Experimental evidence of quint points and non-quantum chirality in a minimalist autonomous electronic oscillator

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Abstract Recent intensive simulations have uncovered remarkable phenomena in stability diagrams of classical oscillators, for instance, quint points, parameter rings, and chiral structures of non-quantum origin. So far, their experimental observation has remained elusive. Here, using a simple electronic circuit, we report the experimental detection of five phases of oscillation spread around a quint point, an exceptional point where five oscillatory modes meet. This finding corroborates predictions of non-quantum chirality in the control parameter space of nonlinear oscillators governed by rate equations.

1 Introduction

The availability of high-performance and high-throughput computer clusters has opened the possibility of performing model simulations and optimizations which were impossible until recently. For instance, it has become possible to search systematically for adequate laser operational conditions by tuning material properties over wide parameter ranges and geometries [1–3], or to search for suitable conditions to conduct laboratory investigation of complex (bio)chemical mechanisms to develop, e.g., new drugs and treatments [4–6].

Recently, systematic search for operational conditions in systems as diverse as a chemical reaction and an electronic circuit uncovered a number of unsuspected new features in stability diagrams of such classical oscillators. For example, the ubiquitous presence of exceptional *quint points* [7], which are points where five distinct phases meet, parameter *rings* [8] along which system stability thrives even in the presence of small parameter fluctuations, and innovative *non-quantum chiral structures* [9–11]. As discussed in these references, non-quantum chirality refers to chirality which arises from oscillators governed by rate equations, and not by quantum equations of motion as is the case for standard chirality. Despite the apparent omnipresence of these features in distinct physical systems, they have not yet been observed experimentally.

The aim of this paper is to report the experimental observation in a simple electronic circuit of five distinct oscillation phases distributed around a predicted *quint point*, an exceptional point where five different modes of oscillation meet. The electronic circuit provides the realization of a minimalist flow discussed recently [11]. This observation is significant because it provides the first experimental corroboration of predictions of non-quantum chirality (Fig. 1) for an oscillator governed by classical rate equations. As it is well known, chirality is eminently a property of quantum phenomena, associated with the spatial geometry of the atoms composing molecules, with (bio)chemical interactions, and with optical activity and related phenomena from the perspective of molecular scattering of polarized light [12–14].

2 The minimalist three-dimensional flow

Chiral points were reported for a trigonometrically driven ruthenium-catalyzed Belousov–Zhabotinsky reaction [7]. Apart from the trigonometric drive, the model of this reaction contains a rational function and, therefore, is relatively time-consuming to be investigated numerically. This drawback motivated us to seek a very simple model, involving as few as possible arithmetic operations and satisfying two conditions: to significantly reduce the computational workflow, and to be conveniently accessible to experimental

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Fig. 1 Sketch of the first three levels of the mirror symmetric duo seen when tuning two independent parameters p and q

investigation. A minimalist flow fulfilling these conditions which is a suitable normal form to investigate the inner details of the cascading sketched in Fig. 1 is the following, obtained by properly linearly shifting variables of a quadratic function [11, 15]:

$$\dot{x} = y, \quad \dot{y} = z, \quad \dot{z} = -a - y - bz + x^2. \quad (1)$$

Here, a and b are freely tunable parameters. These equations represent arguably the simplest flow displaying the aforementioned new dynamical features, as discussed in the remainder of the paper. As a significant computational bonus, Eq. (1) is autonomous, meaning that there is no need to evaluate repeatedly time-dependent functions during numerical integrations.

There is no known theoretical framework to solve analytically coupled nonlinear differential equations, like Eq. (1), that would allow one to use such solutions to draw multiple phase boundaries delimiting oscillations of a distinct nature, e.g., having an arbitrary number of spikes per period, their phase volumes, or their relative orientation in phase diagrams. Hence, we resort to high-performance multi-scale numerical computation to obtain stability charts summarizing with high-resolution the behavior of millions of individual oscillations, showing how they evolve and reveal the many spike-doubling and spike-adding cascades seen in Figs. 3 and 4.

Briefly, selected $a \times b$ windows of interest are covered with a mesh of $1200 \times 1200 = 1.44 \times 10^6$ equidistant points. For each point of the mesh, the temporal evolution of Eq. (1) is determined using a fourth-order Runge–Kutta algorithm with fixed time-step 0.005. Integrations start from an arbitrarily selected initial condition, $(x, y, z) = (0.003, 0.02, 0.83)$, and proceed by following the attractor [7–10] horizontally from left to right. See these references for details.

3 The autonomous electronic circuit

In the last decades, the physical realizations of theoretical dynamical models have been studied extensively for confirming their feasibility and using them in practical applications [16]–[21]. Therefore, in this section, a circuit realization of the theoretical system of Eq. (1) is presented. For this purpose, the general design methodology according to the operational amplifiers is applied.

Figure 2 shows a sketch of the electronic circuit, which represents the flow in Eq. (1), as well as its implementation on a breadboard, and the experimental setup for acquisition of signals and phase portraits with a digital oscilloscope (SIGLENT SDS 1104X-E). A digital multimeter (FLUKE 187) has been used to adjust the voltage V_a . The circuit involves simple electronic components, namely two inverting amplifiers (U1, U2), three integrators (U3–U5), and one analog multiplier (U6) of type AD633. The operational amplifiers are of type TL084. The circuit obtained is not complicated, not expensive, and may be also used for didactic purposes, to investigate the aforementioned innovative effects in the dynamics of complex classical oscillators.

According to Kirchhoff's laws, the circuit in Fig. 2 is governed by the equations

$$\dot{x} = \frac{1}{RC} y, \quad (2)$$

$$\dot{y} = \frac{1}{RC} z, \quad (3)$$

$$\dot{z} = \frac{1}{RC} (-V_a - y - \frac{R}{R_b} z + x^2), \quad (4)$$

where x, y, z represent output voltages of the integrators (U1–U3). After a time rescaling $\tau = t/(RC)$ the above equations reduce to Eq. (1), where $a = V_a$ and $b = R/R_b$. Here, we use $R = R_1 = 10 \text{ k}\Omega$, $R_2 = 90 \text{ k}\Omega$, $R_b =$ variable resistor, $C = 10 \text{ nF}$ and $V_a = 15 \text{ V}$ is a DC voltage applied from a voltage source. Although R_2 does not appear explicitly in the above equations, it is important to define the output of the multiplier AD633 ($= U6$):

$$W = \frac{x^2}{10 \text{ V}} + z = \frac{x^2}{10 \text{ V}} + \frac{R_2}{R_1 + R_2} W \quad (5)$$

that is $W = \frac{x^2}{10 \text{ V}} \frac{R_1 + R_2}{R_1}$. For $R_1 = 10 \text{ k}\Omega$ and $R_2 = 90 \text{ k}\Omega$, one has $W = x^2/(1 \text{ V})$.

Fig. 2 Schematic representation of the electronic circuit used and experimental setup to measure the temporal evolutions of x , y , z , in Eqs. (2)–(4). The control parameter space of the circuit has an intricate but hierarchically organized distribution of oscillations spikes as illustrated in Fig. 3b

4 Results

Figure 3 compares three stability diagrams, obtained by counting spikes per period of the x , y , z oscillations. Such diagrams are the so-called *isospike diagrams* [9–11], obtained by counting the number of spikes per period of the oscillations. Clearly, the diagrams obtained by counting spikes of x and z display similar phases, which differ only by the number of spikes per period among corresponding phases. However, the diagram obtained by counting spikes of y is markedly distinct from the other two, showing a much richer variation of spikes. In this diagram, it is possible to recognize the presence of several quint points [7], points where five distinct phases meet. For example, two such points are located at the center of the two circles. Several additional quint points are not difficult to identify inside the left box in Fig. 3b. As indicated in Fig. 1, parabola-like phases arise from spikes additions and are characterized by oscillations with odd number of spikes, while their adjacent phases contain even numbers of spikes.

Figure 4a and b shows magnifications of the pair of boxes seen in Figure 3b. Figure 4a shows a detailed view of the anticlockwise non-quantum chiral structure while Fig. 4b highlights the location of an anticlockwise, A, and clockwise, C, chiral duo. Figure 4c shows details from the box in Fig. 4b. These structures A and C should be compared with the schematic one indicated in Fig. 1. Clearly, Fig. 4 shows that Eq. (1) indeed contains quint points as well as a chiral pair. Furthermore, Fig. 4 shows unambiguously that the network of quint points and the chiral pair are critically needed to provide the necessary complex but regular “bridges” that interconnect the several phases of oscillations characterized by distinct number of spikes per period.

Until now, chiral structures of non-quantum origin [9–11] were only reported to mediate oscillations along closed parameter paths, namely along parameter rings [8]. In contrast, Fig. 4 shows that chiral structures may also occur in “open” stability regions, not involving parameter rings. Several additional features may be recognized directly in Fig. 4b and 4c, or in magnifications of it (not shown here). Manifestly, the spikes unfolding is seen to “spill over” into the stripes of periodic oscillations embedded in the black phase corresponding to chaotic oscillations. What about the structure of these stripes?

An additional feature observed in Fig. 4 is the remarkable existence of several quint points located in certain “shrimp-shaped” regions [22–25]. For example, roughly at the center of Fig. 4b there is a large shrimp whose main isospike phases involve oscillations with 16 and 17 spikes per period. As it is not difficult to see, these two phases converge to a quint point located on the right side of the shrimp. Furthermore, all shrimps *embedded* in the black chaotic phase give rise to stripes, which also contain quint points, in fact many such points. Two presence of quint points along the stripes emanating from two of the largest shrimps are indicated by series of small circles. Along the stripe whose main phase has oscillations with seven spikes per period, there are three regions characterized by color changes, a fact that indicates the presence of lattices of quint points. Similarly, along the stripe with six small circles one finds networks of quint points which, as indicated by the letters A and C, form an anticlockwise and clockwise non-quantum chiral pair. Similar features exist in nearby shrimps, as may be recognized from suitable magnifications (not shown here).

As illustrated in Figs. 3 and 4, patterns defining quint points and non-quantum chirality become visible in diagrams depicting spikes variation for large sets of solutions of the equations of motion. Such diagrams can be obtained experimentally, analogously as already done for another experiment, about discontinuous spirals of oscillations [26]. To construct such diagrams is certainly an efficient method to detect any type of patterns eventually formed in the control parameter space by large sets of stable periodic solutions of the equations of motion. Although feasible in principle, this methodology is normally a rather challenging task, requiring the ability to fine-tune parameter variations, and that circuit components remain constant over significantly long time intervals without temperature fluctuations [26]. Fortunately, such complications may be easily bypassed. Since the nature of the temporal evolution of oscillations around quint points is known, it suffices to determine the number of spikes for just a few representative signals, checking the relative order among adjacent phases, e.g., $5 \rightarrow 6$, or $10 \rightarrow 11 \rightarrow 12$, etc., in Fig. 4a. Thus, by simply comparing a few temporal signals one can identify unambiguously non-quantum chirality. This is what we do next.

Figure 5 presents a comparison of computed and measured temporal evolution of y signals. Dots are added to some spikes to help visualize the period length. In Fig. 5, “peak” refers to the number of spikes and T to the oscillation period. Although some spikes

Fig. 3 Comparison of stability diagrams obtained by counting spikes of **a** x , **b** y , and **c** z . In panel **b**, A and C refer to anticlockwise and clockwise chiral structures (see Fig. 1). Numbers denote the number of spikes per period, black marks chaotic oscillations, and pink indicates parameters leading to divergence. Manifestly, stability boundaries obtained by counting spikes of y are very distinct and much richer from those obtained by counting spikes of x and z . Dots and circles are added in panels **a** and **c** to facilitate comparisons. The temporal evolution of y for the five points in **b** indicated by dots is given in Fig. 5

Fig. 4 Stability diagrams obtained by counting spikes of y for the flow in Eq. (1). (a) Magnification of the left box in Fig. 3b, illustrating anticlockwise spikes complexification around quint points [7] located at circle centers. Compare with Fig. 1. Numbers denote the number of spikes per period, black marks chaotic oscillations, pink denotes divergence. Parabola-like phases arise from spikes additions and are characterized by odd number of spikes, e.g., $11 = 5 + 6$, $21 = 10 + 11$, etc. Coordinates of the five representative dots around the salient quint point are given in Fig. 5. (b) Magnification of the right box in Fig. 3b, illustrating clockwise spikes complexification inside the rectangle (see Fig. 1). Circle centers mark quint points, which form a discrete lattice. (c) Magnification showing details of the clockwise structure inside the rectangular box of panel (b). See text

Fig. 5 Comparison of computed and measured temporal evolution of y signals. Dots are added to some spikes to help visualization. Although some spikes are small in the scale of the panels, they are perfectly measurable. Peak refers to the number of spikes, while T is the oscillation period. The five (a, b) values above are marked as dots in Fig. 4a and b

are small in the scale of the panels, they are measurable with no difficulty. In particular, note that time-series containing an even number of spikes display more subtle variations of local maxima. For example, a cursory glance at the panel displaying 12 spikes per period in Fig. 5 may give the impression of a temporal series having just six rather than twelve spikes per period, something that closer inspection reveals not to be true. As may be seen from Fig. 5, there is a quite satisfactory agreement between simulations and experiments.

5 Conclusions and outlook

A minimalist flow capable of producing quint points and non-quantum chirality has been proposed and experimentally verified using the electronic circuit in Fig. 2 and described by Eqs. (2)–(4). Such chiral structures mediate families of stable oscillations characterized by different numbers of spikes (local maxima) per period. Phase diagrams displaying the chiral structures reported here are observed through computer simulations because of the complete absence of any framework to obtain analytical solutions for systems of nonlinear sets of coupled differential equations.

Non-quantum chirality is not a property of isolated trajectories but, instead, arises in specific regions of the control parameter space as a macroscopic collective property due to an assemblage of a large number of distinct trajectories. As it is clear from the phase diagrams in Figs. 3 and 4, chirality cannot be found using either the time-honored Lyapunov diagrams, period length diagrams, or any other similar type of diagrams. Clearly, the isospikes diagrams shown in Figs. 3 and 4 may be also used to analyze arbitrary models governed by complex sets of ordinary differential equations.

The chirality observed in the electronic circuit revises the present knowledge about the topology of the control space of a prototypic electronic oscillator, and draws attention to hitherto unsuspected new properties of generic oscillators governed by purely classical (that is, not quantum) equations of motion. Until now, the chiral dynamic systems presently known involve rings [8] in control parameter space, a fact that may give the impression that the presence of rings is a necessary condition for non-quantum chirality. In contrast, as clearly seen from Figs. 3 and 4, rings are not at all necessary for the presence of non-quantum chirality. Lastly, since in hindsight quint points are now known to exist in a simple oscillator containing a memristor, as may be seen in Figs. 6 and 8 of Ref. [27], an enticing open question is whether non-quantum chirality may be also found in other electronic circuits involving combinations of mem-elements, as it is typical for neuromorphic circuits with artificial intelligence applications [28].

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Author contributions JACG devised the flow, computed all bitmaps, and wrote the paper. CKV designed and constructed the electronic circuit and did all experimental measurements. Both authors analyzed the data and reviewed the manuscript.

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Data Availability All needed data are contained in the paper.

Conflict of interest There are no conflicts of interest to declare.

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