




A dynamical study on stochastic reaction diffusion epidemic model with nonlinear incidence rate

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Abstract The current study deals with the stochastic reaction–diffusion epidemic model numerically with two proposed schemes. Such models have many applications in the disease dynamics of wildlife, human life, and others. During the last decade, it is observed that the epidemic models cannot predict the accurate behavior of infectious diseases. The empirical data gained about the spread of the disease shows non-deterministic behavior. It is a strong challenge for researchers to consider stochastic epidemic models. The effect of the stochastic process is analyzed. So, the SIR epidemic model is considered under the influence of the stochastic process. The time noise term is taken as the stochastic source. The coefficient of the stochastic term is a Borel function, and it is used to control the random behavior in the solutions. The proposed stochastic backward Euler scheme and the proposed stochastic implicit finite difference scheme (IFDS) are used for the numerical solution of the underlying model. Both schemes are consistent in the mean square sense. The stability of the schemes is proven with Von-Neumann criteria and schemes are unconditionally stable. The proposed stochastic backward Euler scheme converges toward a disease-free equilibrium and does not converge toward an endemic equilibrium but also possesses negative behavior. The proposed stochastic IFD scheme converges toward disease-free equilibrium and endemic equilibrium. This scheme also preserves positivity. The graphical behavior of the stochastic SIR model is much similar to the classical SIR epidemic model when noise strength approaches zero. The three-dimensional plots of the susceptible and infected individuals are drawn for two cases of endemic equilibrium and disease-free equilibriums. The efficacy of the proposed scheme is shown in the graphical behavior of the test problem for the various values of the parameters.

1 Introduction

The reaction–diffusion models are used in ecology, and virology to explain various biological phenomena. Such models describe the population or concentration distributed in space under the effect of two phenomena. In population models or concentration models, the diffusion term explains the random motion of the individuals, and their reproduction is described by the reaction term. In epidemiology, reaction–diffusion models are frequently used to understand the phenomena of various diseases such as how the disease starts, how it is affected by the movement of different species, how it can be controlled, and how much time is required to eradicate it from the species. Some researchers worked on the numerical solutions of the epidemic models. A mathematical model was proposed by Wang and Wang [1] for the simulation of the hepatitis B virus infection and introduced a random motion of the

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virus in the deterministic model proposed by Nowak et al. [2], and they assumed that concentration gradient is proportional to the population flux of the virus. Suryanto et al. proposed a nonstandard FDS for the numerical approximation of the SIR epidemic model having a saturated incidence rate. The results obtained with the given scheme are dynamically consistent with the continuous model [3]. Naik et al. considered the SIR epidemic model to have a Crowley–Martin-type functional response and Holling type-II treatment rate. They used the Homotopy analysis method is used for the analytical solutions of the given model. The authors discussed the stability of the model and conclude that it has two equilibria, one is disease free and the other is endemic [4].

Nauman et al. [5] considered the following epidemic model having nonlinear incidence rate as follows:

$$\frac{\partial S}{\partial t} = d_1 \frac{\partial^2 S}{\partial x^2} + \lambda - \eta S - \frac{\beta SI}{1 + c_1 S + c_2 I + c_3 SI}, \quad (1)$$

$$\frac{\partial I}{\partial t} = d_2 \frac{\partial^2 I}{\partial x^2} - (\eta + \delta + \sigma)I + \frac{\beta SI}{1 + c_1 S + c_2 I + c_3 SI}, \quad (2)$$

$$\frac{\partial R}{\partial t} = d_3 \frac{\partial^2 R}{\partial x^2} - \eta R + \sigma R, \quad (3)$$

with initial condition

$$S(x, 0) = \alpha_1(x), \quad (4)$$

$$I(x, 0) = \alpha_2(x), \quad (5)$$

and homogeneous Neumann boundary conditions(B.Cs)

$$\frac{\partial S(0, t)}{\partial x} = 0, \quad \frac{\partial S(l, t)}{\partial x} = 0, \quad (6)$$

$$\frac{\partial I(0, t)}{\partial x} = 0, \quad \frac{\partial I(l, t)}{\partial x} = 0. \quad (7)$$

Where $S(x, t)$, $I(x, t)$, and $R(x, t)$ represent the susceptible, infected, and recovered individuals, respectively, at any space variable x and time t . The parameters λ , η , δ , σ represent the recruitment rate, natural death rate, the death rate due to disease, and recovery rate of the infected individuals, respectively. The d_a , $a = 1, 2$ are the diffusion coefficients, β is the infection parameter, c_a , $a = 1, 2, 3$ are the arbitrary positive constants, $\frac{\beta SI}{1 + c_1 S + c_2 I + c_3 SI}$ is the nonlinear modified incidence rate. The nonlinear modified incidence rate becomes bilinear if $c_a = 0$, $a = 1, 2, 3$, and the saturated incidence if $c_a = 0$, $a = 1, 2$ or $a = 2, 3$. The stability analysis of the above model has been carried out, and it has two equilibria, one is the disease-free equilibrium is $(\lambda/\eta, 0)$ and the other is endemic equilibrium (S^*, I^*) and $S^* = \frac{2(e' + c_2\lambda)}{\beta - c_1 e' + c_2 \eta - c_3 \lambda + \sqrt{d'}}$

$I^* = \frac{\lambda - \eta S^*}{e'}$ where $e' = \eta + \delta + \alpha$, $d' = (\beta - c_1 e' + c_2 \eta - c_3 \lambda)^2 + 4c_3 \eta (e' + c_2 \lambda)$. The reproduction number is given as $R_0 = \frac{\beta \lambda}{(\eta + c_1 \lambda)(\eta + \delta + \alpha)}$. As Eqs. (1) and (2) are independent of the state variable R , it is quite natural to consider the system of Eqs.

(1,2) instead of (1,2,3) because R does not affect the susceptible and infected individuals.

Modeling of physical phenomena is an interesting work. For this purpose, partial differential equations (PDEs) are used to describe the physical behavior [6–9]. The solution of the PDEs is an active area of research, and various techniques are applied for the solutions of PDEs [10–16]. Forty years ago, it was a faith that medical science has made tremendous progress in reducing the mortality rate of humans because it is due to improvements in nutrition, drugs, and vaccines. Even then, infectious diseases have been a severe threat to the human and livestock population. The classical epidemic models fail to describe the true behavior of the diseases. So, it is the need of the hour to consider the epidemic models under some stochastic process. So, new literature is required for cases. various researchers are working on the dynamics of stochastic partial differential equations. The authors used the finite difference scheme to analyze the different aspects of various physical phenomena [17–19]. Macías-Díaz et al. analyzed the stochastic epidemiological model with a nonstandard finite difference scheme [20]. Raza et al. developed a nonstandard finite difference for the study of the stochastic smoking model and discussed various dynamics [21]. Nauman et al. worked on the numerical approximation of the stochastic fractional epidemic model [22]. Raza et al. found the solution of the stochastic dengue epidemic model [23]. Alkhazzan et al. analyzed the SVIR epidemic model and discussed its various dynamics [24].

The main contributions of this manuscript are as given below:

- (i) The epidemic models cannot predict the accurate behavior of the infectious disease. So, stochastic epidemic models are preferred. We are considering the stochastic epidemics model.
- (ii) Numerical approximation of the underlying model is carried out by two schemes.
- (iii) The stability and consistency of the schemes are analyzed in the mean square sense.
- (iv) The given model has two equilibrium points: one is disease free and the other is endemic equilibrium.
- (v) The numerical simulations of the test problem are drawn for various choices of the parameters.
- (vi) The MATLAB 2015a is used for the graphical behavior of the test problem.

In structure of article, Sect. 2 contains the stochastic epidemic model. The Sect. 3 deals with the two numerical schemes for the given model. The analysis of both schemes is carried out. Section 4 deals with the test problem, and its graphical behavior and discussion are carried out for endemic equilibrium and disease-free equilibrium as well. The last Sect. 5 related to the conclusion of this article.

2 Stochastic epidemic model

During the last decade, it is observed that the epidemic models cannot predict the accurate behavior of the infectious disease. The empirical data gained about the spread of the disease show the non-deterministic behavior [25]. So, various stochastic models are proposed to describe the propagation of the disease in human life such as stochastic semi-mechanistic models [26], the stochastic SIS epidemic models with nonlinear incidence rate [27], the stochastic SIRS epidemic models having saturated incidence rate [28], and the SIS nonlinear stochastic epidemiological models having incidence rate and double epidemic hypothesis[29]. It is quite natural to consider the epidemic under the effect of some perturbation.

We are taking the stochastic version of the system of the Eqs. (1,2) under the effect of time noise as follows:

$$\frac{\partial S}{\partial t} = d_1 \frac{\partial S}{\partial x^2} + \lambda - \eta S - \frac{\beta SI}{1 + c_1 S + c_2 I + c_3 SI} + \alpha_1 S \dot{B}_1(t), \tag{8}$$

$$\frac{\partial I}{\partial t} = d_2 \frac{\partial^2 I}{\partial x^2} - (\eta + \delta + \sigma)I + \frac{\beta SI}{1 + c_1 S + c_2 I + c_3 SI} + \alpha_2 I \dot{B}_2(t), \tag{9}$$

with initial conditions

$$S(x, 0) = \alpha_1(x), \tag{10}$$

$$I(x, 0) = \alpha_2(x), \tag{11}$$

and homogeneous Neumann boundary conditions(B.Cs)

$$\frac{\partial S(0, t)}{\partial x} = 0, \frac{\partial S(L, t)}{\partial x} = 0, \tag{12}$$

$$\frac{\partial I(0, t)}{\partial x} = 0, \frac{\partial I(L, t)}{\partial x} = 0. \tag{13}$$

Where $\alpha_a, a = 1, 2$ are the noise strength of the time noise, $\dot{B}_a(t), a = 1, 2$ is the time noise and we consider the standard Wiener process for stochastic behavior of the epidemic model. Various researchers are working on the numerical study of SDEs. Cai et al. worked on the solution of the stochastic epidemic model analytically and numerically [30]. Meng et al. proposed a stochastic eco-epidemiological model with time delay and a general incidence rate. They discussed the unique existence of a positive global solution with positive initial conditions. They also established the stochastic analysis of the model [31]. The authors considered a stochastic hepatitis B model and derived the different conditions for the extinction and persistence of the disease [32]. In [33], the authors used the numerical technique for the study of the *Covid – 19* model. The authors worked on the numerical approximations of the stochastic partial differential equations [34]. Zho et al. worked on the stochastic SIR model and discussed the different dynamics of the model. They show that the model has a unique global positive solution and the solution oscillates around the equilibrium points under some conditions [35]. Some more work on the numerical solutions of the differential equations can be seen [36–41]. We are using two numerical schemes namely, one is the proposed stochastic backward Euler scheme and the other is the proposed stochastic implicit FDS. The analysis of the schemes and after that numerical simulations will be carried out for the efficacy of our findings.

3 Numerical schemes

We have divided the $[0, L] \times [0, T]$ into $N \times M$ grid points, and the stepsizes are $l = T/M, m = L/N$, respectively. The grid points are $x_a = am, a = 0, 1, 2, 3, 4, \dots, N,$
 $t_b = bl, b = 0, 1, 2, 3, 4, \dots, M.$

The proposed stochastic backward Euler scheme for Eqs. (8,9) is

$$(1 + 2R_1)S_a^{b+1} - R_1(S_{a-1}^{b+1} + S_a^{b+1}) = (1 - \eta l)S_a^b + \lambda l - \frac{l\beta S_a^b I_a^b}{1 + c_1 S_a^b + c_2 I_a^b + c_3 S_a^b I_a^b} + \alpha_1 S_a^b (B_1^{(b+1)l} - B_1^{bl}) \tag{14}$$

$$(1 + 2R_2)I_a^{b+1} - R_2(I_{a-1}^{b+1} + I_a^{b+1}) = (1 - l((\eta + \delta + \sigma)))I_a^b + \frac{l\beta S_a^b I_a^b}{1 + c_1 S_a^b + c_2 I_a^b + c_3 S_a^b I_a^b} + \alpha_2 S_a^b (B_2^{(b+1)l} - B_2^{bl}) \tag{15}$$

The proposed stochastic implicit finite difference scheme for Eqs. (8,9) is

$$-R_1 S_{a-1}^{b+1} + (1 + 2R_1 + \eta l + \frac{l\beta I_a^b}{1 + c_1 S_a^b + c_2 I_a^b + c_3 S_a^b I_a^b})S_a^{b+1} - R_1 S_{1+1}^{b+1} = S_a^b + \lambda l + \alpha_1 S_a^b (B_1^{(b+1)l} - B_1^{bl}). \tag{16}$$

$$-R_2 I_{a-1}^{b+1} + (1 + 2R_2 + l(\eta + \delta + \sigma)) I_a^{b+1} - R_2 I_a^{b+1} = I_a^b + \frac{l\beta S_a^b I_a^b}{1 + c_1 S_a^b + c_2 I_a^b + c_3 S_a^b I_a^b} + \alpha_2 S_a^b (B_2^{(b+1)l} - B_2^{bl}). \tag{17}$$

where $R_1 = ld_1/m^2$, and $R_2 = ld_2/m^2$. The S_a^b, I_a^b are the numerical approximations of the S, I at the point (am, bl) .

3.1 Consistency of a scheme

Definition [42–44]. In the mean square sense, a SFDS $A|_a^b U|_a^b = F|_a^b$ is consistent with stochastic PDE $AU = F$ at a point (x, t) , if there exist any continuously differentiable function $\Xi = \Xi(x, t)$, then

$$E| |(A\Xi U)|_a^b - [A|_a^b \Xi|_{(am,bl)} - F|_a^b] |^2 \rightarrow 0 \tag{18}$$

as $m \rightarrow 0, l \rightarrow 0$ and $(am, (b + 1)l) \rightarrow (x, t)$.

3.2 Von-Neumann analysis

In this technique, $S|_a^b$ is taken as follow,

$$S|_a^b = \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} e^{iam\psi} \hat{S}|_a^b(\psi) d(\psi), \tag{19}$$

and $\hat{S}|_a^b$ is defined as

$$\hat{S}|_a^b = \frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{\infty} e^{-iam\psi} S|_a^b m,$$

here, ψ is variable, and by replacing values in a given PDE,

$$\hat{S}|_a^{b+1}(\psi) = \hat{S}|_a^b(\psi) g(\psi m, m, l). \tag{20}$$

Now, the necessary and sufficient condition of this method is given below [45],

$$E|g(\psi m, m, l)|^2 \leq 1 + \chi l, \tag{21}$$

where, χ is a constant.

3.3 Consistency of the proposed stochastic backward Euler scheme

The consistency analysis is carried out in the mean square sense.

Theorem 1 *In a mean square sense, the stochastic FD scheme in Eqs. (14,15) for the variables S, I is consistent with Eqs. (8, 9).*

Proof By assumption, $S(x, t)$ and $I(x, t)$ are smooth functions and by using $A(f) = \int_{bl}^{(b+1)l} f dr$ on Eq. (8). We get

$$\begin{aligned} A(S)|_a^b &= S(am, (b + 1)l) - S(am, bl) - d_1 \int_{bl}^{(b+1)l} S_{xx}(am, r) dr \\ &\quad - \int_{bl}^{(b+1)l} \lambda dr + \eta \int_{bl}^{(b+1)l} S(am, r) dr + \beta \int_{bl}^{(b+1)l} \frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \\ &\quad - \alpha_1 \int_{bl}^{(b+1)l} S(am, r) dB_1(r). \end{aligned}$$

By using the proposed scheme on Eq. (8)

$$\begin{aligned} A|_a^b(S) &= S(am, (b + 1)l) - S(am, bl) \\ &\quad - d_1 l \frac{S((i + 1)m, (b + 1)l) - 2S(am, (b + 1)l) + S((i - 1)m, (b + 1)l)}{m^2} \\ &\quad - l\lambda + \eta S(am, bl) \\ &\quad + l \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \\ &\quad - \alpha_1 S(am, bl)(B_1^{(b+1)l} - B_1^{bl}). \end{aligned}$$

The above two equations can be expressed as in the mean square

$$\begin{aligned}
 E|A(S)|_a^b - A|_a^b S|^2 &\leq 4d_1^2 E \left| \int_{bl}^{(b+1)l} (-S_{xx}(am, r) \right. \\
 &\quad \left. + \frac{S((1+1)m, (b+1)l) - 2S(am, (b+1)l) + S((i-1)m, (b+1)l)}{m^2} \right) dr \Big|^2 \\
 &\quad + 4\eta^2 E \left| \int_{bl}^{(b+1)l} (-S(am, r) + S(am, bl)) dr \right|^2 \\
 &\quad + 4\beta^2 E \left| \int_{bl}^{(b+1)l} \left(\frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \right. \right. \\
 &\quad \left. \left. - \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \right) dr \right|^2 \\
 &\quad + 4\alpha_1^2 E \left| \int_{bl}^{(b+1)l} (-S(am, r) + S(am, bl)) dB_1(r) \right|^2.
 \end{aligned}$$

Due to the Itô integrals' square feature, we can

$$\begin{aligned}
 E|A(S)|_a^b - A|_a^b S|^2 &\leq 4d_1^2 E \left| \int_{bl}^{(b+1)l} (-S_{xx}(am, r) \right. \\
 &\quad \left. + \frac{S((1+1)m, (b+1)l) - 2S(am, (b+1)l) + S((i-1)m, (b+1)l)}{m^2} \right) dr \Big|^2 \\
 &\quad + 4\eta^2 E \left| \int_{bl}^{(b+1)l} (-S(am, r) + S(am, bl)) dr \right|^2 \\
 &\quad + 4\beta^2 E \left| \int_{bl}^{(b+1)l} \left(\frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \right. \right. \\
 &\quad \left. \left. - \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \right) dr \right|^2 \\
 &\quad + 4\alpha_1^2 \int_{bl}^{(b+1)l} E|(-S(am, r) + S(am, bl))|^2 dr.
 \end{aligned}$$

$E|A(S)|_a^b - A|_a^b S|^2 \rightarrow 0$ as $a \rightarrow \infty, b \rightarrow \infty$. The scheme for S is consistent (8). Now, the consistency of the (9) is given below

$$\begin{aligned}
 A(I)|_a^b &= I(am, (b+1)l) - I(am, bl) - d_2 \int_{bl}^{(b+1)l} I_{xx}(am, r) dr \\
 &\quad + (\eta + \delta + \sigma) \int_{bl}^{(b+1)l} I(am, r) dr - \beta \int_{bl}^{(b+1)l} \frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \\
 &\quad - \alpha_2 \int_{bl}^{(b+1)l} I(am, r) dB_2(r).
 \end{aligned}$$

By using scheme on Eq. (8),

$$\begin{aligned}
 A|_a^b(I) &= I(am, (b+1)l) - I(am, bl) \\
 &\quad - d_2 l \frac{I((1+1)m, (b+1)l) - 2I(am, (b+1)l) + I((a-1)m, (b+1)l)}{m^2} \\
 &\quad + (\eta + \delta + \sigma)I(am, bl) - l \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \\
 &\quad - \alpha_2 I(am, bl)(B_2^{(b+1)l} - B_2^{bl}).
 \end{aligned}$$

The above two equations can be expressed as in the mean square

$$\begin{aligned}
 E|A(I)|_a^b - A|_a^b I|^2 &\leq 4d_2^2 E \left| \int_{bl}^{(b+1)l} (-I_{xx}(am, r) \right. \\
 &\quad \left. + \frac{I((1+1)m, (b+1)l) - 2I(am, (b+1)l) + I((a-1)m, (b+1)l)}{m^2} \right) dr \Big|^2 \\
 &\quad + 4(\eta + \delta + \sigma)^2 E \left| \int_{bl}^{(b+1)l} (I(am, r) - I(am, bl)) dr \right|^2 \\
 &\quad + 4\beta^2 E \left| \int_{bl}^{(b+1)l} \left(\frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \right. \right. \\
 &\quad \left. \left. - \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \right) dr \right|^2
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \Big) \\
 & + 4\alpha_2^2 E \left| \int_{bl}^{(b+1)l} \left(-I(am, r) + I(am, bl) \right) dB_2(r) \right|^2.
 \end{aligned}$$

Due to the Itô integrals' square feature, we can

$$\begin{aligned}
 E|A(I)|_a^b - A|_a^b I|^2 & \leq 4d_2^2 E \left| \int_{bl}^{(b+1)l} (-I_{xx}(am, r) \right. \\
 & + \left. \frac{I((1 + 1)m, (b + 1)l) - 2I(am, (b + 1)l) + I((a - 1)m, (b + 1)l)}{m^2} \right) dr \Big|^2 \\
 & + 4(\eta + \delta + \sigma)^2 E \left| \int_{bl}^{(b+1)l} (I(am, r) - I(am, bl)) dr \right|^2 \\
 & + 4\beta^2 E \left| \int_{bl}^{(b+1)l} \left(\frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \right. \right. \\
 & - \left. \left. \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \right) \right. \\
 & \left. + 4\alpha_2^2 \int_{bl}^{(b+1)l} E|(-I(am, r) + I(am, bl))|^2 dr. \right.
 \end{aligned}$$

$E|A(I)|_a^b - A|_a^b I|^2 \rightarrow 0$ as $a \rightarrow \infty, b \rightarrow \infty$. The scheme for I is consistent with (9).

3.4 Consistency of proposed stochastic implicit finite difference scheme

The consistency analysis is carried out in the mean square sense.

Theorem 2 *In a mean square sense, the stochastic FD scheme in Eqs. (16, 16) for the variables S, I is consistent with Eqs. (8,9).*

Proof By assumption $S(x, t)$ and $I(x, t)$ are smooth functions and by using the $A(f) = \int_{bl}^{(b+1)l} f dr$ on Eq. (8). We get

$$\begin{aligned}
 A(S)|_a^b & = S(am, (b + 1)l) - S(am, bl) - d_1 \int_{bl}^{(b+1)l} S_{xx}(am, r) dr - \int_{bl}^{(b+1)l} \lambda dr \\
 & + \eta \int_{bl}^{(b+1)l} S(am, r) dr + \beta \int_{bl}^{(b+1)l} \frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \\
 & - \alpha_1 \int_{bl}^{(b+1)l} S(am, r) dB_1(r).
 \end{aligned}$$

By using the proposed scheme- on Eq. (8),

$$\begin{aligned}
 A|_a^b(S) & = S(am, (b + 1)l) - S(am, bl) \\
 & - d_1 l \frac{S((1 + 1)m, (b + 1)l) - 2S(am, (b + 1)l) + S((a - 1)m, (b + 1)l)}{m^2} \\
 & - l\lambda + \eta S(am, (b + 1)l) + l \frac{S(am, (b + 1)l)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \\
 & - \alpha_1 S(am, bl)(B_1^{(b+1)l} - B_1^{bl}).
 \end{aligned}$$

The above two equations can be expressed as in the mean square

$$\begin{aligned}
 E|A(S)|_a^b - A|_a^b S|^2 & \leq 4d_1^2 E \left| \int_{bl}^{(b+1)l} (-S_{xx}(am, r) \right. \\
 & + \left. \frac{S((1 + 1)m, (b + 1)l) - 2S(am, (b + 1)l) + S((i - 1)m, (b + 1)l)}{m^2} \right) dr \Big|^2 \\
 & + 4\eta^2 E \left| \int_{bl}^{(b+1)l} (-S(am, r) + S(am, (b + 1)l)) dr \right|^2 \\
 & + 4\beta^2 E \left| \int_{bl}^{(b+1)l} \left(\frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \right. \right. \\
 & - \left. \left. \frac{S(am, (b + 1)l)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \right) \right.
 \end{aligned}$$

$$+ 4\alpha_1^2 E \left| \int_{bl}^{(b+1)l} \left(-S(am, r) + S(am, bl) \right) dB_1(r) \right|^2.$$

Due to the Itô integrals' square feature, we can

$$\begin{aligned} E|A(S)|_a^b - A|_a^b S|^2 &\leq 4d_1^2 E \left| \int_{bl}^{(b+1)l} (-S_{xx}(am, r) \right. \\ &\quad \left. + \frac{S((1+1)m, (b+1)l) - 2S(am, (b+1)l) + S((i-1)m, (b+1)l)}{m^2} \right) dr \right|^2 \\ &\quad + 4\eta^2 E \left| \int_{bl}^{(b+1)l} (-S(am, r) + S(am, (b+1)l)) dr \right|^2 \\ &\quad + 4\beta^2 E \left| \int_{bl}^{(b+1)l} \left(\frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \right. \right. \\ &\quad \left. \left. - \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, (b+1)l) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \right) \right|^2 \\ &\quad + 4\alpha_1^2 \int_{bl}^{(b+1)l} E|(-S(am, r) + S(am, bl))|^2 dr. \end{aligned}$$

$E|A(S)|_a^b - A|_a^b S|^2 \rightarrow 0$ as $a \rightarrow \infty, b \rightarrow \infty$, so this proposed scheme for S is consistent with stochastic PDE (8).

Now, the consistency of the (9) is given as

$$\begin{aligned} A(I)|_a^b &= I(am, (b+1)l) - I(am, bl) - d_2 \int_{bl}^{(b+1)l} I_{xx}(am, r) dr \\ &\quad + (\eta + \delta + \sigma) \int_{bl}^{(b+1)l} I(am, r) dr - \beta \int_{bl}^{(b+1)l} \frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \\ &\quad - \alpha_2 \int_{bl}^{(b+1)l} I(am, r) dB_2(r). \end{aligned}$$

By using the proposed scheme on Eq. (9)

$$\begin{aligned} A|_a^b(I) &= I(am, (b+1)l) - I(am, bl) \\ &\quad - d_2 l \frac{I((1+1)m, (b+1)l) - 2I(am, (b+1)l) + I((i-1)m, (b+1)l)}{m^2} \\ &\quad + (\eta + \delta + \sigma) I(am, (b+1)l) - l \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \\ &\quad - \alpha_2 I(am, bl)(B_2^{(b+1)l} - B_2^{bl}). \end{aligned}$$

The above two equations can be expressed as in the mean square

$$\begin{aligned} E|A(I)|_a^b - A|_a^b I|^2 &\leq 4d_2^2 E \left| \int_{bl}^{(b+1)l} (-I_{xx}(am, r) \right. \\ &\quad \left. + \frac{I((1+1)m, (b+1)l) - 2I(am, (b+1)l) + I((i-1)m, (b+1)l)}{m^2} \right) dr \right|^2 \\ &\quad + 4(\eta + \delta + \sigma)^2 E \left| \int_{bl}^{(b+1)l} (I(am, r) - I(am, (b+1)l)) dr \right|^2 \\ &\quad + 4\beta^2 E \left| \int_{bl}^{(b+1)l} \left(\frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \right. \right. \\ &\quad \left. \left. - \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \right) \right|^2 \\ &\quad + 4\alpha_2^2 E \left| \int_{bl}^{(b+1)l} \left(-I(am, r) + I(am, bl) \right) dB_2(r) \right|^2. \end{aligned}$$

Due to the Itô integrals' square feature, we can

$$\begin{aligned}
 E|A(I)|_a^b - A|_a^b I|^2 &\leq 4d_2^2 E \int_{bl}^{(b+1)l} (-I_{xx}(am, r) \\
 &+ \frac{I((1+1)m, (b+1)l) - 2I(am, (b+1)l) + I((i-1)m, (b+1)l)}{m^2}) dr|^2 \\
 &+ 4(\eta + \delta + \sigma)^2 E \int_{bl}^{(b+1)l} (I(am, r) - I(am, (b+1)l)) dr|^2 \\
 &+ 4\beta^2 E \int_{bl}^{(b+1)l} \left(\frac{S(am, r)I(am, r)}{1 + c_1 S(am, r) + c_2 I(am, r) + c_3 S(am, r)I(am, r)} \right. \\
 &\quad \left. - \frac{S(am, bl)I(am, bl)}{1 + c_1 S(am, bl) + c_2 I(am, bl) + c_3 S(am, bl)I(am, bl)} \right) \\
 &+ 4\alpha_2^2 \int_{bl}^{(b+1)l} E|(-I(am, r) + I(am, bl))|^2 dr.
 \end{aligned}$$

$E|A(I)|_a^b - A|_a^b I|^2 \rightarrow 0$ as $a \rightarrow \infty, b \rightarrow \infty$. The scheme for I is consistent with Eq. (9).

3.5 Stability of proposed stochastic backward Euler scheme

The linear stability analysis is carried out.

Theorem 3 *The scheme for S, I by Eqs. (14,15) is unconditionally stable.*

Proof We are using the Von-Neumann technique for the stability analysis. So, by linearizing Eq. (14) and keeping the coefficient term frozen, we have

$$(1 + 2R_1)S_a^{b+1} - R_1(S_{a-1}^{b+1} + S_a^{b+1}) = (1 - \eta l)S_a^b + \alpha_1 S_a^b (B_1^{(b+1)l} - B_1^{bl}). \tag{22}$$

By replacing the value of S from Eq. (19)

$$\begin{aligned}
 (1 + 2R_1 - R_1 e^{-im\psi} - R_1 e^{im\lambda}) \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} e^{iam\psi} \hat{S}_a^{b+1}(\psi) d(\psi) &= \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} \\
 e^{iam\psi} \left(1 - \eta l + \alpha_1 (B_1^{(b+1)l} - B_1^{bl}) \right) \hat{S}_a^b(\psi) d(\psi). \\
 (1 + 2R_1 - R_1 e^{-im\psi} - R_1 e^{im\lambda}) \text{hat} S_a^{b+1}(\psi) &= \left(1 - \eta l + \alpha_1 (B_1^{(b+1)l} - B_1^{bl}) \right) \hat{S}_a^b(\psi). \\
 g(\psi m, l, m) &= \frac{1 - \eta l}{1 + 4R_1 \sin^2(\frac{m\psi}{2})} + \frac{\alpha_1}{1 + 4R_1 \sin^2(\frac{m\psi}{2})} (B_1^{(b+1)l} - B_1^{bl}).
 \end{aligned}$$

Using the Wiener process' independent state, the amplification factor can now be expressed as

$$E|g(\psi m, l, m)|^2 \leq \left| \frac{1 - \eta l}{1 + 4R_1 \sin^2(\frac{m\psi}{2})} \right|^2 + \left| \frac{\alpha_1}{1 + 4R_1 \sin^2(\frac{m\psi}{2})} \right|^2 l.$$

As always

$$\left| \frac{1 - \eta l}{1 + 4R_1 \sin^2(\frac{m\psi}{2})} \right|^2 < 1,$$

and let $\left| \frac{\alpha_1}{1 + 4R_1 \sin^2(\frac{m\psi}{2})} \right|^2 = \chi$, so,

$$E|g(\psi m, l, m)|^2 \leq 1 + \chi l. \tag{23}$$

So, scheme for S is stable.

Now, to discuss the stability of the scheme for I , by linearizing Eq. (9) and keeping the coefficient term frozen, we have

$$(1 + 2R_2)I_a^{b+1} - R_2(I_{a-1}^{b+1} + I_a^{b+1}) = (1 - l((\eta + \delta + \sigma)))I_a^b + \alpha_2 S_a^b (B_2^{(b+1)l} - B_2^{bl}). \tag{24}$$

By replacing the value of I from Eq. (19)

$$(1 + 2R_2 - R_2 e^{-im\psi} - R_2 e^{im\psi}) \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} e^{iam\psi} \hat{I}_a^{b+1}(\psi) d(\psi) = \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}}$$

$$\begin{aligned}
 & e^{i\alpha m \psi} \left(1 - (\eta + \delta + \sigma)l + \alpha_2(B_2^{(b+1)l} - B_2^{bl}) \right) \hat{I}_a^b(\psi) d(\psi). \\
 (1 + 2R_2 - R_2 e^{-i\alpha m \psi} - R_1 e^{i\alpha m \psi}) \text{hat } I_a^{b+1}(\psi) &= \left(1 - (\eta + \delta + \sigma)l + \alpha_2(B_2^{(b+1)l} - B_2^{bl}) \right) \hat{I}_a^b(\psi). \\
 g(\psi m, l, m) &= \frac{1 - (\eta + \delta + \sigma)l}{1 + 4R_2 \sin^2(\frac{m\psi}{2})} + \frac{\alpha_2}{1 + 4R_2 \sin^2(\frac{m\psi}{2})} (B_2^{(b+1)l} - B_2^{bl}).
 \end{aligned}$$

Using the Wiener process' independent state, the amplification factor can now be expressed as

$$E|g(\psi m, l, m)|^2 \leq \left| \frac{1 - (\eta + \delta + \sigma)l}{1 + 4R_2 \sin^2(\frac{m\psi}{2})} \right|^2 + \left| \frac{\alpha_2}{1 + 4R_2 \sin^2(\frac{m\psi}{2})} \right|^2 l.$$

As always

$$\left| \frac{1 - (\eta + \delta + \sigma)l}{1 + 4R_2 \sin^2(\frac{m\psi}{2})} \right|^2 < 1,$$

and let $\left| \frac{\alpha_2}{1 + 4R_2 \sin^2(\frac{m\psi}{2})} \right|^2 = \chi$, so

$$E|g(\psi m, l, m)|^2 \leq 1 + \chi l. \tag{25}$$

So, scheme for I is stable.

3.6 Stability analysis of proposed stochastic implicit scheme

The linear stability analysis is carried out.

Theorem 4 *The scheme for S, I by Eqs. (16,16) is unconditionally stable.*

Proof We are using the Von-Neumann technique for the stability analysis. So, by linearizing Eq. (14) and keeping the coefficient term frozen, we have

$$(1 + 2R_1 + \eta l)S_a^{b+1} - R_1(S_{a-1}^{b+1} + S_{a-1}^{b+1}) = (1)S_a^b + \alpha_1 S_a^b (B_1^{(b+1)l} - B_1^{bl}). \tag{26}$$

By replacing the value of S from Eq. (19),

$$\begin{aligned}
 (1 + 2R_1 + \eta l - R_1 e^{-i\alpha m \psi} - R_1 e^{i\alpha m \psi}) \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} e^{i\alpha m \psi} \hat{S}_a^{b+1}(\psi) d(\psi) &= \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} \\
 e^{i\alpha m \psi} \left(1 + \alpha_1 (B_1^{(b+1)l} - B_1^{bl}) \right) \hat{S}_a^b(\psi) d(\psi). \\
 (1 + 2R_1 + \eta l - R_1 e^{-i\alpha m \psi} - R_1 e^{i\alpha m \psi}) \text{hat } S_a^{b+1}(\psi) &= \left(1 + \alpha_1 (B_1^{(b+1)l} - B_1^{bl}) \right) \hat{S}_a^b(\psi). \\
 g(\psi m, l, m) &= \frac{1}{1 + \eta l + 4R_1 \sin^2(\frac{m\psi}{2})} + \frac{\alpha_1}{1 + \eta l + 4R_1 \sin^2(\frac{m\psi}{2})} (B_1^{(b+1)l} - B_1^{bl}).
 \end{aligned}$$

Using the Wiener process' independent state, the amplification factor can now be expressed as

$$E|g(\psi m, l, m)|^2 \leq \left| \frac{1}{1 + \eta l + 4R_1 \sin^2(\frac{m\psi}{2})} \right|^2 + \left| \frac{\alpha_1}{1 + \eta l + 4R_1 \sin^2(\frac{m\psi}{2})} \right|^2 l.$$

As always

$$\left| \frac{1}{1 + \eta l + 4R_1 \sin^2(\frac{m\psi}{2})} \right|^2 < 1,$$

and let $\left| \frac{\alpha_1}{1 + \eta l + 4R_1 \sin^2(\frac{m\psi}{2})} \right|^2 = \chi$, so

$$E|g(\psi m, l, m)|^2 \leq 1 + \chi l. \tag{27}$$

So, proposed for S is stable.

Table 1 Values of parameters for plots

Sr. no.	c_1	c_2	c_3	d_1	d_2	$\eta = \delta$	$\lambda = \sigma$	m
1	0.1	0.02	0.03	0.1	0.5	0.1	0.5	1/100

Now, to discuss the stability of the scheme for I , by linearizing Eq. (16) and keeping the coefficient term frozen, we have

$$(1 + 2R_2 + l(\eta + \delta + \sigma))I_a^{b+1} - R_2(I_{a-1}^{b+1} + I_{a-1}^{b+1}) = I_a^b + \alpha_2 S_a^b (B_2^{(b+1)l} - B_2^{bl}) \tag{28}$$

By replacing the value of I from Eq. (19)

$$(1 + 2R_2 + l(\eta + \delta + \sigma) - R_2 e^{-im\psi} - R_2 e^{im\psi}) \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} e^{iam\psi} \hat{I}_a^{b+1}(\psi) d(\psi) = \frac{1}{\sqrt{2\pi}} \int_{-\frac{\pi}{m}}^{\frac{\pi}{m}} e^{iam\psi} \left(1 + \alpha_2 (B_2^{(b+1)l} - B_2^{bl}) \right) \hat{I}_a^b(\psi) d(\psi).$$

$$(1 + 2R_2 + l(\eta + \delta + \sigma) - R_2 e^{-im\psi} - R_1 e^{im\psi}) \text{hat} I_a^{b+1}(\psi) = \left(1 + \alpha_2 (B_2^{(b+1)l} - B_2^{bl}) \right) \hat{I}_a^b(\psi).$$

$$g(\psi m, l, m) = \frac{1 - (\eta + \delta + \sigma)l}{1 + l(\eta + \delta + \sigma) + 4R_2 \sin^2(\frac{m\psi}{2})} + \frac{\alpha_2}{1 + l(\eta + \delta + \sigma) + 4R_2 \sin^2(\frac{m\psi}{2})} (B_2^{(b+1)l} - B_2^{bl}).$$

Using the Wiener process' independent state, the amplification factor can now be expressed as

$$E|g(\psi m, l, m)|^2 \leq \left| \frac{1}{1 + l(\eta + \delta + \sigma) + 4R_2 \sin^2(\frac{m\psi}{2})} \right|^2 + \left| \frac{\alpha_2}{1 + l(\eta + \delta + \sigma) + 4R_2 \sin^2(\frac{m\psi}{2})} \right|^2 l.$$

As always

$$\left| \frac{1}{1 + l(\eta + \delta + \sigma) + 4R_2 \sin^2(\frac{m\psi}{2})} \right|^2 < 1,$$

and let $\left| \frac{\alpha_2}{1 + l(\eta + \delta + \sigma) + 4R_2 \sin^2(\frac{m\psi}{2})} \right|^2 = \chi$, so

$$E|g(\psi m, l, m)|^2 \leq 1 + \chi l. \tag{29}$$

So, the scheme for I is stable.

4 Numerical results

$$\frac{\partial S}{\partial t} = d_1 \frac{\partial^2 S}{\partial x^2} + \lambda - \eta S - \frac{\beta SI(x, t)}{1 + c_1 S + c_2 I + c_3 SI(x, t)} + \alpha_1 S \dot{B}_1(t), \tag{30}$$

$$\frac{\partial I}{\partial t} = d_2 \frac{\partial^2 I}{\partial x^2} - (\eta + \delta + \sigma)I + \frac{\beta SI(x, t)}{1 + c_1 S + c_2 I + c_3 SI(x, t)} + \alpha_2 I \dot{B}_2(t), \tag{31}$$

with initial conditions as

$$S(x, 0) = \begin{cases} 1.1x & \text{if } 0 \leq x < 0.5 \\ 1.1(1 - x) & \text{if } 0.5 \leq x \leq 1. \end{cases} \tag{32}$$

$$I(x, 0) = \begin{cases} 0.5x & \text{if } 0 \leq x < 0.5 \\ 0.5(1 - x) & \text{if } 0.5 \leq x \leq 1, \end{cases} \tag{33}$$

and having homogeneous Neumann boundary conditions.

4.1 Disease-free equilibrium of test problem

All the figures of disease-free equilibrium are plotted for different values of the parameters given in Table 1. For the given values of the parameters, the reproductive number for the disease-free equilibrium is $R_0 = 0.9286 < 1$ with $\beta = 0.2$. For the system of the Eqs. (8,9), the disease-free equilibrium $DFE = (\lambda/\eta, 0)$ has the value (5,0) for $\beta = 0.2$. Figures 1, 2 are plotted for noise strength equal to zero, i.e., without stochastic behavior. Figure 1 is acquired by using the proposed stochastic backward Euler scheme and the susceptible and infected individuals converge toward the disease-free equilibrium, and the solution has positive values. Figure 2 is acquired by using the proposed stochastic implicit FDS. The susceptible and infected individuals converge toward disease-free

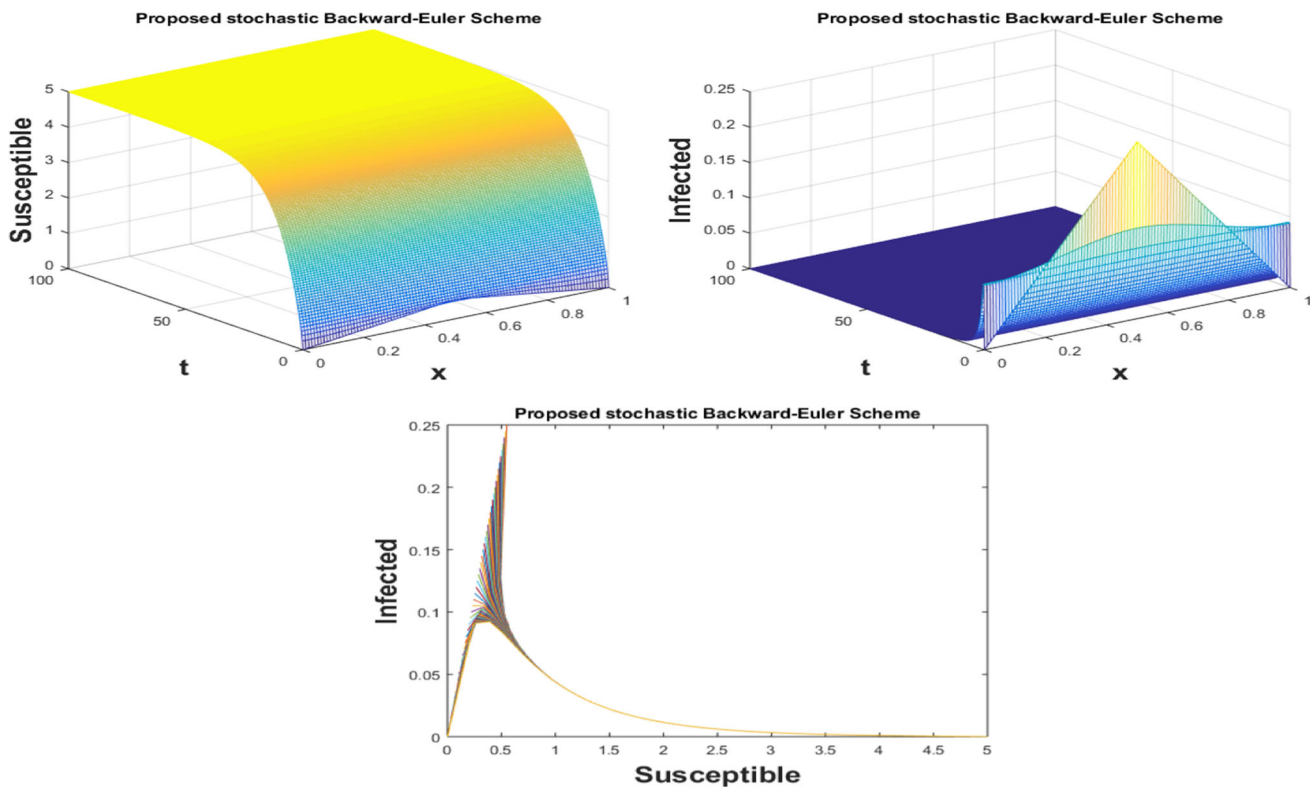


Fig. 1 The graphical behavior of the susceptible and infected individual by proposed stochastic backward Euler scheme for $\alpha_1 = \alpha_2 = 0, \beta = 0.2, l = 0.2$

equilibrium and the solution preserves the positivity and boundedness. Positivity and boundedness are the important dynamics of the model. Figures 3, 4 are drawn for the noise strength have to value 0.5, and stochastic behavior is observed. Figure 3 is acquired by the proposed stochastic Backward Euler scheme, and it converges toward DFE and the solutions also preserve the positivity. Figure 4 is acquired by the proposed stochastic IFD scheme and the susceptible and infected individuals converge toward DFE. So, both schemes converge toward DFE with and without stochastic phenomena and preserve positivity.

4.2 Endemic equilibrium ($\beta = 5, \dots$) of the problem

All the figures of endemic equilibrium are plotted for different values of the parameters given in Table 1. For the given values of the parameters, the reproductive number for the endemic equilibrium is $R_0 = 35.2143$ with $\beta = 5$. For the system of the Eqs. (8,9), the endemic equilibrium $EE = (S^*, I^*)$ has the value (0.1440, 0.6937) for $\beta = 5$. Figures 5, 6 are drawn for noise strength having value zero, i.e., without stochastic behavior. Figure 5 is drawn by the proposed stochastic backward Euler scheme. The susceptible and infected individuals do not converge toward the endemic equilibrium. The solutions possess negative behavior, and it is meaningless in the disease dynamics. Figure 6 is plotted by the proposed stochastic IFD scheme and the susceptible and infected individuals converge toward (S^*, I^*) . The solutions preserve positivity and boundedness as well. Figures 7, 8 are drawn for the noise strength having a value equal to 0.5. Figure 7 is drawn by the proposed stochastic backward Euler scheme. The susceptible and infected individuals do not converge toward the endemic equilibrium and also possess negative behavior and it is meaningless. Figure 8 is plotted by the proposed stochastic IFD scheme and the susceptible and infected individuals converge toward (S^*, I^*) . The solutions preserve positivity. So, the proposed stochastic backward Euler scheme does not have the required results because the given scheme has negative solutions with and without stochastic behavior. But the proposed stochastic IFD scheme has convergent behavior as well preserves positivity.

4.3 Graphical behavior

The plots are drawn for the efficacy of our findings.

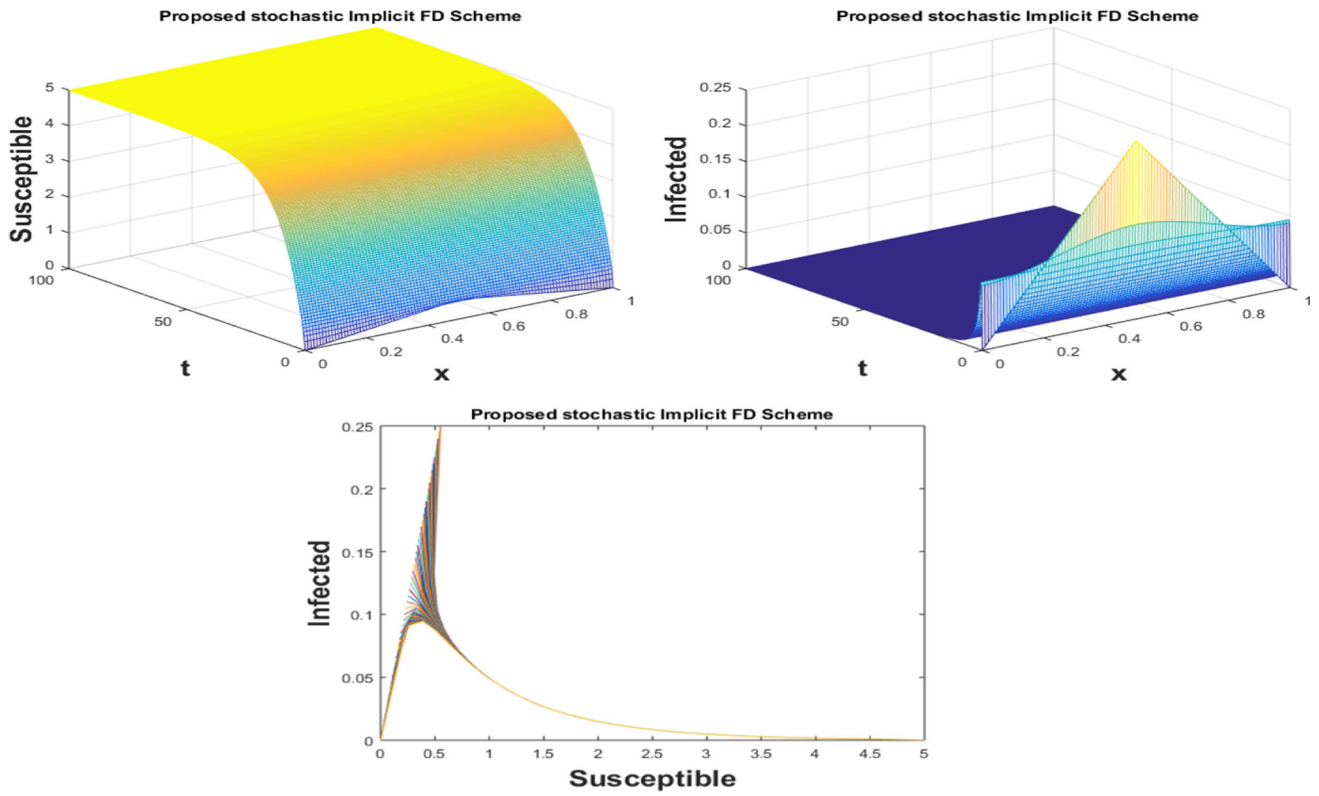


Fig. 2 The graphical behavior of the susceptible and infected individual by proposed stochastic IFD scheme $\alpha = 0, \beta = 0.2, l = 100/500$

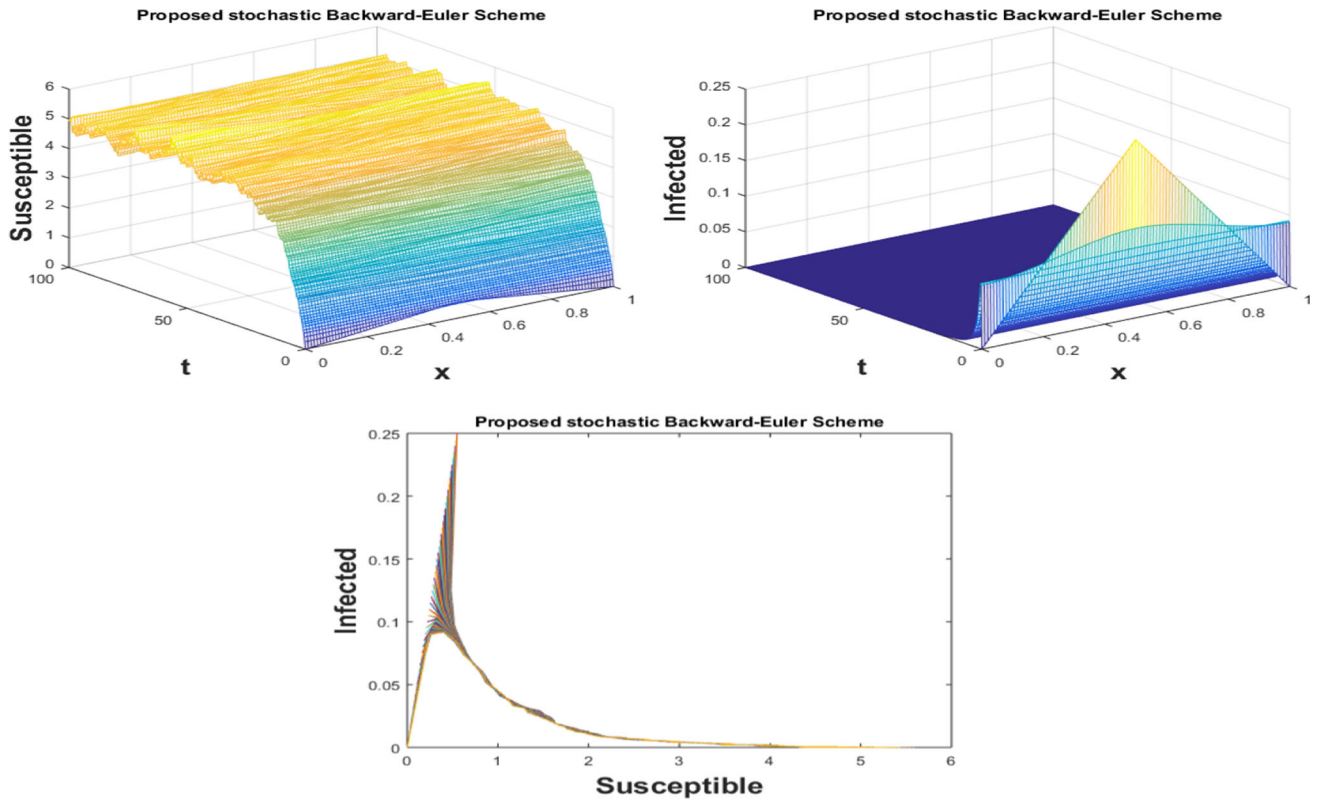


Fig. 3 The graphical behavior of the susceptible and infected individual by proposed stochastic backward Euler scheme $\alpha = 0.5, \beta = 0.2, l = 0.2$

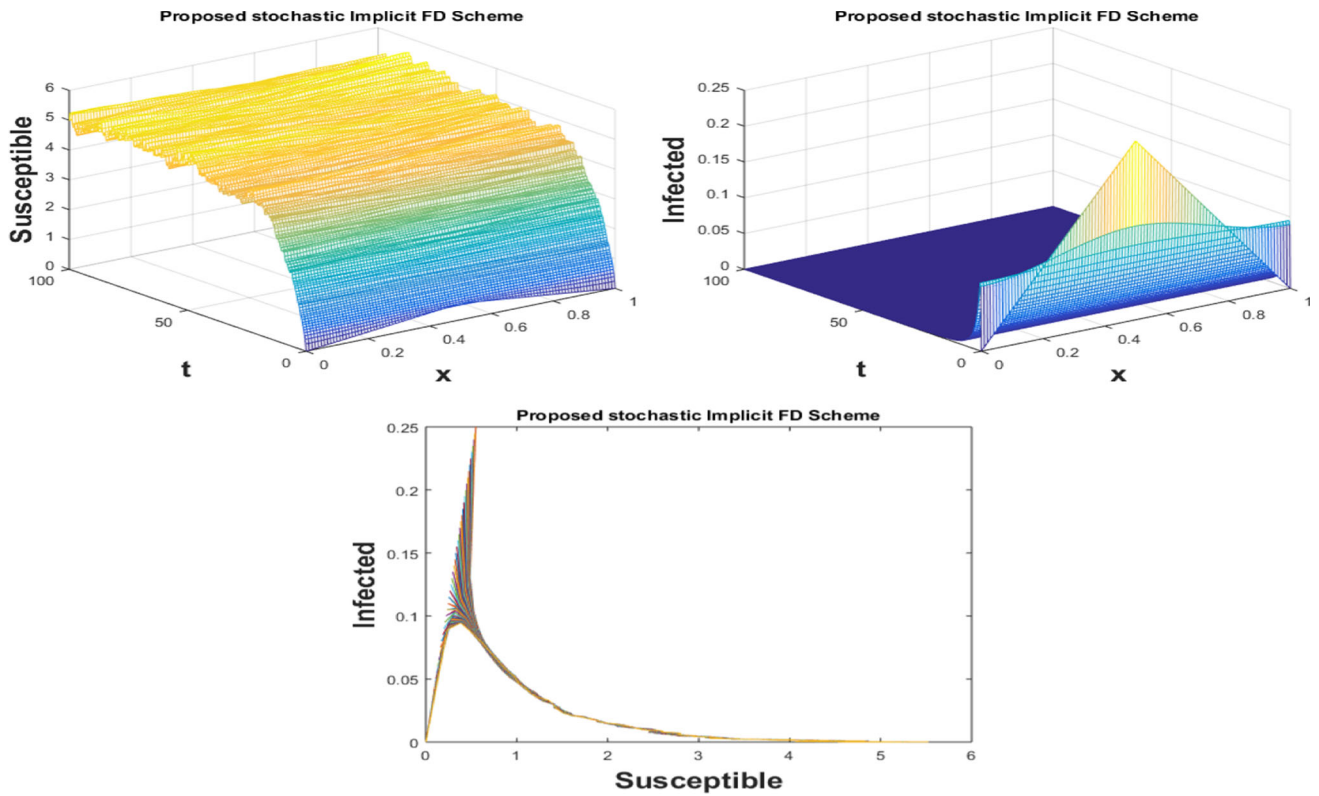


Fig. 4 The graphical behavior of the susceptible and infected individual by proposed stochastic IFD scheme $\alpha=\alpha=0.5, \beta = 0.2, l = 0.2$

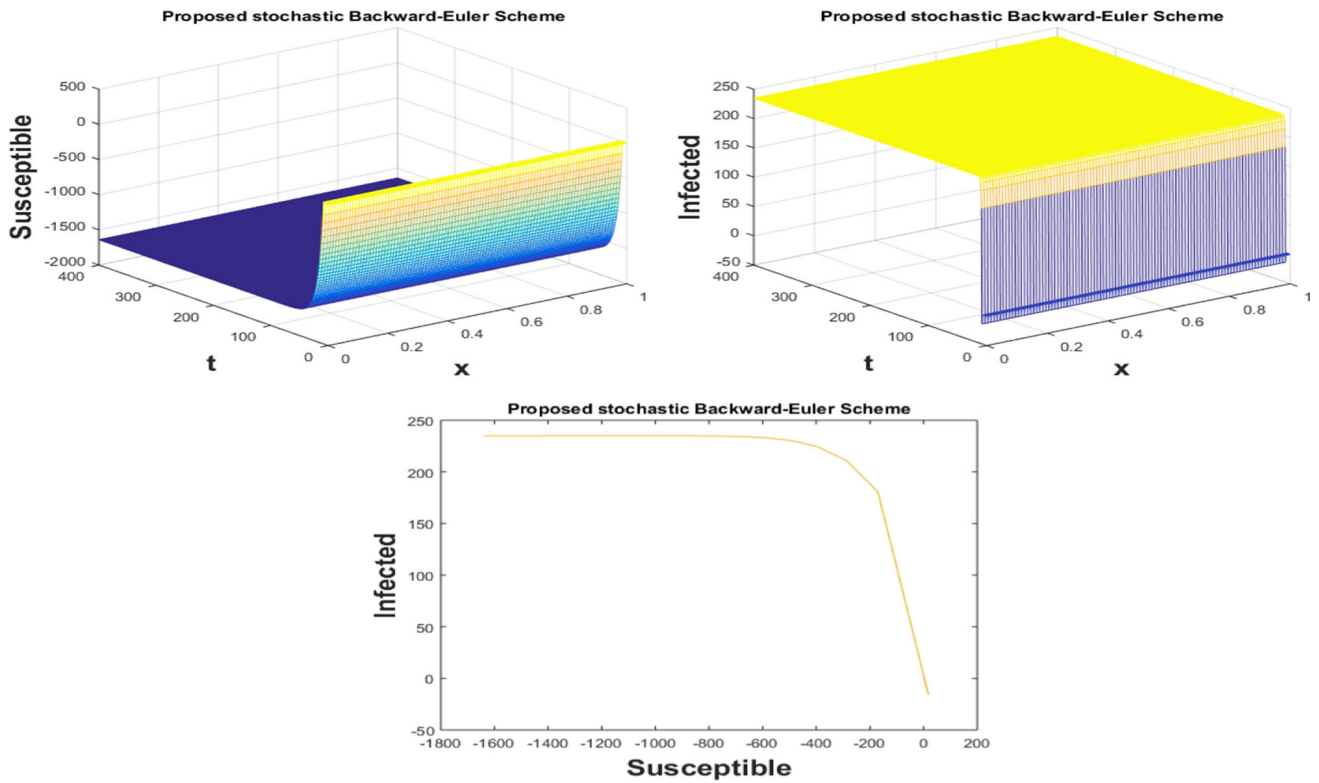


Fig. 5 The graphical behavior of the susceptible and infected individual by proposed stochastic backward Euler scheme $\alpha=\alpha=0, \beta = 5, l = 0.8$.

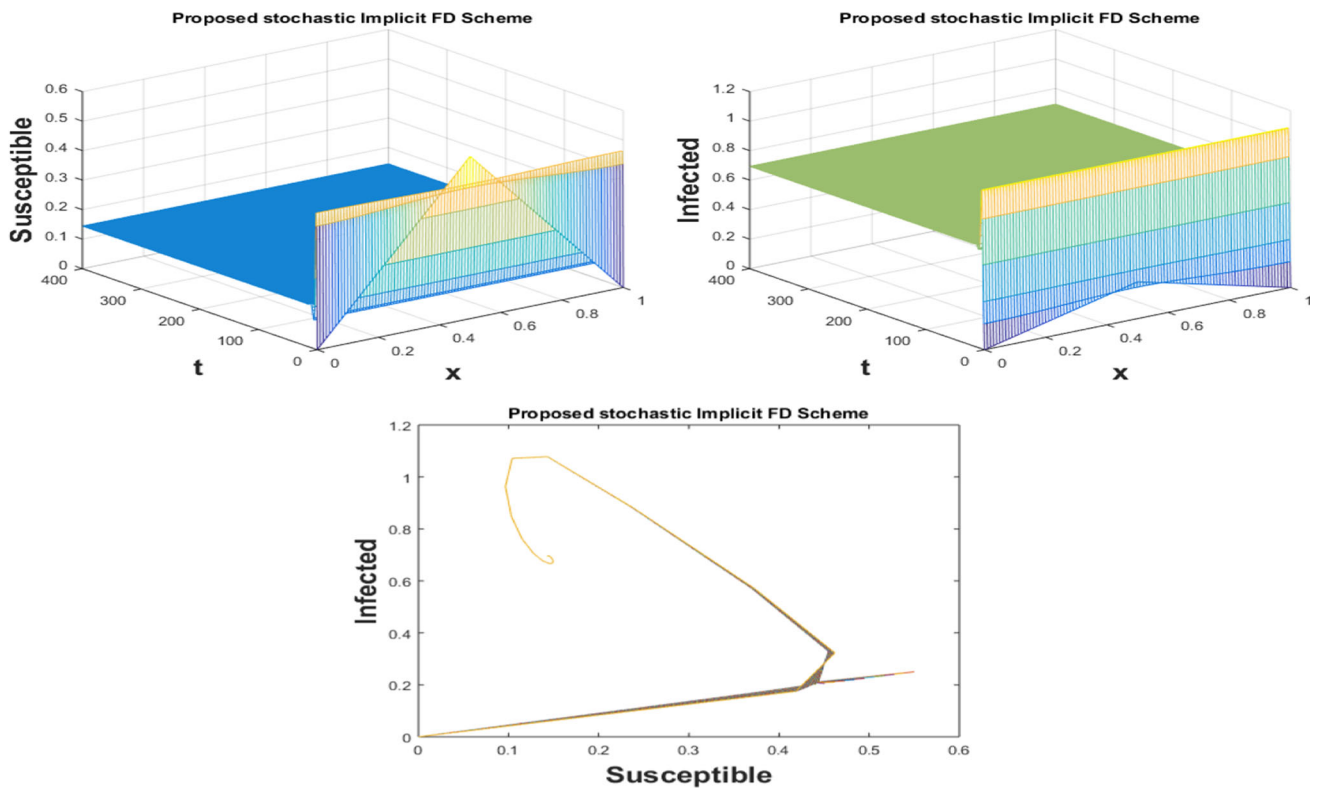


Fig. 6 The graphical behavior of the susceptible and infected individual by proposed stochastic IFD scheme for $\alpha_1 = \alpha_2 = 0, \beta = 5, l = 0.8$

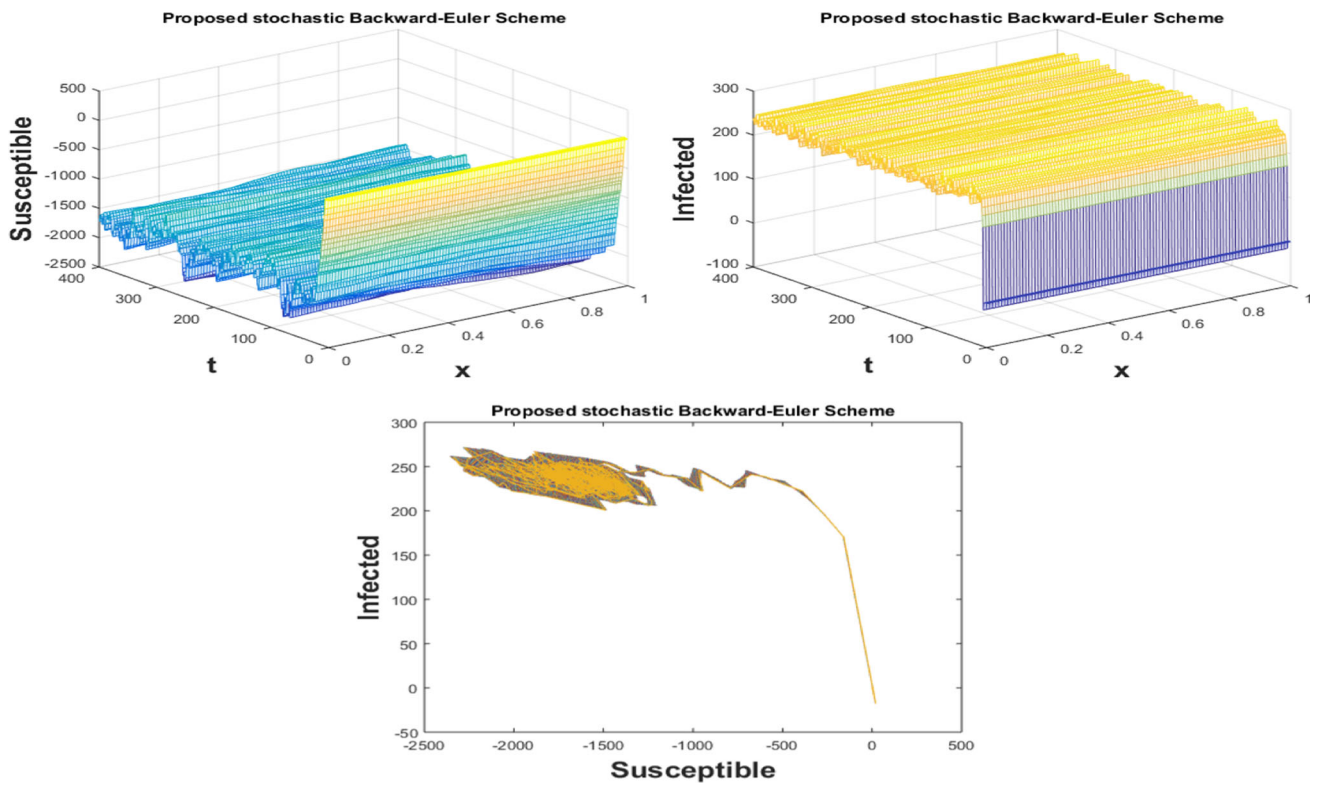


Fig. 7 The graphical behavior of the susceptible and infected individual by proposed stochastic backward Euler scheme $\alpha_1 = \alpha_2 = 0.5, \beta = 5, l = 0.8$.

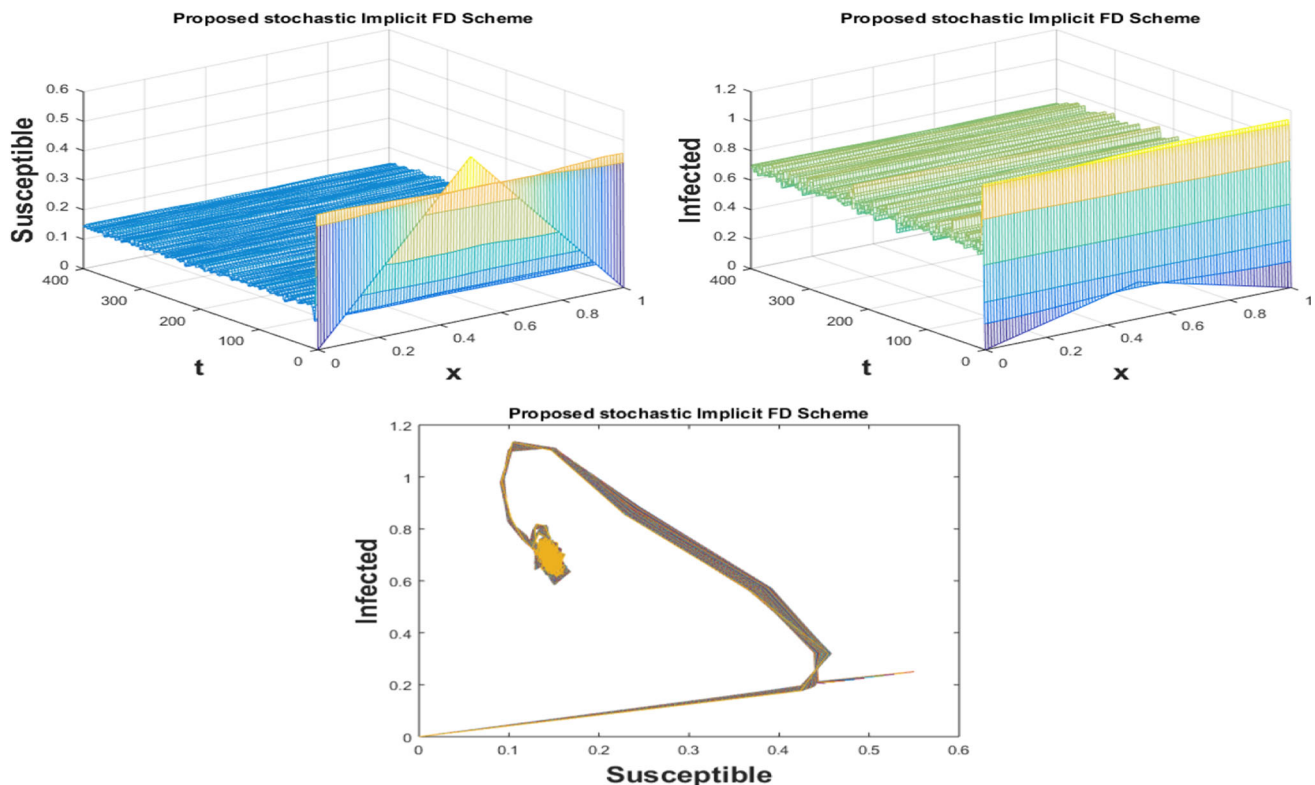


Fig. 8 The graphical behavior of the susceptible and infected individual by proposed stochastic IFD scheme $\alpha=0.5$, $\beta = 5$, $l = 0.8$

5 Conclusion

The stochastic reaction–diffusion epidemic model has been numerically investigated by two time-efficient novel schemes. The proposed stochastic backward Euler scheme is consistent with the given system of equations. It is also unconditionally stable, and its stability analysis is carried out by using the von-Neumann criteria. The suggested stochastic IFD approach is compatible with the provided equation system. Additionally, it is unconditionally stable, and the von Neumann criteria are used to analyze its stability. The system has two equilibriums: an endemic equilibrium and a disease-free equilibrium. For the disease-free equilibrium, both schemes converge toward the equilibrium and preserve positivity. For the endemic equilibrium, the proposed stochastic backward Euler scheme did not show the convergent behavior toward the endemic equilibrium and it also shows the negative behavior of the solution and which is meaningless in the population dynamics. On the other hand, the proposed stochastic IFD scheme shows the convergent behavior toward the endemic equilibrium and also preserves positivity. The graphical behavior of the test problems shows that the proposed stochastic IFD scheme has an advantage over the proposed stochastic backward Euler scheme. The graphical results for the test problem are drawn that show the efficacy of our proposed stochastic IFD scheme.

Data availability Data sharing not applicable to this article.

Declarations

Competing interests The authors declare no competing interest.

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