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Building cubic gravity with healthy and viable scalar and tensor perturbations

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Abstract We investigate sufficient conditions under which cubic gravity is healthy and viable at the perturbation level. We perform a detailed analysis of the scalar and tensor perturbations. We impose the requirement that the two scalar potentials, whose ratio is the post-Newtonian parameter γ, should deviate only minimally form general relativity. Additionally, concerning tensor perturbations we impose satisfaction of the LIGO-VIRGO and Fermi Gamma-ray Burst observations, and thus we result to a gravitational-wave equation with gravitational-wave speed equal to the speed of light, and where the only deviation from general relativity appears in the dispersion relation. Furthermore, we show that cubic gravity exhibits an effective Newton's constant that depends on the model parameter, on the background evolution, and on the wavenumber scale. Hence, by requiring its deviation from the standard Newton's constant to be within observational bounds we extract the constraints on the single coupling parameter β.

1 Introduction

Theories of gravity with higher-order invariants arise naturally as an effective description of a complete String Theory [1], and since they can improve the renormalizability properties of general relativity [2] they have attracted the interest of the literature [3]. On the other hand, one may have an additional motivation of cosmological origin [4–7], since when applied at a cosmological framework such theories may lead to new effective sectors of gravitational origin, that can drive inflation or late-time acceleration, or alleviate the H_0 and S_8 cosmological tensions [8,9].

In order to construct higher-order theories of gravity one starts from the Einstein–Hilbert Lagrangian and includes extra higher-order terms, such as in $f(R)$ gravity [10–12], in $f(G)$ gravity [13,14], in Lovelock gravity [15,16], etc. Alternatively, but not equivalently, one can construct higher-order theories of gravity in the torsional formulation, resulting in $f(T)$ gravity [17–19], in $f(T, T_G)$ gravity [20,21], in $f(T, B)$ gravity [22,23], etc.

In general such gravitational constructions involve extra degrees of freedom, which may be problematic, giving rise to various pathologies, such as ghost and Laplacian instabilities. Hence, one needs to focus on subclasses of these theories that are free from pathologies. We stress here that this has to hold around all backgrounds, and at all orders in perturbation theory, since a well-behaved background evolution does not necessarily guarantee well-behaved perturbations (as for instance was the case in the initial versions of Hořava–Lifshitz [24,25], of new nonlinear massive gravity [26], of entropic-force dark energy [27], etc).

Nevertheless, theoretical consistency is a necessary but not sufficient condition for the acceptance of a particular theory, since observational and experimental viability should also be obtained. Therefore, every theory should satisfy the bounds acquired by Solar System experiments [28], as well as be in agreement with various datasets of cosmological observations, such as Supernovae Type I (SNIa), Baryonic Acoustic Oscillations (BAO), direct Hubble constant measurements with cosmic chronometers (CC), Cosmic Microwave Background (CMB) shift temperature and polarization, redshift space distortion ($f\sigma_8$) and Large Scale Structure measure-

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ments [29], etc. Finally, since modified theories of gravity may predict gravitational wave speed c_{GW} different than the speed of light c, one must guarantee that she can satisfy the LIGO-VIRGO [30] and Fermi Gamma-ray Burst Monitor [31] observations, which require $|c_{GW}/c - 1| \leq 4.5 \times 10^{-16}$ [32].

One interesting class of higher-order gravity is the one that contains as a Lagrangian the cubic combination of curvature terms P [33], which was then extended to $f(P)$ gravity [34]. In [35–38] the authors reduced the possible coefficients in order to obtain second-order field equations that allow for spherically symmetric black hole solutions, while in [39] an extra cubic correction, which is trivial for a spherically symmetric metric, was added in order to lead to second-order field equations in a cosmological background too. The resulting cubic and $f(P)$ gravities prove to have interesting cosmological applications, and thus they have attracted extensive investigation [33,34,40–57].

However, despite the significant research on cubic gravity, the scalar and tensor perturbation analysis has not been performed. Thus, in the present work we are interested in performing such an analysis, and additionally we desire to extract conditions on the model parameters that allow for healthy and viable theories at the perturbation level. The plan of the work is as follows: In Sect. 2 we present cubic gravity and we provide the basic requirement in order to have well-defined cosmological behavior at the background level. Then in Sect. 3 we perform a detailed scalar and tensor perturbation analysis, extracting the conditions corresponding to absence of instabilities as well as to gravitational-wave speed equal to the speed of light. Finally, in Sect. 4 we summarize and conclude.

2 Cubic gravity

In this section we briefly review cubic modified gravity. This theory of gravity is based on adding corrections to the action of General Relativity, constructed from cubic combinations of the Riemann tensor. A general such combination is [15]

$$\begin{aligned} P =& \beta_1 R_{\mu\ \ \nu}^{\ \rho\ \ \sigma} R_{\rho\ \ \sigma}^{\ \gamma\ \ \delta} R_{\gamma\ \ \delta}^{\ \mu\ \ \nu} + \beta_2 R_{\mu\nu}^{\rho\sigma} R_{\rho\sigma}^{\gamma\delta} R_{\gamma\delta}^{\mu\nu} \\ &+\beta_3 R^{\sigma\gamma} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho}{}_{\gamma} + \beta_4 R R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ &+\beta_5 R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} + \beta_6 R_\mu^\nu R_\nu^\rho R_\rho^\mu \\ &+\beta_7 R_{\mu\nu} R^{\mu\nu} R + \beta_8 R^3, \end{aligned} \tag{1}$$

where β_i's are eight coefficients. Adding the above invariant in the Einstein–Hilbert Lagrangian, we can write the action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} (R - 2\Lambda) + \alpha P + \mathcal{L}_m \right], \tag{2}$$

where α is a possible coupling parameter, $\kappa = 8\pi G$ is the gravitational constant, and where for completeness we have also added the cosmological constant Λ, as well as the matter Lagrangian $\mathcal{L}_m$.

Varying the action with respect to the metric $g_{\mu\nu}$ we obtain the general field equations, namely [34].

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa \left(T_{\mu\nu} + \alpha H_{\mu\nu} \right), \tag{3}$$

with $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}}$ the usual energy-momentum tensor, and where the form of $H_{\mu\nu}$ is presented in Appendix A.

Let us now focus on a cosmological background, namely we consider a flat Friedmann–Robertson–Walker (FRW) background spacetime metric of the form

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \tag{4}$$

where $a(t)$ is the scale factor. In this case, the cubic invariant takes the simple form [34]

$$P = 6\tilde{\beta} H^4 \left(2H^2 + 3\dot{H} \right), \tag{5}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and with dots denoting time derivatives, since under the assumptions of being neither topological nor trivial and to lead to second-order field equations, it has only one free parameter, namely

$$\tilde{\beta} = -\beta_1 + 4\beta_2 + 2\beta_3 + 8\beta_4, \tag{6}$$

(note that the four β_i in (6) can still be chosen arbitrarily). Moreover, for the matter sector we consider the standard perfect fluid, whose energy-momentum tensor is $T_{\mu\nu} = (\rho_m + p_m) u_\mu u_\nu - p_m g_{\mu\nu}$.

The two Friedmann equations of cubic gravity in the case of FRW metric become

$$3H^2 = \kappa \left(\rho_m + \rho_{cub} \right), \tag{7}$$

$$3H^2 + 2\dot{H} = -\kappa \left(p_m + p_{cub} \right), \tag{8}$$

where we have defined

$$\rho_{cub} \equiv 6\beta H^6 + \frac{\Lambda}{\kappa}, \tag{9}$$

$$p_{cub} \equiv -6\beta H^4 (H^2 + 2\dot{H}) - \frac{\Lambda}{\kappa}, \tag{10}$$

and where we have merged α and $\tilde{\beta}$ in the sole parameter $\beta \equiv \alpha\tilde{\beta}$. Hence, the cubic terms give rise to an effective sector of geometric origin with the above energy density and pressure, and with effective equation-of-state parameter

$$w_{cub} \equiv \frac{p_{cub}}{\rho_{cub}}. \tag{11}$$

3 Perturbation analysis

In the previous section we presented cubic gravity and we extracted the general field equations. Additionally, we applied them in a cosmological framework, and we provided the background Friedmann equations. Although the Friedmann equations do not contain higher-order time-derivatives and thus the theory is well-defined at the background level, this does not guarantee that instabilities and contradictions with observations will not appear at the perturbation level. Hence, in this section we proceed to a detailed investigation of the perturbations around a cosmological background. As usual, we will investigate the scalar and tensor perturbations separately, however we will do that simultaneously since this will give rise to the necessary constraints on the model parameters.

3.1 Scalar perturbations

Let us start by the examination of scalar perturbations. We consider the usual perturbed metric of isentropic perturbations in the Newtonian gauge [58–64]

$$ds^2 = -a(\eta)^2 (1+2\phi)\, d\eta^2 + a(\eta)^2 \delta_{ij}\, (1-2\psi)\, dx^i dx^j,$$

where for convenience we use the conformal time η (with $dt = a d\eta$), with $a(\eta)$ the corresponding scale factor, and with ϕ,ψ the first-order scalar perturbations. Furthermore, concerning the perturbations of the matter sector, we write

$$\delta T^0_0 = -\delta\rho, \tag{12}$$
$$\delta T^i_j = \delta p \delta^i_j. \tag{13}$$

Inserting the above into the general field equations (3), and transforming as usual to Fourier space, we find that the time-time component of (3) is

$$\begin{aligned}2\psi + \frac{\kappa\alpha k^2}{a^4}\Big\{\Big[&(8\beta_3+48\beta_4+8\beta_5+12\beta_6\\ &+40\beta_7+144\beta_8)\,\mathcal{H}^2+(48\beta_2+32\beta_3+80\beta_4+20\beta_5\\ &+24\beta_6+56\beta_7+144\beta_8)\,\mathcal{H}'\Big]\phi\\ &-\Big[(8\beta_3+32\beta_4+12\beta_5+24\beta_6+72\beta_7+288\beta_8)\,\mathcal{H}^2\\ &+(12\beta_1+8\beta_3+32\beta_4+24\beta_5+12\beta_6\\ &+72\beta_7+288\beta_8)\,\mathcal{H}'\Big]\psi\Big\} = -\frac{\kappa a^2\delta\rho}{k^2},\end{aligned} \tag{14}$$

with k the wavenumber and where $\mathcal{H} = a'/a$ is the conformal Hubble function, with primes denoting conformal-time derivatives. We mention that since we are interested in calculating the corrections to the gravitational potential, we have kept only the leading terms in the $k \gg \mathcal{H}$ regime.

Additionally, the non-diagonal space-space component equation is found to be

$$\begin{aligned}\phi-\psi-\frac{\kappa\alpha k^2}{a^4}\Big\{\Big[&(4\beta_3+16\beta_4+6\beta_5\\ &+12\beta_6+36\beta_7+144\beta_8)\,\mathcal{H}^2\\ &+(6\beta_1+4\beta_3+16\beta_4+12\beta_5\\ &+6\beta_6+36\beta_7+144\beta_8)\,\mathcal{H}'\Big]\phi\\ &-\Big[(6\beta_1+48\beta_2+40\beta_3+112\beta_4\\ &+34\beta_5+36\beta_6+100\beta_7+288\beta_8)\,\mathcal{H}^2\\ &+(8\beta_3+48\beta_4+12\beta_5+18\beta_6\\ &+68\beta_7+288\beta_8)\,\mathcal{H}'\Big]\psi\Big\}=0.\end{aligned} \tag{15}$$

3.2 Tensor perturbations

We continue with the consideration of tensor perturbations around a flat FRW metric, namely we consider

$$ds^2 = -a(\eta)^2 d\eta^2 + a(\eta)^2 \left(\delta_{ij} + h_{ij}\right) dx^i dx^j. \tag{16}$$

As usual, the tensor h_{ij} is divergenceless ($\partial^i h_{ij} = 0$) and traceless ($h^i{}_i = 0$). The general equation for the tensor perturbations around a flat FRW background, in the case of cubic gravity, whose general field equations are given in (3), is given in Eq. (B1) in Appendix B.

3.3 Viability conditions

In the above subsections we examined the scalar and tensor perturbations in cubic gravity. Thus, we can now use them in order to extract conditions on the model parameters in order for the theory to be healthy and viable. In particular, we know that every viable modified gravity is a correction on top of general relativity, since the latter must always be recovered at a particular limit of the parameters of the modified gravity (in our case general relativity is recovered for $\alpha = 0$ or equivalently $\beta = 0$). Obviously, $\beta = 0$ is a sufficient condition that no problematic features are present, however the goal of the present work is to obtain non-trivial versions of the theory, i.e. with non-zero β_i parameters, that still satisfy the desired requirements, namely we want to find minimal non-zero deviations from general relativity that are viable.

A first requirement comes from the correction in the Poisson's law. In the case of general relativity we have $\Phi_{eff} \equiv \frac{\phi+\psi}{2} = -\frac{\kappa a^2\delta\rho}{2k^2}$, which is the quantity that determines the light bending [65,66], with $\phi = \psi = -\frac{\kappa a^2\delta\rho}{2k^2}$. Nevertheless, in general in a modified gravity theory the post-Newtonian parameter $\gamma \equiv \psi/\phi$ is different than 1, however this deviation should be quite small in order to pass the observational tests [28].

A second requirement is that the tensor perturbations, namely the gravitational waves, should propagate at the speed of light c, in order to be in agreement with LIGO-VIRGO [30] and Fermi Gamma-ray Burst Monitor [31] observations, which require $|c_{GW}/c - 1| \leq 4.5 \times 10^{-16}$ [32].

Observing the forms of (14) and (15) one sufficient condition to achieve γ close to one is to choose the model parameters in order to make all new terms apart from one equal to zero (making all of them zero gives back general relativity). Furthermore, concerning tensor perturbations, starting from (B1) we make the standard approximation $k^2 \sim \mathcal{H}^2$, while we consider $\mathcal{H}' \sim \mathcal{H}^2$ and ${h^{(4)}}_{ij} \sim \mathcal{H}^3 {h^{(1)}}_{ij}$, up to ${h^{(1)}}_{ij}$ term, and we impose the same approximations for the h_{ij} term. Under the above considerations we result to

$$\begin{aligned}\beta_1 &= \frac{14}{39}\beta_3 + 8\beta_4 - \frac{34}{39}\beta_5\\ \beta_2 &= -\frac{11}{78}\beta_3 - \frac{1}{26}\beta_5\\ \beta_6 &= \frac{2}{39}\beta_3 + 8\beta_4 - \frac{1}{13}\beta_5\\ \beta_7 &= -\beta_3 - 8\beta_4 - \frac{1}{2}\beta_5,\\ \beta_8 &= \frac{199}{936}\beta_3 + \frac{11}{9}\beta_4 + \frac{121}{1404}\beta_5.\end{aligned} \tag{17}$$

Note that (6) under (17) gives $\tilde{\beta} = \frac{14}{13}\beta_3 + \frac{28}{39}\beta_5$. Hence, a theory with (17), plus the background constraint (6), corresponds to a viable non-trivial minimal deviation from general relativity.

One can clearly see that under conditions (17), equations (14) and (15) provide the potentials ϕ and ψ as

$$\begin{aligned}\phi &= \frac{\left[-2 + \frac{7a^4}{14a^4 + 3k^2\kappa\beta(5\mathcal{H}^2 - 7\mathcal{H}')}\right]\kappa a^2\delta\rho}{3k^2}\\ \psi &= -\frac{\left[1 + \frac{7a^4}{14a^4 + 3k^2\kappa\beta(5\mathcal{H}^2 - 7\mathcal{H}')}\right]\kappa a^2\delta\rho}{3k^2}.\end{aligned} \tag{18}$$

Thus, we do verify that the corrections to the gravitational potentials due to cubic terms depend on the single parameter β and are minimal, satisfying the observed bounds [28]. Hence, the post-Newtonian parameter γ mentioned above becomes

$$\gamma \equiv \frac{\psi}{\phi} = \frac{1}{2} + \frac{1}{2 - \frac{4}{7}\mu\left(2H^2 + 7\dot{H}\right)}, \tag{19}$$

where we have introduced the quantity $\mu \equiv \frac{8\pi G k^2 \beta}{a^2}$ for convenience. In the limit $\beta \to 0$ we obtain $\gamma \to 1$ as expected.

Additionally, we can immediately see that under these conditions the gravitational-wave propagation equation becomes

$$h''_{ij} + 2\mathcal{H}\left(1 + \beta_P\right) h'_{ij} + k^2 h_{ij} = 0, \tag{20}$$

where

$$\beta_P = -\frac{3\kappa\beta\left(153\mathcal{H}^4 - 488\mathcal{H}^2\mathcal{H}' + 235\mathcal{H}'^2\right)}{14a^4}. \tag{21}$$

As we observe, under conditions (17), the gravitational waves in cubic gravity propagate at the speed of light, and thus the theory is viable. However, the cubic terms affect the dispersion relation through the term β_P, a feature that appears in other viable modified theories of gravity too [67–70]. Lastly, we mention that (20) can be re-written in terms of cosmic time, using $\mathcal{H} = aH$, $\mathcal{H}' = a^2(H^2 + \dot{H})$ (note that this will bring coefficient changes between (21) and (23) below) as

$$\ddot{h}_{ij} + 3H\left(1 + \beta_P\right)\dot{h}_{ij} + \frac{k^2}{a^2}h_{ij} = 0, \tag{22}$$

where now

$$\beta_P = \frac{1}{7}\left(100H^4 + 18H^2\dot{H} - 235\dot{H}^2\right)\kappa\beta. \tag{23}$$

We proceed by focusing on the scalar perturbations. As we showed, in cubic gravity the Poisson's law is modified according to (18), which implies that we obtain an effective gravitational constant. Using that $\kappa = 8\pi G$, with G the Newton's constant, we find that the effective Newton's constant is given as

$$G_{eff} \equiv \frac{1}{3}G\left[4 - \frac{7a^4}{7a^4 + 12\pi G k^2\beta\left(5\mathcal{H}^2 - 7\mathcal{H}'\right)}\right], \tag{24}$$

in which case one recovers $\phi = -4\pi G_{eff} a^2\delta\rho/k^2$. Note that in terms of cosmic time we have

$$G_{eff} \equiv \frac{1}{3}G\left[4 + \frac{1}{\frac{12k^2\pi G\beta(2H^2 + 7\dot{H})}{7a^2} - 1}\right]. \tag{25}$$

Hence, in cubic gravity, as it is typical in modified gravity theories, we obtain an effective Newton's constant G_{eff} that is in general different than G, and the deviation depends on the model parameter β as well as on the specific background Hubble function evolution, namely on $H(t)$. For $\beta = 0$, in which case cubic gravity recovers General Relativity, we obtain $G_{eff} = G$ as expected. Note that although the time-dependence of the effective Newton's constant is typical in modified gravity, the scale-dependence appears only in subclasses of them [17,62]. According to observations we obtain $G_{eff}/G = 1.09 \pm 0.2$ at 1σ confidence level [71–74], and thus this deviation should satisfy

$$0.89 \lesssim \frac{G_{eff}}{G} \lesssim 1.29. \tag{26}$$

Now the quantity $\mu \equiv \frac{8\pi G k^2 \beta}{a^2}$ defined above, using the redshift $1 + z = a_0/a$ (with the present scale factor set to $a_0 = 1$) becomes

$$\mu\left(z\right) = 8\pi G\beta k^2\left(1 + z\right)^2. \tag{27}$$

Thus, we can express the effective Newton's constant as $G_{eff} = \frac{1}{3}G\left\{4 + [\frac{3}{14}\mu(2H^2 + 7\dot{H}) - 1]^{-1}\right\}$. Therefore, we conclude that if $0 < \mu(2H^2 + 7\dot{H}) < 14/3$ then we obtain $G_{eff} < G$, otherwise $G_{eff} > G$, while for $\mu = 0$, i.e. $\beta = 0$, we recover $G_{eff} = G$.

In Fig. 1 we depict the **late-time** evolution of the normalized effective gravitational constant G_{eff}/G as a function of the redshift, in the scenario at hand for various choices of β in units $8\pi G = 1$ and $H_0 = 1$ (the subscript "0" marks

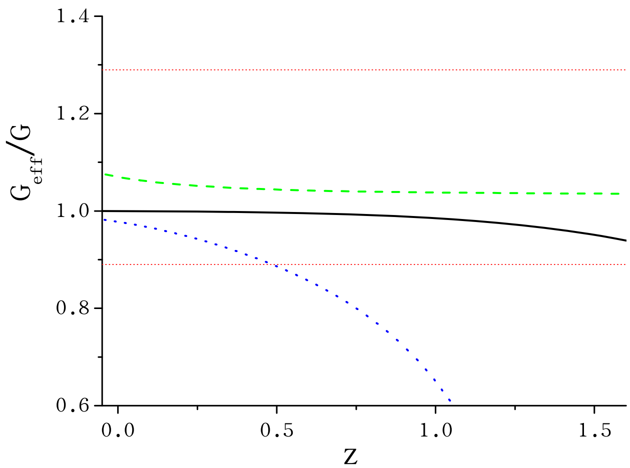

Fig. 1 The evolution of the normalized effective gravitational constant G_{eff}/G as a function of the redshift, at a scale $k = 10^{-3}$ Mpc^{-1}, and for $\beta = -0.0001$ (black solid), $\beta = -0.005$ (green dashed), and $\beta = 0.5$ (blue dotted) in units where $8\pi G = 1$ and $H_0 = 1$, where we have set the present matter density parameter $\Omega_{m0} \equiv 8\pi G\rho_{m0}/(3H_0^2) \approx 0.31$ [75]. The horizontal red dotted lines mark the observational bounds on G_{eff}/G [71–74] given in (26)

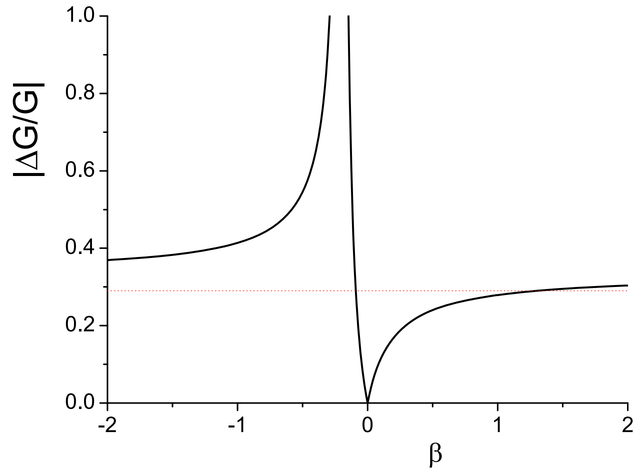

Fig. 2 The normalized difference of the effective gravitational constant $|\Delta G/G|$ as a function of the model parameter β, in units where $8\pi G = 1$ and $H_0 = 1$, at present time ($a_0 = 1$ and $\dot{H}(z=0) \approx -H_0^2\,(1+q_0)$ with $q_0 \approx -0.503$ the current deceleration parameter [76]). The horizontal red dotted line marks the observational bound $|\Delta G/G| \lesssim 0.29$ [71–74]

the value of a quantity at present), at a reference scale $k = 10^{-3}$ Mpc^{-1}, on top of the observational bounds. For completeness, in Fig. 2 we depict $\left|\frac{\Delta G}{G}\right|$, where $\Delta G \equiv G_{eff} - G$, as a function of the model parameter β, at $k = 10^{-3}$ Mpc^{-1} at present time (where $a_0 = 1$ and $\dot{H}(z=0) \approx -H_0^2\,(1+q_0)$ with $q_0 \approx -0.503$ the current deceleration parameter [76]). From these figures we deduce that a viable theory should have $-0.01 \lesssim \beta \lesssim 0.1$. Restoring natural units (where $G \approx 6.7\times10^{-39}$ GeV, $k = 10^{-3}$ Mpc$^{-1} \approx 6.4\times10^{-42}$ GeV, and $H_0 = 1.4\times10^{-42}$ GeV) we obtain that -10^{200} GeV$^{-2} \lesssim \beta \lesssim 10^{201}$ GeV^{-2}. Note that this window is in agreement with the bounds obtained in higher-order corrections to Einstein gravity through causality and unitarity considerations on the graviton scattering [77]. Lastly, note also that there is a specific value of β in which G_{eff} diverges, as expected from the form of (25). Finally, we have checked that changing the reference scale k leads to the same qualitative features.

Note that a varying G_{eff} (of course inside the observational bounds), and in particular a G_{eff} smaller than G by a suitable amount, is known to be one of the mechanisms that can alleviate the H_0 and σ_8 cosmological tensions, since "weaker" gravity can lead to faster expansion and smaller matter clustering (see [8] for various models with this property). Thus, the aforementioned property in the scenario at hand could be useful towards the tensions alleviation too.

Let us mention here that the above analysis focuses on late times, while at very early times the constraints are typically stronger. If we want to extend the analysis up to very early times, namely up to the Big Bang Nucleosynthesis (BBN) epoch ($z \sim 10^9$), then we should also consider the radiation sector, which was neglected above since we focused on late times. However, we note that the BBN constraints on cubic gravity were examined in [54]. Definitely, in the end of the day, all constraints from various investigations should be used simultaneously.

In summary, we have extracted the conditions required for a healthy and viable cubic gravity, in order for scalar and tensor perturbations to be in agreement with observations, and we extracted the constraints on the single parameter β.

4 Conclusions

Cubic gravity is a higher-order modified gravity whose Lagrangian P is built by cubic curvature terms under the theoretical requirement to lead to second-order field equations at four dimensions. Since cubic and $f(P)$ gravity are known to have interesting cosmological phenomenology, in the present work we investigated the conditions under which the theory is healthy and viable at the perturbation level.

We performed a detailed analysis of the scalar and tensor perturbations. We imposed the requirement that the two scalar potentials, whose ratio is the post-Newtonian parameter γ, should deviate only minimally form the general relativity result. Additionally, concerning the tensor perturbations we imposed the condition that the obtained gravitational-wave speed should satisfy the LIGO-VIRGO and Fermi Gamma-ray Burst observations. Thus, we resulted to a gravitational-wave equation with gravitational-wave speed equal to the speed of light, and where the only deviation from general relativity appears in the dispersion relation.

Furthermore, we showed that cubic gravity exhibits an effective Newton's constant that depends on the model parameter, on the background evolution, and on the wavenumber scale. Hence, by requiring its deviation from the standard Newton's constant to be within observational bounds we extracted the constraints on the single coupling parameter β.

In summary, in this work we constructed non-trivial versions of cubic gravity, namely with non-zero parameters, that satisfy the viable observational requirements. This is a necessary addition to its known interesting cosmological phenomenology. Clearly, these are not the only classes of theories that have this property, since there could be more complicated theories, namely with less constraints and thus more parameters, that share this property. Thus, we extracted theories that deviate form general relativity at the level of the action, but have minimal deviations (but still non-zero) at the level of scalar and tensor perturbations.

It would be interesting to apply the results of the present work in order to study the primordial gravitational waves and the primordial black holes in the case of cubic gravity. Such an analysis could be useful in order to extract unique observational signatures of cubic gravity, and distinguish this theory from other modified theories of gravity. Additionally, we could extend the viability investigation in the case of non-linear $f(P)$ gravity and examine whether the extra degrees of freedom alter the results, especially those related to the gravitational wave propagation. Furthermore, although we have shown that there are no instabilities in our approximated expressions, other pathologies could be present, and thus a full stability analysis of tensor perturbations should be performed too. Finally, it would be interesting to compare the obtained cosmological constraints with constraints arising from spherically symmetric solutions. These studies lie beyond the scope of the present work and are left for future projects.

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Data availability This manuscript has no associated data or the data will not be deposited.

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Appendix A: The general field equations of cubic gravity

Varying the action of cubic gravity (2) with respect to the metric we obtain the general field equations as [34].

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa\left(T_{\mu\nu} + \alpha H_{\mu\nu}\right), \tag{A1}$$

with $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}$ the energy-momentum tensor, and where the $H_{\mu\nu}$ tensor reads as

$$\begin{aligned}
H_{\mu\nu} = {} & 2\left(\beta_1 + 4\beta_3 + 40\beta_4 - \beta_5 + \beta_6\right) g_{\mu\nu} R_{\alpha}{}^{\gamma} R^{\alpha\beta} R_{\beta\gamma} \\
& + 2\beta_7 R_{\alpha\beta} R^{\alpha\beta} R_{\mu\nu} - 2\left(2\beta_3 + \beta_5\right) R_{\alpha\beta} R_{\mu}{}^{\alpha} R_{\nu}{}^{\beta} \\
& - \left(\frac{9}{2}\beta_1 + 9\beta_3 + 88\beta_4 + \beta_7\right) g_{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} R_{\mu\nu} \\
& - 8\beta_4 R_{\mu}{}^{\alpha} R_{\nu\alpha} R + 6\beta_8 R_{\mu\nu} R^2 + 2\beta_4 R_{\mu\nu} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \\
& + \left(\frac{5}{8}\beta_1 + \frac{5}{4}\beta_3 12\beta_4 - \beta_8\right) g_{\mu\nu} R^3 \\
& + \left(3\beta_1 + 6\beta_3 + 64\beta_4 + \beta_5 - 3\beta_6\right) g_{\mu\nu} R^{\alpha\gamma} R^{\beta\delta} R_{\alpha\beta\gamma\delta} \\
& + \left(\frac{8}{3}\beta_1 + \frac{3}{4}\beta_3 + 7\beta_4\right) g_{\mu\nu} R R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \\
& - \frac{1}{2}\left[\beta_1 + 2\beta_2 + 3\left(\beta_3 + 8\beta_4\right)\right] g_{\mu\nu} R_{\alpha\beta}{}^{\epsilon\zeta} R^{\alpha\beta\gamma\delta} R_{\gamma\delta\epsilon\zeta} \\
& + 4\left(2\beta_4 + \beta_7\right) R R^{\alpha\beta} R_{\mu\alpha\nu\beta} + 2\left(2\beta_3 + 3\beta_6\right) R^{\beta\gamma} \\
& \times R_{(\nu}{}^{\alpha} R_{\mu)\beta\alpha\gamma} + \left[4\left(\beta_3 + \beta_6\right) - 6\beta_1\right] R_{\alpha}{}^{\gamma} R^{\alpha\beta} R_{\mu\beta\nu\gamma} \\
& - \beta_3 R_{\nu}{}^{\alpha} R_{\alpha\beta\gamma\delta} R_{\mu}{}^{\beta\gamma\delta} - 24\beta_2 R_{\nu}{}^{\alpha} R_{\alpha\gamma\beta\delta} R_{\mu}{}^{\beta\gamma\delta} \\
& + \left(6\beta_1 - 4\beta_3 + 4\beta_5\right) R^{\alpha\beta} R_{\alpha\gamma\beta\delta} R_{\mu}{}^{\gamma}{}_{\nu}{}^{\delta} \\
& + 4\beta_4 R R_{\mu}{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} + 6\beta_2 R_{\beta\gamma\delta\epsilon} R_{\mu}{}^{\alpha\beta\gamma} R_{\nu\alpha}{}^{\delta\epsilon} \\
& + 2\left(6\beta_2 + \beta_3\right) R^{\alpha\beta} R_{\mu\alpha}{}^{\gamma\delta} R_{\nu\beta\gamma\delta} \\
& + 2\left(12\beta_2 + \beta_3 + \beta_5\right) R^{\alpha\beta} R_{\mu}{}^{\gamma}{}_{\alpha}{}^{\delta} R_{\nu\beta\gamma\delta} \\
& + \left(\beta_3 - 6\beta_2\right) R_{\alpha\gamma\delta\epsilon} R_{\mu}{}^{\alpha\beta\gamma} R_{\nu\beta}{}^{\delta\epsilon} \\
& + 9\beta_1 R_{\alpha\delta\gamma\epsilon} R_{\mu}{}^{\alpha\beta\gamma} R_{\nu\beta}{}^{\delta\epsilon} - \beta_3 R_{\mu}{}^{\alpha} R_{\alpha\beta\gamma\delta} R_{\nu}{}^{\beta\gamma\delta} \\
& - 24\beta_2 R_{\mu}{}^{\alpha} R_{\alpha\gamma\beta\delta} R_{\nu}{}^{\beta\gamma\delta} + 2\left(12\beta_2 + \beta_3 + \beta_5\right) R^{\alpha\beta} \\
& \times R_{\mu\alpha}{}^{\gamma\delta} R_{\nu\gamma\beta\delta} + 4\beta_3 R^{\alpha\beta} R_{\mu}{}^{\gamma}{}_{\alpha}{}^{\delta} R_{\nu\gamma\beta\delta} \\
& - 2\left[3\beta_1 + 2\left(\beta_3 + \beta_5\right)\right] R^{\alpha\beta} R_{\mu}{}^{\gamma}{}_{\alpha}{}^{\delta} R_{\nu\delta\beta\gamma} \\
& + \left(\beta_3 - 6\beta_2\right) R_{\beta\gamma\delta\epsilon} R_{\mu}{}^{\alpha\beta\gamma} R_{\nu}{}^{\delta}{}_{\alpha}{}^{\epsilon} \\
& + 3\beta_1 R_{\beta\delta\gamma\epsilon} R_{\mu}{}^{\alpha\beta\gamma} R_{\nu}{}^{\delta}{}_{\alpha}{}^{\epsilon}
\end{aligned}$$

$$+\left[6\beta_1+4\left(\beta_3-6\beta_2\right)\right]R_{\alpha\delta\gamma\epsilon}R_\mu{}^{\alpha\beta\gamma}R_\nu{}^\delta{}_\beta{}^\epsilon$$
$$+6\beta_1R_{\alpha\epsilon\gamma\delta}R_\mu{}^{\alpha\beta\gamma}R_\nu{}^\delta{}_\beta{}^\epsilon$$
$$+2\left(4\beta_4+\beta_7\right)R\Box R_{\mu\nu}+\left(\beta_5+2\beta_7\right)R_{\mu\nu}\Box R$$
$$+\left(\beta_7+12\beta_8\right)g_{\mu\nu}R\Box R+2\left(\beta_3+8\beta_4+\beta_5\right.$$
$$\left.+2\beta_7\right)\nabla_\alpha R_{\mu\nu}\nabla^\alpha R$$
$$+\left[\frac{3}{4}\beta_6+2\left(\beta_7+6\beta_8\right)\right]g_{\mu\nu}\nabla_\alpha R_{\mu\nu}\nabla^\alpha R$$
$$-2\left(\beta_3+\beta_5+\frac{3}{2}\beta_6+2\beta_7\right)R_{\alpha(\nu}\nabla^\alpha\nabla_{\mu)}R$$
$$+2\left(2\beta_3+\beta_5\right)R^{\alpha\beta}\nabla_\beta\nabla_\alpha R_{\mu\nu}+2(2\beta_3+3\beta_6)R_{(\mu}{}^\alpha\Box R_{\nu)\alpha}$$
$$+2\left(2\beta_5-2\beta_3-3\beta_6\right)R^{\alpha\beta}\nabla_\beta\nabla_{(\mu}R_{\nu)\alpha}$$
$$+2\beta_3R^{\alpha\beta\gamma\ delta}\nabla_\beta\nabla_{(\mu}R_{\nu)\alpha\gamma\delta}$$
$$-2\left(3\beta_1+12\beta_2+2\beta_3+\beta_5\right)\nabla_\alpha R_{\nu\beta}\nabla^\beta R_\mu{}^\alpha$$
$$+\left(24\beta_2+8\beta_3+6\beta_6\right)\nabla_\beta R_{\nu\alpha}\nabla^\beta R_\mu{}^\alpha$$
$$+\left(2\beta_7+3\beta_6-\beta_5\right)g_{\mu\nu}R_{\alpha\beta}\nabla^\beta\nabla^\alpha R$$
$$+\left(8\beta_4+2\beta_3-3\beta_1\right)R_{\mu\alpha\nu\beta}\nabla^\beta\nabla^\alpha R$$
$$+2\left(\beta_5+2\beta_7\right)g_{\mu\nu}R^{\alpha\beta}\Box R_{\alpha\beta}$$
$$+2\left(3\beta_1+\beta_5\right)R_{\mu\alpha\nu\beta}\Box R^{\alpha\beta}+2\beta_5R^{\alpha\beta}\Box R_{\mu\alpha\nu\beta}$$
$$+\left(3\beta_6-4\beta_5-2\beta_3\right)g_{\mu\nu}\nabla_\beta R_{\alpha\gamma}\nabla^\gamma R^{\alpha\beta}$$
$$-2\left(2\beta_3-6\beta_1\right)\nabla_\beta R_{\alpha(\mu\nu)\gamma}\nabla^\gamma R^{\alpha\beta}$$
$$+2\left[\beta_3+2\left(\beta_5+\beta_7\right)\right]g_{\mu\nu}\nabla_\gamma R_{\alpha\beta}\nabla^\gamma R^{\alpha\beta}$$
$$+4\left(3\beta_1+\beta_5\right)\nabla_\gamma R_{\mu\alpha\nu\beta}\nabla^\gamma R^{\alpha\beta}$$
$$-2\left(\beta_3-6\beta_1\right)\nabla_\beta R_{\alpha(\mu\nu)\gamma}\nabla^\gamma R^{\alpha\beta}$$
$$+2\left[\beta_3+2\left(\beta_5+\beta_7\right)\right]g_{\mu\nu}\nabla_\gamma R_{\alpha\beta}\nabla^\gamma R^{\alpha\beta}$$
$$+4\left(3\beta_1+\beta_5\right)\nabla_\gamma R_{\mu\alpha\nu\beta}\nabla^\gamma R^{\alpha\beta}$$
$$-2\left(\beta_3-6\beta_1\right)R_{\beta\gamma\alpha(\nu}\nabla^\gamma\nabla^\beta R_{\mu)}{}^\alpha$$
$$-4\left(12\beta_2+\beta_3\right)R_{\alpha\gamma\beta\nu}\nabla^\gamma\nabla^\beta R_\mu{}^\alpha$$
$$-4\left(3\beta_1+\beta_3+\beta_5\right)R_{\beta\alpha\gamma(\nu}\nabla^\gamma\nabla_{\mu)}R^{\alpha\beta}$$
$$+6\beta_1R^{\alpha\beta\gamma\delta}\nabla_\delta\nabla_\beta R_{\mu\alpha\nu\gamma}+2\beta_3R_{(\nu}{}^{\alpha\beta\gamma}R_{\mu)\alpha\beta\gamma}$$
$$+12\beta_2\nabla_\alpha R_{\nu\delta\beta\gamma}\nabla^\delta R_\mu{}^{\alpha\beta\gamma}-6\beta_1\nabla_\gamma R_{\nu\beta\alpha\delta}\nabla^\delta R_\mu{}^{\alpha\beta\gamma}$$
$$+2\beta_3\nabla_\delta R_{\nu\alpha\beta\gamma}\nabla^\delta R_\mu{}^{\alpha\beta\gamma}$$
$$+2\left(2\beta_3+8\beta_4+\beta_5\right)g_{\mu\nu}R_{\alpha\gamma\beta\delta}\nabla^\delta\nabla^\gamma R^{\alpha\beta}$$
$$+\frac{1}{2}\left(\beta_3+8\beta_4\right)g_{\mu\nu}\nabla_\epsilon R_{\alpha\beta\gamma\delta}\nabla^\epsilon R^{\alpha\beta\gamma\delta}$$
$$+\left(6\beta_1-2\beta_3+2\beta_5-3\beta_6\right)\nabla^\beta R_\nu{}^\alpha\nabla_\mu R_{\alpha\beta}$$
$$-2\left(\beta_3+8\beta_4+\frac{3}{2}\beta_6+2\beta_7\right)\nabla^\alpha R\nabla_{(\mu}R_{\nu)\alpha}$$
$$+\beta_3\nabla^\delta R_\nu{}^{\alpha\beta\gamma}\nabla_\mu R_{\alpha\delta\beta\gamma}$$
$$+4\left(\beta_3+\beta_5\right)\nabla^\gamma R^{\alpha\beta}\nabla_{(\mu}R_{\nu)\alpha\beta\gamma}$$
$$+\left(6\beta_1-2\beta_3+2\beta_5-3\beta_6\right)\nabla^\beta R_\mu{}^\alpha\nabla_\nu R_{\alpha\beta}$$
$$+2\left[3\beta_1+2\left(\beta_5+\beta_7\right)\right]\nabla_\mu R^{\alpha\beta}\nabla_\nu R_{\alpha\beta}$$
$$-\left[\frac{1}{2}\beta_5+2\left(\beta_7+6\beta_8\right)\right]\nabla_\mu R\nabla_\nu R$$
$$-4\beta_4\nabla_\mu R^{\alpha\beta\gamma\delta}\nabla_\nu R_{\alpha\beta\gamma\delta}+\beta_3\nabla^\delta R_\mu{}^{\alpha\beta\gamma}\nabla_\nu R_{\alpha\delta\beta\gamma}$$
$$-4\left(\beta_5+\beta_7\right)R^{\alpha\beta}\nabla_\nu\nabla_\mu R_{\alpha\beta}$$
$$-2\left(2\beta_4+\beta_7+6\beta_8\right)R\nabla_\nu\nabla_\mu R-4\beta_4R^{\alpha\beta\gamma\delta}\nabla_\nu\nabla_\mu R_{\alpha\beta\gamma\delta}. \tag{A2}$$

Appendix B: The general equation of tensor perturbations

The general equation for the tensor perturbations (16) around a flat FRW background, in the case of cubic gravity whose general field equations are given by (A1), is given by

$$-2\alpha\kappa\left\{\left[3\beta_1-4\beta_3-24\beta_4+\beta_5-6\left(\beta_6+\beta_7\right)\right]\mathcal{H}^2\right.$$
$$\left.-3\left(8\beta_2+4\beta_3+8\beta_4+2\beta_5+\beta_6+2\beta_7\right)\mathcal{H}'\right\}h^{(4)}{}_{ij}$$
$$-4\alpha\kappa\left\{2\left[-3\beta_1+4\beta_3+24\beta_4-\beta_5+6\left(\beta_6+\beta_7\right)\right]\mathcal{H}^3\right.$$
$$+2\left(3\beta_1+24\beta_2+8\beta_3+7\beta_5-3\beta_6\right)\mathcal{H}\mathcal{H}'$$
$$\left.-3\left(8\beta_2+4\beta_3+8\beta_4+2\beta_5+\beta_6+2\beta_7\right)\mathcal{H}''\right\}h^{(3)}{}_{ij}$$
$$+\left\{a^4+2\alpha\kappa\left\{3\beta_1-4\left(3\beta_2+2\beta_3-3\beta_4+\beta_5\right.\right.\right.$$
$$\left.+6\beta_6+9\beta_7+27\beta_8\right)\mathcal{H}^4-\left(3\beta_1-24\beta_2+26\beta_3\right.$$
$$\left.+192\beta_4+22\beta_5+36\beta_6+96\beta_7+216\beta_8\right)\mathcal{H}^2\mathcal{H}'$$
$$-3\left(16\beta_2+8\beta_3+12\beta_4+5\beta_5+5\beta_6+12\beta_7+36\beta_8\right)\mathcal{H}'^2$$
$$+2k^2\left\{\left[-3\beta_1+12\beta_2+8\beta_3+24\beta_4+\beta_5+6\left(\beta_6+\beta_7\right)\right]\mathcal{H}^2\right.$$
$$\left.+\left(12\beta_2+8\beta_3+24\beta_4+4\beta_5+3\beta_6+6\beta_7\right)\mathcal{H}'\right\}$$
$$-\left(3\beta_1+72\beta_2+26\beta_3+24\beta_4+16\beta_5-6\beta_6\right)\mathcal{H}\mathcal{H}''$$
$$\left.\left.+3\left(8\beta_2+4\beta_3+8\beta_4+2\beta_5+\beta_6+2\beta_7\right)\mathcal{H}'''\right\}\right\}h''{}_{ij}$$
$$+\left\{2a^4\mathcal{H}-2\alpha\kappa\left\{6\left[5\beta_1-4\left(\beta_2+2\beta_3+7\beta_4+4\beta_6\right.\right.\right.\right.$$
$$\left.\left.+5\beta_7+9\beta_8\right)\right]\mathcal{H}^5+2\left[-63\beta_1+2\left(-24\beta_2+7\beta_3+60\beta_4\right.\right.$$
$$\left.\left.-28\beta_5+54\beta_6+30\beta_7\right)\right]\mathcal{H}^3\mathcal{H}'+2\left(39\beta_1+72\beta_2+26\beta_3\right.$$
$$\left.+12\beta_4+55\beta_5-36\beta_6+24\beta_7+108\beta_8\right)\mathcal{H}\mathcal{H}'^2$$
$$+\left(39\beta_1+120\beta_2+62\beta_3+96\beta_4+76\beta_5+84\beta_7\right.$$
$$\left.+216\beta_8\right)\mathcal{H}^2\mathcal{H}''+\left(-15\beta_1-24\beta_2+2\beta_3+48\beta_4\right.$$
$$\left.-8\beta_5+36\beta6+60\beta_7+216\beta_8\right)\mathcal{H}'\mathcal{H}''$$
$$-k^2\left\{4\left[3\beta_1-12\beta_2-8\beta_3-24\beta_4-\beta_5-6\left(\beta6+\beta_7\right)\right]\mathcal{H}^3\right.$$
$$-12\left(\beta_1+\beta_5-\beta_6\right)\mathcal{H}\mathcal{H}'$$
$$+2\left(12\beta_2+8\beta_3+24\beta_4+4\beta_5+3\beta6+06\beta_7\right)\mathcal{H}''$$
$$-\left[3\beta_1+2\left(12\beta_2+7\beta_3+12\beta_4+5\beta_5\right.\right.$$
$$\left.\left.\left.\left.\left.+3\beta6+6\beta_7\right)\right]\mathcal{H}\mathcal{H}'''\right\}\right\}\right\}h'{}_{ij}$$
$$+k^2\left\{-2\alpha\kappa k^2\left\{\left[3\beta_1-10\beta_3-3\left[8\beta_4+\beta_5+2\left(\beta_6+2\beta_7\right)\right]\right]\mathcal{H}^2\right.\right.$$
$$\left.-2\left(4\beta_3+24\beta_4+2\beta_5+3\beta_6+6\beta_7\right)\mathcal{H}'\right\}+a^4$$
$$-2\alpha\kappa\left\{3\left[3\beta_1-4\left(3\beta_2+2\beta_3+\beta_4+3\beta_6+\beta_7-9\beta_8\right)\right]\mathcal{H}^4\right.$$
$$-\left[27\beta_1-2\left(36\beta_2+23\beta_3+24\beta_4+\beta_5+42\beta_6\right.\right.$$
$$\left.\left.+48\beta_7+108\beta_8\right)\right]\mathcal{H}^2\mathcal{H}'$$
$$+\left[\left(12\beta_1+8\beta_3+36\beta_4+19\beta_5+3\beta_6+36\left(\beta_7+3\beta_8\right)\right]\mathcal{H}'^2\right.$$

$$+ (15\beta_1 + 10\beta_3 + 24\beta_4 + 20\beta_5 + 6\beta_6 + 24\beta_7)\,\mathcal{H}\mathcal{H}'' - (3\beta_1 + 2\beta_3 + 4\beta_5 + 3\beta_6 + 6\beta_7)\,\mathcal{H}'''\}\}\,h_{ij} = 0. \quad \text{(B1)}$$

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