

## Focus Point on Modelling Complex Real-World Problems with Fractal and New Trends of Fractional Differentiation

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The complexities of nature have led researchers to develop more sophisticated and complex mathematical operators to capture, or accurately replicate observed realities [1–7]. In the recent decades, it has been widely accepted that the differential operators, based on the concept of rate of change, are not always suitable candidates for modelling complex real-world problems, as they cannot capture heterogeneous behaviours [8–11]. Specific examples are provided by long-range, random walk, anomalous diffusion, non-Markovian processes, fractal behaviours, and groundwater flowing in non-conventional media. To accurately replicate these natural processes, the concept of non-local differential operators was suggested as well as that of local differential operators along with power law setting. Non-local operators are grouped into three classes, including fractional differential operators with singular and non-local kernel, fractional differential operators with non-singular and local kernel, and fractional differential operators with non-singular and non-local kernel. Those with singular kernel were recently introduced and have been applied in several fields of science, technology and engineering, with great success [12–18]. One of their key properties is the crossover behaviour, described for given-range random walk, deterministic and stochastic behaviour. For instance, those with Mittag-Leffler kernel have crossover behaviour from waiting time distribution, mean square displacement to density probability distribution. These operators do not impose singularities to any model, especially those with no singularities, as in the case of singular kernel operators. The *Focus Point* was devoted to creating a discussion and to collecting the latest innovative results, where non-singular kernel differential operators and local operators, based on power law setting, are applied to more complex problems arising in physics and other related topics. We invited top researchers in the field and received many submissions from all the continents. Of the total submitted papers, only 22 papers were accepted, providing a well-balanced representation of all continents. Sonal suggested a new numerical scheme for solving ordinary differential equations with power law setting differential operator [1]. A new numerical method for partial differential equations based on the two-step Laplace transform Adams-Bashforth approximation was applied by Jain and Alkahtani [2,3]. Owolabi presented the model of a dynamical system using the Atangana-Baleanu differential operator [4], and Ahmed *et al.* suggested a modified exponential rational function method for non-linear fractional differential equations [5]. Allwright applied a power-law setting differential operator to the advection dispersion equation and suggested a new numerical scheme [6]. Karaagac presented an analysis of the cable equation using the Atangana-Baleanu differential operator [7]. Atangana and Alqahtani studied a tumour model with intrusive morphology, progressive phenotypical heterogeneity and memory [19]. Goufo presented a replicator-mutator dynamical model using the Caputo-Fabrizio differential operators. The dynamical spread of Ebola with a crossover waiting-time differential operator was suggested and studied by Koca, while a cubic isothermal autocatalytic chemical system was presented by Saad [20,21]. Some unsteady Couette flows of an Oldroyd-B fluid with non-integer derivatives were presented within the scope of non-local operators by Zafar *et al.* [22]. A revolutionary study within the field of fractional calculus and application was presented by Atangana and Gomez with the title *Decolonisation of fractional calculus rules: Breaking commutativity and associativity to capture more natural phenomena* [23]. Mishra presented a remark on the fractional differential equation involving I-function [24].

Published papers under this *Focus Point* have already attracted the attention of several researchers within the field. We believed with no doubt the focus point has benefited the field of applied mathematics and we are looking forward to receiving citations from its published papers.

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