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Comment

Comment on: Magnetic field measurements in Rb vapor by splitting Hanle resonances under the presence of a perpendicular scanning magnetic field

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Abstract. In a recent article in this journal Grewal and Pattabiraman reported on the splitting of ground state Hanle resonances (recorded with linearly polarized light) by a transverse field. They claimed a "linearly proportional" dependence on the transverse field strength and supported this observation with results from numerical simulations. In this comment we argue that the splitting occurs only beyond a certain threshold field value and that it has a strong non-linearity near threshold. We base this claim on our previously published algebraic expressions for the line shapes and support this by experimental evidence.

Recently Grewal and Pattabiraman have investigated the ground state Hanle effect in ^{87}Rb under excitation with resonant linearly-polarized laser light [1]. Hanle resonances were recorded by scanning the magnitude of a magnetic field oriented parallel to the light polarization from negative to positive values (sketch of the experiment in Fig. 1). They found that the resonances have the shape of a two-peaked structure when an additional transverse static field is applied during the scan, a phenomenon that they referred to as splitting of the Hanle resonance. They also found that this splitting is "linearly proportional" to the magnitude of the transverse field, a fact that they substantiated by theoretical results based on numerical solutions of the Liouville equation.

It seems that the authors of [1] have not been aware that algebraic expressions for the Hanle resonance lineshapes (observed with linearly polarized light) in arbitrarily oriented magnetic field had been previously derived by Breschi and Weis [2]. The same paper also reported on an experimental observation (Fig. 7 in [2]) of the splitting effect studied in [1]. The aim of this comment is to show that those analytical results yield lineshapes that are compatible with the observations of Grewal and Pattabiraman. We also show that the analytical approach gives an exact expression for the relation between the line splitting and the transverse field magnitude. Moreover, it allows inferring the critical field value beyond which the splitting occurs, and to describe the nonlinear dependence of the splitting on the field magnitude expected in small transverse fields.

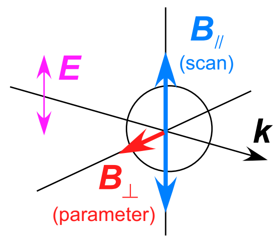

Fig. 1. Experimental geometry for observing "split" ground state Hanle resonances.

As discussed in reference [2], it is convenient to express the magnetic fields B_i in terms of dimensionless variables $\beta_i = \gamma_F B_i/\gamma$. The gyromagnetic ratio of the investigated ground state hyperfine level F is γ_F and γ is the spin relaxation rate, assuming equality of all three alignment relaxation rates $\gamma_q^{(2)}$. In this sense we refer to the scanned magnetic field component along the light polarization as $\beta_\parallel$, while the transverse field is given by $\beta_\perp$. With these definitions, it was shown in reference [2] that the lineshapes (measured as power changes δP of the transmitted light beam) are given by

$$\delta P(\beta_\parallel;\beta_\perp) \propto \frac{1}{4} - 3\,\frac{\beta_\parallel^2+\beta_\parallel^4}{1+\beta_\parallel^2+\beta_\perp^2} + \frac{3}{4}\,\frac{1+8\beta_\parallel^2+16\beta_\parallel^4}{1+4\beta_\parallel^2+4\beta_\perp^2}\,. \quad (1)$$

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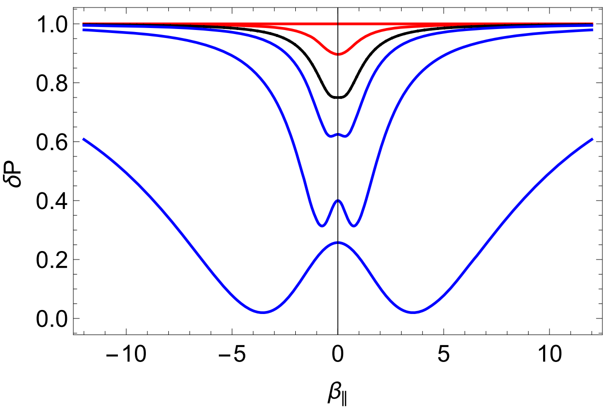

Fig. 2. Hanle resonance lineshapes $\delta P(\beta_\parallel)$ as given by equation (1) for various applied transverse fields $\beta_\perp = \left(0, 0.2, \frac{1}{2\sqrt{2}}, 0.5, 1, 2\right)$, from top to bottom. A distinct line splitting appears only for $\beta_\perp > \frac{1}{2\sqrt{2}}$.

Figure 2 shows a set of Hanle resonance curves calculated from equation (1) which exhibit the line splitting phenomenon occuring above a certain threshold value for $\beta_\perp$.

Equation (1) shows that the split lineshape results from a superposition of two structures of opposite sign and of widths differing by a factor of 2, whose physical origin are zero-field level crossing signals of $|\Delta M| = 1$ and $|\Delta M| = 2$ coherences of the ground state alignment, respectively.

One easily shows that the field values $\beta_\parallel^\pm$ at which the two minima of the resonances in Figure 2 occur are located at

$$\beta_\parallel^\pm = \pm\frac{1}{2}\sqrt{-1 - \beta_\perp^2 + 3\sqrt{\beta_\perp^2 + \beta_\perp^4}}\,, \qquad (2)$$

so that the splitting is given by $\Delta\beta_\parallel = \beta_\parallel^+ - \beta_\parallel^-$. The $\beta_\perp$-dependence of the positions $\beta_\parallel^\pm$ of the two minima is shown in Figure 3. The figure illustrates the region $|\beta_\perp| < (2\sqrt{2})^{-1}$ in which the resonance is not split, since equation (2) has imaginary values in that region.

The dashed lines represent the asymptotic ($\beta_\perp \gg 1$) behaviour $\beta_\parallel^\pm \approx \pm\beta_\perp/\sqrt{2}$ of equation (2), which corresponds to the linear dependence observed by Grewal and Pattabiraman [1]. The asymptotic splitting thus reads

$$\Delta\beta_\parallel = \beta_\parallel^+ - \beta_\parallel^- \approx \sqrt{2}\beta_\perp \quad \Leftrightarrow \quad \Delta B_\parallel \approx \sqrt{2} B_\perp\,, \qquad (3)$$

when expressed in relative or absolute field units, respectively.

We have repeated the experiment of Grewal and Pattabiraman with an apparatus similar to theirs, the main difference being the use of ^{133}Cs in a paraffin-coated cell, rather than ^{87}Rb in a vacuum/buffer gas cell. The laser frequency was locked to the 4→3 hyperfine component of the D_1 transition. The transverse field dependence of the line splittings (inferred from fits) are shown as data points in Figure 4. They are in good agreement with the model predictions presented above and illustrate

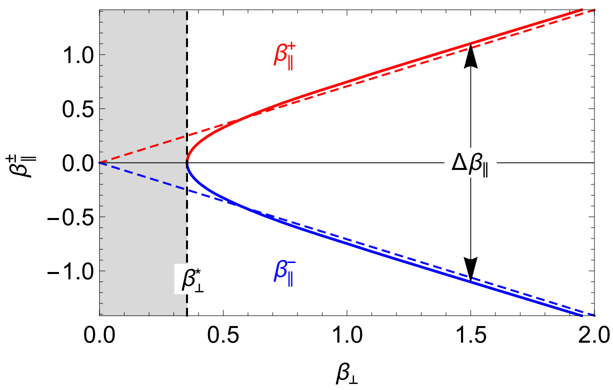

Fig. 3. Dependence of the positions $\beta_\parallel^\pm$ of the two minima in the lineshapes of Figure 2 on the transverse field $\beta_\perp$. The grey zone marks the region in which the line is not split. The dashed red lines mark the linear asymptotic behavior given by equation (3).

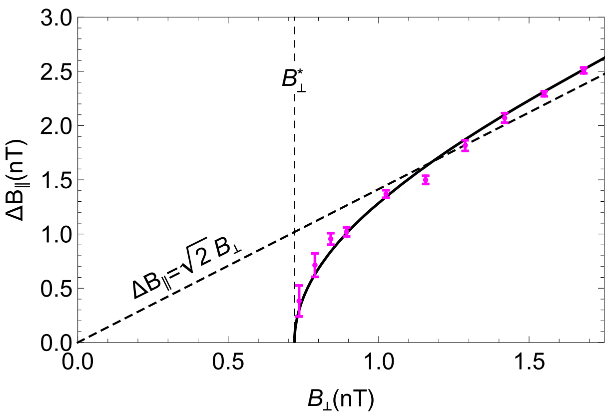

Fig. 4. Our measurements of the line splitting near threshold. Both $\Delta B_\parallel$ and $B_\perp$ were measured (after coil calibration and remnant field compensation) in absolute field units. Note that the theoretical curve does not represent a fit, but rather reflects equation (2), expressed in absolute field units using a numerical value of γ, inferred from an auxiliary experiment.

well the nonlinear behavior near the threshold transverse field value

$$\beta_\perp^* = \frac{1}{2\sqrt{2}} \quad \Leftrightarrow \quad B_\perp^* = \frac{\gamma}{2\sqrt{2}\,\gamma_F}\,. \qquad (4)$$

The asymptotic behaviour – both in the dimensionless $\Delta\beta_\parallel(\beta_\perp)$ and in the abolute $\Delta B_\parallel(B_\perp)$ representations – is a universal behaviour in the sense that it does neither depend on the relaxation rate γ, nor on the gyromagnetic ratio γ_F. We illustrate this by showing in Figure 5 our own data and the data extracted from Figures 6b and 7 of reference [1], together with the anticipated asymptotic behaviour on an absolute scale. The data of Grewal and Pattabiraman obey the general trend of the asymptotic behavior predicted by the algebraic model. We note that a change of the slope $\mathrm{d}\Delta B_\parallel/\mathrm{d}B_\perp$ in equation (3) implies a vertical translation of the asymptote line in the logarithmic representation of Figure 5.

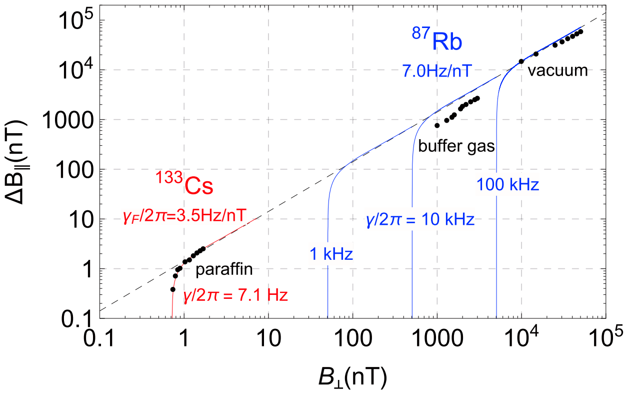

Fig. 5. Comparison – in absolute field units – of our own data from a paraffin-coated Cs vapour cell with the data from Rb vapour in a buffer gas cell and in a vacuum cell [1]. All error bars are omitted. The dashed line represents the asymptotic behaviour of equation (3).

Not knowing the relaxation rate of the experiments from [1], we cannot predict the threshold value for the used Rb cells, and can thus not predict below which transverse field their splitting will deviate from the linear dependence. In Figure 5 we therefore illustrate several nonlinear scenarios by showing the corresponding dependencies for a selection of relaxation rates γ.

With the above said, we feel that the statement "*The sensitivity of this technique is determined by the smallest detectable change in the separation of the two peaks.*" made in Section 4 of [1] is not correct, since the method will fail for transverse fields below the threshold field, where there is no line splitting. For measuring transverse fields, it is much more advantageous to scan directly an applied transverse field, as shown by Breschi and Weis [2].

In conclusion we have shown that the linear proportionality advocated in [1] for the splitting of GSHE resonances by a transverse field is only valid in an asymptotic limit for large transverse fields $B_\perp \gg \gamma/\gamma_F$. Moreover, we have shown that their proposed method for measuring transverse fields is not applicable for fields below a certain threshold field value that is determined by the relaxation rate γ and the gyromagnetic ratio γ_F.

Grewal and Pattabiraman have used numerical density matrix solutions of the Liouville equation of motion for obtaining the linear dependence that is at the center of their paper. On the other hand, it is often possible to derive algebraic expressions for problems in nonlinear magneto-optical spectroscopy of atoms. The present comment is an example demonstrating that such an algebraic approach yields a better insight into details of the problem at hand.

We add a final remark concerning numerical and algebraic approaches. The algebraic approach of Breschi and Weis [2] yields an explicit expression for the proportionality constant in equation (1). This constant describes the absolute signal magnitude and depends on the atomic number density, the sample length, the resonant absorption cross section and the degree of alignment created by optical pumping. In the algebraic approach, the latter parameter (depending on light intensity) may not be predicted in a simple way, and must be determined experimentally. However, since the problem at hand merely deals with line shapes (under identical light intensity conditions), the absolute scale is irrelevant, and was therefore not addressed. Numerical approaches, on the other hand, which treat all involved processes (pumping, precession, relaxation) simultaneously are able, in principle, to predict the detected signal in terms of absolute power levels.

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Author contribution statement

All authors have contributed in equal parts to this work. A.W. has done the algebraic calculations and has written most of the text. Y.S. and Z.D.G. have performed the measurements and have contributed to the text. Y.S. has done the data analysis and prepared the corresponding graphs.

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