



Scale-separated $\text{AdS}_3 \times S^1$ vacua from IIA orientifolds

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Abstract We study supersymmetric AdS_3 flux vacua of massive type-IIA supergravity on anisotropic G_2 orientifolds. Depending on the value of the F_4 flux the seven-dimensional compact space can either have six small and one large dimension such that the “external” space is scale-separated and effectively four-dimensional, or all seven compact dimensions small and parametrically scale-separated from the three external ones. Within this setup we also discuss the Distance Conjecture (including appropriate D4-branes), and highlight that such vacua provide a non-trivial example of the so-called strong Spin-2 Conjecture.

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1 Introduction

Deducing the existence of an effectively lower-dimensional theory of gravity from ten-dimensional (or eleven-dimensional) string/M-theory is both pivotal for realistic model building and also presents an interesting theoretical question on its own right. In particular, supersymmetric AdS vacua with scale separation are interesting for a variety of reasons: for example, in some cases such vacua can be uplifted to de Sitter [1], while in principle they are also of interest in holography.

The status of scale-separated AdS remains however unclear, even though one would expect supersymmetry to provide an advantage. For instance, the classical constructions with scale separation that appeared in [2] (see also [3–7] for closely related work) invoke type-IIA supergravity with Romans mass and (smeared) O6-planes, the latter possibly being an essential ingredient of scale separation in any case [8–10]. The consistency of each of these ingredients or their combination has also been under debate [11–13], while various steps in resolving or at least understanding some of the intricacies have been taken in [14–29]. More recently, an analysis of the holographic duals was performed in [30–35], and some extensions with anisotropies are discussed in [36,37], where also massive type-IIA compactifications on toroidal orbifolds are presented and the possibility to have a T-dual orientifold description within massless type-IIA is highlighted. Controlled type-IIB vacua with scale separation are not known classically [38,39], and the only known examples are the ones that require quantum corrections [1] (and have a series of open issues [40–43]), while there seems to be a scaling argument behind this difficulty [44]. Arguments from supergravity also indicate the difficulty in constructing scale-separated AdS with extended supersymmetry [45–48].¹ In the mean time AdS_3 constructions with mini-

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¹ See also [49,50] for restrictions on scale separation in higher dimensional AdS spaces.

mal supersymmetry and scale separation (but still with the same ingredients) have appeared in the literature [51–54].² For a pedagogical and up-to-date review of the status of flux vacua with moduli stabilization and scale separation see e.g. [58, 59].

In this work we will assume that smeared O6-planes are a consistent ingredient of flux compactifications and that there does exist an actual solution in string theory that is described in the low energy regime by such an approximation. In our constructions we will therefore build on the G2 orientifolds of [51] which originally give rise to scale-separated AdS₃ vacua with a seven-dimensional (almost fully) isotropic compact internal space. However, our aim is to create a large anisotropy in the internal space such that: 1. Six of the internal dimensions, always maintaining large volume (to have a good supergravity approximation), become considerably small with respect to the typical length scale of the non-compact three-dimensional (AdS₃) external space, L_{AdS} ; 2. Only one becomes large and comparable in magnitude to the same L_{AdS} . In this way the actual external space is four-dimensional whereas the six-dimensional internal one is scale-separated (not parametrically though) from the external one. When instead the F_4 flux is made parametrically large, one recovers a seven-dimensional small internal compact space, albeit anisotropic, together with the AdS₃ external space.

2 Massive type-IIA on G2 orientifolds

We will work within the general setup discussed in [51, 54]; therefore, here, we only need to mention the salient features of such flux compactifications and establish our conventions. (Let us also note right away that we fix $\alpha' = 1$).

The setup is massive type-IIA supergravity compactified on a G2 space, which we take to be a toroidal orbifold $X_7 = T^7/(Z_2)^3$ with coordinates $\{y_i\}_{i=1,\dots,7}$. The Z_2 involutions generate a group $\Gamma = \{\Theta_\alpha, \Theta_\beta, \Theta_\gamma\}$, and are defined as

$$\begin{aligned} \Theta_\alpha &: (y^1, \dots, y^7) \rightarrow (-y^1, -y^2, -y^3, -y^4, y^5, y^6, y^7), \\ \Theta_\beta &: (y^1, \dots, y^7) \rightarrow (-y^1, -y^2, y^3, y^4, -y^5, -y^6, y^7), \\ \Theta_\gamma &: (y^1, \dots, y^7) \rightarrow (-y^1, y^2, -y^3, y^4, -y^5, y^6, -y^7); \end{aligned} \tag{2.1}$$

one should also include in Γ all their combinations, e.g. $\Theta_{\alpha\beta} = \Theta_\alpha\Theta_\beta$, etc.. Since the compact space is seven-dimensional, a three-dimensional non-compact space is left, which in [51, 54] is AdS₃. Eventually the ten-dimensional Einstein-frame metric involved in the compactification has

² Recent work on type-II AdS₃ flux vacua with minimal or extended supersymmetry but without scale separation can be found, for example, in [55–57].

the form

$$ds_{10}^2 = (\text{Vol}(X_7))^{-2} ds_3^2 + \sum_{i=1}^7 r_i^2 dy_i^2, \quad i = 1, \dots, 7, \tag{2.2}$$

and $\text{Vol}(X_7) = \prod_{i=1}^7 r^i$. Such G2 compactification would in principle preserve four real Killing spinors,³ but, because one also includes mutually supersymmetric O2/O6-planes in these constructions, the number of supersymmetry is further reduced by half. The O2-planes fill the non-compact space, whereas the O6-planes have three dimensions that fill the non-compact space while the remaining four wrap 4-cycles in the seven-dimensional compact space as follows:

$$\begin{pmatrix} \text{O6}_\alpha & : & \times & \times & \times & \times & - & - & - \\ \text{O6}_\beta & : & \times & \times & - & - & \times & \times & - \\ \text{O6}_\gamma & : & \times & - & \times & - & \times & - & \times \\ \text{O6}_{\alpha\beta} & : & - & - & \times & \times & \times & \times & - \\ \text{O6}_{\beta\gamma} & : & - & \times & \times & - & - & \times & \times \\ \text{O6}_{\gamma\alpha} & : & - & \times & - & \times & \times & - & \times \\ \text{O6}_{\alpha\beta\gamma} & : & \times & - & - & \times & - & \times & \times \end{pmatrix}, \tag{2.3}$$

where the “ \times ” indicates a direction parallel to the given O6-plane and the “ $-$ ” a direction orthogonal to it. The O6-planes would be localized in a full string theory solution, but here we consider them to be smeared along those directions. The full construction works nicely making the O6-planes actually the images of the O2-planes under the Z_2 involutions.

Due to the various ingredients the only fluxes that can have non-zero background values are the F_4 flux, the H_3 flux and the Romans mass F_0 , which are properly expanded on the basis of harmonic forms with the correct parities. The harmonic three-forms

$$\Phi_i = \{dy^{127}, -dy^{347}, -dy^{567}, dy^{136}, -dy^{235}, dy^{145}, dy^{246}\} \tag{2.4}$$

provide a proper basis for the H_3 background flux and the harmonic four-forms Ψ_i , which are defined by

$$\int_{X_7} \Phi_i \wedge \Psi_j = \delta_{ij}, \tag{2.5}$$

provide the proper basis for the F_4 flux. In the mean time the only allowed moduli are the dilaton and the seven radii r^i of the toroidal orbifold or, equivalently, the volumes of the seven 3-cycles Φ_i , which we will indicate as s^i and are defined as

$$\begin{aligned} s^1 &= r^1 r^2 r^7, s^2 = r^3 r^4 r^7, s^3 = r^5 r^6 r^7, s^4 = r^1 r^3 r^6, \\ s^5 &= r^2 r^3 r^5, s^6 = r^1 r^4 r^5, s^7 = r^2 r^4 r^6. \end{aligned} \tag{2.6}$$

³ More concretely, one can think of this in terms of gravitini such that the counting is valid also when supersymmetry is spontaneously broken.

All the other closed string scalars are automatically truncated due to the orbifold/orientifold action. We ignore the open string moduli here: we assume that, being compact, they will be stabilized independently at their supersymmetric positions.

In our analysis it is favorable to work with the moduli x , y (not to be confused with the y^i coordinates of the internal space) and \tilde{s}^a (with $a = 1, \dots, 6$), which have the following properties: the moduli x and y are a combination of the overall compact space volume $\text{Vol}(X_7)$ and the dilaton ϕ , namely

$$\begin{aligned} x &= -\frac{3\sqrt{7}}{8}\phi + \frac{1}{2\sqrt{7}}\log[\text{Vol}(X_7)], \\ y &= -\frac{3}{2}\log[\text{Vol}(X_7)] - \frac{1}{8}\phi. \end{aligned} \tag{2.7}$$

Then, since we treat the volume $\text{Vol}(X_7)$ as an independent modulus, one of the seven moduli s^i is made redundant and can be eliminated by working with the six \tilde{s}^a scalars that are deduced from the relations

$$\begin{aligned} s^i &= \text{Vol}(X_7)^{3/7}\tilde{s}^i \quad \text{with } i = 1, \dots, 7, \\ \tilde{s}^7 &= \prod_{a=1}^6 \frac{1}{\tilde{s}^a} \quad \text{with } a = 1, \dots, 6. \end{aligned} \tag{2.8}$$

To recap, we have six independent ‘‘shape’’ moduli \tilde{s}^a (with $a = 1, \dots, 6$), while a seventh shape modulus, \tilde{s}^7 , is given in terms of the other six.

Having discussed the moduli, let us turn to the tadpole conditions. The contribution of the O2-planes to the tadpole will be cancelled by appropriately distributed D2-branes, whereas the O6-plane contribution will be cancelled by fluxes. Therefore, from the Bianchi identity $dF_6 = 0$ we have

$$0 = \int_{X_7} H_3 \wedge F_4 \tag{2.9}$$

due to vanishing net D2 charge. When integrating over each i th 3-cycle, the Bianchi identity $dF_2 = 0$ is given by

$$0 = \int_i H_3 \wedge F_0 - 2\pi \times 16, \tag{2.10}$$

where we have taken into account that $N_{O6} = 2^3$ per 3-cycle. This setup has been analyzed extensively in [51,54]: here, we refrain from getting into more details and refer the reader directly to the aforementioned articles.

With these ingredients the direct dimensional reduction of the ten-dimensional Einstein-frame action down to three dimensions produces the bosonic sector

$$\begin{aligned} \frac{\mathcal{L}_3}{\sqrt{-\det[g_{\mu\nu}^{(3)}]}} &= \frac{1}{2}R_3 - \frac{1}{4}(\partial x)^2 - \frac{1}{4}(\partial y)^2 - \frac{1}{4} \\ &\times \sum_{i=1}^7 \frac{1}{(\tilde{s}^i)^2} \partial \tilde{s}^i \partial \tilde{s}^i - V(x, y, \tilde{s}), \end{aligned} \tag{2.11}$$

where $V(x, y, \tilde{s})$ indicates the three-dimensional scalar potential. The latter can also be described by a superpotential that can be found in [51] and will be presented momentarily. When deducing the metric of the moduli space of (2.11) one should not forget the restrictions (2.8).

3 AdS₃ × S¹ scale-separated vacua

3.1 Supersymmetric conditions

In this section, always referring to the base of harmonic forms, we work with the specific flux choices

$$\begin{aligned} H_3 &= h \sum_{i=1}^7 \Phi_i, \quad F_4 = f (\Psi_1 + \Psi_2 + \Psi_3 - 3\Psi_4) \\ &\quad + q (\Psi_5 + \Psi_6 - 2\Psi_7), \end{aligned} \tag{3.1}$$

such that $H_3 \wedge F_4 \equiv 0$ and the O2-tadpole cancels indeed with D2-branes. We also include a Romans mass $F_0 = m$. Let us note that the parameters f and q can be made arbitrarily large and are not bounded by the tadpoles. Moreover, as far as our analysis is concerned, we are going to assume, consistently both with the tadpole cancellation condition and flux quantization, that $f, q > 0$ and that $m, h < 0$. This choice of signs for the fluxes is forced upon us, among other reasons, due to moduli stabilization. We could also have $f, q < 0$ and $m, h > 0$, which is the same setup, but we could not achieve moduli stabilization in our numerical examples if we assumed $m > 0$ and $h < 0$ (or vice-versa).

In order to achieve scale separation in the large volume and weak coupling regime we are going to use the large f limit, while we are going to take q large to make the seventh radius, i.e. r_7 , considerably bigger than the remaining compact dimensions. In that case four dimensions will be large and external (in the sense that the r_7 KK-modes can not be ignored for a consistent low-energy EFT) while the other six will be made considerably smaller with a clear separation of the corresponding scales.

The flux quantization conditions, for $\alpha' = 1$, are

$$\begin{aligned} h &= (2\pi)^2 K, \quad m = (2\pi)^{-1} M, \quad f = (2\pi)^3 N, \\ q &= (2\pi)^3 Q, \end{aligned} \tag{3.2}$$

and from the cancellation of the O6-tadpole without any D6-branes we simply have $KM = 16$. As we noted earlier, in our setup we have $m, h < 0$, which means that $K, M < 0$; we will therefore sometimes denote $-K = |K|$ and $-M = |M|$.

For this setup the three-dimensional scalar potential can be written in terms of a real superpotential P that was derived in [51]. Furthermore, in [53] it was explicitly verified that minimizing this superpotential corresponds to solving the full ten-dimensional supersymmetry equations in the bispinor formalism. In terms of the \tilde{s}^a moduli (with $a = 1, \dots, 6$)

the superpotential is

$$\begin{aligned} \frac{P}{(2\pi)^7} &= \frac{m}{8} \exp\left[\frac{y}{2} - \frac{\sqrt{7}x}{2}\right] + \frac{h}{8} \exp\left[y + \frac{x}{\sqrt{7}}\right] \\ &\times \left(\sum_{a=1}^6 \frac{1}{\tilde{s}^a} + \prod_{a=1}^6 \tilde{s}^a\right) \\ &+ \frac{1}{8} \exp\left[y - \frac{x}{\sqrt{7}}\right] \left[f(\tilde{s}^1 + \tilde{s}^2 + \tilde{s}^3 - 3\tilde{s}^4)\right. \\ &\left. + q\left(\tilde{s}^5 + \tilde{s}^6 - 2\prod_{a=1}^6 \frac{1}{\tilde{s}^a}\right)\right]. \end{aligned} \tag{3.3}$$

Having in mind supersymmetric critical configurations, let us observe that, since on a supersymmetric vacuum the vacuum energy is $\langle V \rangle = -4P^2$, the AdS₃ length scale is characterized by P as $L_{\text{AdS}} = \frac{1}{\sqrt{|V|}} \sim P^{-1}$ (for a constant three-dimensional Planck mass, which here, and from now on, we set to be unit).

A supersymmetric vacuum requires to have $P_x = P_y = P_a = 0$. Once we perform the variations, we implement an ansatz for the various shape moduli \tilde{s}^a , each one associated with the volume of a corresponding 3-cycle, of the form

$$\langle \tilde{s}^1 \rangle = \langle \tilde{s}^2 \rangle = \langle \tilde{s}^3 \rangle = \sigma, \quad \langle \tilde{s}^4 \rangle = \rho, \quad \langle \tilde{s}^5 \rangle = \langle \tilde{s}^6 \rangle = \tau. \tag{3.4}$$

We further define

$$\begin{aligned} \mu &= \frac{m}{f} \exp\left[-\frac{y_0}{2} - \frac{5x_0}{2\sqrt{7}}\right], \quad \chi = \frac{h}{f} \exp\left[\frac{2x_0}{\sqrt{7}}\right], \\ \gamma &= \frac{q}{f}, \end{aligned} \tag{3.5}$$

simply denoting $\langle x \rangle = x_0$ and $\langle y \rangle = y_0$. Let us furthermore remind the reader that $\langle \tilde{s}^7 \rangle = 1/\prod_{a=1}^6 \langle \tilde{s}^a \rangle$. Then, with the above definitions the supersymmetric equations lead to the following independent set of conditions:

$$\begin{aligned} 0 &= -\frac{7\mu}{2} - 3\sigma - 2\gamma\tau + \frac{3\chi}{\sigma} + \frac{2\chi}{\tau} + \frac{\chi + \frac{2\gamma}{\sigma^3\tau^2}}{\rho} \\ &\quad + \rho(\sigma^3\tau^2\chi + 3), \\ 0 &= \frac{\mu}{2} + 3\sigma + 2\gamma\tau + \frac{3\chi}{\sigma} + \frac{2\chi}{\tau} + \frac{\chi - \frac{2\gamma}{\sigma^3\tau^2}}{\rho} \\ &\quad + \rho(\sigma^3\tau^2\chi - 3), \end{aligned} \tag{3.6}$$

and

$$\begin{aligned} 0 &= 1 + \frac{2\gamma}{\rho\sigma^4\tau^2} - \frac{\chi}{\sigma^2} + \rho\sigma^2\tau^2\chi, \\ 0 &= -3 + \frac{2\gamma}{\rho^2\sigma^3\tau^2} - \frac{\chi}{\rho^2} + \sigma^3\tau^2\chi, \\ 0 &= \gamma + \frac{2\gamma}{\rho\sigma^3\tau^3} - \frac{\chi}{\tau^2} + \rho\sigma^3\tau\chi. \end{aligned} \tag{3.7}$$

Once the values of the various parameters have been determined, we can extract

$$\begin{aligned} x_0 &= \frac{\sqrt{7}}{2} \log\left[\frac{f\chi}{h}\right], \\ y_0 &= 2 \log\left[\frac{m}{f\mu}\right] - \frac{5}{2} \log\left[\frac{f\chi}{h}\right], \end{aligned} \tag{3.8}$$

which in turn give the dilaton ϕ and the volume $\text{Vol}(X_7)$ using (2.7). In particular,

$$\begin{aligned} g_s &= e^{\langle\phi\rangle} = \frac{h}{f^{3/4}\chi} \left(\frac{\mu}{m}\right)^{1/4}, \\ \text{Vol}(X_7) &= f^{49/16} \left(\frac{\chi}{h}\right)^{7/4} \left(\frac{\mu}{m}\right)^{21/16}. \end{aligned} \tag{3.9}$$

It is clear from the above expressions that, when f is large enough, the volume can be made large and the coupling can be made weak. This happens because μ and χ are actually fully determined independently from the value of f as we will show momentarily in the next paragraph by analytically solving $P_x = 0 = P_y$. (In our examples we explicitly verify this independence). Of course, one has to check the radii one-by-one and make sure that each one is independently large, in case the space is very anisotropic. We will do this in a while; for the moment, let us simply note that the actual volumes of the 3-cycles are given by $s^i = \text{Vol}(X_7)^{3/7} \tilde{s}^i$.

We can solve the two equations of (3.6), which actually correspond to $P_x = 0 = P_y$, to deduce

$$\begin{aligned} \mu &= \frac{3(\rho - \sigma)}{2} + \frac{\gamma}{\rho\sigma^3\tau^2} - \gamma\tau, \\ \chi &= \frac{9\rho(\rho - \sigma)\sigma^3\tau^2 + 6\gamma(1 - \rho\sigma^3\tau^3)}{4\sigma^2\tau(2\rho\sigma + 3\rho\tau + \sigma\tau + \rho^2\sigma^4\tau^3)}. \end{aligned} \tag{3.10}$$

Substituting then (3.10) into (3.7) will give us three equations with four unknowns, i.e. $(\sigma, \rho, \tau, \gamma)$: as a result, we have one free parameter which can be conveniently chosen to be γ as it controls the relative anisotropy within the F_4 flux components. It is precisely this freedom that will allow us to break scale separation in one compact dimension and thus make the external/large space effectively four-dimensional. The three equations that have to be solved are then

$$\begin{aligned} 0 &= 1 + \frac{2\gamma}{\rho\sigma^4\tau^2} \\ &\quad + \frac{3(\rho\sigma^4\tau^2 - 1)[3\rho\sigma^3\tau^2(\rho - \sigma) + 2\gamma(1 - \rho\sigma^3\tau^3)]}{4\sigma^4\tau(2\rho\sigma + 3\rho\tau + \sigma\tau + \rho^2\sigma^4\tau^3)}, \\ 0 &= -3 + \frac{2\gamma}{\rho^2\sigma^3\tau^2} \\ &\quad + \frac{3\left(\sigma^3\tau^2 - \frac{1}{\rho^2}\right)[3\rho\sigma^3\tau^2(\rho - \sigma) + 2\gamma(1 - \rho\sigma^3\tau^3)]}{4\sigma^2\tau(2\rho\sigma + 3\rho\tau + \sigma\tau + \rho^2\sigma^4\tau^3)}, \\ 0 &= \gamma + \frac{2\gamma}{\rho\sigma^3\tau^3} \end{aligned}$$

$$+ \frac{3(\rho\sigma^3\tau^3 - 1)[3\rho\sigma^3\tau^2(\rho - \sigma) + 2\gamma(1 - \rho\sigma^3\tau^3)]}{4\sigma^2\tau^3(2\rho\sigma + 3\rho\tau + \sigma\tau + \rho^2\sigma^4\tau^3)}. \tag{3.11}$$

Ideally we would like to solve the equations in (3.11) analytically and express the solutions as functions of γ , that is $\sigma = \sigma(\gamma)$, $\rho = \rho(\gamma)$ and $\tau = \tau(\gamma)$, and further deduce from (3.10) $\mu = \mu(\gamma)$ and $\chi = \chi(\gamma)$. However, due to the highly nonlinear polynomial structure of (3.11) it does not seem possible to directly find its analytic solutions; therefore, from now on we will turn to numerical methods.

3.2 Proceeding numerically

As we already wrote, our aim is to achieve a large volume and weak coupling regime where the seventh radius r_7 is made large, namely of the same order of magnitude of the AdS₃ radius (or even bigger), and the remaining six radii r_a remain small. The comparison between the relevant KK-mode scales associated with each of the radii r_i and the AdS length scale L_{AdS} is made by a formula that is derived, for example, in [54]: for the lowest-lying KK-mode of a tower of states related to some radius r_i ,

$$m_{\text{KK},i}^2 = \frac{(2\pi)^2}{(\text{Vol}(X_7))^2 r_i^2} \times \left(\frac{(2\pi)^7}{2}\right)^2 \tag{3.12}$$

and, as a consequence,

$$\frac{L_{\text{KK},i}^2}{L_{\text{AdS}}^2} = \frac{|\langle V \rangle|}{m_{\text{KK},i}^2} = \frac{4}{\pi^2} \left(\frac{P}{(2\pi)^7}\right)^2 (\text{Vol}(X_7))^2 r_i^2, \tag{3.13}$$

where $|\langle V \rangle| = 1/L_{\text{AdS}}^2$. As we are going to see exploring what happens for different values of γ and f , it is indeed possible to achieve

$$\frac{L_{\text{KK},a}^2}{L_{\text{AdS}}^2} \rightarrow 0, \quad \frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2} \gtrsim 1, \quad g_s \ll 1, \quad r_i \gg 1, \tag{3.14}$$

so that the large external space becomes four-dimensional (again in the sense that the r_7 KK-modes cannot be ignored for a consistent EFT) and the small hidden space six-dimensional. In such cases the three-dimensional theory is not a good EFT and should be considered only as a consistent truncation; however, the vacuum equations are still valid because they are deduced from the superpotential (3.3), which also captures the full ten-dimensional supersymmetry conditions/vacuum equations as it is shown in [53].

We would like to emphasize that the anisotropy between the radius r_7 and the other six radii r_a in our constructions is not parametric. This happens because such anisotropy is controlled by the parameter γ that is consistent with moduli stabilization only for a finite range of values. Nevertheless, as we will see, within those values we could find quite a large anisotropy. It is not clear of course whether there could be

another choice of fluxes that can make the anisotropy which singles out r_7 parametric. (Such a setup does exist for the case of parametric scale separation between $r_{2,4,6}$ and the rest of the radii; it was studied in [54]).

We would also like to stress that, once r_7 is made of the same order of magnitude of L_{AdS} , the large external space is not AdS₄, but AdS₃ × S¹, also underlying the fact that we have a supersymmetric vacuum with two (and not four) Killing spinors.

3.2.1 Having $\gamma \sim \mathcal{O}(1)$ and (almost) no anisotropy

For values of $\gamma \sim \mathcal{O}(1)$ it is easy to check that (almost) no anisotropy is generated between the seven radii r_i : the system therefore behaves essentially like in [51]. Explicitly, solving the equations in (3.11) for $\gamma = 1$, we find

$$\sigma \approx 1.95, \quad \rho \approx 0.18, \quad \tau \approx 1.95. \tag{3.15}$$

Then, from (3.9) with the proper numerical values of μ and χ evaluated from (3.10) we can extract the behaviour of the string coupling

$$g_s \approx 2.897 \times \frac{|K|}{N^{3/4}|M|^{1/4}}, \tag{3.16}$$

and the behaviour of the radii

$$\begin{aligned} \frac{r_{1,3}}{\frac{2\pi N^{7/16}}{|M|^{3/16}|K|^{1/4}}} &\approx 0.93, & \frac{r_{2,4}}{\frac{2\pi N^{7/16}}{|M|^{3/16}|K|^{1/4}}} &\approx 0.95, \\ \frac{r_5}{\frac{2\pi N^{7/16}}{|M|^{3/16}|K|^{1/4}}} &\approx 3.04, & \frac{r_6}{\frac{2\pi N^{7/16}}{|M|^{3/16}|K|^{1/4}}} &\approx 0.29, \end{aligned} \tag{3.17}$$

and

$$\frac{r_7}{\frac{2\pi N^{7/16}}{|M|^{3/16}|K|^{1/4}}} \approx 3.04. \tag{3.18}$$

By appropriately choosing the flux units we can end up having large internal volume $\text{Vol}(X_7)$ (it is sufficient to take $N^{7/16}|M|^{-3/16}|K|^{-1/4} \gg 1$) and a small coupling g_s . We also clearly notice that all the seven radii r_i are comparable in magnitude and therefore (almost) no anisotropy is generated yet.

In any case, since

$$\frac{L_{\text{KK},i}^2}{L_{\text{AdS}}^2} \approx 0.21 \times \frac{K^2|M|}{N}, \tag{3.19}$$

for parametrically large N we can get parametric scale separation between the AdS₃ external space and the seven-dimensional internal one. For example, consistently with the tadpoles, one can have $K = -16$, $M = -1$ and $N = 10^4$, or any other parametrically large value of N . The fact that for parametrically large N we recover AdS₃ as external space is a general feature of the construction we are presenting here.

Table 1 This table presents the critical values of σ , ρ and τ as γ takes the values $\gamma = 10^3, 10^6$

γ	σ	ρ	τ
10^3	93.12	0.02	0.06
10^6	4823.5	0.0012	0.0032

3.2.2 Intermediate γ values

To try to detach the seventh radius r_7 from the other six internal radii r_a and make the former comparable in magnitude to the AdS length scale L_{AdS} , we will now increase γ and consider the cases $\gamma \sim \mathcal{O}(10^3)$ and $\gamma \sim \mathcal{O}(10^6)$. This choice opens a window of values of N within which the above scenario can be realized. (When γ becomes smaller than $\mathcal{O}(1)$, then r_5 is detached; we analyze this in the appendix).

For the aforementioned values of γ the equations in (3.11), which determine the properties of the solution and verify moduli stabilization, are solved for the values of σ , ρ and τ that we present in Table 1. For such values, from (3.9) and with the proper μ and χ evaluated thanks to (3.10), we can consequently estimate g_s to be

$$g_s \approx 1.92 \times \frac{|K|}{N^{3/4}|M|^{1/4}} \tag{3.20}$$

(which is roughly the same for both the values of γ under consideration). In addition, the radii take the values that we gather in Table 2. From (3.20) and Table 2 we readily see that the large volume (i.e. large radii) and weak coupling regime can be realized by adopting appropriate choices of the flux units, e.g. by choosing (always in accordance with the tadpole cancellation condition) $K = -16$, $M = -1$ and N sufficiently large.

As already emphasized, the arbitrarily parametrically large N limit always leads to full scale separation. At this stage, however, we would like to understand if for some moderate (but still sufficiently large) value of N only six of the compact dimensions can remain small while the seventh one can become of the same order of magnitude of the AdS₃ length scale (or even larger). This means that we want to investigate whether

$$\begin{aligned} \frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2} &= \frac{4}{\pi^2} \left(\frac{P}{(2\pi)^7} \right)^2 (\text{Vol}(X_7))^2 r_7^2 \gtrsim 1, \\ \frac{L_{\text{KK},a}^2}{L_{\text{AdS}}^2} &\ll 1. \end{aligned} \tag{3.21}$$

More precisely, we have the ratios

$$\frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2} \Big|_{\gamma=10^3} \approx 6.418 \times 10^4 \times \frac{K^2|M|}{N},$$

$$\frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2} \Big|_{\gamma=10^6} \approx 6.412 \times 10^{10} \times \frac{K^2|M|}{N}, \tag{3.22}$$

which offer a wide range of values of N that can be checked. Indeed, when (once more) fixing $K = -16$ and $M = -1$, we find, for $\gamma = 10^3$, the results that are presented in Table 3, and, for $\gamma = 10^6$, we find the values that are reported in Table 4, together with the relative behaviour of the string coupling and the other radii. We observe that, as γ increases, the possibility to decouple the seventh radius r_7 from the other six radii r_a becomes more and more achievable, with a simultaneous extension of the range of the values of N (once K and M have been fixed) we can refer to.

3.2.3 Large γ values and maximal anisotropy

In this subsection we will work with $\gamma \sim \mathcal{O}(10^9)$. This is roughly the largest value that γ can have while keeping moduli stabilization intact. Indeed, from our numerics we see that $\gamma \approx 2.5305 \times 10^9$ appears to (approximately) be the limiting value that γ can take before solutions to the extremization of (3.3) with the ansatz (3.4) cease to exist.

Solving the equations in (3.11) we get the results presented in Table 5, together with

$$g_s \approx 1.92 \times \frac{|K|}{N^{3/4}|M|^{1/4}}, \tag{3.23}$$

and the values presented in Table 6 as far as the radii r_i are concerned. We can also check that a consistent anisotropy, where only six of the compact dimensions remain small while the seventh one becomes of the same order of magnitude of the AdS₃ length scale (or even larger) can be achieved. Focusing, as a matter of illustration, on $\gamma = 10^9$, we search for a realization of

$$\begin{aligned} \frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2} &\approx 6.412 \times 10^{16} \times \frac{K^2|M|}{N} \gtrsim 1, \\ \frac{L_{\text{KK},a}^2}{L_{\text{AdS}}^2} &\approx 10^{-3} \times \frac{K^2|M|}{N} \ll 1. \end{aligned} \tag{3.24}$$

We also choose to keep the same K and M units of flux as before, i.e.

$$K = -16, \quad M = -1, \tag{3.25}$$

so that the tadpole cancellation condition is satisfied and we can simply vary the value of N .

In particular, if we take $N \sim 10^{19}$, we find (always in appropriate string-length units, since $\alpha' = 1$)

$$g_s \sim 10^{-13}, \quad r_a \sim 10^8, \quad r_7 \sim 10^{18}, \tag{3.26}$$

and

$$\frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2} \sim 1, \quad \frac{L_{\text{KK},a}^2}{L_{\text{AdS}}^2} \sim 10^{-19}. \tag{3.27}$$

Table 2 This table shows the values of the radii r_i as γ takes the values $\gamma = 10^3, 10^6$. One can clearly notice that, as γ becomes larger, r_7 becomes bigger than the other six radii r_a (for fixed flux units)

γ	$\frac{r_{1,3}}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_{2,4}}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_5}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_6}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_7}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$
10^3	1.09	0.94	1.88	0.54	2821
10^6	1.086	0.941	1.881	0.543	2.822×10^6

Table 3 This table shows the three interesting regimes one can end up with while changing N for $\gamma = 10^3$, once the other flux units have been fixed, namely $K = -16$ and $M = -1$. When a circumstance like (a) realizes, one has full scale separation; if, instead, one works with cases similar to (b) or (c), then the radius r_7 disentangles from the other

six radii and the external space becomes effectively $\text{AdS}_3 \times S^1$. For the in-between values of N one gets of course intermediate results. Note that N can not be too small in order for the large volume/weak coupling condition to still be satisfied

	N	g_s	r_a	r_7	$\frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2}$	$\frac{L_{\text{KK},a}^2}{L_{\text{AdS}}^2}$
(a)	10^{10}	9.72×10^{-7}	$\mathcal{O}(10^4)$	2.10×10^8	1.64×10^{-3}	$\mathcal{O}(10^{-10})$
(b)	10^7	1.73×10^{-4}	$\mathcal{O}(10^3)$	1.02×10^7	1.64	$\mathcal{O}(10^{-7})$
(c)	10^4	0.03	$\mathcal{O}(10)$	4.98×10^4	1.64×10^3	$\mathcal{O}(10^{-4})$

Table 4 This table shows three interesting regimes one can end up with while changing N for $\gamma = 10^6$, once the other flux units have been fixed, namely $K = -16$ and $M = -1$. One can again see a similar behaviour with respect to the case $\gamma = 10^3$, which is presented in

Table 3: a pattern indeed emerges. For the in-between values of N one gets of course intermediate results, while one can appreciate that the windows for the various regimes cover now a wider range of values of N

	N	g_s	r_a	r_7	$\frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2}$	$\frac{L_{\text{KK},a}^2}{L_{\text{AdS}}^2}$
(a)	10^{19}	1.73×10^{-13}	$\mathcal{O}(10^8)$	1.82×10^{15}	1.64×10^{-6}	$\mathcal{O}(10^{-19})$
(b)	10^{13}	5.46×10^{-9}	$\mathcal{O}(10^6)$	4.32×10^{12}	1.64	$\mathcal{O}(10^{-13})$
(c)	10^7	1.73×10^{-4}	$\mathcal{O}(10^3)$	1.02×10^{10}	1.64×10^6	$\mathcal{O}(10^{-7})$

As we expect, because the characteristic length scale associated with r_7 is comparable to the AdS_3 length scale, the previous relation shows that the external space becomes effectively four-dimensional (that is, the r_7 KK-modes can not be ignored), while the internal space is six-dimensional and it is scale-separated from the large external one.

It is also possible to choose values of N such that a different separation of scales is achieved, of the form

$$L_{\text{KK},7}^2 \gg L_{\text{AdS}}^2 \gg L_{\text{KK},a}^2. \tag{3.28}$$

For example, when $N \sim 10^9$,

$$g_s \sim 10^{-6}, \quad r_a \sim 10^4, \quad r_7 \sim 10^{13} \tag{3.29}$$

and

$$\frac{L_{\text{KK},7}^2}{L_{\text{AdS}}^2} \sim 10^{10} \quad \frac{L_{\text{KK},a}^2}{L_{\text{AdS}}^2} \sim 10^{-9}. \tag{3.30}$$

The four-dimensional external space is again $\text{AdS}_3 \times S^1$, still well scale-separated from the six-dimensional internal one; however, since the radius of the seventh compact dimension is larger than the AdS length scale, one could say that the external space effectively becomes $\text{AdS}_3 \times \mathbb{R}^1$.

Table 5 This table exhibits the critical values of σ, ρ and τ as γ takes the values $\gamma = 10^9, 2.5305 \times 10^9$

γ	σ	ρ	τ
10^9	249833	0.0000555	0.0001666
2.5305×10^9	424673	0.0000373	0.0001119

We note once more that, when the integer N takes parametrically large values, even though the compact space may remain highly anisotropic, scale separation makes the internal space seven-dimensional and the external space AdS_3 .

Moreover, let us observe that our flux choices do not exhaust all the possible options that would give such an anisotropy; we leave a general scan of the various possibilities for future work.

4 Mass spectrum

We now turn to the study of the mass spectrum of the model of interest focusing on the closed string moduli that we con-

Table 6 This table shows the values of the radii r_i as γ takes the values $\gamma = 10^9, 2.5302 \times 10^9$. One can clearly notice that r_7 is much bigger than the other six radii r_a for fixed flux units

γ	$\frac{r_{1,3}}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_{2,4}}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_5}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_6}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_7}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$
10^9	1.086	0.941	1.881	0.543	2.822×10^9
2.5305×10^9	1.086	0.941	1.881	0.543	7.140×10^9

sidered until now. Once we divide the masses of the scalar fields x, y and \tilde{s}^a by the vacuum energy, we see that the analysis does not depend on the specific choice of the units N of the F_4 flux, but only on μ, χ and γ . Therefore here we will collect some results and considerations involving the mass spectrum of the model simply as γ takes the values $\gamma = 1, 10^3, 10^6$ and $\gamma \sim \mathcal{O}(10^9)$. We should remember of course that the relative magnitude of the r_7 KK-mode masses compared to the AdS length scale depends on N . This means that depending on the value of N the three-dimensional theory with only the x, y and \tilde{s}^a is either a consistent truncation of a four-dimensional theory or an actual three-dimensional effective theory.

As anticipated, for the evaluation of the masses we will proceed by considering the AdS₃ construction with the eight moduli x, y and \tilde{s}^a . We will firstly focus on the $\gamma = 10^9$ case since it is the circumstance with the larger window for the values of N and thus covers all the interesting cases.

The scalar potential takes the form

$$\frac{V}{(2\pi)^{14}} = F(\tilde{s}^a) e^{2y - \frac{2x}{\sqrt{7}}} + H(\tilde{s}^a) e^{2y + \frac{2x}{\sqrt{7}}} + C e^{y - \sqrt{7}x} + T(\tilde{s}^a) e^{\frac{3y}{2} - \frac{5x}{2\sqrt{7}}} \tag{4.1}$$

with

$$F = \frac{1}{16} \left[f^2 \left(\sum_{a=1}^3 (\tilde{s}^a)^2 + 9((\tilde{s}^4)^2) \right) + q^2 \left(\sum_{a=5}^6 (\tilde{s}^a)^2 \right) + 4q^2 \prod_{a=1}^6 (\tilde{s}^a)^{-2} \right],$$

$$H = \frac{h^2}{16} \left(\sum_{a=1}^6 (\tilde{s}^a)^{-2} + \prod_{a=1}^6 (\tilde{s}^a)^2 \right) \tag{4.2}$$

and

$$C = \frac{m^2}{16}, \quad T = -\frac{hm}{8} \left(\sum_{a=1}^6 \frac{1}{\tilde{s}^a} + \prod_{a=1}^6 \tilde{s}^a \right). \tag{4.3}$$

From here we evaluate the Hessian matrix and its eigenvalues:

$$\text{Eigen} \left[\frac{\langle V_{IJ} \rangle}{|\langle V \rangle|} \right] = \{8.872 \times 10^8, 3.204 \times 10^7, 2.604 \times 10^7, -8.038,$$

$$4.152, 1.720 \times 10^{-10}, 6.409 \times 10^{-11}, 6.399 \times 10^{-11}\}, \tag{4.4}$$

where the eight indices I, J run over the x, y and \tilde{s}^a . Moreover, once the fields have been properly normalized, we get

$$m^2 L_{\text{AdS}}^2 = \text{Eigen} \left[\langle K_{IJ} \rangle^{-1} \frac{\langle V_{IJ} \rangle}{|\langle V \rangle|} \right] \approx \{49.192, 8, 8, 8, 3.934, 2.095, 1.778, -0.998\}, \tag{4.5}$$

the matrix K_{IJ} being

$$K_{IJ} = 2 \times \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & G_{ab} \end{bmatrix}, \quad \text{where } G_{ab} = \frac{1 + \delta_{ab}}{4 \tilde{s}^a \tilde{s}^b}. \tag{4.6}$$

Let us importantly observe that, once the modes have been canonically normalized, the tachyon respects the Breitenlohner–Freedman (BF) bound, which for AdS₃ is

$$m^2 \geq \langle V \rangle = -\frac{1}{L_{\text{AdS}}^2}. \tag{4.7}$$

In the case at hand we see that the canonically normalized tachyon mass is $(m^2 L_{\text{AdS}}^2)_{\text{tachyon}} \simeq -0.998 > -1$. Interestingly, the possibly generic existence of a tachyonic scalar mode within specific AdS setups was investigated in [25] within the scope of the so-called AdS-TCC (or ATCC).

In Table 7 we present the normalized masses of the scalars x, y and \tilde{s}^a as the parameter γ takes also the values 1 (when the seven-dimensional internal space is almost isotropic), $10^3, 10^6$, repeating the $\gamma = 10^9$ for comparison, and including also the limiting case $\gamma = 2.5305 \times 10^9$. A negative mass eigenvalue is present in all these cases and its value is always above the Breitenlohner–Freedman bound. Looking at the behaviour of this mass, one could suspect that for some value of γ around 2.5305×10^9 the BF bound may be saturated by the tachyonic scalar, giving precisely $(m^2 L_{\text{AdS}}^2)_{\text{tachyon}} = -1$, while above such critical value of γ supersymmetric vacua of (3.3) within (3.4) cease to exist. However, without an analytic solution for (3.11), we can not verify this explicitly.

Table 7 This table exhibits the (rounded-up numerical values of the) masses of the closed string moduli as the parameter γ takes the values 1, 10^3 , 10^6 , 10^9 and 2.5305×10^9 . The negative eigenvalue remains always slightly above the BF bound

γ	$m^2 L_{\text{AdS}}^2$
1	{49.254, 7.581, 5.435, 5.435, 5.435, 5.435, 2.405, -0.9799}
10^3	{49.19188, 7.99867, 7.99467, 7.99467, 3.93721, 2.09572, 1.78518, -0.997995}
10^6	{49.19179, 8, 7.99999, 7.99999, 3.93391, 2.09457, 1.77779, -0.998051}
10^9	{49.19179, 8, 8, 8, 3.93391, 2.09457, 1.77778, -0.998051}
2.5305×10^9	{49.19179, 8, 8, 8, 3.93391, 2.09457, 1.77778, -0.998051}

5 Swampland considerations

As we have seen by analysing the mass spectrum of the model under consideration, our setup provides a realization of the conjectured negative mass of [25]. We can now discuss some further interesting aspects of the Swampland Program within our construction, related (in particular) to the Distance Conjecture and the Spin-2 Conjecture.

5.1 The Distance Conjecture

Clearly, it is interesting to study the interpolation between the vacua that are characterized by full scale separation and those ones that have a large radius, and see how the KK-modes behave. This can be done by following the setup of [26, 60],⁴ varying the value of N with the use of D4-branes while keeping γ fixed. Specifically, for appropriate (and fixed) values of γ and when N is not parametrically large, we have a large value of r_7 ; by increasing N , we appreciate that full scale separation takes place, while the tower of KK-modes has, as we will see, an exponentially dropping mass to realize the Distance Conjecture.

To be precise, let us remind the reader that we have

$$F_4 = (2\pi)^3 N [(\Psi_1 + \Psi_2 + \Psi_3 - 3\Psi_4) + \gamma (\Psi_5 + \Psi_6 - 2\Psi_7)], \tag{5.1}$$

and $H_3 = (2\pi)^2 K \sum_i \Phi_i$. Similarly to [26], where it is shown that with the inclusion of appropriate D4-branes the value of N can change at the cost of introducing a new scalar in the moduli space, let us consider \tilde{N}_i D4-branes wrapping the three-dimensional external space and a 2-cycle within an internal 3-cycle $\Sigma_{3,i}$.⁵ We furthermore parametrize the metric as

$$ds_{10}^2 = e^{2\alpha v} ds_3^2 + s_i^{\frac{2}{3}} \left[d\psi_i^2 + w_i^2(\psi_i) g_{mn} dy^m dy^n \right]$$

⁴ For another perspective on the evaluation of the distance see also [61].

⁵ We will not discuss here the scaling of the potential to justify the use of such D4-branes within the effective description: it is fairly accepted at this point that D4-branes are admitted and facilitate the flux jump. In any case we do not have more convincing or newer arguments to offer on top of the ones presented in [26].

$$+ e^{2\beta v} ds_{4,i}^2, \tag{5.2}$$

where ψ_i is the transverse coordinate to the 2-cycle along (y^m, y^n) , which the D4-brane occupies within $\Sigma_{3,i}$. Here the real scalar v is a convenient rewriting of the volume modulus with

$$e^{\beta v} = \text{Vol}(X_7)^{1/7}, \quad e^{\alpha v} = \text{Vol}(X_7)^{-1}, \tag{5.3}$$

where the constants α and β take the values $\alpha = \sqrt{7}/4$ and $\beta = -1/(4\sqrt{7})$.

The full D4-brane action in the Einstein frame then takes the form

$$S_{\text{D4},i} = S_{\text{D4},i}^{\text{DBI}} + S_{\text{D4},i}^{\text{CS}} \tag{5.4}$$

with

$$S_{\text{D4},i}^{\text{DBI}} = -|\tilde{N}_i| T_4 \times \int d^5 \xi e^{\frac{\phi}{4}} \sqrt{-\det [g_{AB} + e^{-\frac{\phi}{2}} (2\pi F_{AB} - B_{AB})]} \\ S_{\text{D4},i}^{\text{CS}} = |\tilde{N}_i| T_4 \int \left(\sum_r C_r \wedge e^{-B_2} \right) \wedge e^{2\pi F} \Big|_5, \tag{5.5}$$

the tension of a D4-brane being $T_4 = \frac{1}{(2\pi)^4}$ (as its charge), once $\alpha' = 1$. In the absence of world-volume flux and expressing the B_2 field as

$$B_{2,i} = [B_2]_{mn,i}(\psi_i) dy^m \wedge dy^n \\ = (2\pi)^2 U_i(\psi_i) \epsilon_{mn} \sqrt{\det[g_{pq}]} dy^m \wedge dy^n, \tag{5.6}$$

which has to satisfy $\int_{\Sigma_{3,i}} dB_{2,i} = h = (2\pi)^2 K$ and for which $U_i(\text{sgn}(\tilde{N}_i)\pi f) = \text{sgn}(\tilde{N}_i)fh$ (in order to resolve the anomaly due to the potential appearance of a net D2 charge), we get

$$S_{\text{D4},i} = -\frac{(2\pi)^{21} |\tilde{N}_i| T_4}{8} \int_{\text{AdS}_3} d^3 x \sqrt{-\det[g_{\mu\nu}^{(3)}]} \\ \times \int_{2\text{-cycle}} d^2 y \sqrt{\det[g_{mn}^{(2)}]} \\ \times e^{\frac{\phi}{4}} e^{3\alpha v} \sqrt{1 + \frac{4s_i^{\frac{2}{3}} e^{-2\alpha v}}{(2\pi)^{14}} (\partial\psi_i)^2}$$

$$\sqrt{s_i^{\frac{4}{3}} w_i(\psi_i)^4 + e^{-\phi} (2\pi)^4 U_i(\psi_i)^2 + \dots}, \tag{5.7}$$

or

$$S_{D4,i} = -\frac{(2\pi)^{21} |\tilde{N}_i| T_4 v_{2,i}}{8} \int_{\text{AdS}_3} d^3x \sqrt{-\det[g_{\mu\nu}^{(3)}]} \\ \times e^{\frac{\phi}{4}} e^{3\alpha v} \sqrt{1 + \frac{4s_i^{\frac{2}{3}} e^{-2\alpha v}}{(2\pi)^{14}} (\partial\psi_i)^2} \\ \sqrt{s_i^{\frac{4}{3}} w_i(\psi_i)^4 + e^{-\phi} (2\pi)^4 U_i(\psi_i)^2 + \dots}, \tag{5.8}$$

where we have rescaled the three-dimensional metric $g_{\mu\nu}^{(3)}$ as $g_{\mu\nu}^{(3)} \rightarrow \frac{(2\pi)^{14}}{4} g_{\mu\nu}^{(3)}$ to make contact with the standard conventions for three-dimensional supergravity theories, and the ‘dots’ stand for terms that do not play a role for the upcoming discussion. From the expression above we can extract the field space metric of the position scalar ψ_i that is

$$g_{\psi_i \psi_i} = \frac{(2\pi)^7 |\tilde{N}_i| T_4 v_{2,i}}{2} e^{\frac{\phi}{4} - 7\beta v} s_i^{\frac{2}{3}} \\ \sqrt{s_i^{\frac{4}{3}} w_i(\psi_i)^4 + e^{-\phi} (2\pi)^4 U_i(\psi_i)^2} \\ = \lambda e^{\frac{\phi}{4} - 3\beta v} \tilde{s}_i^{\frac{4}{3}} \sqrt{w_i(\psi_i)^4 + \tilde{s}_i^{-\frac{4}{3}} e^{-\phi - 4\beta v} (2\pi)^4 U_i(\psi_i)^2}. \tag{5.9}$$

Other than $g_{\psi_i \psi_i}$, in order to evaluate the distance between the vacuum where a radius (e.g. r_7) is large and the one where full scale separation is realized, we need the metric components $g_{\phi\phi}$, g_{vv} and $g_{\tilde{s}^a \tilde{s}^b}$, which can be extracted from the dimensionally reduced ten-dimensional action over the G2 space, once the fields ϕ , v and \tilde{s}^a have been canonically normalized:

$$g_{\phi\phi} = \frac{1}{2}, \quad g_{vv} = \frac{1}{2} \quad \text{and} \quad g_{\tilde{s}^a \tilde{s}^b} = \frac{1 + \delta_{ab}}{2\tilde{s}^a \tilde{s}^b} \\ \text{for } a, b = 1, \dots, 6. \tag{5.10}$$

The field space distance of interest is

$$\Delta = \int_{\zeta=0}^{\zeta=1} d\zeta \sqrt{g_{\mathcal{I}\mathcal{J}} \frac{d\varphi^{\mathcal{I}}}{d\zeta} \frac{d\varphi^{\mathcal{J}}}{d\zeta}}, \tag{5.11}$$

where the indices \mathcal{I}, \mathcal{J} run over the fields $\varphi = \{\psi_i, \phi, v, \tilde{s}^a\}$ and $g_{\mathcal{I},\mathcal{J}}$ denotes the corresponding metric element. By exploiting reparametrization invariance we can impose that $\zeta = 0$ corresponds to the circumstance where the radius r_7 is large and $\zeta = 1$ is associated with the realization of full scale separation. When considering the contributions from the relevant 3-cycles $\Sigma_{3,i} \equiv \Phi_i$ for $i = 1, \dots, 7$, and redefining σ, ρ and τ as

$$\sigma = e^{3s}, \quad \rho = e^{3r}, \quad \tau = e^{3t}, \tag{5.12}$$

we get

$$\Delta = \int_0^1 d\zeta \left[\sum_{i=1}^7 \lambda e^{\frac{\phi}{4} - 3\beta v} \tilde{s}_i^{\frac{4}{3}} \sqrt{w_i(\psi_i)^4 + \tilde{s}_i^{-\frac{4}{3}} e^{-\phi - 4\beta v} (2\pi)^4 U_i(\psi_i)^2} \left(\frac{d\psi_i}{d\zeta} \right)^2 \right. \\ \left. + \frac{1}{2} \left(\frac{d\phi}{d\zeta} \right)^2 + \frac{1}{2} \left(\frac{dv}{d\zeta} \right)^2 + 54 \left(\frac{ds}{d\zeta} \right)^2 + 9 \left(\frac{dr}{d\zeta} \right)^2 \right. \\ \left. + 27 \left(\frac{dt}{d\zeta} \right)^2 + 27 \frac{ds}{d\zeta} \frac{dr}{d\zeta} + 54 \frac{ds}{d\zeta} \frac{dt}{d\zeta} + 18 \frac{dr}{d\zeta} \frac{dt}{d\zeta} \right]^{\frac{1}{2}}. \tag{5.13}$$

Keeping in mind that we are simply interested in extracting how Δ scales with N and considering the numerical results of Sect. 3.2, let us massage the previous expression by grouping the 3-cycles $\Sigma_{3,i}$ according to their approximate size: in particular, we are going to set $s_1 = s_2 = s_3 = e^{3\beta v} \sigma = e^{3\beta v + 3s}$ and $s_4 = e^{3\beta v} \rho = e^{3\beta v + 3r} \sim e^{3\beta v + 3t} = e^{3\beta v} \tau = s_5 = s_6 \sim s_7$; we will denote as \hat{s}_1 and \hat{s}_2 the typical sizes of these two classes of 3-cycles. Moreover, we will refer to the position scalars corresponding to such two groups as $\hat{\psi}_1$ and $\hat{\psi}_2$, respectively. Then, if $r \sim t \simeq u$,

$$\Delta \approx \int_0^1 d\zeta \left[\lambda e^{\frac{\phi}{4} - 3\beta v} \times \left(3e^{4s} \sqrt{\hat{w}_1(\hat{\psi}_1)^4 + e^{-\phi - 4\beta v - 4s} (2\pi)^4 \hat{U}_1(\hat{\psi}_1)^2} \left(\frac{d\hat{\psi}_1}{d\zeta} \right)^2 \right. \right. \\ \left. \left. + 4e^{4u} \sqrt{\hat{w}_2(\hat{\psi}_2)^4 + e^{-\phi - 4\beta v - 4u} (2\pi)^4 \hat{U}_2(\hat{\psi}_2)^2} \left(\frac{d\hat{\psi}_2}{d\zeta} \right)^2 \right) \right. \\ \left. + \frac{1}{2} \left(\frac{d\phi}{d\zeta} \right)^2 + \frac{1}{2} \left(\frac{dv}{d\zeta} \right)^2 \right. \\ \left. + 54 \left(\frac{ds}{d\zeta} \right)^2 + 54 \left(\frac{du}{d\zeta} \right)^2 + 81 \frac{ds}{d\zeta} \frac{du}{d\zeta} \right]^{\frac{1}{2}}. \tag{5.14}$$

Let us importantly observe that, since the functions $w_i(\psi_i)$ and $U_i(\psi_i)$ do not scale with N , and because of the scalings with N of e^ϕ and $e^{\beta v}$,

$$\sqrt{\hat{w}_k(\hat{\psi}_k)^4 + e^{-\phi - 4\beta v} \tilde{s}_k^{-\frac{4}{3}} (2\pi)^4 \hat{U}_k(\hat{\psi}_k)^2} \approx \hat{w}_k(\hat{\psi}_k)^2, \tag{5.15}$$

the factor $e^{-\phi - 4\beta v}$, which carries the dependence on N and accompanies $\hat{U}_k(\hat{\psi}_k)^2$, being $\mathcal{O}(N^{-1})$, i.e. consistently suppressed for large N . As a consequence, we obtain

$$\Delta \approx \int_0^1 d\zeta \left[e^{\frac{\phi}{4} - 3\beta v + 4s} \left(\frac{d\hat{\chi}_1}{d\zeta} \right)^2 \right. \\ \left. + e^{\frac{\phi}{4} - 3\beta v + 4u} \left(\frac{d\hat{\chi}_2}{d\zeta} \right)^2 \right]$$

$$\begin{aligned}
 & + \frac{1}{2} \left(\frac{d\phi}{d\zeta} \right)^2 + \frac{1}{2} \left(\frac{dv}{d\zeta} \right)^2 + 54 \left(\frac{ds}{d\zeta} \right)^2 \\
 & + 54 \left(\frac{du}{d\zeta} \right)^2 + 81 \left[\frac{ds}{d\zeta} \frac{du}{d\zeta} \right]^{\frac{1}{2}}, \tag{5.16}
 \end{aligned}$$

where we also exploited the following field re-definitions:

$$\frac{d\hat{\chi}_1}{d\hat{\psi}_1} = \sqrt{3\lambda}|\hat{w}_1(\hat{\psi}_1)| \quad \text{and} \quad \frac{d\hat{\chi}_2}{d\hat{\psi}_2} = \sqrt{4\lambda}|\hat{w}_2(\hat{\psi}_2)|. \tag{5.17}$$

In order to identify the geodesic path we have to recognize (at least approximately) the geometric structure of the scalar field space under consideration. To this end we firstly observe that the second term in the first line of (5.16) is highly suppressed, by $e^{4u-4s} \sim \left(\frac{\rho}{\sigma}\right)^{\frac{4}{3}} \sim \left(\frac{\tau}{\sigma}\right)^{\frac{4}{3}} \ll 1$, with respect to the term that precedes it in the same (5.16)⁶. We are therefore going to ignore such contribution from the open string moduli sector and retain only that associated with the bigger internal 3-cycles, with volumes s_1, s_2 and s_3 . This being established, we have to perform some useful field transformations. We trade s for

$$s = \check{s} - \frac{3}{4}u \tag{5.18}$$

and we further rescale \check{s} and u as

$$\check{s} = \frac{\check{\check{s}}}{\sqrt{108}} \quad \text{and} \quad u = \sqrt{\frac{4}{189}}\check{u}. \tag{5.19}$$

Moreover, we can perform an $O(4)$ transformation via the orthogonal matrix⁷

$$\begin{aligned}
 O &= \begin{pmatrix} -\frac{1}{8} & -\frac{3\beta}{2} & \frac{1}{3\sqrt{3}} & -\frac{1}{\sqrt{21}} \\ \frac{3\beta}{2} & -\frac{1}{8} & -\frac{1}{\sqrt{21}} & -\frac{1}{3\sqrt{3}} \\ -\frac{1}{3\sqrt{3}} & \frac{1}{\sqrt{21}} & -\frac{1}{8} & -\frac{3\beta}{2} \\ \frac{1}{\sqrt{21}} & \frac{1}{3\sqrt{3}} & \frac{3\beta}{2} & -\frac{1}{8} \end{pmatrix} \\
 & \text{with } \det[O] = \left(\frac{1213 + 27216\beta^2}{12096} \right)^2 \tag{5.20}
 \end{aligned}$$

so that

$$\begin{aligned}
 \begin{pmatrix} \phi \\ v \\ \check{s} \\ \check{u} \end{pmatrix} &= \frac{12096}{1213 + 27216\beta^2} \\
 & \times \begin{pmatrix} -\frac{1}{8} & -\frac{3\beta}{2} & \frac{1}{3\sqrt{3}} & -\frac{1}{\sqrt{21}} \\ \frac{3\beta}{2} & -\frac{1}{8} & -\frac{1}{\sqrt{21}} & -\frac{1}{3\sqrt{3}} \\ -\frac{1}{3\sqrt{3}} & \frac{1}{\sqrt{21}} & -\frac{1}{8} & -\frac{3\beta}{2} \\ \frac{1}{\sqrt{21}} & \frac{1}{3\sqrt{3}} & \frac{3\beta}{2} & -\frac{1}{8} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}. \tag{5.21}
 \end{aligned}$$

⁶ This, in accordance to the numerical results of Sect. 3.2 when considering, for instance, the (relevant) cases $\gamma = 10^3, 10^6$ or $\gamma \sim \mathcal{O}(10^9)$.

⁷ As required by the orthogonal group, the matrix O is satisfies $OO^T = (\det[O])^{\frac{1}{2}}\mathbb{1}_4$.

Then, after redefining $\hat{\psi}_1$ to be

$$\hat{\psi}_1 = \frac{1}{\sqrt{2}(\det[O])^{\frac{1}{4}}} \hat{h}_1, \tag{5.22}$$

we get

$$\begin{aligned}
 \Delta &\approx \frac{1}{\sqrt{2}(\det[O])^{\frac{1}{4}}} \\
 & \times \int_0^1 d\zeta \sqrt{e^{-2z_1} \left(\frac{d\hat{h}_1}{d\zeta} \right)^2 + \left(\frac{dz_1}{d\zeta} \right)^2 + \left(\frac{dz_2}{d\zeta} \right)^2 + \left(\frac{dz_3}{d\zeta} \right)^2 + \left(\frac{dz_4}{d\zeta} \right)^2}. \tag{5.23}
 \end{aligned}$$

By further redefining z_1 so that $\hat{h}_2 = e^{-z_1}$ the distance Δ takes the form

$$\begin{aligned}
 \Delta &\approx \frac{1}{\sqrt{2}(\det[O])^{\frac{1}{4}}} \int_0^1 d\zeta \\
 & \sqrt{\frac{1}{\hat{h}_2^2} \left[\left(\frac{d\hat{h}_1}{d\zeta} \right)^2 + \left(\frac{d\hat{h}_2}{d\zeta} \right)^2 \right] + \left(\frac{dz_2}{d\zeta} \right)^2 + \left(\frac{dz_3}{d\zeta} \right)^2 + \left(\frac{dz_4}{d\zeta} \right)^2}. \tag{5.24}
 \end{aligned}$$

The geodesic path can thus be approximated as

$$\hat{h}_1 = l \sin[f(\zeta)] + \hat{h}_{1,0}, \quad \hat{h}_2 = l \cos[f(\zeta)] \tag{5.25}$$

with

$$f(\zeta) = 2 \arctan \left[\sinh \left[\frac{d_1\zeta + d_2}{2} \right] \right] \tag{5.26}$$

and

$$z_2 = d_3\zeta + d_4, \quad z_3 = d_5\zeta + d_6, \quad z_4 = d_7\zeta + d_8, \tag{5.27}$$

so that the geodesic distance is estimated to be

$$\Delta \approx \frac{1}{\sqrt{2}(\det[O])^{\frac{1}{4}}} \sqrt{d_1^2 + d_3^2 + d_5^2 + d_7^2}, \tag{5.28}$$

where the coefficients $\{d_j\}_{j=1,3,5,7}$ are determined by the boundary values of the fields $\hat{h}_1, \hat{h}_2, z_2, z_3$ and z_4 corresponding to the two vacuum configurations we are interpolating between, namely the one where a radius (e.g. r_7) is large (at $\zeta = 0$) and the one where full scale separation is achieved (at $\zeta = 1$). Accounting for the scalings (up to numerical factors)

$$\hat{\psi}_1 \sim N, \quad \phi \sim \log N, \quad v \sim \log N \tag{5.29}$$

we can deduce, again up to numerical factors, the behaviours

$$\hat{h}_1 \sim N \sim \hat{h}_2, \quad z_k \sim \log N \quad \text{for } k = 2, 3, 4 \tag{5.30}$$

so that

$$d_j \sim \log N \quad \text{for } j = 1, 3, 5, 7, \tag{5.31}$$

and eventually

$$\Delta \sim \delta \log N, \tag{5.32}$$

δ being a numerical coefficient that can be crudely estimated to be $\delta \simeq 24.6$. The crucial result here is the logarithmic behaviour, which is precisely the way Δ should scale with N , if the Distance Conjecture

$$\frac{m_{\text{KK}}(\zeta = 1)}{m_{\text{KK}}(\zeta = 0)} \sim e^{-\kappa\Delta}, \quad \kappa \sim \mathcal{O}(1) \tag{5.33}$$

is realized. This happens because we already have

$$\frac{m_{\text{KK}}(\zeta = 1)}{m_{\text{KK}}(\zeta = 0)} \sim N^{-\frac{7}{2}}, \tag{5.34}$$

as one can easily infer from (3.12) accounting for the scaling with N of the volume $\text{Vol}(X_7)$ and the radii r_i .

We conclude that for the highly anisotropic vacua under analysis the Distance Conjecture is realized, always with the inclusion of the appropriate D4-branes. This distance is along the path that connects $\text{AdS}_3 \times S^1$ and fully scale-separated AdS_3 , with the tower of the r_7 KK-modes turning full scale separation *on* and *off*.

5.2 The Strong Spin-2 Conjecture

A further interesting connection of our work with the Swampland Program has to do with the Strong Spin-2 Conjecture [62], which states that in any theory with massive spin-2 fields there has to be a tower of states related to the massive spin-2 modes even in the absence of a massless spin-2 excitation. Indeed, since standard gravity in three dimensions does not have local excitations (i.e. there are no massless gravitons), the only spin-2 states are the ones related to the KK-modes of the graviton. Since they are KK-modes, these states essentially form a tower that has as characteristic mass scale the mass of the lowest one, which also defines the cut-off of the three-dimensional effective theory $\Lambda_{\text{UV}} \sim m_{\text{spin-2}}$. In AdS space having such low cut-off is not an inconsistency per se, if one is protected by supersymmetry; it just means that the system is not inherently three-dimensional. In this way our construction provides a non-trivial example in favour of [62].

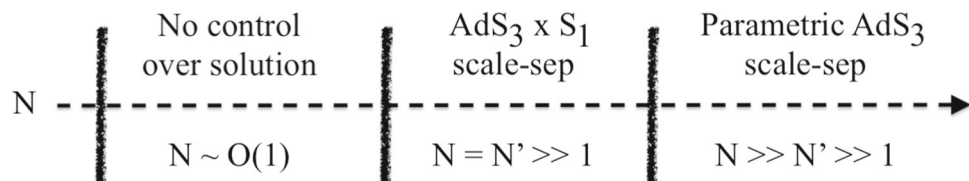
have $\Lambda_{\text{UV}} \ll H$ which signals an inherent inconsistency for de Sitter. Actually, such an inconsistency can be directly deduced from the Higuchi bound [63], which in three dimensions is

$$m_{\text{spin-2}}^2 \geq H^2.$$

Therefore, one can never make a single radius parametrically grow in de Sitter and break scale separation as we did here for the AdS case.

6 Discussion

In this work we have studied classical flux vacua of massive type-IIA supergravity with appropriate numbers of branes and orientifold planes. All the constructions that we have analyzed are characterized by proper flux quantization, closed string moduli stabilization, large volume and weak coupling, while the O6-planes are smeared. In such setup the compact space is a seven-dimensional G2 toroidal orbifold and the non-compact space is AdS_3 . Our aim was to find configurations that induce a considerable anisotropy in the compact space such that one dimension can become large and comparable to the non-compact space length scale, whereas the other six dimensions become considerably smaller than the non-compact space length scale. In this way the original ten-dimensional space reduces to a product of a large four-dimensional external one and a small six-dimensional one, with (non-parametric) scale separation between the two. When, instead, the F_4 flux takes parametrically large values we recover AdS_3 parametrically scale-separated from the seven-dimensional internal space, while preserving (as γ is fixed) the amount of anisotropy between one of the internal dimensions (e.g. $L_{\text{KK},7}$) and the remaining six ones (i.e. $L_{\text{KK},a}$). This behaviour can be schematically described by the figure below for some F_4 flux N (N' being the value that allows one large radius):



This observation also tells us that, even though we have been able to find $\text{AdS}_3 \times S^1$ solutions, we would not be able to find $dS_3 \times S^1$ solutions within a controlled setup. This happens because, if we were able to have dS_3 and to lower the KK-modes such that $m_{\text{spin-2}} \ll H$, then we would also

This happens because such vacua have the interesting property that, while N increases, both $L_{\text{KK},i}$ and L_{AdS} increase; however, their ratio changes proportionally to $L_{\text{KK},i}^2/L_{\text{AdS}}^2 \sim 1/N$, thus always restoring full scale separation at parametrically large N .

Our work leaves a series of open questions for future investigation. Firstly, a full analysis of all types of flux vacua that can be constructed from the G2 orientifold of [51] is missing.

The full list of choices that one can make for the F_4 and the H_3 flux are still not classified; any development in that direction would be welcome. Secondly, in all of these constructions the open string moduli are assumed to be stabilized. However, a careful analysis that checks this and clarifies the full moduli space and superpotential would be interesting. Thirdly, we are still working with a toroidal orbifold here; the explicit study of different G2 spaces or G2-structure manifolds is still an open question (however, see [56] for general conditions on the G2-structure from type-IIB), and the only explicit development providing a superpotential in this direction was the analysis of type-IIB supergravity on co-calibrated G2 toroidal orbifolds with O5/O9-planes [39], while a general superpotential for such type-II constructions was derived in [53]. Finally, as far as the more fundamental issues are concerned, one could try to go to higher orders in the O6-plane backreaction since, for the moment, only the leading order was analyzed in [52].

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Data Availability Statement The manuscript has associated data in a data repository. [Authors’ comment: This is a theoretical investigation and therefore there are no data related to it.]

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A Small γ values with different anisotropy

As a complement to what we discussed in the bulk of the paper, it is interesting to investigate what happens when γ takes small values. For the sake of the presentation, we will focus on two representative values of γ , namely $\gamma = 10^{-3}$ and $\gamma = 5 \times 10^{-6}$ (being conscious of the fact that for $\gamma \sim 4 \times 10^{-6}$ moduli stabilization of (3.3) within the ansatz (3.4) ceases to exist, and that, since $q = \gamma f, \gamma N \in \mathbb{Z}$, which is satisfied in our examples.).

The equations in (3.11) are solved by the values of σ, ρ and τ that are reported in Table 8; as a consequence, we also get that the string coupling is given by

Table 8 This table presents the critical values of σ, ρ and τ as γ takes the values $\gamma = 10^{-3}, 5 \times 10^{-6}$

γ	σ	ρ	τ
10^{-3}	0.212	0.033	268.4
5×10^{-6}	0.047	0.007	11811.8

$$g_s \approx 71.33 \times \frac{|K|}{N^{3/4}|M|^{1/4}}, \quad \text{for } \gamma = 10^{-3} \tag{A.1}$$

or

$$g_s \approx 1.0085 \times 10^3 \times \frac{|K|}{N^{3/4}|M|^{1/4}}, \quad \text{for } \gamma = 5 \times 10^{-6}, \tag{A.2}$$

and that the radii take the values that Table 9 exhibits. We see once again that, after fixing K and M consistently with the tadpole conditions (e.g. $K = -16$ and $M = -1$), we can end up in the desired weak string coupling and large internal volume regime by taking a sufficient amount of N units of the F_4 flux.

Besides the possibility to recover full scale separation by taking N parametrically large, we would like to understand once more if for some moderate (but still sufficiently large) value of N we could effectively obtain a four-dimensional external space, $\text{AdS}_3 \times S^1$, scale-separated from a six-dimensional internal one. This means that, in light of our previous discussion and the results presented in Table 9, we want to investigate whether

$$\frac{L_{\text{KK},5}^2}{L_{\text{AdS}}^2} = \frac{4}{\pi^2} \left(\frac{P}{(2\pi)^7} \right)^2 (\text{Vol}(X_7))^2 r_5^2 \gtrsim 1, \tag{A.3}$$

$$\frac{L_{\text{KK},i \neq 5}^2}{L_{\text{AdS}}^2} \ll 1.$$

More precisely, we have the ratios

$$\frac{L_{\text{KK},5}^2}{L_{\text{AdS}}^2} \Big|_{\gamma=10^{-3}} \approx 9 \times 10^4 \times \frac{K^2|M|}{N}, \quad \frac{L_{\text{KK},5}^2}{L_{\text{AdS}}^2} \Big|_{\gamma=5 \times 10^{-6}} \approx 3.6 \times 10^9 \times \frac{K^2|M|}{N}, \tag{A.4}$$

which offer quite a wide range of values of N that can be checked. Indeed, when fixing $K = -16$ and $M = -1$, we find, for $\gamma = 10^{-3}$, the results that are presented in Table 10, and, for $\gamma = 5 \times 10^{-6}$, we end up with the values that are reported in Table 11, together with the relative behaviour of the string coupling and the other radii.

Taking now also into account the behaviour of the system for $\gamma > 1$, we can observe that for given K, M and N , as we vary the value of γ from $\gamma \ll 1$ to $\gamma \gg 1$, the length of the radius r_5 decreases while the radius r_7 increases in magnitude. As a result, if we start having scale-separated $\text{AdS}_3 \times S^1(r_5)$ with $r_5 \gg r_7$, by increasing γ we pass through

Table 9 This table shows the values of the radii r_i as γ takes the values $\gamma = 10^{-3}, 5 \times 10^{-6}$. One can clearly notice that, as γ becomes smaller, r_5 becomes bigger than the other six radii r_a (with $a \neq 5$) and r_7 , for fixed flux units

γ	$\frac{r_{1,3}}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_{2,4}}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_5}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_6}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$	$\frac{r_7}{\frac{2\pi N^{7/16}}{ M ^{3/16} K ^{1/4}}}$
10^{-3}	0.402	0.469	36.37	0.005	0.029
5×10^{-6}	0.207	0.242	265.2	0.0002	0.001

Table 10 This table shows the three interesting regimes one can end up with while changing N for $\gamma = 10^{-3}$, once the other flux units have been fixed, namely $K = -16$ and $M = -1$. When a circumstance like (a) realizes, one has full scale separation; if, instead, one works with cases similar to (b) or (c), then the radius r_5 disentangles from the other

six radii and the external space becomes effectively $\text{AdS}_3 \times S^1$. For the in-between values of N one gets of course intermediate results. Note that N can not be too small in order for the large volume/weak coupling condition to still be satisfied

	N	g_s	$r_{i < 5}$	$r_{i > 5}$	r_5	$\frac{L_{\text{KK},5}^2}{L_{\text{AdS}}^2}$	$\frac{L_{\text{KK},i < 5}^2}{L_{\text{AdS}}^2}$	$\frac{L_{\text{KK},i > 5}^2}{L_{\text{AdS}}^2}$
(a)	10^{10}	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^4)$	$\mathcal{O}(10^{2 \pm 3})$	2.7×10^6	2.3×10^{-3}	$\mathcal{O}(10^{-7})$	$\mathcal{O}(10^{-11})$
(b)	10^7	$\mathcal{O}(10^{-3})$	$\mathcal{O}(10^3)$	$\mathcal{O}(10^{1 \pm 2})$	1.3×10^5	2.3	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-8})$
(c)	10^5	$\mathcal{O}(10^{-1})$	$\mathcal{O}(10^2)$	$\mathcal{O}(10^{0 \pm 1})$	1.8×10^4	2.3×10^2	$\mathcal{O}(10^{-2})$	$\mathcal{O}(10^{-6})$

Table 11 This table shows the three interesting regimes one can end up with while changing N for $\gamma = 5 \times 10^{-6}$, once the other flux units have been fixed, namely $K = -16$ and $M = -1$. One can again see a

similar behaviour with respect to the case $\gamma = 10^{-3}$, which is presented in Table 10

	N	g_s	$r_{i < 5}$	$r_{i > 5}$	r_5	$\frac{L_{\text{KK},5}^2}{L_{\text{AdS}}^2}$	$\frac{L_{\text{KK},i < 5}^2}{L_{\text{AdS}}^2}$	$\frac{L_{\text{KK},i > 5}^2}{L_{\text{AdS}}^2}$
(a)	10^{18}	$\mathcal{O}(10^{-10})$	$\mathcal{O}(10^7)$	$\mathcal{O}(10^{4 \pm 5})$	6.3×10^{10}	9.2×10^{-7}	$\mathcal{O}(10^{-14})$	$\mathcal{O}(10^{-19})$
(b)	10^{12}	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^5)$	$\mathcal{O}(10^2)$	1.5×10^8	0.92	$\mathcal{O}(10^{-7})$	$\mathcal{O}(10^{-12})$
(c)	10^6	$\mathcal{O}(10^{-1})$	$\mathcal{O}(10^2)$	$\mathcal{O}(10^{-1 \pm 0})$	3.5×10^5	9.2×10^5	$\mathcal{O}(10^{-2})$	$\mathcal{O}(10^{-7})$

Table 12 This table exhibits the (rounded-up numerical values of the) masses of the closed string moduli as the parameter γ takes the values 10^{-3} and 5×10^{-6} . The negative eigenvalue remains always slightly above the BF bound

γ	$m^2 L_{\text{AdS}}^2$
10^{-3}	{49.4952, 7.9987, 7.996, 5.8683, 3.6456, 3.6456, 2.3506, -0.999965}
5×10^{-6}	{49.4958, 8, 8, 5.8683, 3.6428, 3.6428, 2.3504, -0.99996}

a regime where $r_5 \sim r_7$ and end up into the regime where $r_5 \ll r_7$ with scale-separated $\text{AdS}_3 \times S^1(r_7)$. This means that, while we are varying γ , there is a tower of KK-modes becoming light and simultaneously another tower of KK-modes becoming heavy. Here, we are not going to precisely discuss a mechanism that allows to vary γ and realize the Distance Conjecture, but it can be done probably with actual jumping fluxes, in analogy with [27], or with the use of D4-branes similarly to [26], and actually following the same steps as in Sect. 5.

As we did in Sect. 4, we can also evaluate the normalized masses for the moduli x, y and \tilde{s}^a . Their numerical values are reported in Table 12, where we again notice the presence of a negative mass mode that is consistent with the BF bound.

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