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Comment

Comment on: Two-qutrit entanglement witnesses and Gell-Mann matrices

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Abstract. Recently, Jafarizadeh et al. [Eur. Phys. J. D **47**, 283 (2008)] has constructed several two-qutrit entanglement witnesses based on the Gell-Mann matrices by using the linear programming method, moreover they claimed that all W_2 given by equation (16) are entanglement witnesses. In this comment, we would like to point out that there exit some W_2 given by equation (16) are not entanglement witnesses in general.

In 2008, based on the Gell-Mann matrices by using the linear programming method, Jafarizadeh et al. [1] has constructed the following entanglement witnesses

$$\begin{aligned} W_2 = 2I_3 \otimes I_3 - \frac{3}{2}\big[&(-1)^{i_1}\lambda_1 \otimes \lambda_1 + (-1)^{i_2}\lambda_2 \otimes \lambda_2 \\ &- \lambda_3 \otimes \lambda_3 + (-1)^{i_4}\lambda_4 \otimes \lambda_4] \qquad (1) \\ &+ (-1)^{i_5}\lambda_5 \otimes \lambda_5 + (-1)^{i_6}\lambda_6 \otimes \lambda_6 \\ &+ (-1)^{i_7}\lambda_7 \otimes \lambda_7 - \lambda_8 \otimes \lambda_8\big] \qquad (2) \end{aligned}$$

(cf. Eq. (16) in Ref. [1], p. 286). Where $i_1, i_2, i_4, \ldots, i_7 \in \{0, 1\}$, and

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}, \qquad (3)$$

$$\lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \qquad (4)$$

Jafarizadeh et al. [1] claimed that the following key fact to hold:
Fact. All W_2 given by equations (1) and (2) are entanglement witnesses (cf. Eq. (16) in Ref. [1], pp. 286-287).

Unfortunately, above **Fact** is incorrect in general, it can be illustrated in the following:

Now choose

$$i_1 = i_4 = i_6 = 1, \quad i_2 = i_5 = i_7 = 0.$$

From equations (1) and (2), through direct calculations, we can obtain

$$\begin{aligned} W_2 = 2I_3 \otimes I_3 + \frac{3}{2}\big[&\lambda_1 \otimes \lambda_1 - \lambda_2 \otimes \lambda_2 + \lambda_3 \otimes \lambda_3 + \lambda_8 \otimes \lambda_8 \\ &\lambda_4 \otimes \lambda_4 - \lambda_5 \otimes \lambda_5 + \lambda_6 \otimes \lambda_6 - \lambda_7 \otimes \lambda_7\big]. \end{aligned}$$

That is

$$W_2 = \begin{bmatrix} 4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 4 \end{bmatrix}. \qquad (5)$$

In this case, it is easy to verify that W_2 given by equation (5) be Hermitian positive definite operator, obviously, W_2 given by equation (5) is not entanglement witnesses.

In conclusion, above analysis shows that there exit some W_2 given by equations (1) and (2) are not entanglement witnesses, therefore, above **Fact** given by Jafarizadeh et al. [1] is incorrect in general.

References

1. M.A. Jafarizadeh, Y. Akbari, N. Behzadi, Eur. Phys. J. D **47**, 283 (2008)

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