Eur. Phys. J. Spec. Top.
https://doi.org/10.1140/epjs/s11734-022-00603-9

THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

Check for updates

Editorial

Bursting oscillations, bifurcation delay and multi-stability in complex nonlinear systems

Zhouchao Wei[1], Rajagopal Karthikeyan[2], and Youhua Qian[3,a]

[1] School of Mathematics and Physics, China University of Geosciences, Hubei 430074, Wuhan, People's Republic of China
[2] Center for Nonlinear Systems, Chennai Institute of Technology, Chennai, Tamil Nadu 600069, India
[3] College of Mathematics and Computer Science, Zhejiang Normal University, Zhejiang 321004, Jinhua, People's Republic of China

© The Author(s), under exclusive licence to EDP Sciences, Springer-Verlag GmbH Germany, part of Springer Nature 2022

Abstract This special issue presents 32 contributions on bifurcation, chaos, and bursting oscillations. Some of them are devoted to dynamical behaviors, some to modeling and stability analysis. Bifurcation and chaos are important in the research of nonlinear dynamical behaviors, and bursting oscillations at multiple time scales also have a significant influence on these systems. This leads to many conclusions that can be used to guide practical engineering. A large number of new methods have been proposed for practical model-building and analysis, including scaling, adaptive control, and deep learning methods.

1 Introduction

Many phenomena in engineering and neuroscience can be described by differential dynamical equations. These equations cannot bey directly solved analytically, but we can study the relevant properties by approximate analytical and numerical methods. The bifurcations that occur in the systems (fold bifurcations, Hopf bifurcations, pitchfork bifurcations, etc.)are of great importance and can cause changes in the stability of the systems. The phenomenon and control of chaos are also vital topics in the field of nonlinear dynamics. In recent years, with the development of science and technology, the study of multi-time-scale systems has become a hot issue. Some scholars have proposed different classifications for the oscillation modes and the mechanisms behind them.

We divide these contributions into two main areas: bifurcation, chaos, and bursting oscillation; modeling and analysis. This classification is not necessarily accurate, but is given as a reference for the reader. On bifurcation phenomena, scholars use normal forms and central manifold theory to reduce systems and study the dynamical behavior near complex bifurcation points. In terms of chaotic phenomena, scholars use chaotic attractors, Lyapunov exponents, attractor basins, Melnikov method, and other analytical methods to explore the stability of equilibrium points, bifurcation phenomena, chaotic phenomena, and so on. Regarding the bursting oscillation phenomenon, scholars use the fast–slow analysis method to study the mechanism behind the bursting oscillation phenomenon of the system, and follow the naming protocol proposed by Izhikevich to characterize different types of bursting oscillation phenomena. Some scholars also seek to combine bursting oscillations with practical applications.

In the modeling process, scholars combine actual models, including neural networks, lesion models, and new material models, and analyze them through bifurcation and stability to guide the actual situation. Models can be analyzed and optimized using time history or phase diagrams, Poincare maps, bifurcation diagrams, and spectrograms.

2 Bifurcation, chaos, and bursting oscillation

This section focuses on the study of bifurcation, chaos, and bursting oscillation phenomena and the mechanisms behind them.

Liu et al. [1] analyze the double-Hopf bifurcation of the aerosol–cloud–precipitation system using the normal and central manifold forms, and then obtain the stable equilibrium, stable periodic, and quasi-periodic solutions. These solutions can reveal cloud and rain phenomena. For example, stable equilibrium corresponds to the actual system and indicates the formation and depletion of clouds (drizzle), and the stable periodic solution indicates that thicker clouds are consumed by stronger rain (moderate drizzle). The authors suggest that a double-Hopf bifurcation analysis will mitigate the effects of natural hazards.

Zhou et al. [2] discuss the stability and Hopf bifurcation of the system using stability theory, central

[a] e-mail: qyh2004@zjnu.edu.cn (corresponding author)

Published online: 31 May 2022

manifold theory, and normal form theory, and perform numerical simulations. A robust controller is then added to fine-tune the control parameters, whereby the Hopf bifurcation is achieved or delayed at a certain point.

Zhang et al. [3] study the bifurcation characteristics of cubic discrete chaotic systems, investigate the cause of symmetry-breaking bifurcation by solving the periodic bifurcation solution of the system, and obtain the critical value for the recovery of symmetry-breaking bifurcation. The authors add a nonlinear controller to control the bifurcation of the system and change the position of the bifurcation point. The delay or advance of the period-doubling bifurcation and the emergence of symmetric fracture bifurcation are then achieved.

Lai et al. [4] study the Jacobi stability of a resonant nonlinear Schrödinger (RNS) system using Kosambi–Cartan–Chern (KCC) theory, discussing the Jacobi stability of three equilibrium points and the focusing trend of the trajectory around the equilibrium point. The numerical results also show that the system is quasi-periodic and chaotic under periodic perturbations.

Ramakrishnan et al. [5] propose a megastable oscillator with a vast number of coexisting limit cycles that spread on a surface. The dynamical characteristics of this oscillator are studied by analytical methods including bifurcation, attractor basin, and Lyapunov exponents.

Wang et al. [6] propose a symmetric oscillator with multiple stability and study the dynamical properties of the oscillator, including chaotic attractors, Lyapunov exponents, bifurcation, and attractor basin. Crucially, the authors build the circuit to prove its feasibility.

Chen et al. [7] investigate chimeric states in a network of identical oscillators with symmetric coexisting attractors in ring and multiplex topologies. Chimerism, cluster synchronization, and complete synchronization states are examined by choosing different initial conditions from the basin of attraction.

Zhang et al. [8] report a rare class of two-dimensional rational memristive maps in which all attractors are hidden. Taking a quadratic memristor as an example, and using numerical tools such as phase diagrams, basins of attraction, bifurcation diagrams, and Lyapunov exponents, they find that these maps can generate periodic, chaotic, quasi-periodic, and hyperchaotic solutions, among others.

Wen et al. [9] study the bifurcation and chaos threshold of the Duffing oscillator with fractional-order delayed feedback control, which demonstrates functions of displacement and velocity feedback. Exploiting the Melnikov method, the necessary conditions for the chaotic analytical solution are obtained and verified numerically. Finally, the authors find that the increase in fractional-order delayed feedback gain will suppress chaos generation.

Dong et al. [10] analyze plates with a fixed support in the center and four free edges. Partial differential control equations for the bistable plate motion are established using Hamilton's principle. By solving the nonlinear static equations, two stable configurations and one unstable configuration can be determined. This is called supercritical pitchfork bifurcation. The two potential energy wells are determined by the stable configurations so that the effect of threshold can be studied. This dynamic snap-through phenomenon found experimentally in Ref. [10] is associated with bursting oscillations.

Chen et al. [11] study bistable asymmetric laminated square plates with time delay. The system exhibits pitchfork and fold/fold bursting oscillation. The mechanism behind it and the effect of time delay are illustrated.

Chen et al. [12] discuss the phenomenon of bursting oscillations in a multi-stable nonlinear energy harvester and numerically simulate multiparameter bifurcation and transformed phase diagrams to explain them. It is hoped that the bursting oscillations can be used to improve energy harvesting.

Lin et al. [13] explain the bursting oscillations of a piezoelectric energy harvester with magnets (PEHM) and its mechanism using the fast–slow analysis method. The authors evaluate the effect of bursting oscillations on energy harvesting efficiency using the average output voltage value as a measurement index. The results show that the excitation frequency close to the bursting frequency can lead to larger instantaneous output power.

Ma et al. [14] investigate new bursting patterns caused by different types of hysteresis loops in parametric and externally driven nonlinear oscillators. The rupture behavior in the form of "fold/turning point-turning point/fold" generated by the "fold/turning point" hysteresis loop is studied in detail using the fast–slow analysis method. A large number of types of bursting oscillations are obtained, and the underlying mechanisms are explained.

Wei et al. [15] use fast–slow analysis to study a parametrically and externally excited jerk circuit system and find correlated bursting oscillations generated by a supercritical Hopf bifurcation. As the parametric excitation amplitude increases, a large number of phenomena including period-doubling and inverse period-doubling cascades can be observed in orbits of periods 2, 4, and 8, leading to complex bursting patterns with multi-period active states.

Zhang et al. [16] consider a four-dimensional Chay–Cook model with multiple time scales and change the time scale of a variable in the model from fast to medium-slow to slow to control it directly. The new definition of the topological type of bursting proposed in the work is an extension and improvement of the existing classification and provides a dynamical basis for further study of the complexity of information encoding in neuronal systems.

3 Modeling and analysis

In this section, the actual modeling and related dynamical analysis articles are summarized. These works aim to guide practical engineering by applying dynamical analysis approaches.

Zhou et al. [17] analyze the nonlinear dynamics of two first-order 2:3 external and 3:2 internal mean motion resonances (MMRs) of a three-body system. The Poincaré diagram of the planar circular confinement three-body model is calculated to find the semi-long axis and phase, which has a significant effect on the steady state of the resonant orbit. Stable and chaotic domains are also obtained through the resonance space.

Sun et al. [18] study the nonlinear dynamics of circular mesh antennas. They first equate it to a cylindrical shell structure, and then carry out a discrete analysis using the third-order Galerkin method. Based on the energy-phase method of Haller and Wiggins, the geometry of three jump pulses in six-dimensional phase space is described by the extended energy-phase method.

Wang et al. [19] propose a new noncircular cross-section elastic rod model, and explain these inherent conformational principles by analytical methods using the concept of effective bending stiffness. The bifurcation and stability analysis also reveals the plain and flexural conformations of the rod.

Wang et al. [20] consider the Bass model based on the classic durable consumer goods marketing network model, deriving degree-based mean-field theory. For the annealing network setting, they establish its balance, stability, and aggressiveness. Numerical results show that the method can optimize costs.

Xu et al. [21] study the space–time fractional equation. Multiple solitary and periodic solutions are obtained by new fractional scaling transformations of fractional nonlinear systems which transform different time and space orders into integer orders. The authors expect that the exact solutions of fractional nonlinear wave systems can be treated in a similar way.

Tian et al. [22] construct a nonlinear dynamic vibration absorber (DVA) with variable frequency and damping, and evaluate its performance using a novel force transferability. A two-step optimization method for nonlinear DVA parameters over a wide frequency range is also designed to overcome the limitations of the traditional single optimization method for broadband damping. Finally, time history, phase, Poncaré, and frequency spectrum diagrams are analyzed to prove the effectiveness and superiority of the two-step optimization method.

Doubla et al. [23] investigate the Hopfield neural network (HNN) model with a hyperbolic-type memristor. Equilibrium point analysis shows that the instability line of the system is associated with external stimuli. The coexisting attractors are then analyzed using bifurcation diagrams, Lyapunov exponents, and phase diagrams. Finally, the authors design a new electronic circuit of a hyperbolic memristor, which enables the computer simulation of the whole system to be designed for future engineering applications.

Parkavi et al. [24] propose a method to generate a class of nonlinear ordinary differential equations (ODEs) which comprise simple harmonic oscillator equations with either amplitude-independent or integral-dependent oscillation frequency characteristics. They then use an example to illustrate the theory.

Shi et al. [25] study adaptive consistent tracking control of perturbed multi-intelligent systems (MASs) by a discontinuous protocol approach. Using algebraic graph theory, differential inclusion theory, and matrix theory, sufficient consistency conditions are obtained for the mathematical model under consideration. Finally, the usability of the theoretical analysis is demonstrated by an example.

Reddy et al. [26] show the application of entropy generation for gold-blood pseudoplastic nanofluid flow in electrodynamically and electro-conductively heated microchannels. The dimensional form of the momentum and heat equations are transformed into dimensionless form using long-wavelength and small Reynolds number approximations. The effects of parameters including radiation, Weissenberg number, Helmholtz–Smoluchowski velocity, and Joule heating are analyzed through homotopy perturbation method (HPM) calculations.

Sivasaravanababu et al. [27] study seizure features expressed in EEG data using a deep convolutional variational autoencoder (DCVAE) based on deep learning and preprocessed data using a tunable Q-factor wavelet transform (TQWT). Subsequently, these violent EEG features associated with seizures are applied as input to a stacked bidirectional long- and short-term memory (SB-LSTM) model for automatic seizure detection. The method demonstrates high accuracy and sensitivity.

Ramakrishnan et al. [28] consider an exponential flux memristor-based Morris–Lecar neuron model for one-, two-, and three-layer neural networks subjected to the effects of low-frequency electromagnetic fields (MLELFs) and noise variance on helical wave suppression. The multilayer neural network is found to have a substantial influence on low noise variance in helical wave suppression.

Karami et al. [29] propose a two-dimensional megastable oscillator with a square-wave function, which has an unstable equilibrium point. Based on this equilibrium, the innermost attractor is self-excited, while the others are hidden. The authors also study the basin of attraction of the limit cycles number function and its chaotic dynamics under sinusoidal forces. Various dynamics of the forced oscillator generated by varying the amplitude and frequency of the forcing term are discussed.

Tian et al. [30] analyze the fundamental dynamics of chaotic systems with a hidden attractor and a line equilibrium. Moreover, the infinity dynamics of the system are studied based on Poincaré compactification theory, and the type of equilibria at infinity are also analyzed and proved. A fixed-time synchronization observer is proposed, and the master system synchronizes with the slave system at settling time.

Wang et al. [31] investigate the fractional-order Colpitts oscillator on the basis of the multi-step fractional differential transform method (MFDTM) algorithm and consider bifurcation diagrams of the fractional-order system with the corresponding integer-order system. A field-programmable gate array (FPGA) implementation scheme is proposed for the fractional-order Colpitts oscillator based on the MFDTM algorithm.

Yuan et al. [32] study the first and second comparison theorems of tempering fractional differential equations, and analyze the continuous dependence of the equation solution on the parameters. Two examples are then given to support the theoretical analysis.

4 Discussion and outlook

We hope that the content of this volume will further advance the study of bifurcations, chaos, and bursting oscillations. This volume provides an in-depth study of these phenomena, and combined with actual engineering, medical, aerospace, and other models, it provides new ideas and schemes for optimizing parameter design at the same time. In addition to classical analytical methods, scholars have combined cutting-edge methods from different disciplines, including nanotechnology, adaptive control, and deep learning methods. On this basis, we propose several suggestions for future research directions. Firstly, although the phenomenon of bursting oscillations is very extensive, it has not yet been supported by a profound theory, and it also has not yet received widespread deep theoretical support. It is important to establish a mathematical definition of bursting oscillations and the theory behind them. Second, although bifurcation, chaos, and bursting phenomena can be used to guide practical engineering, they are not closely integrated with actual models. The use of theoretical analysis for precise control is an important step in the application direction.

We wish to thank all the authors for their enthusiastic response and hard work, and for supporting our idea to present their recent achievements and progress in this area of scientific research. We hope that this volume will contribute to constructive discussions on theoretical advances and potential applications for bifurcation, chaos, and bursting oscillation in engineering and mechanics.

Acknowledgements The authors gratefully acknowledge the support of the National Natural Science Foundation of China (NNSFC) through Grant Nos. 12172333 and 12172340, the Natural Science Foundation of Zhejiang Province through Grant Nos. LY20A020001 and LY20A020003, and the Fundamental Research Funds for the Central Universities, China University of Geosciences Wuhan (CUGGC05).

References

1. X. Liu, L.J. Pei, W.Y. Wang et al., Double Hopf bifurcation and stability of Koren-Feingold cloud-rain system with rain production delay. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00412-6
2. L.Q. Zhou, A. Kabbah, Hopf bifurcation and its control in a 3D autonomous system. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00488-8
3. L. Zhang, Q. Han, Symmetry breaking bifurcation analysis and control of a cubic discrete chaotic system. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00479-9
4. T. Lai, C.S. Feng, Y.J. Liu et al., Geometric analysis and onset of chaos for the resonant nonlinear Schrödinger system. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00398-1
5. B. Ramakrishnan, A. Ahmadi, F. Nazarimehr et al., Oyster oscillator: a novel mega-stable nonlinear chaotic system. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00368-7
6. Z. Wang, D. Veeman, M. Zhang et al., A symmetric oscillator with multi-stability and chaotic dynamics: bifurcations, circuit implementation, and impulsive control. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00371-y
7. M.S. Chen, D. Veeman, Z. Wang et al., Chimera states in a network of identical oscillators with symmetric coexisting attractors. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00480-2
8. L.P. Zhang, Y. Liu, Z.C. Wei et al., Hidden attractors in a class of two-dimensional rational memristive maps with no fixed points. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00483-z
9. S.F. Wen, H. Qin, Y.J. Shen et al., Chaos threshold analysis of Duffing oscillator with fractional-order delayed feedback control. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-021-00369-6
10. T. Dong, W. Zhang, Y.Z. Liu, Dynamic snap-through phenomena and nonlinear vibrations of bistable asymmetric cross-ply composite-laminated square plates with two potential wells under center base excitation. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00375-8
11. Y.N. Chen, D.J. Zhang, B.W. Lin, Pitchfork and fold/fold bursting of time delay duffing systems with parametric excitation and external excitation. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00394-5
12. Z.Y. Chen, W.A. Jiang, L.Q. Chen et al., Bursting analysis of multi-stable nonlinear mechanical oscillator and its application in energy harvesting. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00377-6
13. B.W. Lin, Y.H. Wang, Y.H. Qian, Bursting oscillation and its mechanism of the flow-induced vibration piezoelectric energy harvester with magnets by low-frequency excitation. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00481-1
14. X.D. Ma, H.Q. Zhao, Q.S. Bi, Novel bursting patterns induced by hysteresis loops in a one-degree-of-freedom nonlinear oscillator with parametric and external excitations. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00485-x

15. M.K. Wei, X.J. Han, X.D. Ma et al., Bursting patterns with complex structures in a parametrically and externally excited Jerk circuit system. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00427-7
16. E. Zhang, L. Yu, Z.Q. Yang, New topological classification of bursting in multi-time-scale Chay-Cook model. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00508-7
17. Y. Zhou, W. Zhang, Analysis on nonlinear dynamics of two first-order resonances in a three-body system. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00428-6
18. Y. Sun, W. Zhang, M.H. Yao et al., Multi-pulse chaotic dynamics and global dynamics analysis of circular mesh antenna with three-degree-of-freedom system. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00366-9
19. Z.X. Wang, W. Wang, Q.C. Zhang, New effective bending rigidity and structural instability analysis of noncircular cross-section elastic rod model. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00370-z
20. Y. Wang, W.Q. Luo, J.L. Ma et al., Marketing consumer durables on random networks with dynamic advertising rate and individual consciousness. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00482-0
21. H. Xu, W.H. Cheng, J.F. Cui, Multiple-soliton and periodic solutions to space-time fractional Whitham-Broer-Kaup equations. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00374-9
22. R.L. Tian, T. Wang, X.W. Yang et al., New force transmissibility and optimization for a nonlinear dynamic vibration absorber. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00425-9
23. I.S. Doubla, B. Ramakrishnan, Z.N. Tabekoueng et al., Infinitely many coexisting hidden attractors in a new hyperbolic-type memristor-based HNN. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-021-00372-x
24. J.R. Parkavi, R. Mohanasubha, V.K. Chandrasekar et al., A class of isochronous and non-isochronous nonlinear oscillators. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00484-y
25. L.L. Shi, W.L. Guo, W.N. He, Distributed adaptive discontinuous consensus tracking control for disturbed multi-agent systems based on input-observers. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-021-00381-w
26. S.R.R. Reddy, H.T. Basha, P. Duraisamy, Entropy generation for peristaltic flow of gold-blood nanofluid driven by electrokinetic force in a microchannel. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-021-00379-4
27. S. Sivasaravanababu, V. Prabhu, V. Parthasarathy et al., An efficient epileptic seizure detection based on tunable Q-wavelet transform and DCVAE-stacked Bi-LSTM model using electroencephalogram. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00380-x
28. B. Ramakrishnan, A. Karthikeyan, A. Srinivasan et al., Effect of noise variance in spiral wave suppression for a multi-layered neuron model with flux coupling modelled using a memristor. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00478-w
29. M. Karami, R. Ramamoorthy, A.M.A. Ali et al., Jagged-shape chaotic attractors of a megastable oscillator with spatially square-wave damping. Eur. Phys. J. Spec. Top. (2021). https://doi.org/10.1140/epjs/s11734-021-00373-w
30. H.G. Tian, Z. Wang, H. Zhang et al., Dynamical analysis and fixed-time synchronization of a chaotic system with hidden attractor and a line equilibrium. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00553-2
31. H.H. Wang, D.L. Zhan, X.M. Wu et al., Dynamics of a fractional-order Colpitts oscillator and its FPGA implementation. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00487-9
32. L.G. Yuan, S. Zheng, Z.C. Wei, Comparison theorems of tempered fractional differential equations. Eur. Phys. J. Spec. Top. (2022). https://doi.org/10.1140/epjs/s11734-022-00486-w