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Erratum

Erratum to: Evidence for the maximally entangled low x proton in Deep Inelastic Scattering from H1 data

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In this erratum we would like to report an error in the evaluation of the HSS gluon distribution, used for the determination of entanglement entropy in [1]. As explained in Eq. (7) of [1], the overall normalization of the HSS unintegrated gluon density $\mathcal{F}(x, \boldsymbol{k}^2)$, is related to a certain overall constant $\mathcal{C}$, see Eq. (7) of that paper. As explained in the appendix of [1], the original fit [2], uses a certain scale choice of an overall running coupling constant, which is unnatural for the determination of the seaquark distribution. The latter has been obtained using the expression

$$x\Sigma(x, Q) = \int_0^\infty \frac{d\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2} \int_0^\infty d\boldsymbol{k}^2 \times \int_0^1 dz \Theta\left(Q^2 - \frac{\boldsymbol{\Delta}^2}{1-z} - z\boldsymbol{k}^2\right) \tilde{P}_{qg} \times \left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right) \mathcal{F}\left(\frac{x}{z}, \boldsymbol{k}^2\right),$$

$$\tilde{P}_{qg}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right) = \frac{\alpha_s 2n_f}{2\pi} T_F \left(\frac{\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2 + z(1-z)\boldsymbol{k}^2}\right)^2 \times \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right]. \quad (1)$$

Note that Eqs. (4), (5) of [1] suffered of two typos which have been corrected in the above formula. In particular the argument of the unintegrated gluon distribution has been given as x instead of x/z on the LHS of this equation and the expression for the splitting function lacked an overall factor of $\boldsymbol{\Delta}^2$.

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For the numerical evaluation in [1], the above corrected formulas had been used.

In [1] it has been argued, based on general arguments related to collinear factorization, that the overall running coupling constant in Eq. (1) is to be evaluated at the hard scale Q^2, while the original fit [2] used a different convention. To account for that, we prepared in [1] a refit of the 2013 HSS fit with an overall running coupling constant evaluated at the hard scale Q^2. This refit lead then to a modified normalization constant $\mathcal{C} = 4.31$, as reported in the appendix of [1]. For the numerical study, this new value $\mathcal{C} = 4.31$ has been used for the evaluation of the seaquark distribution, while the evaluation of the integrated gluon distribution, Eq. (3) of [1] used erroneously the original value of $\mathcal{C} = 2.39$. Since the gluon distribution dominates the determination of the number of partons in Eq. (2) of [1], this erroneous factor affects our numerical result for the parton number and therefore our result for entanglement entropy.

This mistake has been already corrected in a follow up paper [3]. The mistake was difficult to spot since the formulas that we used did not account for the fact that only charged hadrons were measured. As discussed in [3], this mismatch requires to introduce a correction to the number of partons, if the resulting entanglement entropy is to be compared to the entropy of charged hadrons. This factor can be estimated from isospin symmetry. At low x the proton's partonic content is dominated by gluons and therefore one can assume that gluons split with the same rate into light quark flavors. The latter form then predominantly positive, negative and neutral pions. Since gluons do not prefer any specific quark flavor, the rate of production of π^+, π^-, π^0 is identical. Since only charged pions are measured, it is needed to correct our prediction for the hadron multiplicity by a factor $2/3$, which corresponds to accounting for charged pions only. Numerically $\ln 2/3 \simeq -0.41$, while $\ln 2.39/4.31 \simeq -0.59$, which made it difficult to spot the error in the comparison to data,

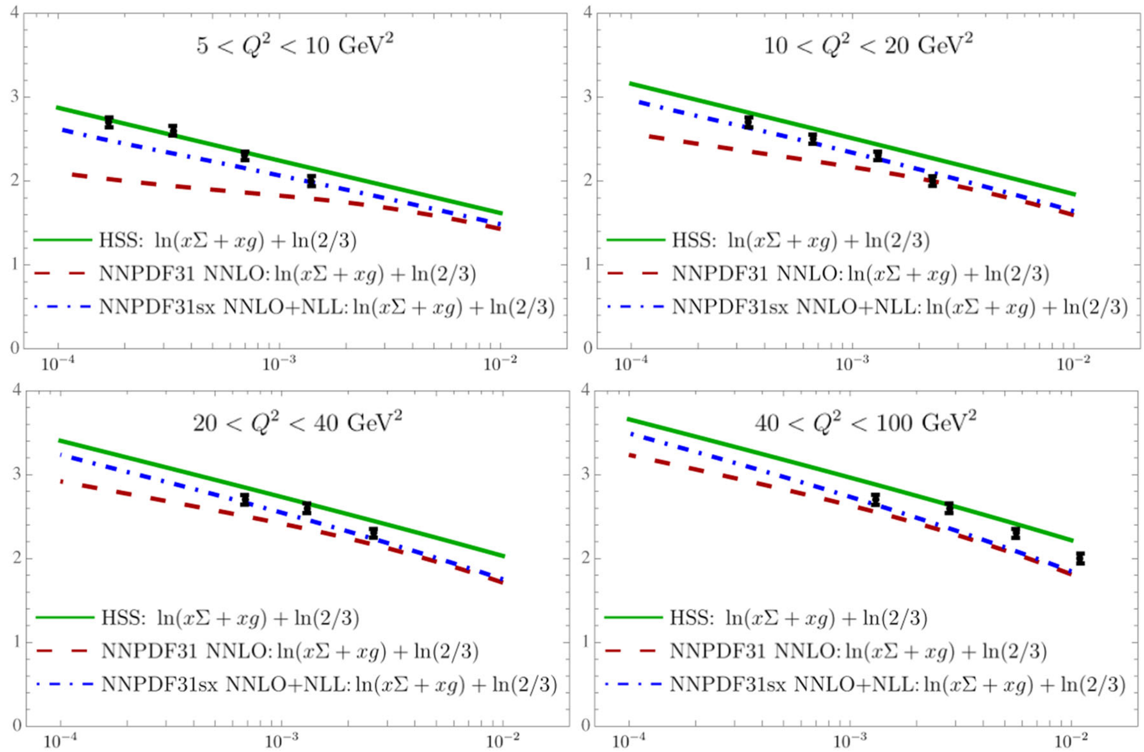

Fig. 1 Partonic entropy versus Bjorken x, as given by Eqs. (3) and (2). We further show results based on the gluon distribution only as well as on quarks and gluons together. Results are compared to the final state hadron entropy derived from the multiplicity distributions measured at H1 [4]

since furthermore the quark contribution has been calculated with the correct normalization constant. The number of partons in the corrected formulas, was already reported in [3] and reads:

$$\left\langle n\left(\ln\frac{1}{x}, Q\right)\right\rangle = \frac{2}{3}\left[xg(x, Q) + x\Sigma(x, Q)\right], \qquad (2)$$

where $g(x, Q)$ ($\Sigma(x, Q)$) denotes the gluon (seaquark) distribution function at the factorization scale Q. To calculate entropy for the H1 Q^2 bins, we employ the following averaging procedure,

$$\bar{S}(x)_{Q_2^2, Q_1^2} = \ln\frac{1}{Q_2^2 - Q_1^2}\int_{Q_1^2}^{Q_2^2} dQ^2 \left\langle n\left(\ln\frac{1}{x}, Q\right)\right\rangle. \qquad (3)$$

The corrected results are shown in Fig. 1.

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