Erratum to: The Dirac equation in $(1 + 2)$ -dimensional Gürses space-time backgrounds

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After publication of paper [1], in which we investigated the relativistic quantum dynamics of spin- $\frac{1}{2}$ particles in $(1+2)$ -dimensional Gürses space-time backgrounds and obtained the energy eigenvalues, we have observed errors in the energy eigenvalues of spin- $\frac{1}{2}$ particles, which are presented here in their correct version with references to the original-article equation numbers.

The three-dimensional Gürses metric in [1] is

$$ds^2 = -dt^2 + dr^2 + r^2 (1 - \Omega^2 r^2) d\phi^2 - 2 \Omega r^2 dt d\phi, \quad (1)$$

where $\mu, \nu = 0, 1, 2$.

The Dirac equation for a free Fermi field of mass m in curved space-time can be written as

$$[i \gamma^\mu(\mathbf{x}) \nabla_\mu - m] \Psi = 0, \quad (11)$$

where ∇_μ is given by

$$\nabla_\mu = \partial_\mu + \Gamma_\mu. \quad (12)$$

Here Γ_μ is the spinorial affine connection. The second term in eq. (11) becomes [1]

$$\gamma^\mu(\mathbf{x}) \Gamma_\mu(\mathbf{x}) = \frac{1}{2} \begin{pmatrix} i \Omega \sigma^3 \frac{\sigma^1}{r} \\ -\frac{\sigma^1}{r} & -i \Omega \sigma^3 \end{pmatrix}. \quad (20)$$

We write the Dirac spinor as

$$\Psi(t, r, \phi) = e^{-i E t} e^{j \phi} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad (22)$$

where E is the total energy, and $j = l + \frac{1}{2}$ with $l = 0, \pm 1, \pm 2, \dots$

Substituting the ansatz (22) into Eq. (11), we obtain the following differential equations:

$$\left(E - \frac{\Omega}{2} \sigma^3 - m \right) \psi_1 = \left[\left(\Omega E r + \frac{j}{r} \right) \sigma^2 - \frac{i}{2r} \sigma^1 \right] \psi_2 - i \sigma^1 \psi'_2, \quad (24)$$

$$\left(E - \frac{\Omega}{2} \sigma^3 + m \right) \psi_2 = \left[\left(\Omega E r + \frac{j}{r} \right) \sigma^2 - \frac{i}{2r} \sigma^1 \right] \psi_1 - i \sigma^1 \psi'_1, \quad (25)$$

After decoupling, we obtain the following second-order differential equation:

$$\psi_s'' + \frac{1}{r} \psi_s' + \left[\lambda - \Omega^2 E^2 r^2 - \frac{j^2}{r^2} \right] \psi_s = 0, \quad (26)$$

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where

$$\begin{aligned}\lambda &= E^2 + \frac{\Omega^2}{4} - m^2 - 2\Omega E(j+s), \\ \tilde{j}^2 &= j^2 - js + \frac{1}{4} = \left(j - \frac{s}{2}\right)^2.\end{aligned}\quad (27)$$

In eq. (27) in the original work [1], the term λ was incorrect and we have corrected it here via eq. (27). Thus eqs. (32), (34), (42), (43) in [1] are incorrect, and we present them here in their correct version.

We employ the change of variable $x = \Omega E r^2$, we obtain [2]

$$\psi_s''(x) + \frac{1}{x} \psi_s'(x) + \frac{1}{x^2} (-\xi_1 x^2 + \xi_2 x - \xi_3) \psi_s(x) = 0, \quad (29)$$

where

$$\xi_1 = \frac{1}{4}, \quad \xi_2 = \frac{\lambda}{4\Omega E}, \quad \xi_3 = \frac{\tilde{j}^2}{4}. \quad (30)$$

Therefore, the second-degree energy eigenvalue is

$$E_{n,l}^2 - 2\Omega E_{n,l} (2n+1 + |\tilde{j}| + j+s) - m^2 + \frac{\Omega^2}{4} = 0,$$

with the energy spectrum

$$\begin{aligned}E_{n,l} &= \Omega \left(2n+1 + \left|j - \frac{s}{2}\right| + j+s\right) \pm \sqrt{\Omega^2 \left(2n+1 + \left|j - \frac{s}{2}\right| + j+s\right)^2 + m^2 - \frac{\Omega^2}{4}} \\ &= \Omega \left(2n+1 + \left|l + \frac{1}{2}(1-s)\right| + l + \frac{1}{2} + s\right) \pm \sqrt{\Omega^2 \left(2n+1 + \left|l + \frac{1}{2}(1-s)\right| + l + \frac{1}{2} + s\right)^2 + m^2 - \frac{\Omega^2}{4}}.\end{aligned}\quad (32)$$

The correct expression of the energy eigenvalue compared to eq. (32) can be found in [1], where $n = 0, 1, 2, 3, \dots$, $l = 0, \pm 1, \pm 2, \dots$ and $s = \pm 1$.

For massless Dirac fermions, that is, $m = 0$, we have the following energy eigenvalues:

$$E_{n,l} = \Omega \left[\left(2n+1 + \left|l + \frac{1}{2}(1-s)\right| + l + \frac{1}{2} + s\right) + \sqrt{\left(2n+1 + \left|l + \frac{1}{2}(1-s)\right| + l + \frac{1}{2} + s\right)^2 - \frac{1}{4}} \right]. \quad (34)$$

For $s = 1$, the energy eigenvalue becomes

$$E_{n,l} = \Omega \left[\left(2n + |l| + l + \frac{5}{2}\right) + \sqrt{\left(2n + |l| + l + \frac{5}{2}\right)^2 - \frac{1}{4}} \right]. \quad (42)$$

For $s = -1$, the energy eigenvalues are

$$E_{n,l} = \Omega \left[\left(2n + |l+1| + l + \frac{1}{2}\right) + \sqrt{\left(2n + |l+1| + l + \frac{1}{2}\right)^2 - \frac{1}{4}} \right]. \quad (43)$$

References

1. F. Ahmed, Eur. Phys. J. Plus **134**, 518 (2019).
2. A.F. Nikiforov, V.B. Uvarov, *Special Functions of Mathematical Physics* (Birkhauser, Basel, 1988).