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Effects of a thermal bath and the accelerated motion on collective transitions of two atoms in an entangled state

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Abstract Transition rates of a uniformly accelerated atom coupled to a vacuum scalar field are identical with those of one being static in a thermal bath at a temperature proportional to the proper acceleration a, i.e., $T_U = \frac{a}{2\pi}$. We discuss in this paper whether there exists such an equivalence in the electromagnetic field case if more atoms are involved. To be specific, we explore the similarities and distinctions between effects of a thermal bath and those of the uniformly accelerated motion by comparing the transition rates of a two-atom system initially in the symmetric or antisymmetric entangled state [$|\psi_\pm\rangle$] in two cases, i.e., two static atoms in a thermal bath [the thermal case] and two atoms uniformly accelerated in vacuum [the acceleration case]. We discover that, for some particular orientations of atomic dipole moments, coherent radiation which happens in the acceleration case never occurs in the thermal case. The rates in the acceleration case with a low acceleration which means a low T_U can be smaller or larger and even equal to their counterparts in the thermal case; while the rates in the acceleration case with a high acceleration which means a high T_U are always much larger than their counterparts in the thermal case. Our results suggest that effects of a thermal bath and those of the uniformly accelerated motion on transition properties of a two-atom system in $|\psi_\pm\rangle$ are not necessarily distinctive, but can be equivalent if the acceleration is relatively low, depending on the value of the interatomic separation.

1 Introduction

Fluctuations of quantum fields are ubiquitous in nature and they are responsible for various phenomena. For an example, when an atom in an excited state is perturbed by a fluctuating field in vacuum, it may transition to the ground state and meanwhile emits a radiation field, and this process is spontaneous emission. When more atoms interact with a common radiation field, each atom no longer radiates independently and coherent radiation may take place. As a result, the atomic collective transition properties may differ obviously from those of atoms individually coupled to the field. Particularly, Dicke showed that the spontaneous radiation rate of an ensemble of identical atoms, which are confined in a volume with its size much smaller than the transition wavelength of the atoms, is abnormally large [1]. This phenomenon, dubbed as superradiance, is rooted in the symmetric correlation between the states of atoms. By contrast, when identical atoms are in antisymmetric correlation, subradiance may happen, meaning that the emission of radiation is greatly suppressed and the decay of the system is inhibited. Ever since Dicke's seminal work, the collective transitions of various systems in diverse circumstances have attracted much attention [2–14] (just to name a few), and nowadays they play especially important roles in many interesting applications in quantum information science and quantum optics [15–20].

Two identical two-level atoms is one of the simplest systems frequently exploited for the study of collective transition properties of atoms. Recently, there have been some renewed interest in this topic, boundaries effects on collective transition rates of a two-atom system coupled to a vacuum scalar field [21,22] or an electromagnetic field [14,23] for instances. For two identical atoms located near boundaries and in the symmetric or antisymmetric entangled state $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|g_A e_B\rangle \pm |e_A g_B\rangle)$, where $|g_\xi\rangle$ and $|e_\xi\rangle$ with $\xi = A$ or B denote the ground and the excited states of the two atoms, it is found that the spontaneous emission rate of the two-atom system, as compared with their counterparts in an unbounded space, can be greatly enhanced or inhibited, depending on the specific entangled state, the interatomic separation, and the atom-boundary separation. However, atoms in real situations are surrounded by a thermal environment but rather immersed in vacuum. So on one hand,

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one may ask how will thermal fluctuations affect the collective transition properties of a two-atom system?

On the other hand, when referring to thermal effects on atoms, one is often reminiscent of the other phenomenon, i.e., the Fulling–Davies–Unruh (FDU) effect, which affirms that a uniformly accelerated observer perceives the quantum vacuum as a thermal bath at a temperature proportional to the proper acceleration a, i.e., the Unruh temperature $T_U = \frac{a}{2\pi}$ [24–26].[1] The FDU effect seems to suggest an equivalence between the effects of a thermal bath and those of the uniformly accelerated motion. For the case of a single atom in interaction with a massless scalar field, there indeed exists such an equivalence in the sense that the response rate as well as the average variation rate of energy of an atom uniformly accelerated in vacuum is identical to its counterpart of a static one in a thermal bath at a temperature equal to T_U [27,28]. While when more atoms are involved, things seem to be different. For the scalar field case, it has already been showed that analytical expressions of the transition rates of a two-atom system in $|\psi_+\rangle$ or $|\psi_-\rangle$ and uniformly accelerated in vacuum differ from their counterparts in a thermal bath even when the temperature of the bath equals T_U [29]. So it is straightforward for one to expect that thermal effects and the effects of acceleration on a two-atom system in $|\psi_+\rangle$ or $|\psi_-\rangle$ are also always distinctive in the electromagnetic field case.

In this paper, we explore the intrinsic combinations and distinctions between the effects of thermal fluctuations and those of the uniformly accelerated motion in terms of the collective transition properties of two identical atoms in $|\psi_+\rangle$ or $|\psi_-\rangle$. This work is a subsequent one of Ref. [30], in which we have already derived the collective transition rates of two atoms initially in $|\psi_+\rangle$ or $|\psi_-\rangle$, linearly coupled to a vacuum fluctuating electromagnetic field, and synchronously and uniformly accelerated with a constant interatomic separation perpendicular to the acceleration. The paper is organized as follows. We evaluate in Sect. 2, with the time-dependent perturbation theory, the collective transition rates of two static atoms in $|\psi_+\rangle$ or $|\psi_-\rangle$ in a thermal bath. Then we discuss, in Sects. 3 and 4, the intrinsic combinations and distinctions between the effects of a thermal bath and those of the uniformly accelerated motion on the collective transition rates as well as the evolution of energy of the two-atom system. We give a brief summary for our work in Sect. 5.

[1] Throughout the paper, we exploit units that $\hbar = c = k_B = 1$, where $\hbar$, c and k_B are respectively the reduced Plank constant, the velocity of photons, and the Boltzmann constant.

2 Collective transition rates of two static atoms in a thermal bath

We consider that two static identical two-level atoms labelled by A and B are located in a thermal bath at temperature T. We denote the ground and the excited states of atom $\xi(=A, B)$ with energy $-\frac{\omega_0}{2}$ and $+\frac{\omega_0}{2}$ by $|g_\xi\rangle$ and $|e_\xi\rangle$, respectively. Then for the two-atom system, there are three eigenstates whose energy are respectively $-\omega_0, 0, \omega_0$, and we denote them by $|\varepsilon_n\rangle$ with $n = 1, 2, 3$. Obviously, $|\varepsilon_1\rangle = |g_A g_B\rangle$ and $|\varepsilon_3\rangle = |e_A e_B\rangle$, and they are nondegenerate, while the intermediate eigenstate $|\varepsilon_2\rangle$ is degenerate. We suppose that the two atoms are initially prepared in the symmetric or anti-symmetric entangled state, $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|g_A e_B\rangle \pm |e_A g_B\rangle)$, and their trajectories are depicted by

$$
\begin{aligned}
&t_A = t_B = \tau, \quad x_A = x_B = 0, \\
&y_A = 0, y_B = L, \quad z_A = z_B = 0,
\end{aligned} \tag{1}
$$

where τ is the proper time.

The atoms in the thermal bath are inevitably perturbed by the fluctuating electromagnetic field, and the atom-field interaction can be given by

$$
H_I(\tau) = -\boldsymbol{\mu}_A(\tau) \cdot \mathbf{E}(x_A(\tau)) - \boldsymbol{\mu}_B(\tau) \cdot \mathbf{E}(x_B(\tau)), \tag{2}
$$

where $\boldsymbol{\mu}_\xi$ represents the dipole moment of atom ξ, and $\mathbf{E}(x(\tau))$ is the electric field operator which is associated with the vector potential A_μ by

$$
E_i = -\partial_i A_0 - \partial_0 A_i. \tag{3}
$$

Hereafter, $i, j = 1, 2, 3$ are spatial indices. Under the Lorentz gauge,

$$
A_\mu(x) = \int d^3\mathbf{k}\, g_\mathbf{k} \sum_{\lambda=0}^{3} \left[\hat{a}_{\mathbf{k}\lambda} \epsilon_\mu(\mathbf{k}, \lambda) e^{-ik\cdot x} + H.c.\right], \tag{4}
$$

where $g_\mathbf{k} = (2\pi)^{-\frac{3}{2}}(2\omega_k)^{-\frac{1}{2}}$, $\omega_k = |\mathbf{k}|$, λ is the polarization index, $\hat{a}_{\mathbf{k}\lambda}$ is the annihilation operator, “H.c.” denotes the Hermitian Conjugate, and $\epsilon_\mu(\mathbf{k}, \lambda)$ is the polarization vector satisfying $\sum_{\lambda=0}^{3} g_{\lambda\lambda} \epsilon^\mu(\mathbf{k}, \lambda) \epsilon^\nu(\mathbf{k}, \lambda) = g^{\mu\nu}$ with $g^{\mu\nu} = diag(1, -1, -1, -1)$.

We exploit the time-dependent perturbation theory to calculate transition rates of the two-atom system. As the thermal state is a mixed state, we first consider the simple case of atoms in interaction with the field in a pure state $|l_{\omega,\mathbf{k}}\rangle$ with $l \geq 0$, and accordingly we denote the state of “atoms+field” system by $|\psi_\pm, l_{\omega,\mathbf{k}}\rangle$. Then write out the amplitude for the “atoms+field” system to transition from the initial state $|\psi_\pm, l_{\omega,\mathbf{k}}\rangle$ to a final state $|\varepsilon_n, s_{\omega,\mathbf{k}'}\rangle$, take the square of its modulus, and we get

$$P_{|\psi_\pm l_{\mathbf{k}}\rangle\to|\varepsilon_n,s_{\mathbf{k}'}\rangle}=\int_{-\infty}^{\infty}d\tau\int_{-\infty}^{\infty}d\tau'\times\langle l_{\mathbf{k}},\psi_\pm|\left[\mu_A^iE_i(x_A(\tau))+\mu_B^iE_i(x_B(\tau))\right]|\varepsilon_n,s_{\mathbf{k}'}\rangle\times\langle s_{\mathbf{k}'},\varepsilon_n|\left[\mu_A^jE_j(x_A(\tau'))+\mu_B^jE_j(x_B(\tau'))\right]|\psi_\pm,l_{\mathbf{k}}\rangle,\tag{5}$$

which is the transition probability for the "atoms+field" system to transition from $|\psi_\pm,l_{\mathbf{k}}\rangle$ to $|\varepsilon_n,s_{\mathbf{k}'}\rangle$. Hereafter, summation is implied for repeated indices. Considering that the probability for the field to populate in the pure state $|l_{\omega,\mathbf{k}}\rangle$ is $p_l(\omega)=\frac{e^{-\beta l\omega}}{N(\omega)}$ with $\beta=T^{-1}$ and $N(\omega)=\sum_{l=0}^{\infty}e^{-\beta l\omega}$, we next multiply the above probability by $p_l(\omega)$, take the summation of it over all possible values of l, s and $\mathbf{k}$, do some simplifications, and we finally arrive at the following transition probability:

$$P_{|\psi_\pm\rangle\to|\varepsilon_n\rangle}=\sum_\xi\sum_{\xi'}\langle\psi_\pm|\mu_\xi^i(0)|\varepsilon_n\rangle\langle\varepsilon_n|\mu_{\xi'}^j(0)|\psi_\pm\rangle\times\int_{-\infty}^{\infty}d\tau\int_{-\infty}^{\infty}d\tau'e^{-i\Delta\varepsilon\Delta\tau}G_{ij}^\beta\left(x_\xi(\tau),x_{\xi'}(\tau')\right),\tag{6}$$

where $\Delta\tau=\tau-\tau'$, $\Delta\varepsilon$ represents the energy gap between the two states $|\varepsilon_n\rangle$ and $|\psi_\pm\rangle$, and

$$G_{ij}^\beta\left(x_\xi(\tau),x_{\xi'}(\tau')\right)=\sum_{l=0}^{\infty}\int d\mathbf{k}\frac{e^{-\beta l\omega}}{N(\omega)}\langle l_{\mathbf{k}}|E_i(x_\xi(\tau))E_j(x_{\xi'}(\tau'))|l_{\mathbf{k}}\rangle\tag{7}$$

is the two-point correlation function of the field in the thermal state. Dividing the transition probability Eq. (6) by the infinite time interval, we obtain the transition rate of the two-atom system from the state $|\psi_\pm\rangle$ to the state $|\varepsilon_n\rangle$:

$$\mathcal{R}^\beta_{|\psi_\pm\rangle\to|\varepsilon_n\rangle}=\sum_\xi\sum_{\xi'}\langle\psi_\pm|\mu_\xi^i(0)|\varepsilon_n\rangle\langle\varepsilon_n|\mu_{\xi'}^j(0)|\psi_\pm\rangle\times\int_{-\infty}^{\infty}d\Delta\tau e^{-i\Delta\varepsilon\Delta\tau}G_{ij}^\beta\left(x_\xi(\tau),x_{\xi'}(\tau')\right).\tag{8}$$

So to evaluate the transition rates of the two-atom system, we must first derive the two-point correlation function of the field $G_{ij}^\beta\left(x_\xi(\tau),x_{\xi'}(\tau')\right)$.

Using Eqs. (3) and (4) in Eq. (7), and doing some simplifications, we find

$$G_{ij}^\beta\left(x_\xi(\tau),x_{\xi'}(\tau')\right)=\frac{\delta_{ij}}{6\pi^2}\times\int_0^\infty d\omega\,\omega^3\left[\frac{1}{e^{\beta\omega}-1}e^{i\omega\Delta\tau}+\left(1+\frac{1}{e^{\beta\omega}-1}\right)e^{-i\omega\Delta\tau}\right]\tag{9}$$

for $\xi=\xi'$, and

$$G_{ij}^\beta\left(x_\xi(\tau),x_{\xi'}(\tau')\right)=\frac{(-1)^{i-1}\left(2\delta_{ij}-n_in_j-m_im_j\right)}{4\pi^2L^3}\times\int_0^\infty d\omega\left[\frac{1}{e^{\beta\omega}-1}e^{i\omega\Delta\tau}+\left(1+\frac{1}{e^{\beta\omega}-1}\right)e^{-i\omega\Delta\tau}\right]\times\left\{\left[\left(n_in_j+m_im_j\right)\omega^2L^2-1\right]\sin(\omega L)+\omega L\cos(\omega L)\right\}\tag{10}$$

with $\mathbf{n}=(1,0,0)$ and $\mathbf{m}=(0,1,0)$ for $\xi\neq\xi'$.

Insert the above correlation functions Eqs. (9) and (10) into Eq. (8) and do the integrations, we then obtain the transition rates of the static two-atom system. We find that both downward and upward transitions can take place. Concretely, the downward transition rate is

$$\begin{aligned}\mathcal{R}^\beta_{|\psi_\pm\rangle\to|g_Ag_B\rangle}&=\left(|\boldsymbol{\mu}_A|_{ge}^2+|\boldsymbol{\mu}_B|_{ge}^2\right)\frac{\omega_0^3}{6\pi}\left(1+\frac{1}{e^{\omega_0/T}-1}\right)\\&\pm\Big\{\left[\left(\mu_A^x\right)_{ge}\left(\mu_B^x\right)_{ge}+\left(\mu_A^z\right)_{ge}\left(\mu_B^z\right)_{ge}\right]\\&\times\left[\frac{\omega_0^2L^2-1}{2\pi L^3}\sin(\omega_0L)+\frac{\omega_0}{2\pi L^2}\cos(\omega_0L)\right]\\&+\left(\mu_A^y\right)_{ge}\left(\mu_B^y\right)_{ge}\left[\frac{1}{\pi L^3}\sin(\omega_0L)-\frac{\omega_0}{\pi L^2}\cos(\omega_0L)\right]\Big\}\\&\times\left(1+\frac{1}{e^{\omega_0/T}-1}\right)\end{aligned}\tag{11}$$

with $|\boldsymbol{\mu}_\xi|_{ge}^2=\sum_i|\mu_\xi^i|_{ge}^2$, and the upward transition rate is

$$\begin{aligned}\mathcal{R}^\beta_{|\psi_\pm\rangle\to|e_Ae_B\rangle}&=\left(|\boldsymbol{\mu}_A|_{ge}^2+|\boldsymbol{\mu}_B|_{ge}^2\right)\frac{\omega_0^3}{6\pi}\frac{1}{e^{\omega_0/T}-1}\\&\pm\Big\{\left[\left(\mu_A^x\right)_{ge}\left(\mu_B^x\right)_{ge}+\left(\mu_A^z\right)_{ge}\left(\mu_B^z\right)_{ge}\right]\\&\times\left[\frac{\omega_0^2L^2-1}{2\pi L^3}\sin(\omega_0L)+\frac{\omega_0}{2\pi L^2}\cos(\omega_0L)\right]\\&+\left(\mu_A^y\right)_{ge}\left(\mu_B^y\right)_{ge}\left[\frac{1}{\pi L^3}\sin(\omega_0L)-\frac{\omega_0}{\pi L^2}\cos(\omega_0L)\right]\Big\}\\&\frac{1}{e^{\omega_0/T}-1}.\end{aligned}\tag{12}$$

Hereafter, the "±" on the right of equations correspond to $|\psi_\pm\rangle$, respectively. Obviously, both the downward and upward transition rates are crucially dependent on orientations of the atomic dipole moments, the interatomic separation and the temperature of the thermal bath, and both are composed of a term which is separation-independent and cross-terms which are separation-dependent. The separation-independent term in the downward [upward] transition rate is equal to one half of the transition rates of two atoms initially in their excited [ground] states and individually coupled to the thermal field; the cross-terms are interference terms, which are rooted in the quantum correlation between the two atoms mediated by the field. As compared with their counterparts in vacuum, here the temperature-dependence of the transition rates suggests that the collective transitions of the static two-atom system are modified by the thermal bath.

When $T = 0$, the downward transition rate [Eq. (11)] reduces to

$$\begin{aligned}
&\mathcal{R}_{|\psi_{\pm}\rangle\to|g_A g_B\rangle}\\
&= \left(|\boldsymbol{\mu}_A|_{ge}^2 + |\boldsymbol{\mu}_B|_{ge}^2\right)\frac{\omega_0^3}{6\pi}\\
&\quad \pm\Bigg\{\left[\left(\mu_A^x\right)_{ge}\left(\mu_B^x\right)_{ge} + \left(\mu_A^z\right)_{ge}\left(\mu_B^z\right)_{ge}\right]\\
&\quad \times\left[\frac{\omega_0^2L^2-1}{2\pi L^3}\sin(\omega_0 L) + \frac{\omega_0}{2\pi L^2}\cos(\omega_0 L)\right]\\
&\quad + \left(\mu_A^y\right)_{ge}\left(\mu_B^y\right)_{ge}\left[\frac{1}{\pi L^3}\sin(\omega_0 L) - \frac{\omega_0}{\pi L^2}\cos(\omega_0 L)\right]\Bigg\},
\end{aligned} \tag{13}$$

which is the downward transition rate of a static two-atom system in vacuum; while the upward transition rate [Eq. (12)] vanishes, i.e.,

$$\mathcal{R}_{|\psi_{\pm}\rangle\to|g_A g_B\rangle} = 0, \tag{14}$$

suggesting that upward transition in vacuum can never happen for a static two-atom system in the symmetric or antisymmetric entangled state.

We can see from the above results that thermal effects can obviously modify the transition rates of a static two-atom system in the symmetric or antisymmetric entangled state. Particularly, the upward transition $|\psi_{\pm}\rangle \to |e_A e_B\rangle$ which never happens for a static two-atom system in vacuum takes place for one being static in a thermal bath. This result reminds us of effects of the uniformly accelerated motion, which can also induce the upward transition $|\psi_{\pm}\rangle \to |e_A e_B\rangle$ [30]. Besides, let us also note that the transition rates in the thermal case [Eqs. (11) and (12)] are characterized by the factor $(e^{\omega_0/T} - 1)^{-1}$, and this is in similarity to their counterparts of a two-atom system uniformly accelerated in vacuum, which are characterized by $(e^{\omega_0/T_U} - 1)^{-1}$ [30]. So will the effects of a thermal bath and those of the uniformly accelerated motion be equivalent?

3 Comparisons between thermal effects and effects of the uniformly accelerated motion

As we have mentioned previously, analytical expressions of the transition rates of two atoms uniformly accelerated in vacuum with the acceleration perpendicular to the interatomic separation L [the acceleration case] have already been derived in Ref. [30] [see also Appendix. A]. Comparing the rates in the thermal case [Eqs. (11) and (12)] with those in the acceleration case [Eqs. (A2) and (A3)], we find some important similarities as well as obvious distinctions between the effects of a thermal bath and those of the uniformly accelerated motion.

3.1 Similarities

As we can see from the transition rates in the thermal case [Eqs. (11), (12)] and those in the acceleration case [Eqs. (A2), and (A3)], each of the downward and upward transition rates in both cases is composed of a term independent of the interatomic separation L and other terms crucially dependent on L. To be specific, the L-independent term of the downward [upward] transition rates in both cases equals one half of the sum of downward [upward] transition rates of two atoms initially in their excited [ground] state and individually coupled to the field; the L-dependent terms are characterized by oscillatory functions and they are interference terms, which are resulted from the entanglement of the two atoms mediated by the quantum field. To facilitate the comparisons about concrete behaviors of the transition rates in two cases, we next take two atoms equally and isotropically polarizable for an instance, i.e., $|\boldsymbol{\mu}_A|_{ge}^2 = |\boldsymbol{\mu}_B|_{ge}^2 = |\boldsymbol{\mu}|_{ge}^2$. We display the numerical results in Fig. 1.

The black lines in every subfigure correspond to the symmetric entangled state and the red lines correspond to the antisymmetric entangled state. Figure 1a, b respectively exhibit the separation-dependence of the downward and upward transition rates in the thermal case, while Fig. 1c, d display that in the acceleration case. The black lines show that both the downward and upward transition rates of a system initially in $|\psi_+\rangle$ with $\omega_0 L \ll 1$ in both the thermal and acceleration cases are obviously larger than their counterparts of one with two atoms far apart, indicating that constructive interference is resulted in the small separation limit and this is superradiance. The red lines, however, show that the downward and upward transition rates of a system in $|\psi_-\rangle$ with $\omega_0 L \ll 1$ in both the thermal and acceleration cases are vanishing, which is in sharp contrast to their counterparts of one with two atoms far apart, suggesting that destructive interference is resulted in the small separation limit and this is subradiance. As a result, transitions of the two-atom system in $|\psi_-\rangle$ in the small separation limit are inhibited. Note also that in the thermal case, two solid lines which correspond to $T = 0$ and show up in Fig. 1a never appear in Fig. 1b. Similarly, in the acceleration case, two solid lines which correspond to $T_U = 0$ or equivalently $a = 0$ and show up in Fig. 1c never appear in Fig. 1d. The absence of solid lines in both cases is consistent with the conclusion that the spontaneous excitation $|\psi_\pm\rangle \to |e_A e_B\rangle$ can never happen for a static two-atom system in vacuum.

3.2 Distinctions

Despite similarities, there are some obvious distinctions between the effects of a thermal bath and those of the uniformly accelerated motion. First, analytical expressions of the transition rates in the thermal case [Eqs. (11), (12)] and those in the acceleration case [30] [see also Eqs. (A2) and (A3)] seem to be distinctive, even when the temperature in the thermal case coincides with the Unruh temperature, i.e., $T = T_U$. Particularly, cross-terms, which are related with different components of atomic dipole moments [see those terms in Eqs. (A2) and (A3) characterized by $(\mu_x^A)_{ge}(\mu_y^B)_{ge}$ or $(\mu_x^B)_{ge}(\mu_y^A)_{ge}$], contribute to the transition rates in the acceleration case but do not show up in the thermal case [see Eqs. (11) and (12)]. So if dipole moments of two atoms in the acceleration case are respectively oriented along the interatomic separation and the acceleration, both the downward and upward transition rates are separation-dependent, i.e., coherent radiation happens. However, transition rates of two static atoms in the thermal case with dipole moments respectively oriented along and perpendicular to the interatomic separation are always separation-independent, and the downward [upward] transition rate equals one half of the sum of transition rates of two atoms in their excited [ground] states and individually coupled to the field. As a result, transitions happen as if the two atoms were uncorrelated. Furthermore, analytical expressions of the transition rates in two cases still seem to be distinctive even when we assume $(\mu_x^A)_{ge}(\mu_y^B)_{ge} = (\mu_x^B)_{ge}(\mu_y^A)_{ge}$, which means that cross-terms in the acceleration case are vanishing. These distinctions seem to suggest that the effects of a thermal bath and those of the uniformly accelerated motion on the transitions of a two-atom system in $|\psi_+\rangle$ or $|\psi_-\rangle$ are always nonequivalent. However, as we shall explain, this is not always the case.

Assuming that the temperature of the thermal bath T equals the Unruh temperature T_U, i.e., $T = T_U$, we next display in Fig. 2 the temperature- and separation-dependence of the transition rates in both cases. The blue and orange surfaces in every subfigure correspond to the thermal and the acceleration cases, respectively. One can see that transition rates in the acceleration case with a low acceleration which means a low T_U can be larger or smaller than and even equal to their counterparts in the thermal case, depending on the concrete value of the interatomic separation; however, at a relative high acceleration which means a relative high T_U, the transition rates in the acceleration case are always much larger than their counterparts in the thermal case. This is the second distinction we have found between the effects of a thermal bath and those of the uniformly accelerated motion.

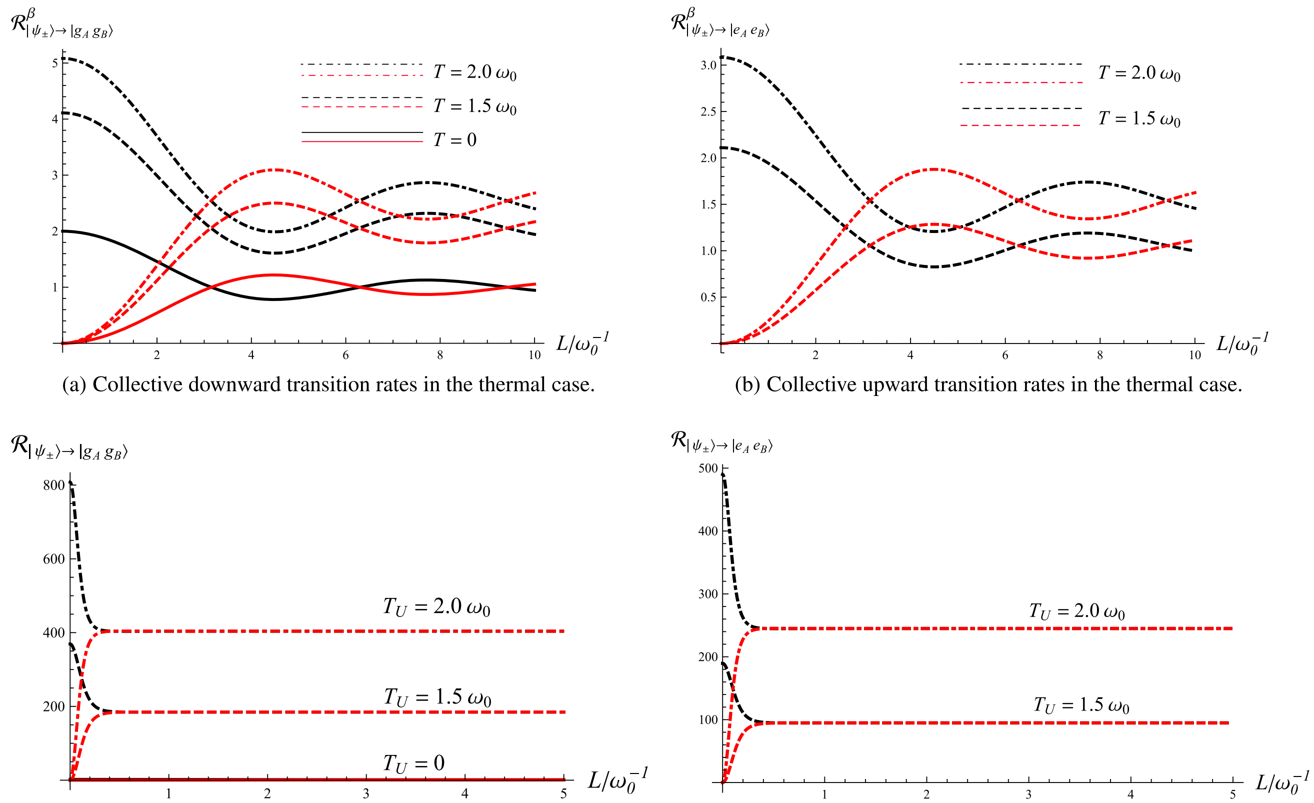
Third, as we can see from Fig. 1a, b, the transition rates in the thermal case always oscillate obviously with the interatomic separation, regardless whether the temperature is low or high. However, as exhibited in Fig. 1c, d, the rates in the acceleration case with a relatively high acceleration varies much more smoothly with the increase of the interatomic separation.

So far, we have demonstrated the important similarities as well as obvious distinctions between the effects of a thermal bath and those of the uniformly accelerated motion on the transition rates of a two-atom system in the symmetric or antisymmetric entangled state, and some comments are now in order. First, the existence of similarities and distinctions between the transition rates in the thermal and acceleration cases can actually be deduced by a comparison between the analytical expressions of the transition rates in two cases. To be specific, let us note that the coefficients f_{ij} and g_{ij} with $i \neq j$ of the oscillatory functions $\sin(\omega_0 L_a)$ with $L_a = \frac{2}{a}\sinh^{-1}(\frac{1}{2}aL)$ and $\cos(\omega_0 L_a)$ of Eqs. (A2) and (A3) [the transition rates in the acceleration case] never appear in the thermal case [see Eqs. (11) and (12)], and they are proportional to a and are thus vanishing when $a \to 0$; while the coefficients f_{ij} and g_{ij} with $i = j$ are similar to their counterparts in the thermal case when $aL \ll 1$ or equivalently $a \ll L^{-1}$. This suggests that the transition rates in the acceleration case may behave similarly as those in the thermal case when $a \ll L^{-1}$, and obvious distinctions between them, as we have shown in this section, show up once this condition is not satisfied. Second, both the downward and upward transitions can happen in both the thermal and acceleration cases, and they respectively result in the decrease and enhancement of energy of a two-atom system being static in a thermal bath or uniformly accelerated in vacuum. Then how about the energy of the two-atom system in two cases? With the transition rates derived, we next consider the average variation rate of energy of the two-atom system in both the thermal and the acceleration cases.

4 Average variation rate of energy

The average variation rate of energy of a two-atom system initially in $|\psi_+\rangle$ or $|\psi_-\rangle$ are related with the transition rates by $\langle\frac{dH}{d\tau}\rangle_\pm = \omega_o\mathcal{R}_{|\psi_\pm\rangle\to|e_Ae_B\rangle} - \omega_o\mathcal{R}_{|\psi_\pm\rangle\to|g_Ag_B\rangle}$, the use of Eqs. (11) and (12) in which then gives rise to

$$\left\langle\frac{dH}{d\tau}\right\rangle_\pm^{th} = -\left(|\boldsymbol{\mu}_A|_{ge}^2 + |\boldsymbol{\mu}_B|_{ge}^2\right)\frac{\omega_0^4}{6\pi}$$
$$\mp\omega_0\left\{\left[\left(\mu_A^x\right)_{ge}\left(\mu_B^x\right)_{ge} + \left(\mu_A^z\right)_{ge}\left(\mu_B^z\right)_{ge}\right]\right.$$



(a) Collective downward transition rates in the thermal case.

(b) Collective upward transition rates in the thermal case.

(c) Collective downward transition rates in the acceleration case.

(d) Collective upward transition rates in the acceleration case.

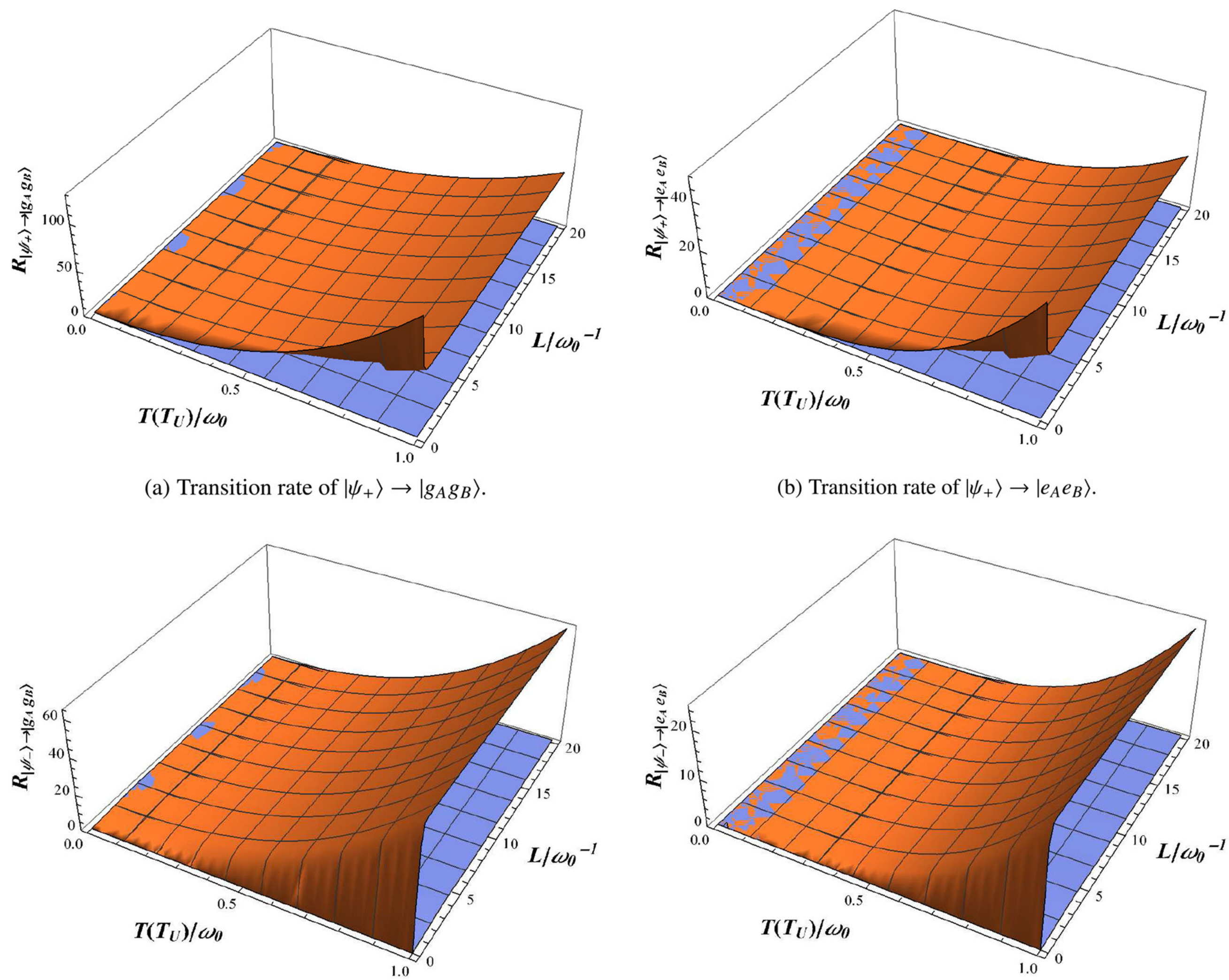
Fig. 1 Separation-dependence of collective transition rates of two atoms in $|\psi_+\rangle$ or $|\psi_-\rangle$. **a**, **b** are respectively the downward and upward transition rates in the thermal case, and **c**, **d** are respectively those in the acceleration case. The atoms are assumed to be equally and isotropically polarizable, the rates are plotted in unit of $\frac{|\boldsymbol{\mu}|^2_{ge}}{3\pi}\omega_0^3$, and "$L/\omega_0^{-1}$" denotes that the interatomic separation is in unit of ω_0^{-1}. The black and red lines correspond to $|\psi_+\rangle$ and $|\psi_-\rangle$, and the solid, dashed and dot-dashed lines correspond to T in the thermal case or T_U in the acceleration case being $0,\ 1.5\omega_0$ and $2.0\omega_0$, respectively

$$\times\left[\frac{\omega_0^2L^2-1}{2\pi L^3}\sin(\omega_0 L)+\frac{\omega_0}{2\pi L^2}\cos(\omega_0 L)\right]$$
$$+\left(\mu_A^y\right)_{ge}\left(\mu_B^y\right)_{ge}\left[\frac{1}{\pi L^3}\sin(\omega_0 L)\right.$$
$$\left.\left.-\frac{\omega_0}{\pi L^2}\cos(\omega_0 L)\right]\right\}, \tag{15}$$

which is also crucially dependent on the orientations of atomic dipole moments and the interatomic separation but not on the temperature of the thermal bath, as a result of perfect cancellation of temperature-dependent terms in both the downward and upward transition rates. Actually, Eq. (15) is accurately the same as its counterpart of one being static in vacuum. Due to this fact, we no longer stress in the following a static two-atom system initially in $|\psi_+\rangle$ or $|\psi_-\rangle$ is in a thermal bath or in vacuum when referring to the average variation rate of its energy.

Noteworthily, Eq. (15) appears to be distinctive from its counterpart of one uniformly accelerated in vacuum in that the latter is not only dependent on the orientations of atomic dipole moments and the interatomic separation but also on the acceleration [or equivalently the Urnuh temperature T_U]. Besides, cross-terms which are characterized by the multiplication of different components of atomic dipole moments and never appear in Eq. (15) contribute in the acceleration case [see Eq. (38) of Ref. [30]]. However, as is shown in the following, *this does not necessarily mean that the variation rate of energy of a two-atom system in two cases are always distinctive*. By contrast, they can be identical!

Figure 3 shows the separation- and (Unruh-)temperature-dependence of the average variation rate of energy of a two-atom system initially in $|\psi_+\rangle$ or $|\psi_-\rangle$ in both the thermal and acceleration cases. Two atoms are still assumed to be equally and isotropically polarizable. The blue and orange surfaces, which respectively correspond to the thermal and acceleration cases, show that the average variation rate of energy of a two-atom system in both cases are negative, meaning that the energy of a two-atom system in both cases always decreases with time. This result is physically understandable: the downward transition rate is always larger than the upward rate [see Eqs. (11) and (12) for the thermal case and Eqs. (A2) and (A3) for the acceleration case], and the down-



(a) Transition rate of $|\psi_+\rangle \to |g_A g_B\rangle$.

(b) Transition rate of $|\psi_+\rangle \to |e_A e_B\rangle$.

(c) Transition rate of $|\psi_-\rangle \to |g_A g_B\rangle$.

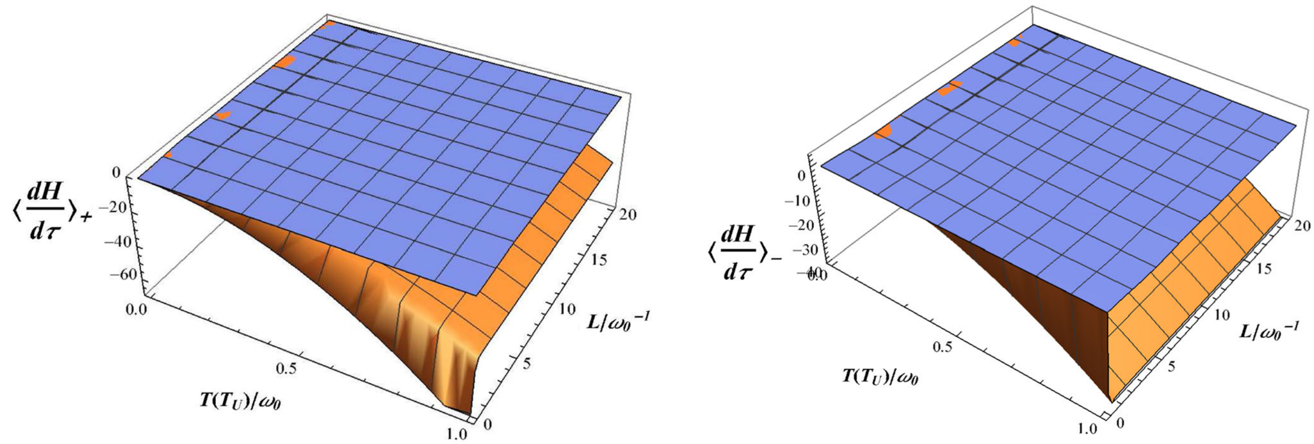
(d) Transition rate of $|\psi_-\rangle \to |e_A e_B\rangle$.

Fig. 2 Separation- and temperature-dependence of the collective transition rates of two atoms in $|\psi_+\rangle$ or $|\psi_-\rangle$. The blue and orange surfaces respectively correspond to a two-atom system being static in a thermal bath and one uniformly accelerated in vacuum. The temperature in the thermal case is set to be equal to the Unruh temperature, the atoms are assumed to be equally and isotropically polarizable, the transition rates are described in unit of $\frac{|\mu|^2_{ge}}{3\pi}\omega_0^3$, and "$T(T_U)/\omega_0$" and "$L/\omega_0^{-1}$" denote that the (Unruh-)temperature and the interatomic separation are in units of ω_0 and ω_0^{-1}, respectively

ward and upward transitions respectively lead to the decrease and increase of energy. Notice also that, at a low (Unruh-)temperature which corresponds to a small acceleration, the average variation rate of energy of the two-atom system in the acceleration case can be smaller or larger than and even equal to its counterpart of one being static in a thermal bath, suggesting that the decrease of energy of a two-atom system in the acceleration case can be slower or quicker than and even the same as its counterpart of two atoms being static [in vacuum or in a thermal bath], depending on the value of the interatomic separation; while at a relative high (Unruh-)temperature which means a relative large acceleration, the average variation rate of energy in the acceleration case is always much larger than its counterpart of a static one, i.e., the energy of a two-atom system in high acceleration always decreases much more quickly than that of a static one (in vacuum or in a thermal bath).

5 Summary

In this paper, we evaluated, with the time-dependent perturbation theory, the transition rates of a static two-atom system in the symmetric or antisymmetric entangled state [$|\psi_+\rangle$ or $|\psi_-\rangle$] and coupled to an electromagnetic field in a thermal bath (the thermal case). By comparing these rates in this ther-



(a) $\langle \frac{dH}{d\tau} \rangle$ of two atoms initially in $|\psi_+\rangle$.

(b) $\langle \frac{dH}{d\tau} \rangle$ of two atoms initially in $|\psi_-\rangle$.

Fig. 3 Separation- and temperature-dependence of the average variation rate of energy of a two-atom system in $|\psi_+\rangle$ or $|\psi_-\rangle$. The atoms are assumed to be equally and isotropically polarizable, the average variation rate of energy is described in unit of $\frac{|\mu|_{ge}^2}{3\pi}\omega_0^4$, and "$T(T_U)/\omega_0$" and "$L/\omega_0^{-1}$" denote that the (Unruh-)temperature and the interatomic separation are respectively in units of ω_0 and ω_0^{-1}. The blue and orange surfaces in each subfigure correspond to the thermal and the acceleration cases, respectively

mal case with their counterparts of one uniformly accelerated in vacuum (the acceleration case), we found some important similarities and obvious distinctions between the effects of a thermal bath and those of the uniformly accelerated motion on the transition properties of the two-atom system.

On one hand, effects of a thermal bath and those of the uniformly accelerated motion are in similarity that they both can induce upward transitions for a two-atom system initially in $|\psi_+\rangle$ or $|\psi_-\rangle$. Particularly, the collective transition rates of two atoms in $|\psi_+\rangle$ in the small separation limit in both cases are abnormally large as compared with those of two atoms far apart, as a result of constructive interference generated by the symmetric correlation between the atoms; while by contrast, the collective transition rates of two atoms in $|\psi_-\rangle$ are vanishing, as a result of destructive interference generated by the antisymmetric correlation between two atoms. On the other hand, important distinctions between the transition rates in two cases also exist: even when we assume the temperature in the thermal case to be equal to the Urunh temperature, the transition rates in two cases can be distinctive; the transition rates of a two-atom system in the thermal case always oscillate obviously with the interatomic separation, no matter at a low or high temperature, while oscillation of the transition rates with respect to the interatomic separation in the acceleration case becomes less obvious when the the acceleration or equivalently the Unruh temperature increases.

We then compared the effects of a thermal bath and those of the accelerated motion on the evolution of energy of a two-atom system in $|\psi_+\rangle$ or $|\psi_-\rangle$. We discovered that the energy of the atom-system in the thermal case decreases at the same speed as that of one in vacuum; while in the acceleration case, the energy of the atom-system in low acceleration which means a low Unruh temperature can decrease slower or more quickly than and even at the same speed as that of a static one (in vacuum or in a thermal bath), depending on the value of the interatomic separation, and in high acceleration which means a high Unruh temperature decreases much more quickly than that of a static one (in vacuum or in a thermal bath).

So as in sharp contrast to the equivalence between the effects of a thermal bath at the Unruh temperature and those of the uniformly accelerated motion in the case of a single atom in interaction with a massless scalar field, the results in the present work concerning the transition properties of a two-atom system in $|\psi_+\rangle$ or $|\psi_-\rangle$ in the electromagnetic field case suggest that the effects of a thermal bath at the Unruh temperature and those of the uniformly accelerated motion are not necessarily always identical or distinctive: they can be distinctive, when the acceleration is relatively large for an instance; they can be equivalent if the (Unruh-)temperature is relatively low (or equivalently the acceleration is relative small), depending on the value of the interatomic separation.

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Appendix A: Transition rates in the acceleration case

In this appendix, we list out the collective transition rates of two atoms, which are initially in $|\psi_+\rangle$ or $|\psi_-\rangle$ and are synchronously and uniformly accelerated in vacuum with the interatomic separation L perpendicular to the proper acceleration a. The trajectories are given by

$$t_\xi(\tau) = \frac{1}{a}\sinh(a\tau), \quad x_\xi(\tau) = \frac{1}{a}\cosh(a\tau),$$
$$y_A = 0,\ \ y_B = L, z_\xi = 0. \tag{A1}$$

Obviously, the separation between two atoms moving along such trajectories remains constant despite that both atoms are in motion.

As a result of the interaction between the atoms and the fluctuating electromagnetic field, both the downward and the upward transitions can happen for the two-atom system, and the rates are respectively [30]

$$\begin{aligned}
&\mathcal{R}_{|\psi_\pm\rangle\to|g_Ag_B\rangle}\\
&= \left(|\boldsymbol{\mu}_A|^2_{ge} + |\boldsymbol{\mu}_B|^2_{ge}\right)\\
&\quad\times\frac{\omega_0^3}{6\pi}\left(1+\frac{a^2}{\omega_0^2}\right)\left(1+\frac{1}{e^{2\pi\omega_0/a}-1}\right)\\
&\quad\pm\Big\{\left(\mu_A^x\right)_{ge}\left(\mu_B^x\right)_{ge}\left[f_{xx}\cos(\omega_0 L_a)+g_{xx}\sin(\omega_0 L_a)\right]\\
&\quad+\left(\mu_A^y\right)_{ge}\left(\mu_B^y\right)_{ge}\left[f_{yy}\cos(\omega_0 L_a)+g_{yy}\sin(\omega_0 L_a)\right]\\
&\quad+\left(\mu_A^z\right)_{ge}\left(\mu_B^z\right)_{ge}\left[f_{zz}\cos(\omega_0 L_a)+g_{zz}\sin(\omega_0 L_a)\right]\\
&\quad+\left[\left(\mu_A^x\right)_{ge}\left(\mu_B^y\right)_{ge}-\left(\mu_A^y\right)_{ge}\left(\mu_B^x\right)_{ge}\right]\left[f_{xy}\cos(\omega_0 L_a)\right.\\
&\quad\left.+g_{xy}\sin(\omega_0 L_a)\right]\Big\}\left(1+\frac{1}{e^{2\pi\omega_0/a}-1}\right)
\end{aligned}\tag{A2}$$

and

$$\begin{aligned}
&\mathcal{R}_{|\psi_\pm\rangle\to|e_Ae_B\rangle}\\
&= \left(|\boldsymbol{\mu}_A|^2_{ge} + |\boldsymbol{\mu}_B|^2_{ge}\right)\times\frac{\omega_0^3}{6\pi}\left(1+\frac{a^2}{\omega_0^2}\right)\\
&\quad\times\frac{1}{e^{2\pi\omega_0/a}-1}\pm\Big\{\left(\mu_A^x\right)_{ge}\left(\mu_B^x\right)_{ge}\left[f_{xx}\cos(\omega_0 L_a)\right.\\
&\quad\left.+g_{xx}\sin(\omega_0 L_a)\right]\\
&\quad+\left(\mu_A^y\right)_{ge}\left(\mu_B^y\right)_{ge}\left[f_{yy}\cos(\omega_0 L_a)+g_{yy}\sin(\omega_0 L_a)\right]\\
&\quad+\left(\mu_A^z\right)_{ge}\left(\mu_B^z\right)_{ge}\left[f_{zz}\cos(\omega_0 L_a)+g_{zz}\sin(\omega_0 L_a)\right]\\
&\quad+\left[\left(\mu_A^x\right)_{ge}\left(\mu_B^y\right)_{ge}-\left(\mu_A^y\right)_{ge}\left(\mu_B^x\right)_{ge}\right]\left[f_{xy}\cos(\omega_0 L_a)\right.\\
&\quad\left.+g_{xy}\sin(\omega_0 L_a)\right]\Big\}\frac{1}{e^{2\pi\omega_0/a}-1},
\end{aligned}\tag{A3}$$

where $L_a = \frac{2}{a}\sinh^{-1}(\frac{aL}{2})$,

$$\begin{cases}
f_{xx} = \frac{\omega_0(1+a^2L^2)}{2\pi L^2\mathcal{N}^2(a,L)},\\
f_{yy} = -\frac{\omega_0\left(1+\frac{1}{8}a^2L^2+\frac{1}{16}a^4L^4\right)}{\pi L^2\mathcal{N}^2(a,L)},\\
f_{zz} = \frac{\omega_0\left(1+\frac{1}{2}a^2L^2\right)}{2\pi L^2\mathcal{N}(a,L)},\\
f_{xy} = -f_{yx} = -\frac{a\omega_0\left(1-\frac{1}{2}a^2L^2\right)}{4\pi L\mathcal{N}^2(a,L)},
\end{cases}$$
$$\begin{cases}
g_{xx} = -\frac{1+\frac{1}{2}a^2L^2+\frac{1}{4}a^4L^4-L^2\omega_0^2\left(1+\frac{1}{4}a^2L^2\right)}{2\pi L^3\mathcal{N}^{5/2}(a,L)},\\
g_{yy} = \frac{1+\frac{5}{8}a^2L^2-\frac{1}{8}\omega_0^2a^2L^4\left(1+\frac{1}{4}a^2L^2\right)}{\pi L^3\mathcal{N}^{5/2}(a,L)},\\
g_{zz} = -\frac{1-\omega_0^2L^2\left(1+\frac{1}{4}a^2L^2\right)}{2\pi L^3\mathcal{N}^{3/2}(a,L)},\\
g_{xy} = -g_{yx} = \frac{a\left(1+a^2L^2+\omega_0^2L^2\left(1+\frac{1}{4}a^2L^2\right)\right)}{4\pi L^2\mathcal{N}^{5/2}(a,L)}.
\end{cases}\tag{A4}$$

and $\mathcal{N}(a, L) = 1 + \frac{1}{4}a^2L^2$.

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