Renormalons and hyperasymptotics in QCD

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Abstract This issue of EPJST contains a set of 11 articles aiming to put in perspective recent developments in the area of renormalons and hyperasymptotics in QCD. Conceptual problems and phenomenological applications of renormalons are reviewed, based on perturbative calculations, lattice QCD, and mathematical ideas from transseries and resurgence theory.

1 Introduction

The perturbative expansion of quantum field theory observables are known to lead to divergent series for most realistic theories.¹ This fact was first noticed in the case of Quantum Electrodynamics (QED) by Freeman Dyson in 1952. To give the divergent series a precise meaning, Dyson proposed to interpret it as asymptotic to the exact function. In QED, due to the very small value of the electromagnetic coupling for the energies of interest, the divergence of the perturbative series does not affect physical predictions. However, even in this case, the interpretation of the series as asymptotic has profound consequences: it provides, for example, new insight into the interplay between perturbative contributions and non-perturbative power corrections, which has implications for the intrinsic limitations of the perturbative expansion in powers of the coupling.

In Quantum Chromodynamics (QCD), due to the much larger value of the strong coupling, α_s , the divergent character of the series can have nonnegligible consequences in phenomenological applications. This is particularly relevant nowadays since, thanks to the advances in higher-order loop computations, several observables are known at $\mathcal{O}(\alpha_s^3)$ or even $\mathcal{O}(\alpha_s^4)$. In QCD, the fact that the perturbation series has zero radius of convergence is inferred from the singularity structure of the correlators at zero coupling, proved by 't Hooft to follow from renormalization-group invariance, analyticity, and unitarity. Alternatively, particular classes of Feynman diagrams predict a factorial increase of the expansion coefficients at large orders.

A factorial increase of the expansion coefficients of some observables was obtained recently also from lattice QCD. Therefore, the divergence of perturbation theory is very well established. Its interpretation as an asymptotic series, however, remains, to this day, a very plausible but yet unproven conjecture. The fact that the perturbative expansions in QCD are divergent strongly affects the renormalization scheme and scale dependence of the truncated expansions, which represents a major source of uncertainty in the theoretical predictions of the standard model (SM). Understanding and taming this behaviour is, therefore, one of the main challenges for precision QCD to match the requirements of future accelerator facilities such as the FCC-ee.

As it is known, there are infinitely many functions having the same asymptotic expansion. Ideally, one would like to recover the expanded function which should have, as much as possible, the properties of the physical amplitude. Borel summation is a useful tool to this end. The large-order behaviour of the expansion coefficients of a function is encoded in the singularities of its Borel transform in the Borel complex plane. In QCD, the dominant singularities in the Borel plane are the so-called infrared (IR) and ultraviolet (UV) renormalons, which are related to contributions of specific momentum regions in the loop integrals.

The existence of IR renormalons has dramatic consequences for the structure of perturbative QCD. These singularities are situated on the positive real axis in the Borel plane and are responsible for an ambiguity of the Borel–Laplace integral by which the function is obtained from its Borel transform. According to the general view, this means that perturbative QCD is not a complete theory and must be supplemented by additional, nonperturbative terms to recover the physical correlators. A popular way to do this is by means of the power corrections in the operator product expansion (OPE). One hopes that the ambiguities of the standard

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¹ References for the material presented in this Introduction can be found in the specific contributions included in the issue.

perturbation theory, which is the dimension-zero term in the OPE, are eventually compensated by those of the higher dimensional OPE terms, although in QCD a rigorous proof of this compensation does not exist. In general, terms that go beyond the OPE and that decrease exponentially in the momentum plane must also be included. These terms are related to the so-called violation of the quark-hadron duality, defined by the standard approach based solely on the OPE and analytic continuation

In recent years, much progress in the study of divergent series was achieved by the mathematical theory of resurgence, which shows how to recover a function from a limited number of coefficients of its asymptotic divergent expansion. Specifically, in the hyperasymptotic approach the expanded functions is recovered by a transseries, i.e. a sequence of truncated series, each of them exponentially small in the expansion parameter of the previous one. These mathematical concepts proved to be useful for the goal of obtaining nonperturbative features from perturbative QCD. The resurgence framework provides a natural understanding for the interconnections between perturbative QCD, higher dimensional OPE terms, and duality violation contributions, for example.

In the present volume, several aspects of the topics introduced above are discussed by renowned specialists in the field. The problem is tackled from different angles: in the continuum as well as on the lattice, for QCD and for related quantum field theories, from a more fundamental view point or a phenomenological approach. Below we give a brief overview of the articles included in this volume.

2 Content of the collection

The first group of contributions to this special issue is devoted to applications of renormalon techniques and the associated OPE for the study of specific observables. In [1], Beneke discusses the quark pole-mass renormalon and the exact structure of the associated divergence in QCD. Different definitions of renormalon-free quark masses are presented, with an up-to-date numerical comparison of their convergence order by order in perturbation theory for the charm-, bottom-, and top-quark mass. Ferrario Ravasio reviews in [2] some aspects of infrared renormalons in collider processes, with emphasis on the role played by the linear renormalons. These renormalons can arise in certain kinematic distributions relevant for collider phenomenology where an OPE is missing, and are responsible for large perturbative uncertainties. In [3], Takaura gives a brief review of the current understanding of renormalons in the static QCD potential and reconsiders the estimation of the normalization constant of the $u = 3/2$ renormalon, which is crucial for renormalon subtraction methods.

Compelling evidence for the factorial increase of perturbative coefficients in QCD and the related singular-

ity structure of the Borel transform in the Borel plane has been provided recently also by lattice calculations. This problem is discussed by Pineda [4] for the plaquette in gluodynamics. The existence of a dominant renormalon from the divergent asymptotic pattern of the coefficients calculated by stochastic perturbation theory is inferred, and its treatment in the spirit of hyperasymptotic theory is shown to increase the precision of the gluon condensate determination. The status of perturbative QCD for the Adler function and the scalar correlator is reviewed by Jamin in [5], where realistic Borel models are used to predict the higher order perturbative coefficients, not yet available from the calculation of Feynman diagrams. The dependence on the renormalization scheme is also discussed in [5], the advantages of the so-called C scheme over the standard $\overline{\text{MS}}$ being emphasized. The problem of renormalization scale dependence is investigated in detail in [6], where Hoang and Regner advocate the existence of different Borel representations of the τ hadronic spectral function moments obtained with two popular prescriptions for the renormalization scale, contour-improved perturbation theory (CIPT) and fixed-order perturbation theory (FOPT), which suggests that the associated OPEs differ. Mariño et al. discuss in [7] the Bethe ansatz for the free energy in two-dimensional integrable models, proving that the ansatz is verified up to very high orders in the coupling constant for the non-linear sigma model and its supersymmetric extension. Moreover, information on the dominant renormalons is obtained with a method based on resurgence theory.

The next group of contributions is devoted to the mathematical techniques of conformal and uniformizing mappings, as alternative ways of recovering the expanded function from its perturbative coefficients. In [8], Caprini investigates the expansions in powers of the variable that maps the Borel complex plane of the QCD correlators onto a unit disk. Using arguments based on convergence and analyticity in the complex coupling plane, it is argued that the modified expansions can provide an alternative to the power corrections in the standard OPE. In [9], Costin and Dunne prove that the method of Padé approximants combined with conformal and uniformizing maps of the Borel plane allows to recover the expanded function to a remarkable good precision. The generic case of Borel transforms with branch points in the complex Borel plane is treated, from a single branch point on the real axis to a general number of singularities in the complex plane.

The volume also contains contributions which illustrate recent progress in the description of the real hadronic world by QCD. In [10], Peris discusses quark-hadron duality violations, using developments in the theory of transseries and hyperasymptotics to set the right mathematical language. Exponentially suppressed terms beyond the OPE, related to duality violations, are proved to stem from branch-point singularities in the Borel plane of a suitable Borel-Laplace transform, and the connection between these singularities and the exact QCD spectrum is discussed in the limit of large number N_c of colors. Finally, Shifman [11] discusses

various sources of nonperturbative effects in Yang–Mills theories at weak coupling and their implications for the validity of the perturbative expansions. The author emphasizes difficulties encountered in applying in a straightforward way the mathematical ideas of transseries and resurgence to asymptotically free Yang–Mills theories like QCD. Also, the properties of the OPE and the compensation of its ambiguities in the frame of exactly solvable models are reviewed.

3 Final remarks

This collection of papers reviews recent developments in QCD, focused on applications of renormalons, conformal mappings and hyperasymptotic theory. Controversial problems related to Borel summation and renormalization scale and scheme dependence, the interpretation of the OPE and alternatives to it, and the usefulness of resurgence and transseries for asymptotically free Yang–Mills theories have been touched. Some conjectures formulated in the contributions to this volume are open to more investigations and to phenomenological confirmation. We hope the issue will stimulate further research in this domain, the ultimate goal being to increase the precision of the SM predictions in the strongly interacting sector.

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