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Editorial

Framework of fractals in data analysis: theory and interpretation

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Abstract This special issue is a compilation of pioneering research articles that explore the robustness of fractal theories to address and analyse the complexity of real-time data under the topic "Framework of Fractals in Data Analysis: Theory and Interpretation".

Data is the core for both industrial and academic research. On performing controlled observations and experiments by experts, research data are obtained with high quality. In the olden days, data was collected through manual logs, whereas in modern times, sensors are used. However, minimal human errors occur in empirical data, so repetitions are carried out to reduce uncertainty. The repetition process is one of the tools that connects data analysis with fractal theory, since fractals are self-similar patterns generated by infinitely repeating a simple procedure. In general, most real-world data exhibits irregular and complex patterns when plotted graphically. To generate new visual conceptions for such real-world objects, exhibiting roughness in their traces, the notions of fractal geometry including fractal dimension and fractal interpolation functions are primarily employed.

The book *Fractals Everywhere*, by Barnsley, is a milestone in the development of fractal theory [1]. He introduced the concept of generating fractals and fractal functions using the iterated function system. Fractal geometry has completely changed the view of natural creations with its precise modelling. For instance, to fit real data containing stock or temperature information and to approximate image components consisting of cloud tops or mountain ranges, classical techniques are insufficient. The aforementioned difficulties can be addressed by using the fractal interpolation functions possessing fractal properties. In Ref. [2], a variety of fractals and their fractal dimension estimation are discussed elaborately. It is illustrated that increasing fractal dimension indicates greater complexity of the considered function. Owing to these advantages, in recent times, fractal functions and several types of fractal dimension have been applied to achieve greater accuracy in image processing, one- and multi-dimensional signal reconstruction as well as computer visualization. Many times, series data have been reconstructed using fractal functions; also, unsurprisingly, the epidemic curves of the recent coronavirus disease 2019 (COVID-19) world pandemic have been reconstructed and analysed by several researchers. Therefore, behind every fractal concept, there is an essential relationship with data interpretation. This special issue acknowledges such a fractal aspect of data analysis with the contribution of the following four sections, consisting of 12 research articles:

– Data fitting via fractals
– Fractal functions and numerical simulations
– Fractal patterns in epidemiological data
– Applications of fractal dimension

Section 1 comprises three articles to explore the data fitting of various datasets utilizing fractal notions. Luor and Liu present a hybrid method of data fitting, considering continuous functions and fractal interpolation functions [3]. Two datasets, viz. Bitcoin USD (BTC-USD) and the NASDAQ 100 Index, are fitted using this hybrid method of interpolation. The empirical errors in the fitting are minimized for optimal values of the parameters determined by the approach of sequential quadratic programming. This paper highlights the importance of the optimization algorithm in the approximation using fractal functions. In Ref. [4], the nature of perturbed tsunami waves is explored using fractal interpolation functions via phase projections and time series plots. The resulting effects of the Coriolis parameter on nonlinear and super-nonlinear

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tsunami waves are investigated, in addition. The fractal versions of the phase transitions confirm the hidden self-similarity of tsunami waves besides their quasiperiodic nature. In Ref. [5], the authors mainly explore the fractal analogue of classical mechanics such as Newton, Lagrange, Hamilton and Appell's mechanics via fractal calculus. Further, they obtain the Langevin equation on fractal curves, namely, the Koch-like curves, by defining the fractal α-velocity and α-acceleration. This work strongly indicates the scope for investigating particle motion in fractal time and space.

Section 2 is devoted to an exploration of new kinds of fractal interpolation functions. In Ref. [6], Vijay and Chand construct a rational quadratic trigonometric spline fractal function with scaling factors as functions, wherein the rational functions in the numerator and denominator are quadratic trigonometric polynomials. Mild conditions are enforced on the scaling parameters to preserve the monotonicity and positivity of the associated dataset. Verifying the convergence of the new fractal function, the shape-preserving nature is illustrated by numerical simulations. In Ref. [7], the authors investigate the rational cubic spline fractal interpolation surface within rectangular and cuboid domains. In respect to perturbation of scaling parameters, the stability of the constructed fractal surface is studied and an upper bound is estimated for the perturbation error. A novel method of generating bivariate fractal interpolation functions over rectangular domains is introduced by Aparna in [8]. In addition, a formula for the vertical scaling parameter is derived to minimize the errors in approximation. A linear relation between two different fractal functions, namely the bivariate fractal interpolation function and the bilinear interpolation function, is presented. Motivated by the α-fractal functions and local fractal functions, a local α-fractal interpolation function is constructed as an advancement in Ref. [9]. Estimating the error between the constructed local fractal function with the provided continuous function and an operator analogue to the fractal operator is proposed, and its boundedness as well as linearity are discussed. Numerical examples approximating the discontinuous function using the local α-fractal function are presented.

The fractal patterns in epidemiological data are explored in Sect. 3. In Ref. [10], structural proteins are used to discuss the relationship between severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) and other kinds of virus. Also, graphical illustrations such as zig-zag curves, protein contact maps and chaos game representations for Middle East respiratory syndrome corona virus (MERS-CoV), SARS-CoV, Bat Cov RaTG13 and SARS Cov-2 structural proteins are explored. Slight variation in the graphs clearly reveals their functional and structural differences, and the changes are observed with the help of an elegant fractal parameter called the fractal dimension. Reference [11] is devoted to the growth of the COVID-19 virus from a fractal point of view in a broader sense. The α-fractal interpolation function is employed to approximate the COVID-19 data for India during a certain time period. The results demonstrate that a higher dimension of the graph of the epidemic curve increases the severity of the distribution of the COVID-19 virus.

Section 4 discusses the application of the fractal dimension in various domains. Liang approximates the prescribed continuous function using the linear fractal interpolation function [12], illustrating that the box dimension of the approximated continuous functions and the approximant interpolation function is the same. This observation of the same dimension helps in the approximation of a self-affine function to preserve its local structure. In Ref. [13], inspired by the Fourier series representation of fractal interpolation functions, the authors present Fourier transforms of various fractal functions. Unlike the Fourier series representation, an explicit expression can be presented for the cases of both periodic and non-periodic functions using Fourier transforms. The fractal dimension is also estimated by providing a formula with the Fourier representation of fractal functions. In Ref. [14], Verma and Kumar use fractal notions to understand the effect of merger and acquisition transactions in the stock market. The fractal dimension is employed to observe stock price fluctuations, and interpolation at different scales can help make better decisions in financial markets.

The above-discussed articles explicitly demonstrate the remarkable scope for implementing fractal concepts in the epidemiological study of various contagious diseases, real-time data analysis using machine learning or artificial intelligence and the computer graphics of complex structures. With this end, the editors of this special issue are pleased to convey their sincere thanks to the authors for their significant contributions, as well as the reviewers for their diligent work in reviewing the manuscripts. We hope that the papers that we chose to include here will increase readers' understanding and help in the advancement of fractal analysis in the field of data analysis. Finally, we would like to thank all of the EPJ ST members for publishing this special issue.

Data availability Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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