



Circuit application of chaotic systems: modeling, dynamical analysis and control

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Published online 28 July 2021

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Abstract This special issue selects some interesting contributions of recent scientific achievements in circuit application of chaotic systems, focusing on showing the new methods and technologies of the modeling, dynamical analysis and control of chaotic systems. This presented letter summarizes the work of the special issue and discusses some thoughts on the future directions of chaotic circuits and systems.

Chaos research arose from the discovery of the famous Lorenz attractor in [1], and has gradually matured after intensive and extensive investigation in the past few decades. The breakthrough of the broad engineering applications of chaos has prompted the chaos generation (or chaotification) to be an important research issue, yielding the appearance of large quantities of chaotic systems [2–7]. Generally speaking, any dynamical system with chaotic solution can be regarded as a chaotic system. There are three main research problems involving chaotic systems that have been enduring, i.e. how to generate special chaotic systems, how to detect and show the complex dynamics of chaotic systems, and how to control the chaotic systems targeting a desired behavior. Although some research progress has been made on these problems, it is far from forming a unified theoretical and technical framework that can solve these problems well. So we organize this special issue to get some inspiration on the new methods and technologies to model, analyze and control chaotic systems.

The study of chaotic circuits has received much attention since the invention of the Chua circuit in [8]. Many nonlinear electronic circuits which can produce chaos have been established. A chaotic circuit is essentially a kind of chaotic system, and scholars usually use the electronic circuits to yield chaotic signals and demonstrate the physical existence of chaotic systems. Thereby the circuit implementation has become an important method for transiting chaotic systems from theory to practice, and it has been a necessary step for the application of chaos to engineering practice. So

far the circuit application of chaotic systems has been greatly developed. Additionally the discovery of memristors has pushed the study of circuit application of chaotic systems to reach a new level and presented some new challenges. Thus it is very necessary to consider new problems encountered in the circuit application of chaotic systems in this special issue.

This special issue is dedicated to present state-of-the-art results on circuit application of chaotic systems in terms of the modeling, dynamical analysis and control design. We believe that such selected papers will be useful references for the readers. Some of the papers present the generation, analysis, realization and application of memristive chaotic systems [9–17] with multistability, where five papers study the integer-order memristive systems and two papers study fractional-order memristive systems. Authors in paper [9] propose an interesting memristive chaotic system with conditional symmetry, attractor growing and amplitude frequency control properties, and show its coexisting of multiple chaotic attractors by bifurcation diagrams and phase portraits, and use the circuit implementation to determine its existence. Authors in paper [10] construct a five-dimensional two-memristor-based Colpitts system which can yield parameter-boosted attractors and initial-boosted coexisting attractors, and study the circuit simulations of the system on PSIM software platform. Authors in paper [11] establish a simple memristive chaotic oscillation circuit by integrating the memristor emulator and second-order RLC circuit. Simulations results illustrate the coexistence of two symmetric attractors of the system. The hardware circuit design of the system is given. Authors in paper [12] analyze the bursting oscillations and bifurcation

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mechanism two types of piecewise-smooth memristor-based Shimizu-Morioka system. These two systems can exhibit different types of bifurcation and motion states. The accuracy of the work is illustrated by the simulation results from the designed fully integrated circuit. Authors in paper [13] construct a new two-memristor-based FitzHugh-Nagumo circuit with chaos and coexisting attractors, and establish the synchronization conditions of the system with unidirectional and bidirectional coupling. Authors in paper [14] study the complex dynamical behaviors and FPGA circuit realization of a five-dimensional memristive hyperchaotic system and consider its pseudorandom number generator. Authors in paper [15] present four fractional-order memristive, meminductive and memcapacitive chaotic systems with hidden multistability. Numerical simulations and experimental circuit results well describe the influence of parameters and initial conditions on the dynamic behaviors of the systems. Authors in paper [16] put forward a time-delayed fractional-order memristive chaotic system and apply it to the design of image encryption algorithm with DNA encoding. Also there are one paper designs a unified asymmetric memristive diode-bridge emulator with current constraints of the parallel bridge arms and implement the emulator via Multisim circuit, Matlab simulations and hardware experiment [17]. It can be used for the construction of chaotic circuit.

There are three papers [18–20] which address improved chaotic oscillators. Paper [18] studies the complex dynamics and circuit experimental implementation of Shinriki's oscillator. It shows that the rupture of symmetry enables the system to yield a pair of symmetric coexisting chaotic attractors. Paper [19] presents a ferroresonant chaotic oscillation circuit and study its chaos control problem by applying backstepping control method. Paper [20] investigates the multistability of Chua's oscillator with piecewise-linear nonlinearity. The effective multistability control method the system is established and the basins of attractions of the coexisting attractors are discussed. Also the image encryption algorithm for the system is designed and its security against various attacks are numerically tested.

There is significant interest in dynamical analysis, circuit implementation, control design and engineering applications of different types of continuous-time chaotic systems, see papers [21–27] in this special issue. Paper [21] designs and implements a new chaotic system with only one unstable equilibrium and two coexisting strange attractors, and investigates the impulsive synchronization control problem of the system. Paper [22] constructs a three-dimensional chaotic system with no equilibrium and different types of hidden multi-scroll attractors. The dynamical properties of the system are verified by simulations and circuit experiment. Paper [23] uses the pulse control approach to generate compounding self-excited and hidden attractors from Sprott D system. The digital oscilloscope results obtained from the FPGA implementation demonstrate the physical existence of the chaos of the system. Paper [24] presents a four-dimensional chaotic system with

absolute value nonlinearities. The symmetry property, antimonotonicity, offset boosting feature, total amplitude control and multistability of the system are analyzed. Also the Pspice circuit realization of the system is done as well. Paper [25] generates a new chaotic system in which the number of equilibrium points varies with the parameters. The existence of hidden attractors and coexisting attractors of the system is illustrated and the corresponding circuit realization is given. Paper [26] studies the chaos control and synchronization of a new chaotic system via linear quadratic regulator method, and analyzes the circuit implementation and pseudo random number generator of the system on FPGA platform. Paper [27] gives a simplest Lorenz-like system and uses the hardware circuit to implement the chaos-based secure communication of the system. Authors in papers [28,29] analyze the complex dynamical behaviors and hardware circuit experimental observation of two-dimensional discrete chaotic maps. Authors in papers [30–32] show the rich dynamics and circuit realization of the stretch-twist-fold flow, semiconductor laser diode and Wilson neuron system.

Moreover several papers address the dynamical analysis and control of special chaotic systems [33–37]. Authors in paper [33] give a class of simplest symmetric chaotic flows with different equilibria and investigate the chaos synchronization of the systems for revealing the collective behaviors of networks of these systems by analyzing the corresponding master stability function. Authors in paper [34] use the chaotification method to yield new hyperchaotic map with complex nonlinear performance and effective resistance to parameter identification. The design of pseudo-random number generator illustrates the effectiveness of engineering application of the system. Authors in paper [35] study the multistability and chaos of fractional-order oscillator with extreme events. Authors in paper [36] consider the complex generalized synchronization control problem of complex-variable chaotic systems and establish the synchronization conditions by using Lyapunov stability theory. Authors in paper [37] study the synchronization of the network of chaotic mobile agents by applying the speed-accelerating method.

The circuit application of chaotic systems mainly contains three aspects: the construction of chaotic circuits, the circuit realization of chaotic systems, the practical control and application realization of chaotic systems on electronic circuits. The papers included in this special issue involve these aspects, and give a broad and close observation on the recent research of chaotic systems. To our best knowledge, we think there are still many problems on the circuit application of chaotic systems to be solved. The generation of complex and special chaotic circuits for practical application need to be further studied, such as high-dimensional chaotic circuits with multistability and hidden attractors, large-scale memristive chaotic neural network circuits, etc. The study of the circuit realization of the control and application of chaotic systems is also relatively rare. It also will be an important topic to use electronic circuits to actually show the multistability and hidden attrac-

tors as they are interesting dynamical behaviors commonly found in chaotic circuits and systems [38–40]. We will continue to carry out the research of chaos theory and application, and collect more interesting work on the corresponding area in future issues.

Finally and importantly, we sincerely thanks all the authors for their high-quality papers in this special issue, and express our sincere gratitude to all the reviewers for their timely and valuable comments of the papers. Also we would like to appreciate all the members of EPJST for their timely and patient help. Moreover we would like to solemnly recommend this special issue to readers and hope this special issue can provide some help for chaos research and popularity.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: There is no data because all obtained results are in the paper. All authors confirm that there is no part of the paper that requires data.]

References

1. E.N. Lorenz, Deterministic nonperiodic flows. *J. Atmos. Sci.* **20**, 130 (1963)
2. G. Chen, T. Ueta, Yet another chaotic attractor. *Int J Bifur Chaos* **9**, 1465 (1999)
3. Q. Lai, S. Chen, Generating multiple chaotic attractors from Sprott B system. *Int J Bifur Chaos* **26**, 1650177 (2016)
4. Q. Lai, B. Norouzi, F. Liu, Dynamic analysis, circuit realization, control design and image encryption application of an extended Lu system with coexisting attractors. *Chaos Solit. Fract.* **114**, 230 (2018)
5. Q. Lai, Z. Wan, P. D. Kamdem Kuate. Modeling and circuit realisation of a new no-equilibrium chaotic system with hidden attractor and coexisting attractors. *Electronics Letters* **56**, 1044 (2020)
6. Q. Lai, P. D. Kamdem Kuate, F. Liu, H. H. C. Iu. An extremely simple chaotic system with infinitely many coexisting attractors. *IEEE Trans. Circ. Syst.-II: Express Briefs* **67**, 1129 (2020)
7. Q. Lai, A unified chaotic system with various coexisting attractors. *Int. J. Bifur. Chaos* **31**, 2150013 (2021)
8. L.Q. Chua, M. Komuro, T. Matsumoto, The double scroll family. *IEEE Trans. Circ. Syst.* **33**, 289 (1986)
9. J. Gu, C.B. Li, T. Lei, S. He, F. Min, A memristive chaotic system with flexible attractor growing. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00125-w>
10. X. Ren, B. Chen, Q. Xu, H. Wu, M. Chen, Parameter and initial offset boosting dynamics in two-memristor-based Colpitts system. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00118-9>
11. C.L. Li, Y. Yang, J. Du, A simple chaotic circuit with magnetic flux-controlled memristor. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00181-2>
12. M. Ma, Y. Fang, Z. Li, Y. Sun, M. Wang, Bursting oscillations and bifurcation mechanism in a fully integrated piecewissmooth chaotic system. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00128-7>
13. Y. Wang, F. Min, Y. Cheng, Y. Dou, Dynamical analysis in dual-memristor-based FitzHugh-Nagumo circuit and its coupling finite-time synchronization. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00121-0>
14. F. Yu, L. Li, B. He, L. Liu, S. Qian, H. Shen, Z. Zhang, S. Cai, Y. Li, Pseudorandom number generator based on a 5D hyperchaotic four-wing memristive system and its FPGA implementation. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00132-x>
15. M. Borah, B.K. Roy, Hidden multistability in four fractional-order memristive, meminductive and memcapacitive chaotic systems with bursting and boosting phenomena. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00179-w>
16. Z. Yang, D. Liang, D. Ding, Y. Hu, Dynamic analysis of fractional-order memristive chaotic system with time delay and its application in color image encryption based on DNA encoding. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00117-w>
17. F. Li, T. Wang, M. Chen, H. Wu, A unified asymmetric memristive diode-bridge emulator and hardware confirmation. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00180-3>
18. L.K. Kengne, R. Kengne, Z.T. Njitacke, T.F. Fozzin, J.R.M. Pone, H.T.K. Tagne, Effects of symmetry-breaking on the dynamics of the Shinriki's oscillator. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00130-z>
19. S. Emiroglu, Y. Uyaroglu, T.E. Gumus, Recursive backstepping control of ferroresonant chaotic oscillations consisting between grading capacitor with nonlinear inductance of voltage transformer. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00150-9>
20. Z.T. Njitacke, M.E. Sone, T.F. Fozzin, N. Tsafack, G.D. Leutcho, C.T. Tchapgá, Control of multistability with selection of chaotic attractor: application to image encryption. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00137-6>
21. J.J. He, B.C. Lai, Investigation and realization of novel chaotic system with one unstable equilibrium and symmetric coexisting attractors. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00124-x>
22. Y. Wu, C.H. Wang, Q. Deng, A new 3D multi-scroll chaotic system generated with three types of hidden attractors. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00119-8>
23. J. Gao, M. Wang, X. Peng1, Y. Zeng. Compounding self-excited and hidden attractors via a non-autonomous approach. *Eur. Phys. J. Spec. Top.* (2021) <https://doi.org/10.1140/epjs/s11734-021-00120-1>
24. G.D. Leutcho, H. Wang, R. Kengne, L.K. Kengne, Z.T. Njitacke, T.F. Fozzin, Symmetry-breaking, amplitude control and constant Lyapunov exponent based on single parameter snap flow. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00136-7>
25. H.Y. Cao, L. Zhao, A new chaotic system with different equilibria and attractors. *Eur. Phys. J.*

- Spec. Top. (2021). <https://doi.org/10.1140/epjs/s11734-021-00123-y>
26. I. Koyuncu, K. Rajagopal, M. Alcin, A. Karthikeyan, M. Tuna, M. Varan, Control, synchronization with linear quadratic regulator method and FFANN-based PRNG application on FPGA of a novel chaotic system. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00178-x>
 27. L. Xiong, L. Qi, S. Teng, Q. Wang, L. Wang, X. Zhang, A simplest Lorenz-like chaotic circuit and its applications in secure communication and weak signal detection. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00177-y>
 28. C. Ma, J. Mou, P. Li, T. Liu, Dynamic analysis of a new two-dimensional map in three forms: integer-order, fractional-order and improper fractional-order. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00133-w>
 29. C. Li, Y. Yang, S. He, X. Yang, J. Du, Self-reproducing dynamics in a two-dimensional discrete map. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00182-1>
 30. C.H. Li, Z.C. Wei, W. Zhang, Periodic solutions and circuit design of chaos in a unified stretch-twist-fold flow. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00127-8>
 31. T.H. Tchinda, Z.T. Njitacke, T.F. Fonzin, H.B. Fotsin, Hidden dynamics of an optically injected laser diode subject to threshold electromagnetic induction: coexistence of multiple stable states. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00134-9>
 32. Q. Xu, Z. Ju, C. Feng, H. Wu, M. Chen, Analogy circuit synthesis and dynamics confirmation of a bipolar pulse current-forced 2D Wilson neuron model. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00183-0>
 33. K. Rajagopal, A. Jafari, S. He, F. Parastesh, S. Jafari, I. Hussain, Simplest symmetric chaotic flows: the strange case of asymmetry in master stability function. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00131-y>
 34. C. Wu, K. Sun, Y. Xiao, A hyperchaotic map with multi elliptic cavities based on modulation and coupling. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00126-9>
 35. A. Ouannas, N. Debbouche, V.T. Pham, S.L. Kingston, T. Kapitaniak, Chaos in fractional system with extreme events. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00135-8>
 36. X. Zhao, J. Liu, F. Zhang, C. Jiang, Complex generalized synchronization in complex-variable chaotic system. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00129-6>
 37. Y. Zhou, J. Zhou, C. Chen, G. Xiao, Speed-accelerating method for the control of mobile chaotic agents. *Eur. Phys. J. Spec. Top.* (2021). <https://doi.org/10.1140/epjs/s11734-021-00122-z>
 38. Q. Lai, Z. Wan, L.K. Kengne, P.D. Kamdem Kuate, C. Chen. Two-memristor-based chaotic system with infinite coexisting attractors. *IEEE Trans. Circ. Sys.-II: Express Briefs*, **68**, 2197 (2021)
 39. Q. Lai, Z. Wan, A. Akgul, O.F. Boyraz, M.Z. Yildiz, Design and implementation of a new memristive chaotic system with application in touchless fingerprint encryption. *Chin. J. Phys.* **67**, 615 (2020)
 40. Q. Lai, Z. Wan, P.D. Kamdem Kuate, H. Fotsin. Coexisting attractors, circuit implementation and synchronization control of a new chaotic system evolved from the simplest memristor chaotic circuit. *Commun. Nonlinear Sci. Numer. Simul.* **89**, 105341 (2020)