 The most recent LCSR results for $B_{(s)} \to P$ form factors at $q^2 = 0$. Their full q^2 dependence is given in the corresponding papers

FF	$f^+(0) = f^0(0)$	$f^T(0)$	References
$B \to \pi$	0.297 ± 0.030	0.293 ± 0.028	[46]
	0.301 ± 0.023	0.273 ± 0.021	[41]
$B \to K$	0.395 ± 0.033	0.381 ± 0.027	[41]
$B_s \to K$	0.364 ± 0.026	0.394 ± 0.023	[47]
	0.336 ± 0.023	0.320 ± 0.019	[41]
$B \to \eta$	$0.168^{+0.041}_{-0.047}$	$0.173^{+0.041}_{-0.035}$	[44]
$B \to \eta'$	$0.130^{+0.036}_{-0.032}$	$0.141^{+0.032}_{-0.030}$	
$B_s \to \eta$	$0.212^{+0.015}_{-0.013}$	$0.225^{+0.019}_{-0.014}$	
$B_s \to \eta'$	$0.252^{+0.023}_{-0.020}$	$0.280^{+0.022}_{-0.016}$	

Table 4 The LCSR results for $B_{(s)} \to V$ transition form factors at $q^2 = 0$ from Ref. [53]

FF	$V(0)$	$A_0(0)$ $A_1(0)$ $A_{12}(0)$	$T_1(0) = T_2(0)$ $T_{23}(0)$
$B \to \rho$	0.327 ± 0.031	0.356± 0.042 0.262 ± 0.026 0.297 ± 0.035	0.272 ± 0.026 0.747 ± 0.076
$B \to \omega$	0.304 ± 0.038	0.328 ± 0.048 0.243 ± 0.031 0.270 ± 0.040	0.272 ± 0.026 0.683 ± 0.090
$B \to K^*$	0.341 ± 0.036	0.356 ± 0.046 0.269 ± 0.029 0.256 ± 0.033	0.282 ± 0.031 0.668 ± 0.083
$B_s \to \phi$	0.387 ± 0.033	0.389 ± 0.045 0.296 ± 0.027 0.246 ± 0.029	0.309 ± 0.027 0.676 ± 0.071
$B_s \to K^*$	0.296 ± 0.030	0.314 ± 0.048 0.230 ± 0.025 0.229 ± 0.035	0.239 ± 0.024 0.597 ± 0.076

2.3 Sum rules with dipion distribution amplitudes

The LCSRs for $B \to \rho$, K^* form factors with the vector meson DAs considered in the previous subsection are derived neglecting the total widths of vector mesons. This is certainly a poor approximation for such a broad resonances as $\rho(770)$ and $K^*(892)$. A more comprehensive approach is to consider the $B \to 2\pi$ and $B \to K\pi$ transitions, where ρ and K^* resonances are only a part, albeit dominant, of the dimeson 2π and $K\pi$ states, respectively. The phenomenology of $B \to 2\pi$ form factors was studied in Ref. [54] were one can find all necessary definitions (see also Ref. [55]).

Here, as an example, we consider the $\bar{B}^0 \to \pi^+\pi^0\ell\nu_\ell$ semileptonic transition in which the $\bar{B}^0 \to \rho^+$ form factors are the resonance parts of the $\bar{B}^0 \to \pi^+\pi^0$ form factors. To avoid lengthy formulas, we take only the vector part of the weak $b \to u$ current, which yields a single form factor defined as:

$$i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu b|\bar{B}^0(p)\rangle = -F_\perp(q^2,\, k^2,\, \zeta)\,\frac{4}{\sqrt{k^2\lambda_B}}\, i\epsilon^{\mu\alpha\beta\gamma}\, q_\alpha\, k_{1\beta}\, k_{2\gamma}, \tag{15}$$

where $k^2 = (k_1 + k_2)^2$ is the squared invariant mass of the dipion state and ζ is an additional angular variable. The $B \to 2\pi$ form factor is then expanded in partial waves corresponding to angular momenta $\ell = 1, 3, 5, \ldots$ of the dipion state, yielding a series in Legendre polynomials in the angular variable. Note that the partial waves with $\ell = 0, 2, 4, \ldots$ are forbidden for the $\pi^+\pi^0$ state on symmetry grounds. The $B \to \rho$ form factor contributes only to the $\ell = 1$ component $F_\perp^{(\ell=1)}(q^2,\, k^2)$ of the $B \to 2\pi$ form factor via dispersion relation in the variable p^2 at fixed q^2:

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}(q^2,\, k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}\,\frac{V^{B\to\rho}(q^2)}{m_B + m_\rho} + \cdots, \tag{16}$$

where $g_{\rho\pi\pi}$ is the strong $\rho\pi\pi$ coupling. In the above relation, the energy-dependent width is inserted in the Breit-Wigner formula and contributions of excited states, such as the $\rho(1450)$ resonance, are indicated by the ellipsis. Although dispersion relation by itself is model-independent and follows from analyticity and unitarity principles, a certain model-dependence is unavoidable in the resonance term. In any case, this relation is the only realistic possibility to extract the $B \to \rho$ form factor from the $B \to 2\pi$ form factor, taking into account not only the ρ width effect, but also the nonresonant background. The latter emerges from the continuum and excited state contributions hidden under the ellipsis in Eq. (16).

To access the $B \to 2\pi$ form factors directly, the method of LCSRs, resembling the one presented in the previous subsections, was suggested in Ref. [56] (see also Ref. [57]). A correlator similar to Eq. (1) was used in which, instead of a single meson state, there is an on-shell state of two pions with a variable invariant mass squared $k^2 \geq 4m_\pi^2$. Applying light-cone OPE yields an expression with a structure resembling Eq. (3). But in this case, the perturbative kernels with a virtual b-quark are convoluted with the vacuum-to-dipion matrix elements of quark-antiquark or quark-antiquark-gluon operators. These matrix elements are parametrized in terms of a set of new objects, the dipion DAs. The latter were introduced and used much earlier [58–62], to describe hard exclusive processes with dimeson states, such as $\gamma^*\gamma \to 2\pi$.

The dipion DAs are also classified by their twist and by the multiplicity of quark and gluon fields. Currently, only the most important two-particle (quark-antiquark) DAs of twist-2 are available. One of them is defined as:

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,\,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int\limits_0^1 du e^{iu(k\cdot x)}\Phi_\parallel^{I=1}(u,\,\zeta,\,k^2), \tag{17}$$

and the second one denoted as $\Phi_\perp^{I=1}$ has a $\sigma_{\mu\nu}$ Dirac structure between quark fields. The index $I = 1$ reflects the isospin of the $\pi^+\pi^0$ state. Full definitions and many important properties of these DAs can be found in Ref. [61]. Both DAs undergo a double expansion in Legendre polynomials—i.e. in partial waves of the dipion state—and in Gegenbauer polynomials. The latter expansion reflects the momentum distribution between quark and antiquark and has the same form as Eq. (6). The coefficients of this double expansion replace Gegenbauer moments a_{2n} in Eq. (6) and are complex valued functions of k^2, with the phase reflecting strong rescattering of the final-state pions. The local limit $x \to 0$ of the matrix element (17) in the isospin symmetry limit, is proportional to the pion electromagnetic form factor in the timelike region which is well measured. However, for the second twist-2 DA this normalization coincides with the timelike form factor of the tensor current which is not directly measurable. A calculation of this hadronic parameter together with a few lowest Gegenbauer functions for both dipion DAs was only performed at small k^2 in Ref. [63], employing the instanton vacuum model of QCD.

The OPE of the correlator for the $\bar{B} \to \pi^+\pi^0$ form factors was obtained in Refs. [56, 57] with a twist-2 accuracy and in the LO. This expansion is valid at sufficiently small dipion invariant masses, $k^2 \ll m_b^2$ and, simultaneously, in the large recoil region $q^2 \ll m_b^2$. The rest of the LCSR derivation is essentially the same as for the sum rules with a single light-meson DAs discussed in Sect. 2.1. In particular, the same hadronic dispersion relation and duality in the channel of the B-meson interpolating current are used. In the resulting LCSRs, the partial wave components $F_\perp^{(\ell)}(q^2,\,k^2)$ with $\ell = 1,\,3,\,..$ are separated from each other. As shown in Ref. [56] in detail, the sum rule for $F_\perp^{(\ell=1)}$, together with its analogs for the $B \to 2\pi$ form factors of the axial weak current, determine the proportion of the $B \to \rho$ channel in the general $B \to 2\pi$ transition. In addition, the ratios of the form factors with $\ell > 1$ with respect to the lowest one with $\ell = 1$ were estimated.

The method of LCSRs with dipion DAs has a considerable potential for further improvement. To increase the precision, one needs detailed studies of the twist expansion for dipion DAs, including the three-particle (quark-antiquark-gluon) DAs. On the other hand, a better knowledge of Gegenbaeur functions for the leading twist-2 DA is necessary. A possibility to gain some information on these universal functions from the $D \to 2\pi\ell\nu$ decays (using the $b \to c$ replacement in the LCSRs) is currently being studied [64]. In the future, in order to extend this method to other important form factors, DAs for the different states of two light pseudoscalar mesons should also be studied. Most important are the dipion states with the spin-parities $J^P = 0^+,\,2^+$ relevant for $B \to \pi^+\pi^-$ form factors, as well as the $K\pi$ and $K\bar{K}$ states needed for the $B \to K^*$ and $B \to \phi$ form factors, respectively. In this respect, let us mention an earlier paper [65] where the form factors of B transitions to the scalar dimeson ($K\pi$ and 2π) were obtained from LCSRs.

2.4 Uses of LCSR form factors in the Standard Model tests

The B-meson transition form factors obtained from the LCSRs with light-meson DAs are mainly used to determine the modulus of the CKM matrix element V_{ub} from the data on exclusive semileptonic $b \to u\ell\nu_\ell$ decays, predominantly using the $B \to \pi\ell\nu_\ell$ decay, but also the $B_s \to K\ell\nu_\ell$, $B \to \rho\ell\nu_\ell$ and $B \to \omega\ell\nu_\ell$ decays.

One way to extract $|V_{ub}|$ is to use the LCSR result for the $B \to \pi$ form factor $f^+_{B\pi}(q^2)$ and integrate the predicted differential $B \to \pi\ell\nu_\ell$ width ($\ell = e,\,\mu$) over the LCSR validity region $0 < q^2 < q^2_{max}$ (see e.g. Refs. [34, 41]). The other way is to extrapolate the form factor up to the zero recoil point of the pion, $q^2 = (m_B - m_\pi)^2$ using the z-parametrization, e.g. the one in Ref. [35]. A combined fit to both LCSR and lattice QCD predictions can also be performed, to achieve the theoretically most accurate form factor in the whole semileptonic region of q^2.

The recent determination of $|V_{ub}|$ using the combined approach to the $B \to \pi$ form factors was performed in Ref. [46]. An independent extraction of $|V_{ub}|$ from $B \to \rho(\omega)\ell\nu_\ell$ decays was performed in Ref. [66] using the LCSR form factors from Ref. [53]. In Ref. [67], the channels with vector mesons were added to the combined analysis of the $B \to \pi\ell\nu_\ell$ decay, employing an advanced statistical tool [68]. Separate process-specific $|V_{ub}|$ values were obtained: $|V_{ub}|_{B\to\pi}= (3.79 \pm 0.15)\cdot 10^{-3}$, $|V_{ub}|_{B\to\rho}= (2.92^{+0.28}_{-0.25})\cdot 10^{-3}$, $|V_{ub}|_{B\to\omega}= (3.00^{+0.38}_{-0.32})\cdot 10^{-3}$, with an overall average:

$$|V_{ub}|= (3.59^{+0.13}_{-0.12})\cdot 10^{-3}. \tag{18}$$

The observed difference between the $|V_{ub}|$ values extracted using $B \to \pi$ and $B \to \rho(\omega)$ form factors demands further investigation, in particular, an update of LCSRs for $B \to V$ form factors is desirable. The point is that the twist-3 NLO contributions to LCSRs for $B \to V$ form factors are not available in analytical form and should be completely recalculated and reassessed [69]. It would also be useful to quantify the nonresonant background for $B \to \rho(\omega)\ell\nu_\ell$ decays, along the lines presented in Ref. [56] as already discussed in Sect. 2.3.

Returning to the CKM matrix elements, quite recently, the combined analysis of the $B_s \to K$ form factors was done in Ref. [47] and used to extract the ratio $|V_{ub}/V_{cb}|$ from the data on the $B_s \to K\ell\nu_\ell$ and $B_s \to D_s\ell\nu_\ell$ decays. A similar analysis was also done in Ref. [70].

The $B \to K^*$ form factors obtained from LCSRs were also extensively used in the exploration of various observables in the $B \to K^*\ell^+\ell^-$ and $B \to K^*\gamma$ FCNC decays. Already in Ref. [71] and after that in Ref. [53] these observables were extended from SM to various models of new physics. An independent determination of Wolfenstein parameters of CKM matrix from a combination of observables in $B_{(s)} \to P\ell^+\ell^-$ decays was suggested in Ref. [41].

In the current tests of the lepton flavour universality (LFU) in semileptonic B decays, the LCSR results for the form factors, combined with the lattice QCD results, were also employed. The main goal was to obtain predictions of LFU ratios, such as R_π in Ref. [46] or R_ρ and R_ω in Ref. [66], where also a possible influence of new physics on the polarizations and asymmetries in $B \to (\rho,\, \omega)\ell\nu_\ell$ decays was examined. Note that for LFU tests involving $B \to P\tau\nu_\tau$ decays, the scalar form factors $f^0_{BP}(q^2)$ are essential. The latter ones, as well as the tensor form factors $f^T_{BP}(q^2)$, are not always available from the lattice QCD, enhancing even more the importance of their LCSR calculations.

Based on the form factors extracted from LCSRs, in Ref. [67] also the effects of possible new physics were analysed in the framework of the Weak Effective Theory (WET). Similarly, the LCSR results for $B \to \pi,\, \rho,\, \omega$ form factors from Refs [18] and [53] were used in Ref. [72] to examine new physics interpretation in the Standard Model Effective Field Theory (SMEFT).

3 Sum rules with B-meson distribution amplitudes

3.1 An alternative method for the B-meson form factors

The underlying idea of this version of LCSRs suggested in Ref. [73] and developed further in Ref. [74] was to swap the meson state and interpolating current in the initial correlator. E.g., for the $B \to \pi$ transition one has to consider, instead of Eq. (1), the B-to-vacuum correlator with the pion interpolating current:

$$F^{(B)}_{\mu\nu}(p,\, q) = i\int d^4x\; e^{ip\cdot x}\langle 0|T\big\{\bar{d}(x)\gamma_\mu\gamma_5 u(x),\, \bar{u}(0)\gamma_\nu b(0)\big\}|\bar{B}(p+q)\rangle. \tag{19}$$

This opens up a possibility to obtain form factors of B-meson transitions into any light or charmed meson by just switching from one interpolating current to another, and, correspondingly, applying quark-hadron duality in that channel. There is no need to introduce a separate set of DAs for a particular light meson. New nonperturbative objects that emerge in the OPE of the correlator (19), after contracting the virtual u-quark fields, are the universal DAs of B meson. However, these DAs can only be systematically defined in the infinite heavy-quark mass limit, in the framework of heavy-quark effective theory (HQET). Hence, we have to replace the B-meson state in the correlator by a state B_v with a definite velocity $v = (p + q)/m_B$, and the b-quark field by an effective HQET field h_v.

The leading twist-2 and subleading twist-3 B-meson DAs,[3] denoted, respectively as ϕ^B_+ and ϕ^B_- have been defined originally in Ref. [76] (see also Ref. [77] and the review [78]):

[3] Note that the concept of twist for the B meson DAs introduced and explained in detail in Ref. [75] differs from the one for the light-meson DAs.

$$\langle 0|\bar{d}_\beta(x)[x,0]h_{v,\,\alpha}(0)|\bar{B}_v\rangle = -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega\, e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\alpha\beta}, \tag{20}$$

where the normalization constant is taken equal to the physical B-meson decay constant (as usually done in LCSRs at LO considered in this subsection). The HQET distribution amplitudes also serve as an indispensable ingredient for the theory descriptions of exclusive B-meson decay matrix elements in the QCD factorization framework [79].

The LCSR derivation starting from the HQET limit of the correlator (19) basically repeats the procedure described in Sect. 2. Hadronic dispersion relation of the correlator in the pion channel (in the invariant variable p^2) is now used, and one has to isolate the pion pole contribution with the help of duality approximation, introducing the effective threshold s_0^π. The resulting sum rule for the $B \to \pi$ vector form factor has a surprisingly simple expression in the lowest twist-3 approximation, after Borel transformation $p^2 \to M^2$:

$$f^+_{B\pi}(0) = \frac{f_B}{f_\pi\, m_B}\int\limits_0^{s_0^\pi} ds e^{-s/M^2}\phi_-^B(s/m_B). \tag{21}$$

The r.h.s. of this sum rule becomes significantly more involved after adding higher-twist (power suppressed) contributions, including those from the quark-antiquark-gluon B-meson DAs. The first calculations for $B \to \pi$ and $B \to K,\, \rho,\, K^*$ form factors in Ref. [74] were superseded by more recent results for these form factors in Refs. [80] and [81]. In both analyses the set of higher-twist DAs worked out in Ref. [75] was taken into account. Note that there is an important difference in the achieved accuracy: in Ref. [80] both two- and three-particle DAs were included (the latter up to twist-six level) whereas in Ref. [81] only the two-particle twist-five contributions were taken into account, which is not consistent from the point of view of HQET equations of motion [75] (see Refs. [82, 83] for further discussions).

The LCSRs described in this section were obtained in the LO, that is, at the zeroth order in α_s, albeit with the corrections beyond the leading power (LP). Therefore, renormalization and scale dependence for B-meson DAs, with their specific evolution discovered in Ref. [84] and discussed in some detail in the next subsection are not fully used here. As a result, the accuracy of OPE in these sum rules is still less than for the LCSRs with light-meson DAs. Also the key parameter of leading-twist DA—the inverse moment $1/\lambda_B = \int_0^\infty d\omega\phi_+^B(\omega)/\omega$—is not yet determined with a sufficient accuracy, in particular, this moment is not yet accessible in lattice QCD (see, however, [85] for interesting discussions in this respect). The input interval of this parameter employed in LCSRs for $B_{(s)}$ meson is usually taken from the two-point QCD sum rules worked out in [86] (see also Ref. [87] and other independent estimates in [88, 89]). The behaviour of the DAs at $\omega \to 0$ is model-independent [75]. The actual form of these DAs at large ω is unimportant at tree level, since in the sum rules such as Eq. (21) the duality interval cuts off the region $\omega > s_0^\pi/m_B$. One popular choice of ω-dependence is the exponential model from Ref. [76] (see also Refs. [84, 90, 91]).

The two additional parameters determining all higher twist DAs in this model are λ_B and λ_H, parametrizing the quark-antiquark-gluon B-to-vacuum matrix elements in HQET. Their estimates from independent two-point sum rules [92, 93] still yield large uncertainty intervals.

Numerically, the form factors of B meson transitions to light mesons obtained from the LCSRs with B meson DAs (see the Tables in the next subsection) agree with the ones calculated from the sum rules with light-meson DAs, but only within their larger uncertainties. Apart from narrowing down the uncertainty intervals of the inputs, there are two important questions to be addressed for a further improvement of this version of LCSRs. The first question, already raised in Ref. [74], is the the size of inverse heavy-quark mass effects which are implicitly neglected when one starts from the correlation function (19) in full QCD with an on-shell B-meson state and then takes the HQET limit of that state. A pragmatic argument in favour of smallness of such correction is a relatively good agreement between form factors obtained with both LCSR methods with light and B-meson DAs. The second question is the size of NLO, $O(\alpha_s)$ corrections to the correlator (19). The answer is found using an alternative formulation of this method to be discussed below.

Finally, we would like to comment on the possibility to extend the method of LCSR with B-meson DAs to D-meson semileptonic form factors, introducing the D-meson DAs. This however implies using HQET for the c-quark and D-meson, hence, considerably limiting the accuracy in phenomenological applications to the D-meson decays.

3.2 Accessing the next-to-leading order with SCET

The method of LCSRs with B-meson DAs was independently formulated [94, 95] in the framework of soft-collinear effective theory (SCET) (see [96] where this theory is introduced in the context of heavy-to-light transitions). To derive the SCET sum rules for our standard example—the $B \to \pi$ form factors—the following definition of the correlator is used (see e.g. [88]):

$$\mathcal{F}_\mu^{(B)}(n\cdot p,\, \bar n\cdot p) = \int d^4x\, e^{ip\cdot x} \langle 0|\mathrm{T}\{\bar d(x)\, \not{n}\gamma_5\, u(x),\ \bar u(0)\,\gamma_\mu\, b(0)\}|\bar B(p_B)\rangle$$
$$= \mathcal{F}^{(B)}(n\cdot p,\, \bar n\cdot p)\, n_\mu + \tilde{\mathcal{F}}^{(B)}(n\cdot p,\, \bar n\cdot p)\, \bar n_\mu, \tag{22}$$

where in the B-meson rest frame the two light-cone vectors n_μ and $\bar n_\mu$ are introduced, such that $n\cdot v = \bar n\cdot v = 1$, $v_\perp = 0$ and $n\cdot\bar n = 2$, and a power-counting scheme for the four-momentum of the pion interpolation current is employed:

$$n\cdot p \sim \mathcal{O}(m_b), \qquad \bar n\cdot p \sim \mathcal{O}(\Lambda_{\mathrm{QCD}}). \tag{23}$$

In this scheme, a generic momentum $P_\mu \equiv (n\cdot P,\, \bar n\cdot P,\, P_\perp)$ is split into the three different momentum modes: hard (h), hard-collinear (hc) and soft (s), with the scaling behavior, respectively, $P_{h,\,\mu} \sim \mathcal{O}(1,\,1,\,1)$, $P_{hc,\,\mu} \sim \mathcal{O}(1,\, \lambda,\, \lambda^{1/2})$, $P_{s,\,\mu} \sim \mathcal{O}(\lambda,\,\lambda,\,\lambda)$. The expansion parameter λ scales as Λ/m_b where Λ is a typical hadronic scale. The momentum transfer q in the large and intermediate recoil region of the pion (accessible to LCSRs) belongs to the hard or hard-collinear mode.

The OPE for a B-to-vacuum correlation function such as the one in Eq. (22) is directly calculated in terms of SCET diagrams. In the leading, zeroth order in α_s the resulting LCSRs in SCET are fully equivalent to the sum rules discussed in the previous subsection. The advantages of SCET are revealed by going beyond the LO approximation, starting from the NLO with one-loop gluon radiative corrections taken into account in the correlator (22). Application of SCET enables one to effectively resum large logarithms emerging in the heavy quark limit and to extend the factorization of the OPE expression to the NLO level. In practice, achieving NLO accuracy is accomplished by invoking a matching procedure, with a two-step transition QCD $\to$ SCET$_{\mathrm{I}}$ $\to$ SCET$_{\mathrm{II}}$. The details of these "embedded" effective theories and their correspondence to the one-loop diagrams are discussed already in Refs. [94, 95]. Further development of both calculational and conceptual aspects of this procedure can be found in Refs. [80, 83, 88, 97–99]. Here we only quote the schematic form of the LP factorization formula for the particular invariant amplitude $\mathcal{F}^{(B)}$ [88, 99]

$$\mathcal{F}^{(B)} = \tilde f_B(\mu)\, m_B \sum_{k=\pm} \mathcal{C}^{(k)}(n\cdot p,\, \mu)\int_0^\infty \frac{d\omega}{\omega - \bar n\cdot p - i0}\; \mathcal{J}^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega},\, \frac{\omega}{\bar n\cdot p}\right)\, \phi_k^B(\omega,\, \mu), \tag{24}$$

where $\tilde f_B(\mu)$ is the HQET decay constant of B-meson. In this formula, $\mathcal{C}^{(k)}$ and $\mathcal{J}^{(k)}$ are, respectively, the hard and jet functions, both stemming from the matching procedure. The B-meson DAs absorbing soft contributions in this formula are, up to the scale dependence, the same as the ones defined in the previous subsection.

Achieving the NLO level in LCSRs demands including into the computational scheme the scale dependence of the B-meson DAs, stemming from the evolution equations. In this direction, there was a lot of progress in recent years. The renormalization-group (RG) evolution equation for $\phi_+^B(\omega,\, \mu)$ determined at one loop [84], was upgraded to the two loops [100, 101]. An explicit solution of this evolution equation was constructed with an integral transform method in Ref. [102] and, independently, with the conformal symmetry technique in Ref. [103] (see also Ref. [104] for the analytical solution of the two-loop RG equation). In addition, the one-loop evolution equation for the twist-three distribution amplitude $\phi_-^B(\omega,\, \mu)$ was constructed within the so-called Wandzura-Wilczek approximation, also including the RG mixing effect generated by the non-vanishing light-quark mass [105]. Then, in Refs. [75, 106], the three-particle higher-twist contribution $\Phi_3(\omega_1,\, \omega_2,\, \mu)$ was included. Furthermore, an explicit solution to the RG equation for $\phi_-^B(\omega,\, \mu)$ was found [107] by exploring the "hidden" symmetries of the evolution kernel of Φ_3 in the large N_c limit.

Employing the RG equations for the HQET B-meson DAs and for the perturbative matching functions, it became possible [80, 88, 99] to carry out an all-order resummation of the parametrically enhanced logarithms appearing in the soft-collinear factorization formulae of the B-meson-to-vacuum correlation functions. After the resummation and all subsequent standard steps in the derivation are done, the LCSR in SCET in the next-to-leading-logarithmic (NLL) approximation is obtained, valid in the large recoil region. For the vector $B \to P$ form

Table 5 The LCSR results for $B_{(s)} \to P$ transition form factors at $q^2 = 0$ from LCSRs with B-meson DAs

FF	$f^+(0) = f^0(0)$	$f^T(0)$	References
$B \to \pi$	0.191 ± 0.073	0.222 ± 0.078	[99]
	0.21 ± 0.07	0.19 ± 0.06	[81]
$B \to K$	0.325 ± 0.085	0.381 ± 0.097	[99]
	0.27 ± 0.08	0.25 ± 0.07	[81]
$B_s \to K$	0.203 ± 0.074	0.260 ± 0.087	[99]

factor f^+_{BP}, $(P = \pi,\, K)$ it has the following form:

$$f_P \, \exp\left[-\frac{m_P^2}{n \cdot p \, \omega_M}\right] \frac{n \cdot p}{m_B} \, f^+_{BP}(q^2) = \left[\hat{U}_2(\mu_{h2},\, \mu)\, \mathcal{F}_B(\mu_{h2})\right] \int_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M}$$
$$\times \left\{ \widetilde{\mathbf{\Phi}}_+^{B,\,\mathrm{eff}}(\omega',\, \mu) + \left[\hat{U}_1(n \cdot p,\, \mu_{h1},\, \mu)\, \widetilde{\mathcal{C}}^{(-)}(n \cdot p,\, \mu_{h1})\right] \widetilde{\mathbf{\Phi}}_-^{B,\,\mathrm{eff}}(\omega',\, \mu) \right.$$
$$\left. + \frac{n \cdot p - m_B}{m_B} \left[\mathbf{\Phi}_+^{B,\,\mathrm{eff}}(\omega',\, \mu) + \mathcal{C}^{(-)}(n \cdot p,\, \mu_{h1})\, \mathbf{\Phi}_-^{B,\,\mathrm{eff}}(\omega',\, \mu)\right] \right\} + \mathcal{O}(\alpha_s^2,\ \Lambda/m_b), \tag{25}$$

where ω_M is the Borel parameter, $\hat{U}_{1,\,2}$ are the evolution functions for for the RG improved hard functions $\mathcal{C}^{(-)}$ and $\widetilde{\mathcal{C}}^{(-)}$. The functions $\mathbf{\Phi}^{\pm}_{B,\,\mathrm{eff}}$ and $\widetilde{\mathbf{\Phi}}^{\pm}_{B,\,\mathrm{eff}}$ are the effective "distribution amplitudes" which encode both the hard-collinear and soft strong interaction dynamics. The explicit expressions for all these functions and further explanatory discussions concerning the formula (25) can be found in Ref. [99]. In the same work, a comprehensive analysis of the power-suppressed contributions to the $B \to \pi$ form factors in the combined LCSR/SCET framework was accomplished, applying non-trivial operator identities due to the HQET equations of motion [75, 108]. Four different sources of the subleading power corrections at tree level were included: i) the higher-order terms from heavy-quark expansion of the hard-collinear quark propagator, ii) the subleading power corrections from the effective matrix element of the SCET$_\mathrm{I}$ weak current, iii) the higher-twist corrections from the two-particle and three-particle HQET distribution amplitudes at twist-six, and (iv) the four-particle twist-five and twist-six contributions in the factorization approximation.

Extension of these analyses to NLO ($\mathcal{O}(\alpha_s)$) accuracy is currently in progress [109], bearing in mind that the higher-twist three-particle HQET DAs also generate LP contributions to the heavy-to-light form factors at $\mathcal{O}(\alpha_s)$ [96]. The one-loop QCD corrections to the short-distance matching coefficients in the SCET sum rules for the $B_{d,\,s} \to \pi,\, K$ form factors at the LP accuracy can shift the corresponding tree-level predictions by an amount of $\mathcal{O}(30\%)$ numerically [99]. Moreover, after these sum rules are improved by the NLL resummation, the perturbative uncertainties from varying the hard and hard-collinear matching scales are considerably pinned down.

The SCET sum rule method with the B-meson DAs is easily extendable to the semileptonic $B \to V$ form factors (with $V = \rho,\ \omega,\ K^*$) at large hadronic recoil, as shown already in Ref. [95]. In the further work [83] it was demonstrated that the active light-quark mass corrections can generate unsuppressed contributions to the longitudinal $B \to V$ form factors in the heavy quark expansion, confirming the earlier observation from the power-counting analysis [110]. The subleading power corrections to these form factors from the two-particle and three-particle HQET DAs fulfilling the "classical" operator identities for the light-cone HQET operators were computed with the same twist-six accuracy as for the $B \to P$ form factors.

Importantly, the two-particle twist-five (off-the-light-cone) contributions to the semileptonic $B \to V$ form factors yield sizeable numerical corrections at the level of (20–30)% of the LP results. Moreover, the long-standing discrepancy between the form-factor ratio $\mathcal{R} = [(m_B + m_V)/m_B]\left[T_1(q^2)/V(q^2)\right]$ predicted using the conventional LCSR method with the light vector meson DAs [52] and the same ratio obtained from the QCD factorization [77] has been clarified in Ref. [83], where a detailed explanation can be found.

Finally, in Tables 5 and 6 we collect the most recent predictions for the form factors of B-meson transitions to light mesons at $q^2 = 0$ obtained from the SCET sum rules at NLO and from the LCSRs at LO. The q^2 dependence is given in the papers quoted in these Tables.

3.3 Obtaining LCSRs for the $B \to 2\pi$ and $B \to K\pi$ form factors

As already explained in Sect. 2.3, an accurate description of semileptonic B decays into broad resonances such as $\rho(770)$ or $K^*(892)$ demands a calculation of the more general $B \to 2\pi$ or, respectively, $B \to K\pi$ form factors.

Table 6 The results for $B \to V$ transition form factors at $q^2 = 0$ from LCSRs with B-meson DAs

FF (Ref. [83])	$V(0)$	$A_0(0)$ $A_1(0)$ $A_{12}(0)$	$T_1(0) = T_2(0)$ $T_{23}(0)$
$B \to \rho$	$0.327^{+0.204}_{-0.135}$	$0.317^{+0.129}_{-0.102}$	$0.287^{+0.180}_{-0.118}$
		$0.249^{+0.155}_{-0.103}$	$0.711^{+0.356}_{-0.250}$
		$0.265^{+0.107}_{-0.086}$	
$B \to \omega$	$0.357^{+0.223}_{-0.148}$	$0.344^{+0.142}_{-0.107}$	$0.312^{+0.197}_{-0.129}$
		$0.270^{+0.170}_{-0.111}$	$0.767^{+0.407}_{-0.266}$
		$0.284^{+0.116}_{-0.087}$	
$B \to K^*$	$0.419^{+0.245}_{-0.157}$	$0.382^{+0.154}_{-0.109}$	$0.361^{+0.211}_{-0.135}$
		$0.306^{+0.180}_{-0.115}$	$0.793^{+0.402}_{-0.258}$
		$0.273^{+0.112}_{-0.077}$	
FF (Ref. [81])	$V(0)$	$A_1(0)$ $A_2(0)$	$T_1(0) = T_2(0)$ $T_{23}(0)$
$B \to \rho$	0.27 ± 0.14	0.22 ± 0.10	0.24 ± 0.12
		0.19 ± 0.11	0.56 ± 0.15
$B \to K^*$	0.33 ± 0.11	0.26 ± 0.08	0.29 ± 0.10
		0.24 ± 0.09	0.58 ± 0.13

Since the method of LCSRs with dipion DAs presented in Sect. 2.3 is not yet accurate enough and since the DAs of the $K\pi$ dimeson state are currently not available, an alternative is to use the LCSRs with B-meson DAs. With this method, it is possible to choose an interpolating light-quark current for an arbitrary state of two pseudoscalar mesons with a given spin-parity and flavour. The first such LCSRs were derived in Ref. [111] for the $\bar{B}^0 \to \pi^+\pi^0$ form factors. The same correlator was used as the one in Eq. (19) introduced originally [74] for the $B \to \rho$ form factor. The idea was to replace the narrow ρ resonance approximation in the channel of interpolating current by a more general intermediate state of two pions staring from the threshold $4m_\pi^2$. The $\pi\pi$ state was inserted in the spectral density and the quark-hadron duality interval $s_0^{2\pi}$ was determined separately, employing the two-point QCD sum rule with the same interpolating currents.

For the P-wave $\bar{B}^0 \to \pi^+\pi^0$ form factor of the vector weak current defined in Eq. (15) the resulting sum rule, after applying duality approximation and Borel transform, has the following expression:

$$\int\limits_{4m_\pi^2}^{s_0^{2\pi}} ds e^{-s/M^2} \kappa_\perp(s, q^2) F_\pi^\star(s) F_\perp^{(\ell=1)}(s, q^2) = f_B m_B \int\limits_0^{\sigma_0^{2\pi}(s_0^{2\pi})} d\sigma e^{-s(\sigma, q^2)/M^2} \frac{\phi_+^B(\sigma m_B)}{1-\sigma} + \cdots, \tag{26}$$

where $\kappa_\perp(s, q^2)$ is a kinematical factor, $F_\pi(s)$ is the pion e.m. form factor in the timelike region. On r.h.s. of the above relation only the leading power contribution of the twist-2 B-meson DA is shown, and the ellipsis denotes power suppressed contributions of higher twist DAs, including the three-particle ones. Since the above sum rule is obtained only at LO, a default scale $\mu \sim M$ is implied for the twist-2 DA. Note that in Eq. (26) the form factor $F_\perp^{(\ell=1)}$ is integrated over the dipion invariant mass. Hence, its direct calculation is only possible in the approximation of a single narrow ρ-resonance, in which case the LCSR for the $B \to \rho$ form factor already considered in Sect. 3.2 is simply restored. To study the effects of the ρ width and to assess the role of the nonresonant background, in Ref. [111] different representations for the $B \to 2\pi$ form factors with $\ell = 1$ with various resonance content were substituted in the sum rule (26), and the parameters were fitted to the r.h.s. calculated from OPE. For the pion e.m. form factor a data-driven resonance representation was used. The main outcome of the numerical analysis was that the cumulative effect of the ρ width and of the nonresonant contributions can alter the $B \to \rho$ form factor calculated in the narrow ρ-width approximation by an appreciable (15–20)% correction. The results are also consistent with the ones from Ref. [56] (see Sect. 2.3) where dipion DAs were used in the LCSR for the same form factor.

The LCSR method was also applied in Ref. [112] and in Ref. [113] for the $B \to K\pi$ form factors with the $K\pi$ state, respectively in the P- and S-wave. These form factors are the most important hadronic inputs for the analysis of the FCNC $B \to K\pi\ell^+\ell^-$ decays. Note that for the S-wave case in Ref. [113] a nontrivial data-driven representation of the $K\pi$ scalar form factors was used in the sum rule relations, whereas in the P-wave case it was sufficient to use a simple overlap of Breit-Wigner resonances.

3.4 The B-meson transitions to charmed mesons

The $B \to D$ and $B \to D^*$ form factors essential for the observables in $B \to D^{(*)}\ell\nu_\ell$ decays are among the best studied hadronic objects in heavy flavour physics, due to powerful HQET methods allowing one to reduce these form factors to universal Isgur-Wise functions. In addition, advanced lattice QCD computations of these form factors are available. However, the region of momentum transfers beyond the zero-recoil point remains largely unexplored and relies on the model-dependent extrapolations of HQET form factors. Reliable estimates of power-suppressed $\sim 1/m_c$ effects in this region are therefore needed.

The method of LCSRs with B-meson DAs is well suited for the calculation of the $B \to D^{(*)}$ form factors in the large recoil region, as shown first in Ref. [114]. Here again we benefit from the universality of this method. Replacing in the underlying correlator (19) the light quark by the c-quark in both weak and interpolating currents, we repeat all main steps of LCSR derivation and arrive at the sum rule for a certain $B \to D$ or $B \to D^*$ form factor. A possibility to assess the $1/m_c$ effects is opened up by expanding the LCSRs in powers of the inverse c-quark mass and comparing the results with the HQET form factors.

In Ref. [81] the LCSRs for the $B \to D$ and $B \to D^*$ form factors were further improved and updated taking into account a more complete set of higher-twist B meson DAs established in Ref. [75]. Furthermore, in Refs. [97, 115] the SCET sum rules with B-meson DAs were applied to the $B \to D^{(*)}$ transition form factors and NLO corrections were computed. In addition in Refs. [98, 115] a variety of the subleading power contributions to these form factors including the higher-twist corrections have been taken into account in these sum rules. The LCSR results were used for the $|V_{cb}|$ determination and for predicting the LFU sensitive ratios $\mathcal{R}(D^{(*)}_{(s)})$.

In Table 7 we collect the $B_{(s)} \to D^{(*)}_{(s)}$ form factors from the most recent analysis of SCET sum rules from Ref. [115] and compared them with the results [81] of conventional LCSRs at LO and with a different treatment of higher-twist effects.

The method of LCSRs with B-meson DAs was also recently used to calculate the form factors of the semileptonic B decays into charmed axial D_1^* and scalar D_0^* mesons, in Refs. [116] and [117], respectively. The derivation of these sum rules, being straightforward on the OPE side (due to a simple replacement of the spin-parity of the interpolation current), turned out nontrivial on the hadronic side. In the case of the $B \to D_1^*$ form factors, a specially designed procedure with a second interpolation current was needed, because the two lowest charmed axial mesons have almost the same mass. Note that, as explained and taken into account in Ref. [117], for the $B \to D_0^*$ transition, one currently faces a problem to choose between two alternative mass patterns of these mesons inferred from the data.

Table 7 $B_{(s)} \to D^{(*)}_{(s)}$ form factors at $q^2 = 0$ from the LCSRs with B-meson DAs

FF	$f^+(0) = f^0(0)$	V(0)	$A_0(0)$ $A_1(0)$ $A_2(0)$	References
$B \to D^{(*)}$	0.586 ± 0.103	0.703 ± 0.160	0.623 ± 0.112 0.704 ± 0.119 0.803 ± 0.186	[115]
	0.65 ± 0.08	0.69 ± 0.13	– 0.60 ± 0.09 0.51 ± 0.09	[81]
$B_s \to D_s^{(*)}$	0.572 ± 0.115	0.671 ± 0.168	0.582 ± 0.120 0.661 ± 0.117 0.765 ± 0.163	[115]

4 Radiative leptonic B decay and related channels

4.1 Power suppressed effects in $B \to \gamma\ell\bar{\nu}_\ell$

At a large energy of the photon, the hadronic amplitude of the radiative leptonic $B \to \gamma\ell\bar{\nu}_\ell$ decay is described by a well studied factorization formula in QCD [118, 119], reformulated in SCET [120, 121]. The LCSR methods play an important role in this analysis, quantifying the power corrections to the form factors of this decay. The inverse moment λ_B of the B-meson DA, discussed in the previous section, directly enters the LP of the factorization formula in the heavy b-quark limit. Therefore, a measurement of the $B \to \gamma\ell\bar{\nu}_\ell$ decay width anticipated at Belle-II [122], will provide us with an accurate value of this nonperturbative parameter.

The hadronic part of the $B \to \gamma\ell\bar{\nu}_\ell$ amplitude is described by the nonlocal B-to-vacuum matrix element:

$$\begin{aligned} T_{\mu\nu}(p,\,q) &= -\,i\int d^4x\, e^{ip\cdot x}\,\langle 0|\mathrm{T}\{j_\mu^{\mathrm{em}}(x),\ \bar{u}(0)\gamma_\nu(1-\gamma_5)b(0)\}|B(p+q)\rangle \\ &= \epsilon_{\mu\nu\rho\sigma}\,p^\rho v^\sigma F_V(n\cdot p) + i[-g_{\mu\nu}v\cdot p + v_\mu p_\nu]F_A(n\cdot p) + \ldots, \end{aligned} \tag{27}$$

where j_μ^{em} is the quark electromagnetic current coupled to a real photon with momentum p and correlated with the weak $b \to u$ current with momentum q transferred to the lepton pair. The relevant part of this hadronic tensor contains the two $B \to \gamma$ form factors F_V and F_A. Here the variable q^2 is customarily replaced with $n\cdot p = 2E_\gamma$, and E_γ is the photon energy in the B-meson rest frame. The light-cone vectors are chosen such that $\bar{n}\cdot p = 0$, and $q^2 = m_B\,(m_B - n\cdot p)$.

The soft-collinear factorization formula for the $B \to \gamma$ form factors, at LP of the expansion in $\Lambda_{\mathrm{QCD}}/m_b$, can be cast in the compact form, equal for both form factors:

$$F_{V,\,A}(n{\cdot}p) \equiv F_{\mathrm{LP}}(n{\cdot}p) = \frac{Q_u\,m_B}{n{\cdot}p}\,\tilde{f}_B(\mu)\,\mathcal{C}_\perp(n{\cdot}p,\,\mu)\int_0^{+\infty}\frac{d\omega}{\omega}\,\mathcal{J}_\perp(n{\cdot}p,\,\omega,\,\mu)\,\phi_B^+(\omega,\,\mu), \tag{28}$$

where the u-quark charge Q_u indicates that a photon emission from the light quark dominates the decay amplitude. At the tree level (LO) the product of the hard and jet functions in Eq. (28) is $[\mathcal{C}_\perp\mathcal{J}_\perp]_{\mathrm{LO}} = 1$, and the integral reduces to the desired inverse moment λ_B. The one-loop expressions for the hard function $\mathcal{C}_\perp$ [123] and for the jet function $\mathcal{J}_\perp$ [120, 121] were obtained with the standard SCET technique. The most advanced analysis in Ref. [124] includes RG resummation of the enhanced logarithms of $m_b/\Lambda_{\mathrm{QCD}}$ at the NLL accuracy.

However, the power-suppressed "soft overlap" contributions to the $B \to \gamma$ form factors cannot be estimated within the perturbative factorization framework. To solve this task, a QCD-based method, combining hadronic dispersion relation with OPE and LCSR in terms of B-meson DAs was suggested in Ref. [125], following the technique originally developed for the $\gamma^*\gamma \to \pi^0$ form factor in Ref. [126] (see also Ref. [28] for a further development).

The main idea is to consider the B-to-vacuum hadronic matrix element (27) at spacelike $p^2 < 0$, so that in terms of SCET the electromagnetic current carries a hard-collinear four-momentum. It is then straightforward to derive factorized expressions of the generalized $B \to \gamma^*$ form factors $F_{V,\,A}^{B\to\gamma^*}(n{\cdot}p,\,p^2)$ in the LP approximation [127]:

$$\begin{aligned} F_{V,\,A}^{B\to\gamma^*}(n{\cdot}p,\,p^2) &\equiv F_{\mathrm{LP}}^{B\to\gamma^*}(n{\cdot}p,\,p^2) \\ &= Q_u\,m_B\,\tilde{f}_B(\mu)\,\mathcal{C}_\perp(n{\cdot}p,\,\mu)\int_0^\infty d\omega\,\frac{\mathcal{J}_\perp(n{\cdot}p,\,p^2,\,\omega,\,\mu)\,\phi_B^+(\omega,\,\mu)}{(n{\cdot}p)\omega - p^2}. \end{aligned} \tag{29}$$

On the other hand, both form factors obey an unsubtracted hadronic dispersion relation in the variable p^2. Taking the vector form factor as an example, this relation reads:

$$F_V^{B\to\gamma^*}(n\cdot p,\ p^2) = \frac{f_\rho\,F_{B\to\rho}(n\cdot p)}{m_\rho^2 - p^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds\,\frac{\mathrm{Im}_s\,F_V^{B\to\gamma^*}(n\cdot p,\,s)}{s - p^2}, \tag{30}$$

where the ground-state contributions from ρ and ω are combined into one resonance term with the narrow-width approximation and with the assumption $m_\rho \simeq m_\omega$ [125]. The numerator in this term contains the decay constant of ρ and the function $F_{B\to\rho}$ which is, up to some normalization factor, equal to the usual $B \to \rho$ vector form factor $V^{B\to\rho}(q^2)$.

The rest of the derivation consists of standard elements of the LCSR technique, already presented in the previous section. The relation (30) at $p^2 < 0$ is equated to the result of QCD calculation $F_{\rm LP}^{B\to\gamma^*}$ given by Eq. (29), and the latter is transformed into a form of dispersion integral with imaginary part $\mathrm{Im}F_{\rm LP}^{B\to\gamma^*}(n\cdot p,\, s)$. After that quark-hadron duality is used to replace the integral on r.h.s. of dispersion relation (30) by an integral over the calculated imaginary part $\mathrm{Im}F_{\rm LP}^{B\to\gamma^*}$. Applying then the standard LCSR technique enables us to express the product of f_ρ and the $B\to\rho$ form factor in terms of the subtracted dispersion integral of the QCD spectral density. Returning to the dispersion relation (30), and substituting this LCSR in the resonance term we notice that the limit $p^2\to 0$ can smoothly be taken, resulting in the desired $B\to\gamma$ form factor for which the following expression is obtained [125]:

$$F_V(n\cdot p) = \frac{1}{\pi}\int\limits_0^{s_0} \frac{ds}{m_\rho^2}\mathrm{Im}F_{\rm LP}^{B\to\gamma^*}(n\cdot p,\, s)e^{-(s-m_\rho^2)/M^2} + \frac{1}{\pi}\int\limits_{s_0}^{\infty}\frac{ds}{s}\mathrm{Im}F_{\rm LP}^{B\to\gamma^*}(n\cdot p,\, s), \tag{31}$$

and analogous expression for the axial form factor F_A. Further analysis of these relations allows one to represent it as a sum of the LO contribution and the needed power-suppressed soft overlap correction. In Ref. [82] this approach was further developed, including higher-twist contributions to B-meson DAs.

An alternative LCSR technique to address the power-suppressed effects in the $B\to\gamma\ell\bar\nu_\ell$ amplitude uses a correlation function with the photon DAs and B-meson interpolation current:

$$\widetilde{T}_\nu(p,\, q) = i\int d^4x\, e^{iq\cdot x}\,\langle\gamma(p)|\mathrm{T}\{\bar u(x)\gamma_\nu(1-\gamma_5)b(x),\; m_b\,\bar b(0)i\gamma_5 u(0)\}|0\rangle. \tag{32}$$

The photon DAs emerging after factorizing this correlator describe long-distance photon emission, the so called hadronic component of the photon. This approach should be put in one category with the LCSRs using light-meson DAs, so that here a central role is played by a twist expansion of photon DAs worked out in Ref. [128]. A key nonperturbative parameter in the leading-twist photon DA is the magnetic susceptibility of the quark condensate [129] which describes the response of the QCD vacuum to an external electromagnetic field. A specific feature of the LCSRs with photon DAs is that, apart from the long-distance photon emission encoded in these DAs, the photon emission at short distances described by triangle heavy-light diagrams also contributes to the correlator. Furthermore, these sum rules are universal with respect to heavy flavour, that is, it is possible to switch to the charmed quark correlator with the same OPE and access also the radiative leptonic $D\to\gamma\ell\nu_\ell$ decay. At leading order the LCSRs with photon DAs were obtained in the early papers [130–132], with an NLO improvement in more recent works [133, 134]. Technically involved NLO gluon radiative corrections to the point-like photon contribution were only recently computed in Ref. [135].

The four-body leptonic $B\to\mu\bar\mu\ell\bar\nu_\ell$ decay with three charged leptons in the final state (free of the helicity suppression) belongs to the rare B-meson decay channels accessible at hadron collider, as opposed to the two-body leptonic decay. The hadronic part of the $B\to\mu\bar\mu\ell\bar\nu_\ell$ decay amplitude, albeit formally described by the same Eq.(27), where the real photon is replaced with a virtual one, is in fact far more complicated than for $B\to\gamma\ell\bar\nu_\ell$ because: (i) there are now three independent form factors, (ii) the timelike photon with $p^2 > 0$ emitted from the light quark generates intermediate hadronic states dominated by resonances with ρ, ω meson quantum numbers. A factorization pattern for this hadronic amplitude is valid if the photon is spacelike, with a virtuality of at least $\mathcal{O}(m_b\Lambda_{\rm QCD})$ (a usual hard-collinear scale). In Ref. [136], applying the two-step matching $\mathrm{QCD}\to\mathrm{SCET_I}\to\mathrm{SCET_{II}}$ for the hadronic tensor, the factorized expressions for the off-shell $B\to\gamma^*$ form factors at the LP accuracy were obtained, including also the NLL resummation of the parametrically enhanced logarithms. Subsequently, the power-suppressed contributions from four distinct sources have been computed with the same factorization method at tree level. The results respect the constraints on the $B\to\gamma^*W^*$ form factors due to QED gauge invariance of the electromagnetic interaction (see also Ref. [137]). The use of these form factors for various decay observables at large timelike p^2, above ground-state resonances, provides to a usual QCD factorization approximation. A complementary study of this process from the hadronic side was also done recently in Ref. [138], employing the vector-meson dominance ansatz and the z-series parametrization for the off-shell $B\to\gamma^*$ form factors. A more systematic approach combining the factorization formulas with dispersion relations and the elements of LCSR technique is a perspective future task.

4.2 Radiative decays of the heavy vector mesons from LCSRs

To access these decays, we consider the vacuum-to-photon correlator (32) and retain only the vector $b\to u$ current, considering it as an interpolation current for the B^* meson. Matching this correlator to a double dispersion relations in both B^* and B channels yields LCSRs for the $B^*B\gamma$ and—after $b\to c$ replacement—for the $D^*D\gamma$ "magnetic"

Table 8 The LCSR predictions of the $B^*B\gamma$ coupling

coupling	Ref. [141] (NLL)	Ref. [142] (NLO)
$g_{B^{*+}B^+\gamma}$	$1.44^{+0.22}_{-0.20}$	$1.44^{+0.27}_{-0.26}$
$g_{B^{*0}B^0\gamma}$	$-0.91^{+0.12}_{-0.13}$	-0.86 ± 0.15
$g_{B_s^{*0}B_s^0\gamma}$	$-0.74^{+0.09}_{-0.10}$	$-0.95^{+0.15}_{-0.16}$

couplings determining the radiative decays $B^* \to B\gamma$ and $D^* \to D\gamma$. An early application of these sum rules can be found in Ref. [139]. A more elaborated LCSR computation of these couplings at LO and in the twist-four approximation was accomplished in Ref. [140]. The result was used to extract the value of the magnetic susceptibility from the measured branching fraction $\mathcal{BR}(D^{*0} \to D^0\gamma)$, confirming an earlier determination with the method of QCD sum rules [128]. Recently, the NLO QCD corrections to the hadronic photon contribution at twist-two in the LCSRs were computed in Ref. [141]. In Ref. [142], the NLO QCD corrections to the short-distance photon emission diagrams in these sum rules have been calculated. In Table 8, we present the two most recent LCSRs predictions. Note that, in contrast to Ref. [142], the complete set of photon DAs at the twist-four accuracy [128] was employed in Ref. [141], where one can find a detailed discussion and comparison with other theory predictions.

4.3 $B_{s,\,d} \to \gamma\gamma$ and $B_{s,\,d} \to \mu\bar{\mu}\gamma$ decays

Among various rare decays of $B_{s(d)}$ mesons mediated by the FCNC $b \to s(d)$ transitions, the double radiative $B_{s(d)} \to \gamma\gamma$ decays, despite a seemingly simple non-hadronic final state, are quite complicated processes from the QCD point of view, with a rich hierarchy of effective operators and several contributing quark topologies. The LP contributions to the two helicity form factors of $B_{s,\,d} \to \gamma\gamma$ have been determined with the QCD factorization approach at $\mathcal{O}(\alpha_s)$ [143] but without including the two-loop $b \to q\gamma$ matrix elements of QCD penguin operators [144, 145]. Factorization properties of the power-suppressed weak annihilation contributions stemming from the current-current operators were explored at two loops in Ref. [146], where the one-loop short-distance functions were also obtained.

A complete NLL computation of the $B_{s,\,d} \to \gamma\gamma$ decay amplitudes at the LP accuracy has been performed in [147], by employing the two-loop RG evolution equation of $\phi_B^+(\omega,\,\mu)$ [100]. In the same paper, subleading-power contributions from five distinct dynamical sources have been established at tree level in a factorized form, expressed via the two- and three-particle B-meson DAs of higher twist. However, as explained in detail in Ref. [147], the power-suppressed "resolved" photon contribution to the $B_{s,\,d} \to \gamma\gamma$ amplitude cannot be computed with the SCET factorization formalism. Here is where the combination of OPE, LCSRs and dispersion relations discussed in the context of the $B \to \gamma\ell\bar{\nu}_\ell$ decay, enters the stage. The following correlator of the effective operator O_7 and the electromagnetic current with a virtual momentum $q^2 < 0$ was considered:

$$\tilde{T}^7_{\alpha\beta}(p,\,q) = 2\overline{m}_b\int d^4x e^{iq\cdot x}\langle 0|\mathrm{T}\big\{j^{\mathrm{em}}_\beta(x),\,\bar{q}_L(0)\sigma_{\mu\alpha}p^\mu b_R(0)\big\}|\bar{B}_q\rangle + [p \leftrightarrow q,\,\alpha \leftrightarrow \beta]. \tag{33}$$

From the results obtained in Ref. [147], we only mention that the power-suppressed soft contribution reveals a destructive interference with the LP effect. Another type of the subleading power contribution to $B_{s,\,d} \to \gamma\gamma$ from the soft gluon radiation off the quark loop has been recently computed [148] in terms of a factorization approach, with a generalized soft function defined by the HQET matrix element of the non-local operator with quark-gluon fields localized on different light-cone directions.

Even more important are the exclusive FCNC $B_{d,\,s} \to \mu\bar{\mu}\gamma$ decays. In particular, the angular distributions of these radiative leptonic decays provide interesting observables linked to the effective couplings in the weak effective $b \to q\ell\bar{\ell}$ Lagrangian. Albeit with an additional suppression by the QED coupling α_{em}, the $B_{d,\,s} \to \mu\bar{\mu}\gamma$ widths do not suffer from the helicity suppression inherent to the purely leptonic $B_{d,\,s} \to \mu\bar{\mu}$ decays.

A systematic investigation of the $B_{d,\,s} \to \mu\bar{\mu}\gamma$ form factors with an energetic photon has been carried out at LP in the heavy quark expansion with the SCET factorization technique [149], thus going beyond previous works based upon model-dependent approximations. In the same paper, the power-suppressed corrections were also computed stemming from: (i) the photon radiation off the heavy quark, (ii) the subleading terms in the expansion of the hard-collinear quark propagator, and (iii) the weak-annihilation diagrams with insertions of the four-quark operators. However, similar to the situation with $B \to \gamma\ell\bar{\nu}_\ell$ discussed above, there are two power-suppressed soft form factors, which were pragmatically treated with the resonance $\oplus$ factorization ansatz. Evaluating these contributions to the off-shell $B_q(p_B) \to \gamma^*(q)\,\gamma(k)$ form factors in the time-like q^2 region with the help of an OPE-controlled dispersion relation remains an interesting task for the future.

5 A brief guide to other applications

5.1 Form factors of $\Lambda_b \to$ baryon transitions from LCSRs

Apart from many applications to B-meson decays, the method of LCSRs was also used to obtain the form factors of semileptonic heavy-baryon decays at large hadronic recoil. Both versions of LCSRs were applied, based, either (I) on the light-baryon (nucleon or strange hyperon) DAs or (II) on the heavy baryon DAs, the latter defined in HQET.

With the first version of the method, the complete set of the $\Lambda_b \to p$ form factors was computed in Ref. [150]. The vacuum-to-nucleon correlator (in the isospin symmetry limit)

$$\Pi_a(P, q) = i \int d^4z \, e^{iq\cdot z} \langle 0|T\{\eta_{\Lambda_b}(0), j_a(z)\}|N(P)\rangle \tag{34}$$

was used, where j_a is the weak $b \to u$ transition current and η_{Λ_b} is the interpolation current of the Λ_b baryon. For this correlator, the OPE in terms of nucleon DAs was obtained, achieving the twist-6 level. These DAs were worked out in, e.g., Refs. [151, 152], aimed at the studies of LCSRs for the nucleon electromagnetic form factors.

In Ref. [150], the correlator (34) was matched to the hadronic dispersion relation in the variable $(P-q)^2$ and, accordingly, the quark-hadron duality in the Λ_b channel was applied. One of the novelties suggested in that work, was the procedure to eliminate the unwanted "contamination" from the contributions of negative-parity heavy baryons in the resulting sum rules. The advantage of the version I for baryonic LCSRs is the possibility to easily switch to the charm sector by a $b \to c$ replacement in the correlator. In this way, phenomenologically important byproducts—the strong couplings $\Lambda_c N D^{(*)}$ and $\Sigma_c N D^{(*)}$—have also been computed in Ref. [150], applying the technique with a double dispersion relation, similar to the one used for the strong couplings of bottom mesons and discussed in section 2.1. Among other applications of this method, LCSRs for the semileptonic $\Lambda_b \to \Lambda$ form factors were obtained in Ref. [153], employing the Λ-baryon DAs [154] (see Ref. [155] for further discussions).

An alternative version II for the heavy-to-light baryonic form factors was first discussed in Ref. [156] at LO, employing the Λ_b-baryon DAs in HQET worked out in many details in Ref. [157]. The NLO QCD corrections to the ten independent $\Lambda_b \to \Lambda$ helicity form factors were computed in the LCSR framework in Refs. [158, 159]. In particular, in Ref. [159] the factorization-scale independence of the Λ_b-to-vacuum correlator has been verified explicitly at the one loop level. Extending the LCSR technique with the HQET heavy-baryon DAs to the semileptonic $\Lambda_b \to \Lambda_c$ form factors will also appear soon [160].

In the future, the accuracy of heavy baryon form factors from LCSRs can be further increased. In the version I of the method, it is desirable to improve our knowledge of the nucleon and light hyperon DAs (e.g., from lattice QCD). In the version II based on HQET and SCET technique, one should investigate additional perturbative contributions. For example, it is anticipated [161, 162] that LP contributions to the heavy-baryon decay form factors arise from the spectator scattering mechanism with two hard-collinear gluon exchanges. To prove that within the LCSR framework, a computation of the two-loop diagrams in the underlying correlator is necessary.

5.2 Nonleptonic two-body decays of B-meson

Even the simplest two-body weak nonleptonic decays of B meson, such as $B \to \pi\pi$, are characterized by a rich pattern of contributions to the decay amplitude with different quark topologies (emission, exchange, penguin, annihilation etc.). An additional complication is caused by the hadronic final-state interactions in these decays. In the current analyses of nonleptonic decays, QCD is systematically used only in the formation of the effective Hamiltonian, taking into account virtual gluons at the energy-momentum scales between m_W and m_b. A complete QCD-based calculation of hadronic amplitudes relevant for nonleptonic B decays is considerably more challenging than for semileptonic or radiative B decays, which are usually fully factorized into hadronic transition form factors. A systematic approach to nonleptonic B decays, known as the QCD factorization (QCDF) [163, 164], exists in the limit $m_b \to \infty$. Within QCDF, two-body nonleptonic B decays are described with a reduced amount of universal hadronic quantities, such as decay constants, B-meson transition form factors, as well as the light-meson and B-meson DAs. However, despite many successful applications to describe nonleptonic decay widths, the CP asymmetries predicted from QCDF generally differ from the corresponding experimental values, signalling that the power suppressed effects of $O(1/m_b)$ are important in the phenomenological applications. A reliable estimate of these effects demands a QCD-based method employing nonperturbative elements and retaining a finite b quark mass.

LCSRs for the $B \to \pi\pi$ decays were introduced in Ref [165]. The method is based on a vacuum-to-pion matrix element correlating the B-meson- and pion-interpolating currents with an operator of the effective weak Hamiltonian. To avoid "parasitic" contributions of light intermediate hadronic states in the B-meson channel, an auxiliary momentum is attributed to the effective vertex. The sum rule is obtained in three steps, applying: (i) dispersion relation and duality in the second pion channel, (ii) local duality approximation, that is, a transition from a spacelike value of the final-state invariant mass squared to its physical timelike value m_B^2, and (iii) dispersion relation and duality in the B meson channel. Using a finite b-quark mass allows one to quantify the $O(1/m_b)$ corrections, reproducing at the same time the QCDF results at $m_b \to \infty$. In particular, the purely factorizable part is naturally reproduced where the $B \to \pi$ form factor is represented by the LCSR with the pion DAs. In addition, in Ref. [165] a soft-gluon nonfactorizable contribution to the emission topology in $B \to \pi\pi$ was estimated which is a typical power suppressed effect not accessible in QCDF.

Subsequent uses of LCSRs for $B \to \pi\pi$ included the gluonic chromomagnetic operator contribution [166], as well as the charm penguin [167] and the annihilation topology [168] contributions. Importantly, the latter is not divergent at a finite m_b, as it appears to take place in QCDF (see also a recent discussion in Ref. [169]).

Turning to other applications, the B-meson two-body decays into kaon and charmonia were also calculated using LCSRs in Refs. [170, 171]. Quite recently, in Ref. [172] the soft nonfactorizable correction to the $B \to D\pi$ decay was obtained using a version of LCSRs with B meson DAs.

Assessing the perspectives of the LCSRs for nonleptonic decays, we have to take into account that this method has an additional "systematic" uncertainty caused by applying the local duality approximation (the step (ii) in the derivation described above in this subsection). Hence, the resulting sum rules are, in general, less accurate than LCSRs for the B transition form factors presented in the previous sections. Still, the method has certain perspectives, mainly because the amplitudes calculated from LCSRs involve power suppressed contributions not accessible or inherently divergent in QCDF. A complete analysis of B decays into two pseudoscalar mesons, such as $B_{(s)} \to K\pi$, $\bar{K}K$ is one of the perspective applications, especially in view of certain tensions between data and QCDF results.

5.3 Nonlocal effects in $B \to K^{(*)}\ell^+\ell^-$ decays

The $B \to K^{(*)}\ell^+\ell^-$ decays generated by the $b \to s\ell^+\ell^-$ transitions with flavour-changing neutral currents (FCNC) provide sensitive tests of the flavour sector in SM (for a recent review, see e.g., Ref. [173] in this volume). LCSRs were used to provide hadronic input in these decays. For the simplest decay $B \to K\ell^+\ell^-$, the full decay amplitude in SM is described by the following formula:

$$A(B(p+q) \to K(p)\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{\pi}V_{tb}V_{ts}^*\bigg[\bar{\ell}\gamma_\mu\ell\, p^\mu\bigg(C_9 f_{BK}^+(q^2) + \frac{2(m_b+m_s)}{m_B+m_K}C_7 f_{BK}^T(q^2) + \sum_{i=1,\,2,\,\dots,\,6,\,8} C_i\ \mathcal{H}_i^{(BK)}(q^2)\bigg) + \bar{\ell}\gamma_\mu\gamma_5\ell\, p^\mu C_{10} f_{BK}^+(q^2)\bigg], \tag{35}$$

where genuine FCNC contributions sensitive to new physics are given by the terms proportional to the effective coefficients $C_{7,\,9,\,10}$. The hadronic parts of these contributions are reduced to the $B \to K$ form factors $f_{BK}^{+,\,T}$ obtained from lattice QCD or from LCSRs. The remaining terms in Eq. (35) are expressed in terms of hadronic matrix elements

$$\langle K(p)|i\int d^4x\, e^{iqx}\, T\{j_\mu^{em}(x),\, O_i(0)\}|B(p+q)\rangle = p_\mu \mathcal{H}_i^{(BK)}(q^2), \tag{36}$$

where O_i are the contributing effective operators[4] and j_μ^{em} is the quark e.m. current. The $B \to K^*\ell\ell$ decay has a more rich kinematical structure with three invariant amplitudes analogous to Eq. (35), each of them containing hadronic contributions similar to the one in Eq. (36).

The nonlocal hadronic matrix elements (36) are known as "charm loops", since they are dominated by a weak $b \to c\bar{c}s$ transition followed by a lepton pair emission via virtual photon: $\bar{c}c \to \gamma^* \to \ell^+\ell^-$. Depending on the momentum transfer q, the intermediate $c\bar{c}$ pair either forms a virtual charm loop (at $q^2 \ll 4m_c^2$) or transforms to an on-shell charmonium resonances (at $q^2 \geq m_{J/\psi}^2$). In the region far below these resonances, there is still a possibility to use QCD factorization, as it was done in Ref. [175]. However, with this method two problems arise: the $\bar{c}c$ threshold explicitly enters observables instead of hadronic thresholds and the soft gluon exchanges between the charm loop and the rest of the hadronic transition are not accessible.

In Ref. [176] a new method to obtain the charm loop contributions in $B \to K^{(*)}\ell^+\ell^-$ was suggested. The idea was to calculate the amplitudes (36) at spacelike q^2, using light-cone OPE for the charm loop and including the soft-gluon contributions. For the latter, LCSRs with three-particle B-meson DAs were used. The result of the OPE was then fitted at $q^2 < 0$ to a dispersion relation in the q^2-variable, containing the J/ψ and $\psi(2S)$ poles and a

[4] See e.g., Ref. [174] for definitions of these operators and their Wilson coefficients.

certain ansatz for the integral over the spectral density of excited $\bar{c}c$ hadronic states. Finally, this relation provides an estimate of nonlocal amplitudes at timelike q^2 up to charmonium threshold.

In Ref. [174] a complete calculation of the amplitudes (36) for $B \to K\ell^+\ell^-$ was done, followed by an update and inclusion of B_s modes in Ref. [41]. In these analyses, certain contributions were estimated using QCD factorization, since their sum rule calculation is technically not feasible. More recently, in Refs. [177, 178] along with a further update of nonlocal effects in $B \to K\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$decays, the LCSR for the soft-gluon contribution was recalculated with an updated set of higher-twist B-meson DAs, yielding a significantly smaller effect.

Note that nonlocal effects in exclusive FCNC decays are not yet accessible in QCD on the lattice. Hence OPE- and LCSR-based calculations combined with hadronic dispersion relations will remain the only available continuum QCD tool. This combined approach, however, still has the following problems deserving dedicated studies: (i) the calculation in the spacelike q^2 region is strictly speaking not a regular OPE, because the charm loop contribution even at $q^2 \ll 4m_c^2$ is complex valued, hence contains intermediate $\bar{c}cs\bar{q}$ hadronic states (where $\bar{q}$ is a spectator quark in the B-meson); (ii) the usual three-particle DAs of B meson are kinematically not well suited for the correlators used for estimating soft-gluon contributions (see recent discussion in [148]). In this situation, an alternative method of calculation is desirable, e.g. the use of LCSRs with the kaon, K^* and ϕ DAs.

5.4 Decays of B meson into dark matter particles

The method of LCSRs can be easily extended to B-meson transition form factors and other hadronic matrix elements emerging due to new interactions beyond SM. If a new particle is coupled to quarks and gluons via an effective pointlike interaction (e.g., due to heavy mediators), the corresponding local operator can be inserted in the vacuum-to-hadron correlator instead of an SM current, resulting in a sum rule for the B decay amplitude into a hadron and a new particle. A recent example is the calculation of the B-meson decay rate into a proton and dark antibaryon in Ref. [179]. This type of decays is predicted in the B-mesogenesis scenario introduced in Ref. [180], where one can find all necessary details concerning the new physics aspects and the quark-level interactions with dark particles.

In Ref. [179] the effective coupling between three quarks and dark antibaryon was correlated with the B-meson interpolating current in the vacuum-to-nucleon correlator. The OPE with nucleon DAs was then used to the leading twist-3 accuracy, and LCSR was obtained after matching the OPE with the dispersion relation in the B-meson channel. A nontrivial difference is observed between various versions of B-mesogenesis model in which the effective interaction differs only by interchange of b and d quarks. The $B \to$ nucleon effective form factors calculated from LCSRs were used to predict the branching ratios of the B decays into a proton and dark-matter antibaryon. An important upgrade of this calculation will include the nucleon DAs up to twist-6 [181]. Recently, the same LCSR method was used in Ref. [182] also for the B decay modes into other baryons and dark antibaryons. The predicted decay rates are within the reach of the Belle-II experiment, where a final-state proton (or other baryon) and a missing energy-momentum in the B decay serve as a signature.

6 Conclusion

The method of light-cone sum rules, after many years of successful development and plenty of topical applications, has become a standard QCD-based tool to calculate hadronic matrix elements for b-quark decays in a form of approximate analytic expressions with an improvable accuracy. In this review, we presented both main versions of the LCSR technique, based on the light-meson and B-meson DAs. In particular, the B-meson transition form factors and other exclusive B-decay amplitudes at large recoil of the final state are calculable with LCSRs, successfully complementing the lattice QCD calculations. A broad variety of other prospective applications, from the heavy baryon form factors to the B-decays into dark matter were also overviewed. We collected references to all essential papers on the LCSR applications to heavy hadron physics. Having in mind the vast amount of literature in this field, it is possible that some of the relevant publications are still overlooked.

We hope that this review will become useful, especially for young researchers who enter the field of QCD methods in flavour physics, providing guidance to their future works on new interesting applications of QCD light-cone sum rules.

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